The $\mathsf{SU(5)}$ Grand Unification Theory Revisited

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Abstract

We review the minimal Grand Unification Theory (GUT) based on the $\mathsf{SU(5)}$ gauge group. We construct all the necessary and useful tools to study its phenomenology. The same tools will be used with next to minimal extensions that cure unification and provide small neutrino masses through seesaw mechanisms. We make explicit statement that GUT models based on $\mathsf{SU(5)}$ are not ruled out and are very predictive.

Keywords: Grand Unification Theory (GUT), $\mathsf{SU(5)}$, Neutrino Mass, Proton Decay, B-Test

1 Introduction

The Standard Model of Particle Physics (SM)[1, 2, 3] is many times considered the great scientific discovery after Quantum Mechanics and Relativity as a consistent, predictive and renormalizable implementation of gauge theories in a quantum field theory (QFT) framework.

Nevertheless the SM is not without flaws and still faces challenges as it turns 50 years old. Some of the limitations of the SM are the massless nature of neutrinos which contradicts experimental evidence, no *a priori* Yukawa or family structure, scale dependence of the gauge couplings seems to converge them at some higher energy scales, no incorporation of gravity, no clear physics to constrain the Higgs potential parameters, and other naturality problems such as the scalars masses dependence on the scale, the hierarchical mass spectrum throughout the families, and the vacuum density energy contribution.

Physics Beyond the Standard Model (BSM) is necessary to solve these problems. A BSM class of the theories is the Grand Unification Theories (GUTs) where we give more structure to the gauge part of the theory, mainly by embedding the SM gauge group

$$\mathsf{G_{SM}} = \mathsf{SU(3)_c} \times \mathsf{SU(2)_L} \times \mathsf{U(1)_Y} \tag{1}$$

into a larger group. If the larger group is a simple group or a product of identical simple groups the couplings are unified when such group is the effective gauge symmetry. The gauge group is eventually broken by a Higgs-like mechanism and $\mathsf{G_{SM}}$ will be the surviving subgroup of the larger group.

A GUT then consists of a theory with a larger gauge group that embeds the SM gauge group and a scalar sector that breaks the group into the SM. In this work we will work with the simpler model which is the one based on $\mathsf{SU(5)}$[4], it has the same rank (rank=4) as $\mathsf{G_{SM}}$ and so the last is a maximal subgroup of the former. Other rank=4 groups could have been chosen but this enables us to create a minimal model, i.e. a model with the same matter content as the SM.

2 Minimal $\mathsf{SU(5)}$ Framework

The minimal model is built as a gauge theory based on the $\mathsf{SU(5)}$ group. As the SM gauge group is a maximal subgroup of $\mathsf{SU(5)}$ we can chose a basis in which the $\mathsf{SU(3)}$ and $\mathsf{SU(2)}$ quantum numbers are not overlapped.

The gauge bosons are associated with the generators through the adjoint representation, $\mathbf{24} = (\mathbf{8}, \mathbf{1}, 0) \oplus (\mathbf{1}, \mathbf{3}, 0) \oplus (\mathbf{3}, \mathbf{2}, 5/3) \oplus (\overline{\mathbf{3}}, \mathbf{2}, -5/3) \oplus (\mathbf{1}, \mathbf{1}, 0)$, and so besides the SM gauge fields one has new leptoquark vector bosons which we will denote as X_μ and X_μ^c for its conjugate.

For matter fields we start by noting that the fundamental representation is $\mathbf{5}= (\mathbf{3}, \mathbf{1}, -2/3) \oplus (\mathbf{1}, \mathbf{2}, 1)$ where the quantum numbers are ordered as $\mathsf{SU(3)}$, $\mathsf{SU(2)}$ and hypercharge, and the last was chosen as to respect the SM equivalent representation.

The $\mathsf{SU(5)}$ generator that will be identified with the hypercharge is the diagonal one that does not come from either $\mathsf{SU(3)}$ or $\mathsf{SU(2)}$ Cartan subalgebras which is $\lambda^{24} = 3/\sqrt{15}\,\mathrm{diag}(2/3, 2/3, 2/3, -1, -1)$. One concludes that we can not use the fundamental representation, $\mathbf{5}$, since the hypercharge readings are off by a sign, we will then use the antifundamental representation $\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1}, 2/3) \oplus (\mathbf{1}, \mathbf{2}, -1)$ and one identifies the fields

$$\overline{\mathbf{5}}_F = (d_1^c, d_2^c, d_3^c, e^-, -\nu)^T = (d_i^c, \epsilon_{ij} L^j)^T \ , \qquad (2)$$

the overall normalization factor of the hypercharge-like generator is residual and it is hidden in the hypercharge coupling constant. This is easily understood: when the larger group is broken we split the subgroups within it and so the coupling will eventually evolve down the energy scale to the SM as different couplings. For example in our case $\mathsf{SU(5)}$ breaks into three groups that will be identified with the SM subgroups and so the couplings will be then identified as

$$\begin{aligned} \alpha_U =& \alpha_1 = \alpha_2 = \alpha_3 \\ & \to \alpha_U = k_1 \alpha_y = k_2 \alpha_w = k_3 \alpha_s \ , \end{aligned} \qquad (3)$$

and the overall factors can be systematically computed as $k_i = \mathrm{Tr}\left\{T_i^2\right\}/\mathrm{Tr}\left\{T^2\right\}$, where T_i stands for the SM subgroups' generators and T are the unified group generators. $\mathsf{SU(5)}$ belongs to what is called the canonical class where $k_i \propto (5/3, 1, 1)$.

