GLOBAL STABILITY OF CABLE-STAYED DECKS

Roberto Ribeiro de Mendonça Feijóo

Department of Civil Engineering, Architecture and Mining, Instituto Superior Técnico,
Technical University of Lisbon – PORTUGAL
October 2011

***Keywords*:**

Elastic stability

Cable-stayed bridges

Linear analysis

Non-linear analysis

ABSTRACT

The elastic global stability analysis of cable-stayed decks is considered in this research. The bending instability in this type of elements is studied, as well as the influence of some specific aspects in the design of the deck, towers and stays arrangement. The linear and non-linear analysis models considered are presented, and the nonlinearities associated to cable-stay bridges included in this research are defined. The linear elastic stability of the deck is evaluated based on a model, using the analogy of a beam on an elastic foundation (BEF). A simplified approach still using the BEF is developed. To compare the results, geometrical non-linear elastic analysis made by the finite elements software *SAP2000* and *ANSYS* are used. The bending stability analysis of a 420 m main span length deck is performed. The influence of some design aspects in the global stability of the bridge are evaluated by a parametric study that considers: The deck live load pattern; the stays arrangement; the towers height and geometry; the stiffness of the deck; the connection between deck and towers; and the intermediate piers on the lateral spans.

1 INTRODUCTION

Cable-stayed structures pose as an efficient and visually appealing solution for bridges of varied span length. In the last 50 years these types of bridges have enjoyed a great development thanks to the progress of construction techniques and analysis models.

The global stability of cable-stayed bridges decks has not received much attention from either designers or researchers, because few studies have shown large load factors against it [8,9]. However, the susceptibility to buckling of the deck on a cable-stayed bridge has increased, because of the use of bigger span lengths and reduced deck depth.

The slanted stays drive vertical forces to the bridge towers that lead to compression on the decks. In the cases of slender decks, if the compressive force is big enough, it may cause global instability.

In this research, the evaluation of cable-stayed decks is performed. As is standard in this type of investigation, the physical nonlinearity of materials is not accounted for, therefore it is assumed that the behaviour of steel and concrete remains elastic for large load factors. This way, the time dependent effects of concrete, just like the physical nonlinearity of the connection between steel and concrete for composite decks, are not considered in this study. The geometrically nonlinear effects related to this type of structure are the only relevant ones for this type of analysis.

Accounting for what was previously mentioned, the following objectives are set:

- Conception of a model to evaluate the elastic stability of a cable-stayed deck based on the analogy of a beam on an elastic foundation.

- Development of parametric studies to assess the influence of several conception traits on the global stability of cable-stayed decks. Traits such as: the cable-stay arrangement; the height and geometry of the towers; the distance between stays; the influence of the load pattern; the connection between deck and towers; and the intermediate piers on the lateral spans.

Fig. 1 *Vasco da Gama cable-stayed bridge, in Lisbon*

2 CASE OF STUDY

The superstructure of a cable-stayed bridge is composed of towers/piers, stays and a deck. There can be many different types of arrangements for each of these structural elements. This makes it so this type of bridge can be used in small, medium and large spans. In this research the most common configuration was taken into account - a configuration similar the one of the *Vasco da Gama* bridge (*Fig. 1*) with three spans, towers in the shape of an *H*, and a twin girder deck with side suspension. The stays are arranged in a semi-fan layout and the lateral spans have two intermediate piers. The model used in this research is a plain model and simulates half of the deck (*Fig. 2*). The main span is 420 m long and the two lateral spans are 194.7 m long with two intermediate piers which divide them in three smaller spans (one measuring 60.125 m and the other two 72.1875 m). There are two towers, 150 m long each (100 m above the deck), and four planes of stays arranged in a semi-fan layout of 4x16 cables. The deck is supported at the intermediate and end piers but not at the towers. The distance between stays, at deck level, is 13.125 m. The deck is simulated by a bar element possessing the same stiffness as half the deck shown in *Fig*. 3.

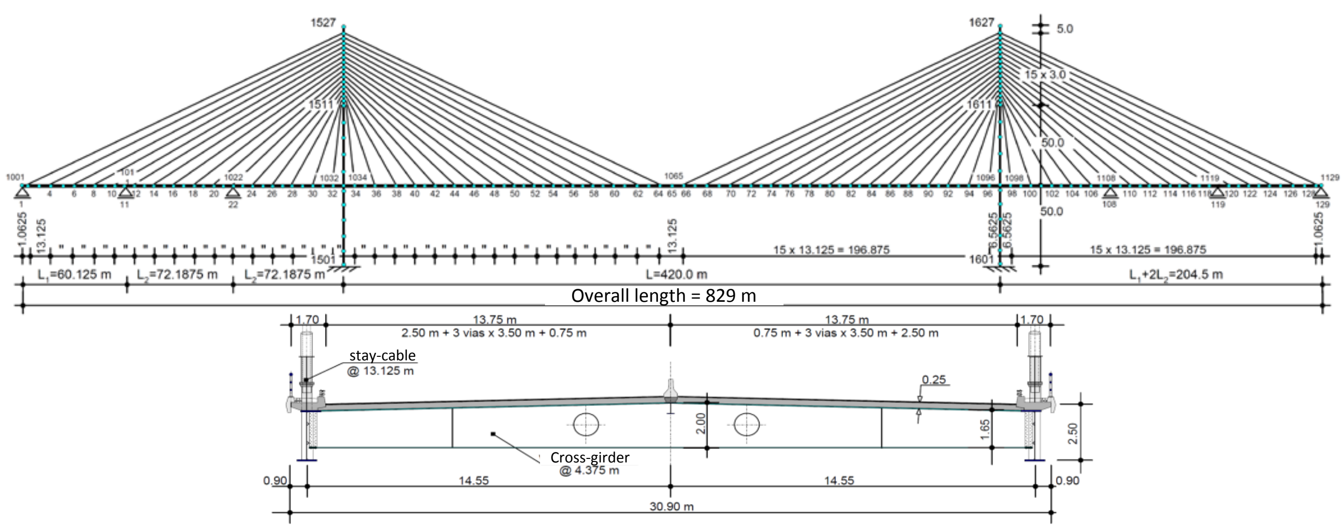

Fig. 2 *420 m main span model used and deck cross-section considered [3]*

Four more models were used in this research with the following main span lengths: 577.5 m; 735 m; 892.5 m; and 1050 m. They were all based on the initial model (with 420 m main span length), differing from it in that they include higher towers and a greater number of stays, although the distance between stays remains the same at deck and tower level. Each new stay has 1.5 cm^2 more than the last one.

In the elastic stability analysis the only necessary parameter to define is Young's modulus, since it is assumed an elastic behavior to both concrete and steel. The value of Young's modulus for steel is E_a = 210 GPa and for the stay cable's steel E_e = 195 GPa. The concrete's Young's modulus is E_{co} = 36 GPa, which refers to a C45/55 concrete.

Both the live and the permanent loads are defined as uniform throughout the deck. The first's value is 171 kN/m and the latter's is 54kN/m [3]. Both loads are relative to half of the deck.

3 LINEAR STABILITY OF CABLE-STAYED DECKS: BEAM-COLUMN ON AN ELASTIC FOUNDATION ANALOGY

The buckling analysis of a cable-stayed deck can be done based on the buckling model of a beam on an elastic foundation (BEF). The main span of a cable-stayed bridge can be considered as a simple supported beam with bending stiffness *EI* and elastically supported along the span by the cables, with variably vertical stiffness K_v, given by *eq*. 1, increasing towards the towers. For vertical loads q this beam is subjected to increasing compressive forces introduced by the cables (*eq*. 2, *Fig*. 3). Since the cables are closely spaced (a = 13.125 m) a continuous vertical restraint β_i can be envisaged for the beam on elastic foundation (*eq*. 3). The cables stiffness and axial forces distribution are functions of the staying configuration (*Fig*. 3).