The remaining matter fields are in the antisymmetric $\mathbf{10} = (\overline{\mathbf{3}}, \mathbf{1}, -4/3) \oplus (\mathbf{3}, \mathbf{2}, 1/3) \oplus (\mathbf{1}, \mathbf{1}, 2)$ and the fields are

$$\mathbf{10}_F = \begin{pmatrix} (\epsilon_3)^{ijk} u_k^c & q \\ -q^T & (\epsilon_2)^{ij} e^c \end{pmatrix} \ . \qquad (4)$$

The scalar part of the theory must have at least two different representations, one that breaks into the SM and other that breaks the SM. A representation that contains the SM Higgs is the fundamental $\mathbf{5}$ while we will use the adjoint to break $\mathsf{SU(5)}$. We identify the fields in the adjoint scalar as $\mathbf{24}_H = \Sigma_O \oplus \Sigma_T \oplus \Sigma_X \oplus \Sigma_{X^c} \oplus \Sigma_S$. The most general potential for the adjoint is

$$\begin{aligned} V(\mathbf{24}_H) = & -\frac{\mu^2}{2}\ \mathrm{Tr}\left\{\mathbf{24}_H^2\right\} + \frac{a}{4}\ \mathrm{Tr}\left\{\mathbf{24}_H^2\right\}^2 + \\ & + \frac{b}{4}\ \mathrm{Tr}\left\{\mathbf{24}_H^4\right\} + \frac{c}{3}\ \mathrm{Tr}\left\{\mathbf{24}_H^3\right\} \ , \end{aligned} \qquad (5)$$

so one can diagonalize the adjoint scalar with a global $\mathsf{SU(5)}$ symmetry to study the breaking patterns using a diagonal form. The SM is retrieved when the scalar field acquires a vev $\langle \mathbf{24}_H \rangle = v \lambda^{24}$. After the spontaneous symmetric breaking, the SM's gauge bosons remain massless, while the leptoquark vector bosons get the mass

$$M_X^2 = \frac{5}{6} g_5^2 v^2 \ , \qquad (6)$$

and so we will usually refer of these vector boson masses and GUT scale mass as equivalent quantities.

The Σ_X and its conjugated fields are would-be Goldstone bosons that are eaten up as longitudinal degrees of freedom of the leptoquark vector bosons. The other fields of the adjoint scalar get the masses

$$m^2(\Sigma_O) = \frac{1}{3} v^2 b \ , \ m^2(\Sigma_T) = \frac{4}{3} v^2 b \ , \ m^2(\Sigma_S) = 2\mu^2 \ , \qquad (7)$$

and so only the singlet part is unconstrained regarding the other fields of the same representation.

The Yukawa part of the theory is obtained with the coupling through Yukawas between the scalar in the fundamental representation, $\mathbf{5}_H$, and the matter fields

$$\mathcal{L}_Y = \overline{\mathbf{5}}_F Y_5 \mathbf{10}_F \mathbf{5}_H^* + \frac{1}{8} \epsilon_5 \mathbf{10}_F Y_{10} \mathbf{10}_F \mathbf{5}_H + \ \mathrm{h.c.} \ , \qquad (8)$$

if one develops these products into the SM fields and compare them to SM Yukawas ($Y_u q \epsilon_2 \phi^* u$, $Y_d q \phi d$, $Y_e L \phi e$) one finds that they are constrained in the $\mathsf{SU(5)}$ framework having the relations

$$Y_e = Y_d^T \ , \ Y_u = Y_u^T \ . \qquad (9)$$

The first prediction is the more striking since it states that at GUT scale the diagonal entries of the down-quarks and charge leptons are the same and so if one runs the masses (or more precisely the Yukawas) from the SM to GUT scale one should have equal values. This fails badly, even for the third family where one gets best results for this prediction.

Also from the Yukawa sector one can deduce new baryon and lepton number violating feynman rules (but $(B-L)$ conserving) mediated by the colour triplet that is embedded within $\mathbf{5}_H$. By integrating out the colour triplet one gets an effective proton decay operator

$$\mathcal{L}_{d=6}^T \sim \frac{1}{m_T^2} Y_{10} Y_5 \ qqqe \ , \qquad (10)$$

and so for naturally small Yukawas and heavy colour triplet, m_T, we might have a narrow decay width and so a large proton lifetime.

The proton decay is not restricted to the Yukawa sector: the new gauge fields will induce proton decay operators through new Feynman rules of the form Xql. By integrating out the fields one will get the effective operator for proton decay

$$\mathcal{L}_{d=6}^X \sim \frac{1}{m_X^2} \alpha_5 qqqe \ , \qquad (11)$$

where α_5 is the unified coupling, m_p the proton mass and M_X the new vector bosons mass. We will discuss proton decay further below in this section. Also we note that there is an overall antisymmetric colour structure (it is the only way to create an invariant with three fundamental representations) and so (10) will be further diminished through symmetry cancellations on the Yukawa matrices.

We can compare the decay widths with experimental bounds from super-Kamiokande. For that we need to compute the values for α_5 and M_X and we will get them by studying the unification.

If the theory is to be a consistent GUT one must unify the SM group into the $\mathsf{SU(5)}$ by evaluating the couplings up the GUT scale and determine the energy at which they meet. For that we use the Renormalization Group Equations (RGE) for the couplings which read

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{4\pi} \ln\left(\frac{\mu_2^2}{\mu_1^2}\right) \ , \qquad (12)$$

where $\mu_2 > \mu_1$ and

$$b_i = \frac{1}{3} \sum_R s(R) t_i(R) \prod_{j \neq i} dim_j(R) \ , \qquad (13)$$

where R is a field in some representation, $t_i(R)$ is the Dynkin of the representation in which the field is[1], the last term accounts for a dimensions that the field is in concerning the other gauge groups[2], and finally $s(R)$ is given by

$$s(R) = \begin{cases} 1 & \text{for } R \text{ scalar}, \\ 2 & \text{for } R \text{ chiral fermion}, \\ -11 & \text{for } R \text{ gauge boson}. \end{cases} \qquad (14)$$

One finds out that they do not meet at any scale, although each of the curves meet the others eventually but not all at the same point. The error is not scandalously wrong but is sufficient to say that a minimal setup does not unify the SM into an $\mathsf{SU(5)}$ based GUT. Other way of saying this is that the $\mathsf{SU(5)}$ GUT does not return the SM. To do so we start by considering that the unification happens and run down (12) to the SM. One can solve the set of equations for the unified coupling and the GUT scale, one gets $\alpha_5 \simeq 1/40$ and $M_X \simeq 7 \times 10^{14}$ GeV. After that we consider the dependence of the Weak angle by the couplings and therefore the scale

$$\sin^2\theta_W(\mu) = \frac{\alpha_y(\mu)}{\alpha_w(\mu) + \alpha_y(\mu)} \ , \qquad (15)$$

and we run down to the SM. One gets $\sin^2\theta_W(\mu) \simeq 0.208$ while the experimental value lays at $\sin^2\theta_W(\mu) \simeq 0.231$. Note that it errs by about 10% which is not awfully bad, in fact is quite interesting the result is so close since we are doing a minimal setup. Also we note that one can compute the weak angle at the unification scale, since the couplings equal apart from the group coefficients k_i one concludes

$$\sin^2\theta_W(\Lambda_{GUT}) = \frac{1}{1 + k_1/k_2} = \frac{3}{8} \ . \qquad (16)$$

We now use (11) and our estimate $\alpha_5 \sim 1/40$, to compute what is the mass of the new vector bosons such that the prediction lays inside the experimental bounds of super-Kamiokande. The decay width of said effective operator is estimated by

$$\Gamma_{pd} \sim \alpha_5^2 m_p^5 / M_X^4 \ , \qquad (17)$$

and by evaluating it such that $\tau_{pd} > 10^{34}$ years one gets $M_X > 4 \times 10^{15}$ GeV which is in agreement with our first estimate on the GUT scale.

This means that the minimal setup predicts similar values for the GUT scale through two different ways: by computing the GUT scale from RGE and from the proton decay experimental bounds. This is yet another coincidence that one must note to motivate further study of this theory.

Also note that we studied the unification issue with a mix of low energy input and group theoretical coefficients. One can in fact systematize the study of unification through the B-Test[5]. By algebraic manipulation of (12) one can deduce

$$B = \frac{B_{23}}{B_{12}} = \frac{\sin^2\theta_W - \frac{k_2}{k_3}\frac{\alpha}{\alpha_s}}{\frac{k_2}{k_1} - \left(1 + \frac{k_2}{k_1}\right)\sin^2\theta_W} \ , \qquad (18)$$

where $B_{ij} = B_i - B_j$ with

$$B_i = b_i + \sum_I b_i^I r^I \ , \ r^I = \frac{\ln(\Lambda_{GUT}/M_I)}{\ln(\Lambda_{GUT}/M_Z)} \ . \qquad (19)$$

We see in (18) that the LHS depends only on group theoretical results and the particle content of the model, while the RHS depends only on group theoretical coefficients and low energy observables. By computing the RHS within the framewokr of $\mathsf{SU(5)}$ one concludes that a theory which unifies into the $\mathsf{SU(5)}$ must accomplish a B-Test of about $B \simeq 0.72$ while the SM B-Test value is $B^{SM} \simeq 0.53$ and so it fails to unify. One can easily deduce that new fields which increases the fraction B_{23}/B_{12} will help unification, specially if they are light since the r^I factor weights how much they contribute for the running of the couplings. Note that we need to add new particles to unify the SM into an $\mathsf{SU(5)}$ based GUT and so we will need to depart from minimality. Fortunately we will add new fields in a quasi-minimal way and we will gain new predictions and features which enrich the theory.