An energy method for buckling analysis, based on the deformation energy of the beam and foundation, and the energy of the compressive forces, allows, by solving an eigenvalues problem, to determine the instability modes and the respective critical loads [10]. However, *Klein* [2] proposed a simplified method to estimate the first buckling load factor based on the existence of a section along the main span, where the ratio between the vertical elastic restraint due to the cables and the compressive load introduced by them is a minimum (*Fig*. 4). This critical section defines the buckling mode of the "actual" BEF, in a way that the same behavior is attained by an "equivalent" BEF with constant elastic stiffness and axial force of the critical section.

The buckling axial load $N_{i,cr}$ of the equivalent BEF is given with sufficient accuracy, for a high number of half-waves of the buckling modes, by the *Engesser* formula (*eq*. 4). The buckling vertical load (q_{cr}) of the actual BEF is $N_{i,cr}/N_i$ (*eq*. 5) where N_i is the compressive load of the critical section due to the vertical load q. Using the same model, the number n of half waves of the buckling mode, and the equivalent buckling length L_{cr} are given by *eq*. 6 and *eq*. 7, correspondingly.

Table 1 presents the results for both methodologies when determining the critical load for all decks' lengths considered in this work, when the decks are entirely loaded.

Table 1 *BEF model and Klein's simplified method results when the deck is entirely loaded*

Main span length (m)	420	577.5	735	892.5	1050
Klein simplified method					
$N_{o,cr}$ [kN]	403442	384159	336412	312865	286908
BEF model					
$N_{o,cr}$ [kN]	407688	382575	343702	313647	289258
Deviation (%)					
	0.99	0.41	2.17	0.25	0.82

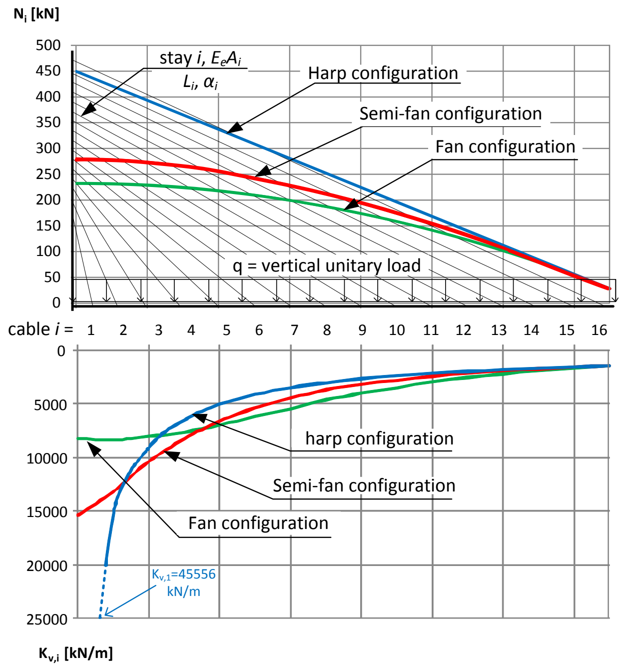

Fig. 3 *Axial force and cables vertical stiffness for a 420 m long span deck with 100 m high towers*

$$K_{v,i} = \frac{E_e A_i}{l_i} sin^2\alpha_i \quad (1) \qquad q_{cr} = \frac{N_{i,cr}}{N_i} = \frac{2\sqrt{EI\beta_i}}{\sum_{j=i}^{n^o\,tir} \frac{a}{tan\alpha_j}} \quad (5)$$

$$N_i = \sum_{j=i}^{n^o\,tir} \frac{q\,a}{\tan\alpha_j} \quad (2) \qquad n^4 \approx \frac{\beta_i L^4}{\pi^4 EI} \quad (6)$$

$$\beta(x) = \frac{K_{v,i}}{a} \quad (3) \qquad L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{i,cr}}} \approx \frac{L}{n\sqrt{2}} \quad (7)$$

$$N_{i,cr} = 2\sqrt{EI\beta_i} \quad (4)$$

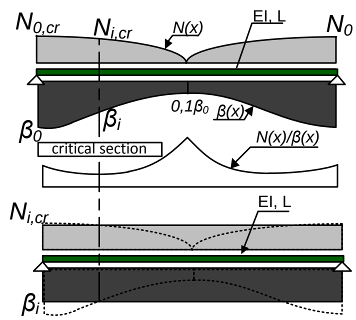

Fig. 4 *Equivalent beam on an elastic foundation model*

4 BEF MODEL MODIFICATION

When an increasing uniform load is applied all over the deck, the analysis of the main span is accurate enough using just the BEF model. However, if only the main span is fully or partially loaded, a large influence of tower and lateral span displacements occur, and the BEF's results are not accurate. To be able to use this model when only the main span is loaded, the towers displacements must be considered in the definition of the BEF model. These displacements decrease the foundation's stiffness value to half of it.

The following hypotheses are considered in this modification:

- the configuration of the stays is symmetric;
- the tower's bending stiffness is not considered, since it is very inferior in comparison with the stay's stiffness;
- the deformability control of the tower is done by the lateral stays;

$$\Delta_{1,v} = \frac{L_i}{E_i A_i}\frac{T}{sin\,\alpha_i} \quad (8) \qquad \Delta_{2,h} = \frac{L_i}{E_i A_i}\frac{T}{cos\,\alpha_i} \quad (10)$$

$$\Delta_{2,v} = \frac{\Delta_{2,h}}{tg\,\alpha_i} \quad (9) \qquad K_v = \frac{Tsin\alpha_i}{\Delta_{1,v} + \Delta_{2,v}} = \frac{E_e A_i}{2L_i} sin^2\alpha_i \quad (11)$$

$$N_{cr}{}' = 2\sqrt{EI\beta'} = 2\sqrt{EI\frac{\beta}{2}} = \frac{2\sqrt{EI\beta}}{\sqrt{2}} = \frac{N_{cr}}{\sqrt{2}} \;\rightarrow\; q'_{cr} = \frac{q_{cr}}{\sqrt{2}} \quad (12)$$

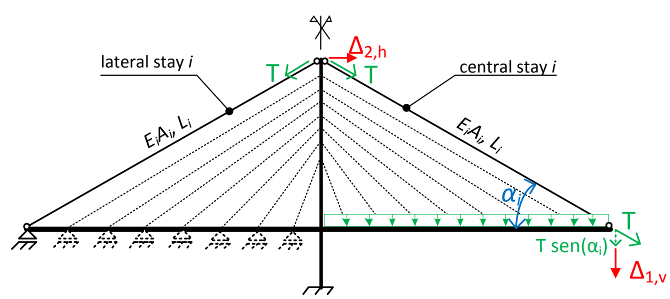

Fig. 5 *Effect of the tower displacements on the foundation's stiffness of the BEF model*

- supports on all lateral stays are considered at deck level.

Equations (8) and (10) show the vertical displacement on the deck ($\Delta_{1,v}$), and the horizontal displacement on the tower ($\Delta_{1,h}$) induced by a pair of stays, when only the main span is loaded, as illustrated in *Fig.5*. The horizontal displacement on the tower is related to a vertical displacement on the deck ($\Delta_{2,v}$) as shown in *eq. 9* when taking into account only small displacements. Knowing the total displacement of the deck, the stiffness assigned by a pair of stays (lateral and central stay) is given by *eq. 11*, which shows that the foundation's stiffness is now half of what it used to be when the deck was entirely loaded (*eq. 1*).

The critical load obtained with *Klein*'s simplified method can also be modified to consider the effect previously explained. The relation between the critical load of an entire loaded deck (q_{cr}), and the one of a deck that is only loaded at the main span (q'_{cr}) is $\sqrt{2}$ (*eq. 12*).