[1] Note that for an abelian group this equals the squared eigenvalue of the field in respect to that group's generator.

[2] Ultimately it counts the multiplicity of the field due to other gauge symmetries.

One should also account the inevitable new fields that appear in the minimal model due the fact $\mathbf{24}_H$ is a large representation which embeds many degrees of freedom. The contributions for the B-Test can be found in Table 1, and although one has two fields contributing for unification, X_μ and Σ_T, one must keep in mind that X_μ must be as high as the GUT scale to consolidate proton decay bounds, and unfortunately Σ_T can not be responsible for unification alone without making it too light as it would already had been at colliders reach.

We now have all the tools to study $\mathsf{SU(5)}$ based GUT models. We still have to solve two problems: Yukawa relations between the down-quarks and charged-leptons, and unification. We will use the B-Test to control the unification issue, the Yukawa part will be solved explicitly by changing the Yukawa sector of the theory. While we do this we will need to constrain new particles masses through proton decay bounds.

3 Extensions and Seesaw Mechanism

We divide the extended models in two classes: the non-renormalizable models[6] and the renormalizable counterparts[7]. We do this since the wrong Yukawa relations prediction is the most problematic consequence of the minimal model based on $\mathsf{SU(5)}$. Proton decay is a problem due to unobservable proton decay events, but we point out that there is some parametric freedom to keep it with agreement with experimental bounds.

We start by the non-renormalizable extension, which assumes an higher energy scale theory which contributes with non-renormalizable terms to the different Yukawas, for the minimal setup we get the new contributions

$$
\begin{aligned}
\Delta\mathcal{L}_Y =& \overline{\mathbf{5}}_F Y_5^{(1)} \mathbf{10}_F \left(\frac{\Phi}{\Lambda}\mathbf{5}_H\right)^* + \overline{\mathbf{5}}_F Y_5^{(2)} \left(\frac{\Phi}{\Lambda}\mathbf{10}_F\right)\mathbf{5}_H^* + \\
&+ \frac{1}{8}\epsilon_5 \mathbf{10}_F Y_{10}^{(1)} \mathbf{10}_F \left(\frac{\Phi}{\Lambda}\mathbf{5}_H\right) \\
&+ \frac{1}{8}\epsilon_5 \mathbf{10}_F Y_{10}^{(2)} \left(\frac{\Phi}{\Lambda}\mathbf{10}_F\right)\mathbf{5}_H + \text{ h.c. } , \qquad (20)
\end{aligned}
$$

and so after the $\mathsf{SU(5)}$ breaking the Yukawa sector effectively changes and the SM equivalent Yukawas will be

$$
Y_e = Y_5 - \sqrt{\frac{3}{5}}\frac{v}{\Lambda}Y_5^{(1)} - \sqrt{\frac{3}{5}}\frac{v}{\Lambda}Y_5^{(2)} \ , \qquad (21)
$$

$$
Y_d = Y_5^T - \sqrt{\frac{3}{5}}\frac{v}{\Lambda}Y_5^{(1)} + \frac{2}{\sqrt{15}}\frac{v}{\Lambda}Y_5^{(2)} \ , \qquad (22)
$$

$$
\begin{aligned}
Y_u =& -\frac{1}{2}(Y_{10} + Y_{10}^T) + \frac{3}{2\sqrt{15}}\frac{v}{\Lambda}(Y_{10}^{(1)} + Y_{10}^{(1)T}) - \\
&- \frac{1}{4\sqrt{15}}\frac{v}{\Lambda}(2Y_{10}^{(2)} - Y_{10}^{(2)T}) \ , \qquad (23)
\end{aligned}
$$

and now we have enough parameters to fit experimental data and to avoid the wrong predictions. Of course that by not adding new fields we did not solve the unification issue, so we now need to decide which fields we add. For that note that another problem remains in the minimal $\mathsf{SU(5)}$: the neutrino has no mass. So we could use this as a good motivation for new fields. Recall that a neutrino mass at the SM level is determined through an effective Weinberg operator where a field respecting the SM symmetries was integrated out. As we want the new fields to contribute to the running we want them to be lighter than the GUT scale and so they must respect the SM symmetries hence they will also be responsible for Weinberg effective operators. We have only three option, each defines a seesaw mechanism:

1) Type-I Seesaw: Add a fermion singlet.
2) Type-II Seesaw: Add a scalar $\mathsf{SU(2)}$ triplet.
3) Type-III Seesaw: Add a fermion $\mathsf{SU(2)}$ triplet.

Note that the mechanisms are not mutually exclusive. The Type-I seesaw will not be of interest by itself since a singlet does not alter the couplings running.

By looking into the representations we note that the $\mathbf{24}$ has singlet and an $\mathsf{SU(2)}$ triplet. So consider the adjoint fermionic[8]

$$
\mathbf{24}_F = \frac{1}{\sqrt{2}}\begin{pmatrix}\Psi_O & \Psi_{X^c} \\ \Psi_X & \Psi_T\end{pmatrix} + \frac{1}{\sqrt{2}}\Psi_S\lambda_{24} \ , \qquad (24)
$$

which will have the following self couplings and couplings with the scalar adjoint

$$
\begin{aligned}
\mathcal{L}_{\mathbf{24}} =& m_F \ \mathrm{Tr}\left\{\mathbf{24}_F^2\right\} + \lambda_F \ \mathrm{Tr}\left\{\mathbf{24}_F^2\mathbf{24}_H\right\} + \\
&+ \frac{1}{\Lambda}\left(a_1 \mathrm{Tr}\left\{\mathbf{24}_F^2\right\}\mathrm{Tr}\left\{\mathbf{24}_H^2\right\} + a_2(\mathrm{Tr}\left\{\mathbf{24}_F\mathbf{24}_H\right\})^2 + \right. \\
&\left. + a_3 \mathrm{Tr}\left\{\mathbf{24}_F^2\mathbf{24}_H^2\right\} + a_4 \mathrm{Tr}\left\{\mathbf{24}_F\mathbf{24}_H\mathbf{24}_F\mathbf{24}_H\right\}\right) \ , \qquad (25)
\end{aligned}
$$

and so the masses of the split fields after the $\mathsf{SU(5)}$

Table 1: B-Test contributions from minimal $\mathsf{SU(5)}$

	X_μ	T	Σ_O	Σ_T
B_{23}	$-\frac{11}{3}r_{X_\mu}$	$-\frac{1}{6}r_T$	$-1r_{\Sigma_O}$	$\frac{2}{3}r_{\Sigma_T}$
B_{12}	$\frac{-22}{3}r_{X_\mu}$	$\frac{1}{15}r_T$	0	$-\frac{2}{3}r_{\Sigma_T}$

symmetry breaking are

$$m_S^F = m_F - \frac{1}{\sqrt{15}}v\lambda_F + \frac{v^2}{\Lambda}\left[2a_1 + 2a_2 + \frac{7}{15}a_3 + \frac{7}{15}a_4\right] , \tag{26}$$

$$m_T^F = m_F - \frac{3}{\sqrt{15}}v\lambda_F + \frac{v^2}{\Lambda}\left[2a_1 + \frac{3}{5}a_3 + \frac{3}{5}a_4\right] , \tag{27}$$

$$m_O^F = m_F + \frac{2}{\sqrt{15}}v\lambda_F + \frac{v^2}{\Lambda}\left[2a_1 + \frac{4}{15}a_3 + \frac{4}{15}a_4\right] , \tag{28}$$

$$m_X^F = m_F - \frac{1}{2\sqrt{15}}v\lambda_F + \frac{v^2}{\Lambda}\left[2a_1 + \frac{13}{30}a_3 - \frac{2}{5}a_4\right] . \tag{29}$$

The new fermionic representation also couples to the fundamental Higgs, i.e. to the SM Higgs, through the Yukawa sector, we have the new contributions

$$\begin{aligned}\mathcal{L}_{Y\mathbf{24}} =& y_0^i\overline{\mathbf{5}}_F^i\mathbf{24}_F\mathbf{5}_H + \frac{1}{\Lambda}\overline{\mathbf{5}}_F^i\left(y_1^i\mathbf{24}_F\mathbf{24}_H + y_2^i\mathbf{24}_H\mathbf{24}_F\right.\\ &\left.+\, y_3^i\,\mathrm{Tr}\left\{\mathbf{24}_F\mathbf{24}_H\right\}\right)\mathbf{5}_H + \text{ h.c.} ,\end{aligned} \tag{30}$$

and one can collect the terms that couple the fermionic triplet and scalar fields to the leptonic doublet, one gets

$$\mathcal{L}_{Y_L} = \epsilon_2 L^i\left(y_S^i\Psi_S + y_T^i\Psi_T\right)H + \text{ h.c.} \tag{31}$$

where the $y^i_{T/S}$ are linear combinations of the y_a^i, $a = 0, .., 3$.

Now, by considering their masses to be

$$\mathcal{L}_{m_{T/S}} = -\frac{m_T^F}{2}T^0T^0 - \frac{m_S^F}{2}\Psi_S\Psi_S + \text{ h.c.} , \tag{32}$$

one can integrate them out and get a neutrino mass of the form

$$m_\nu^{ij} = -\frac{v^2}{2}\left(\frac{y_T^iy_T^j}{m_T^F} + \frac{y_S^iy_S^j}{m_S^F}\right) . \tag{33}$$

So a light mass for neutrinos appears naturally. Note that the new Yukawas are vectors, so we can rotate them into a specific direction in the family space and so we can at most give mass to two different neutrinos.

The B-Test contributions from the new fields are presented in Table 2 and one can conclude we expect light fermion triplet so the theory unifies the gauge couplings. Actually as we have other fields contributing negatively to unification we will eventually have a TeV scale masses for the fermionic triplet and for the scalar triplet from the $\mathbf{24}_H$. In this case the octet's mass must be between 10^5 and 10^8 GeV so it does not spoil unification.