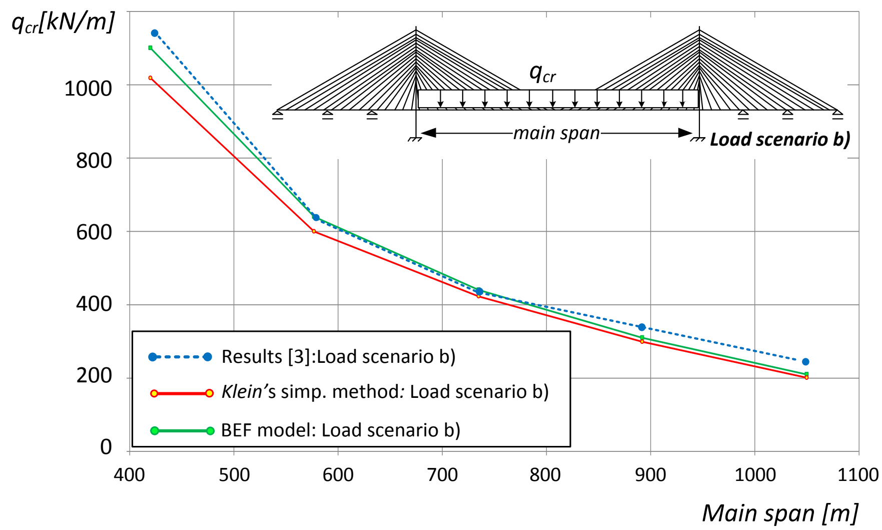

Fig. 6 *Buckling loads for load scenario b) when considering the modifications on both BEF model and Klein's simplified method*

Fig. 6 shows that the results considering this modification, for both BEF model and *Klein*'s simplified method, agree with those obtained by *Pedro* [3] with a nonlinear analysis.

5 PARAMETRIC STUDY

5.1 Constructive process influence

The inclusion of the constructive process on a nonlinear stability analysis induces an initial compressive force in the deck which may affect its critical load. The *SAP2000* software simulates a cantilever constructive method in simple fashion. The insertion of an initial compressive force in the deck is due to the pulling force on the stays during the constructive process, which allows for control of the span deformation. Table 2 shows the results for a nonlinear analysis of a fully loaded deck of the 420 m main span model. It's possible to conclude that the inclusion of the constructive process in the stability analysis does not alter the results in any significant way.

5.2 Load pattern influence

The buckling of the deck is dependent on the load vector [8; 3]. Three load patterns were considered using the nonlinear *SAP2000* model, and both BEF and *Klein*'s simplified method (Fig 7). The results show that when the bridge's deck is loaded at the main span, the buckling load is reduced. When only half of the main span is loaded, the critical load decreases to half the value of the load scenario *a)*.

Table 2 *Constructive process influence on the stability analysis of the deck*

Analysis Type	$N_{máx,inicial}$ [kN]	N_{cr} [kN]	q_{cr} [kN/m]
With Constr. Process	47905	346186	1445
Without Constr. Process	42904	329229	1442

To be able to use the BEF model in load scenario c), another hypothesis has to be admitted: The foundation's stiffness and compressive force distribution are regarded as the same as in load scenario a) and b) in this model. The maximum vertical stiffness is provided by the stay closer to the load pattern border (*Fig.8*). The BEF model considering this hypothesis and the foundation's stiffness modification explained earlier lead to results that agree with those obtained by *Pedro* [3] using a nonlinear analysis for load scenario *c)*.

5.3 Stays arrangement influence

Different stay's arrangements change the distribution of the compressive force and foundation's stiffness, as shown in *Fig.3*. A nonlinear analysis using *SAP2000* for three models, each one with a different stay configuration (fan, semi-fan and harp configurations), shows that when a bridge has a fan arrangement, the critical load of the deck is higher than in any other case (50% higher that the semi-fan case). On the other side the harp arrangement gives the lowest critical load (almost 40% lower that the semi-fan case). In fact, the fan arrangement is the one that provides the highest "vertical support" throughout most of the deck's length, while the harp arrangement provides the lowest.