As the triplet is required to be light it is of interest to point out collider signatures. It is immediate to conclude that the fermionic triplet decays into Z and W mostly, although it can decay through Yukawa couplings into a H. All these modes have also a lepton in the final state

$$T^0/T^\pm \to (W^\pm l^\mp,\ Z\nu,\ h\nu)/(W^\pm\nu,\ Zl^\pm,\ hl^\pm) , \tag{34}$$

but it can also decay through the SM Higgs. Also we note that the decay width is proportional to the new Yukawa couplings

$$\Gamma(\Psi_T) \sim |y_T|^2(m_T^F)^2 , \tag{35}$$

so one has a field observable at the LHC which might give us direct measurements of the Yukawas responsible for neutrino masses.

Instead of the $\mathbf{24}_F$ one could add a scalar representation with an embedded $\mathsf{SU(2)}$ triplet that would couple with the leptonic triplet in the Yukawa sector. We can use the $\mathbf{15} = (\mathbf{6}, \mathbf{1}, -4/3) \oplus (\mathbf{3}, \mathbf{2}, 1/3) \oplus (1, \mathbf{3}, 2)$[9] symmetric representation whose fields we will identify as

$$\mathbf{15}_H = \begin{pmatrix}\phi_{\mathbf{6}} & \phi_q\\ \phi_q^T & \Delta\end{pmatrix} , \tag{36}$$

where the triplet part we use the traditional notation used when studying the Type-II seesaww $\Delta = \vec{\Delta}\cdot\vec{\tau}$. The contributions to the scalar potential are extensive

Table 2: B-Test contributions from $\mathbf{24}_F$

	Ψ_O	Ψ_X	Ψ_T
B_{23}	$-2r_{\Psi_O}$	$-\frac{2}{3}r_{\Psi_X}$	$\frac{4}{3}r_{\Psi_T}$
B_{12}	0	$\frac{2}{3}r_{\Psi_X}$	$-\frac{4}{3}r_{\Psi_T}$

but not hard to deduce, they read

$$\begin{aligned}\Delta V = &-\frac{\mu_{15}^2}{2}\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{15}_H\right\} + \frac{a_{15}^2}{4}\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{15}_H\right\}^2 + \\ &+\frac{b_{15}^2}{4}\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{15}_H\mathbf{15}_H^\dagger\mathbf{15}_H\right\} + c_2\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{24}_H\mathbf{15}_H\right\} + \\ &+c_3^*\mathbf{5}_H^\dagger\mathbf{15}_H\mathbf{5}_H^* + c_3\mathbf{5}_H\mathbf{15}_H^\dagger\mathbf{5}_H + \\ &+b_1\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{15}_H\right\}\mathrm{Tr}\left\{\mathbf{24}_H^2\right\} + \\ &+b_3\mathbf{5}_H^\dagger\mathbf{5}_H\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{15}_H\right\} + b_5\mathbf{5}_H^\dagger\mathbf{15}_H\mathbf{15}_H^\dagger\mathbf{5}_H + \\ &+b_6\mathrm{Tr}\left\{\mathbf{5}_H\mathbf{15}_H^\dagger\mathbf{24}_H\mathbf{24}_H\right\} + b_7\mathrm{Tr}\left\{\mathbf{15}_H^\dagger\mathbf{24}_H\mathbf{15}_H\mathbf{24}_H\right\} \ .\end{aligned} \tag{37}$$

The new Yukawa contributions, which we will need to perform the seesaw mechanism, we have the new contributions to the Yukawa sector

$$\Delta\mathcal{L}_Y = \overline{\mathbf{5}}_F Y_{15}\mathbf{15}_H\overline{\mathbf{5}}_F + \frac{1}{\Lambda}(\overline{\mathbf{5}}_F\mathbf{5}_H)Y_{15}^{(1)}(\overline{\mathbf{5}}_F\mathbf{5}_H) + \text{ h.c. } . \tag{38}$$

Finally the seesaw part of the Lagrangian is

$$\mathcal{L}_{seesaw} = -M_\Delta^2\mathrm{Tr}\left\{\Delta^\dagger\Delta\right\} + Y_{15}L\Delta L + c_3 H\Delta^\dagger H + \text{ h.c.} , \tag{39}$$

and so after performing the seesaw mechanism calculations we get the neutrino mass matrix

$$m_\nu \simeq \frac{Y_{15}c_3}{M_\Delta^2}v_w^2 \ . \tag{40}$$

We expect a naturally light mass for neutrinos. Also we now have a matrix structure instead of a vector structure and so all the neutrinos can be massive.

The remaining fields from the $\mathbf{15}_H$ will play a role in unification, their contributions to the B-Test are presented in Table 3 and one now has two fields favourable for unification. But alas the ϕ_q mediates proton decay through B and L violating Feynman rules of the form $d^c Y_{15}\phi_q L$. On the other hand remember that scalar mediated proton decay through Yukawa sector is not as bad as it is in the vector bosons mediated case due to naturally small Yukawas assumption and the antisymmetric colour structure that suppresses parts of the Yukawa matrices.

We turn our attention to the renormalizable version of these theories. The rationale is simple: since we do not have any more freedom to change the Yukawa couplings as we did in the non-renormalizable fashion, we must add a new scalar representation to give new contributions to the masses through SM-like Yukawas, The new scalar representation is expected to contribute as the SM Higgs at the SM scale, i.e. we will consider SU(5) representation with a SM Higgs-like representation embedded within it.

The $\mathbf{45} = (\mathbf{8},\mathbf{2},1/2)\oplus(\overline{\mathbf{6}},\mathbf{1},-1/3)\oplus(\mathbf{3},\mathbf{3},-1/3)\oplus(\overline{\mathbf{3}},\mathbf{2},-7/6)\oplus(\mathbf{3},\mathbf{1},-1/3)\oplus(\overline{\mathbf{3}},\mathbf{1},4/3)\oplus(\mathbf{1},\mathbf{2},1/2)$ representation has a part that breaks like the SM Higgs. We will identify its fields as $\mathbf{45}_H = \Phi_1\oplus\Phi_2\oplus\Phi_3\oplus\Phi_4\oplus\Phi_5\oplus\Phi_6\oplus H_2$ and it couples in the Yukawa sector through

$$\Delta\mathcal{L}_Y = \mathbf{10}_F Y_{45}\overline{\mathbf{5}}_F\Phi_{\mathbf{45}}^* + \epsilon_5\frac{1}{8}\mathbf{10}_F Y_{\mathbf{45}}'\mathbf{10}_F\Phi_{\mathbf{45}} + \text{ h.c. } . \tag{41}$$

The structure of the $\mathbf{45}$ makes it difficult to visualize how it acquires a vev but the Electromagnetic and Colour invariant vev is such that[10]

$$\begin{gathered}\langle(\Phi_{\mathbf{45}})_1^{15}\rangle = \langle(\Phi_{\mathbf{45}})_2^{25}\rangle = \langle(\Phi_{\mathbf{45}})_3^{35}\rangle = v_{45} \ , \\ \sum_{i=1}^{3}\langle(\Phi_{\mathbf{45}})_i^{i5}\rangle = -\langle(\Phi_{\mathbf{45}})_4^{45}\rangle \ , \ \langle(\Phi_{\mathbf{45}})_5^{55}\rangle = 0 \ ,\end{gathered} \tag{42}$$

and we note that since we obtain this representation through $\mathbf{10}\otimes\overline{\mathbf{5}} = \mathbf{5}\oplus\mathbf{45}$ so it is antisymmetric in the upper indexes. So computing the mass matrices for the down quarks and charged leptons one cures the theory from the bad prediction

$$M_e = Y_5 v_W + 2Y_{45}v_{45} \tag{43}$$

$$M_d = Y_5^T v_W - 6Y_{45}^T v_{45} \ . \tag{44}$$

The new fields will contribute to the couplings running through the B-Test contributions presented in Table 4 where we have omitted the SM-like Higgs for two reasons: 1) it is well known that a light Higgs favours unification and 2) it will be at SM scale and so it contributes as just like another fixed Higgs at the SM.

Table 3: B-Test contributions from $\mathbf{15}_H$

	ϕ_6	ϕ_q	Δ
B_{23}	$-\frac{5}{6}r_{\phi_6}$	$\frac{1}{6}r_{\phi_q}$	$\frac{2}{3}r_{\Delta}$
B_{12}	$\frac{8}{15}r_{\phi_6}$	$-\frac{7}{15}r_{\phi_q}$	$-\frac{1}{15}r_{\Delta}$

	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6
B_{23}	$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$
B_{12}	$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$

Table 4: B-Test contributions from $\mathbf{45}_H$

We also note we will have new proton decay contributions from new feynman rules, namely by integrating out from the rules

$$qY_{45}\Phi_3 q \ , \ qY_{45}\Phi_3^* L \ , \tag{45}$$

and so Φ_3 will be constrained by proton decay bounds although it is a good field to consider light due the B-Test.

We see we have many new fields that contribute to the unification effort, also many that do not. But a study of these fields' mass will eventually show that unification can be achieved with only these fields by letting the bad ones be heavy while the good ones are light. So it seems we solved the unification and the wrong Yukawa prediction with only one new representation. But as we introduce a renormalizable approach for completeness sake we should also consider extensions that complete it even more, namely through seesaw mechanisms that give neutrinos small masses.

Then we consider the previous cases of adding $\mathbf{24}_F$ or $\mathbf{15}_H$. All the calculations are similar and so this last part of the section will be brief. Consider for example the case of the $\mathbf{24}_F$[11], now the new Yukawa contributions read only renormalizable terms

$$\Delta\mathcal{L}_Y = h^i \overline{\mathbf{5}}^i_F \mathbf{24}_F \mathbf{45}_H + \text{ h.c.} \ , \tag{46}$$

and the neutrino masses will be a result of a combination from contributions from either the original SM Higgs and the new that comes from the $\mathbf{45}_H$. After integrating out the singlet and triplet fermion fields we get the new neutrino mass matrix

$$M_\nu^{ij} = \frac{a^i a^j}{m_T^F} + \frac{b^i b^j}{m_S^F} \ , \tag{47}$$

where now the structure is given not only by Yukawa couplings but mass contributions from different Yukawas and the two vev

$$a^i = \frac{1}{\sqrt{2}} y^i v_W - 3h^i v_{45} \ , \ b^i = \frac{\sqrt{15}}{2}\left(\frac{y^i v_W}{5\sqrt{2}} + h^i v_{45}\right) \ . \tag{48}$$

We expect unification constraints to be less strict since we have more fields affecting the running of the couplings. On the other hand the renormalizable masses are more constrained with each other, since the non-renormalizable terms extends the freedom to fit the masses that respect proton decay bounds and unification constraints. Note that the fields favourable to unification whose masses are not constrained by each other are Σ_T, Ψ_T and Φ_3. This means that we lost some comfort zone in splitting the masses but we have more independent masses to fit unification demands. In fact there are viable values that leave the fields outside LHC reach but the unification constraints are usually easier to satisfy with LHC searchable fields.

Finally we turn to the model with a new scalar field in a $\mathbf{15}$[12] representation. As we saw this model induces a natural Type-II seesaw trough an $\mathsf{SU(2)}$ triplet embedded in the $\mathbf{15}$. The renormalizable version of the model with an extra $\mathbf{15}_H$ differs little from the non-renormalizable version. The main problematic aspect is that we need to consolidate two distinct Yukawa mediated proton decay channels due the Φ_3 and ϕ_q interactions in the Yukawa sector.

As one can deduce easily no other new prediction will emerge than the ones we already discussed. The main phenomenological consequence of working with $\mathbf{15}_H$ in a renormalizable model is that we get many unrelated fields contributing for the running, namely we have many favourable contributions from Σ_T, Φ_3, Δ and ϕ_q. The mass spectrum of the listed fields is easily found as sparse since unification is easily accommodated with so many degrees of freedom.

Before we head to the conclusions section we want to refer the supersymmetric versions (SUSY GUT) of these theories. In the minimal setup we find that the scalar channel for proton decay is the most dangerous contribution. This is so because in SUSY we must account for the supersymmetric partners and so we can derive a $d = 5$ proton decay operator of the form

$$\mathcal{L}_{d=5} \propto \frac{1}{m_T} Y_{10} Y_5 q q \tilde{q} \tilde{l} \ , \tag{49}$$

of course since this operator acts at the hadronic scale we must dress it so that the spartners are not final states of the process. This will lead to a full $d = 6$ operator, but what makes it so dangerous is that it is only suppressed by an inverse power of the triplet scalar mass instead of two inverse powers as in the non-SUSY case. This will make the mass of this field more stressed than in the non-SUSY case and one must play carefully with the parametric freedom regarding the symmetry cancellations due the antisymmetric nature of the colour structure of the process and the Yukawa couplings. It has been possible to maintain this theory alive [13] but proton decay bounds are narrowing the viability of the minimal theory. Nonetheless unification is achieved out-of-the-box and it is well known that SUSY solves hierarchy problems regarding the scale dependence of scalars masses.

As SUSY unifies relatively easily the gauge couplings it was regarded as a natural extension of $\mathsf{SU(5)}$ GUTs, but we already showed that unification can be achieved with other interesting extensions. Also note that SUSY does not solve wrong Yukawa predictions, and since proton decay bounds are threatening the viability of such theories nowadays one does not consider necessarily the inclusion of SUSY. Note however that SUSY is an interesting theory by itself and that SUSY and GUTs are not mutually exclusive and, all and all the realization of both theories would bring new and exciting physics. Also we want to point out that it is possible the suppress proton decay in SUSY GUT, for example it was shown that in a non-renormalizable SUSY GUT based on $\mathsf{SU(5)}$ [14] the non-renormalizable terms can in fact help loosen the constraints from proton decay. For a full review and systematic studies on proton decay see [15].

4 Discussion

The conclusion of this work is simple: $\mathsf{SU(5)}$ based GUT models are consistent, realistic and predictive. One can choose from either renormalizable and non-renormalizable approaches, but the inclusion of next-to-minimal fields will bring not only natural seesaw mechanisms but predictions on light particles that can be seen in the current generation of collider experiments. Proton decay is found to be manageable although experimental evidence would ease the stress that these theories suffer since it would be a very distinctive signature for B and L violating processes, which these models naturally are.

While we hope next generation proton decay experiments detect it, we are still confident that GUT is by itself an interesting and consistent set of theories. We support this statement by looking at the *coincidences* and the positive predictions, for example within a GUT framework is natural to quantize electric charge, the GUT scale and the experimental bounds are naturally compatible, the gauge sector gets structure compared to the SM, the Yukawas and family structure is revisited through tree level relations, and so on.

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