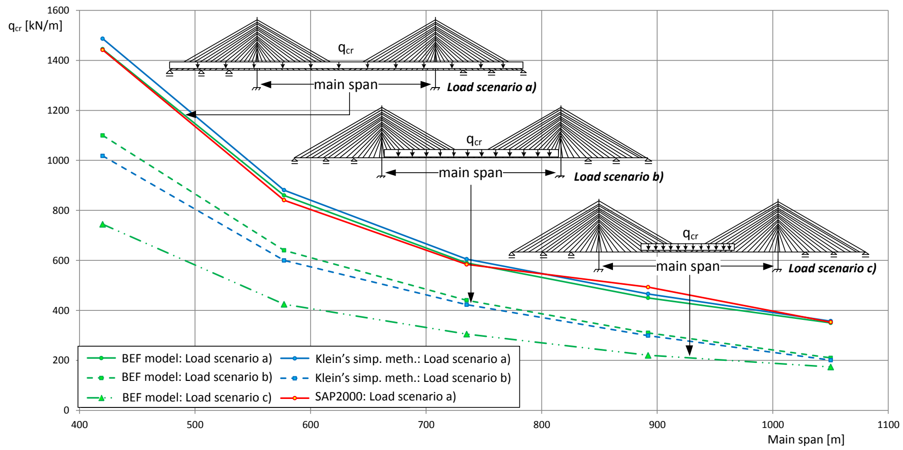

Fig. 7 *Buckling loads for different load patterns*

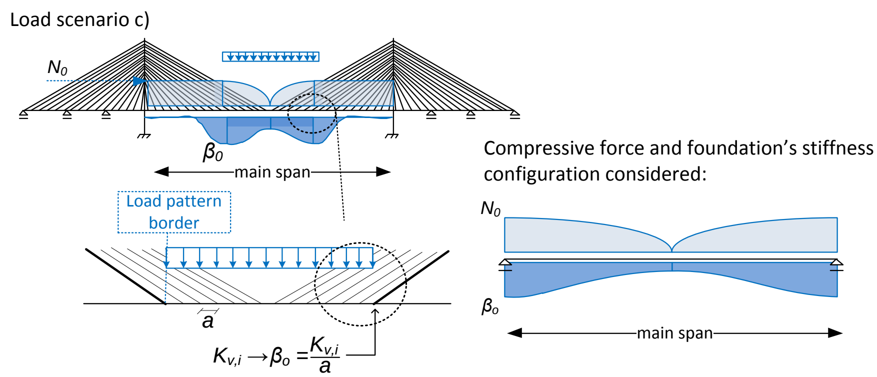

Fig. 8 *Hypothesis considered in the BEF model for load scenario c)*

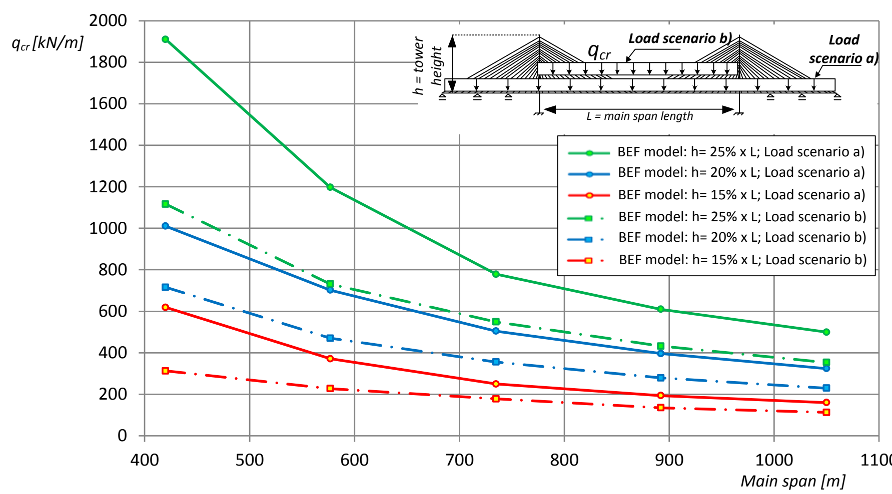

Fig. 9 *Buckling loads for different tower heights*

5.4 Towers height and stays spacing influence

The height of each of the bridge's towers has direct impact on the critical load of the deck. As the towers become higher, so does the critical load of the deck, since the stays become more slanted and induce greater "vertical support" to the deck (similarly to the fan arrangement). As tower height becomes lower, with less slanted stays, the "vertical support" becomes smaller and the compressive forces on the deck increase in value (similarly to the harp arrangement).

Doubling the spacing between stays does not affect the stability of the deck (*Fig. 10*), therefore, replacing a stay during the lifetime of the bridge will not risk its stability [10].

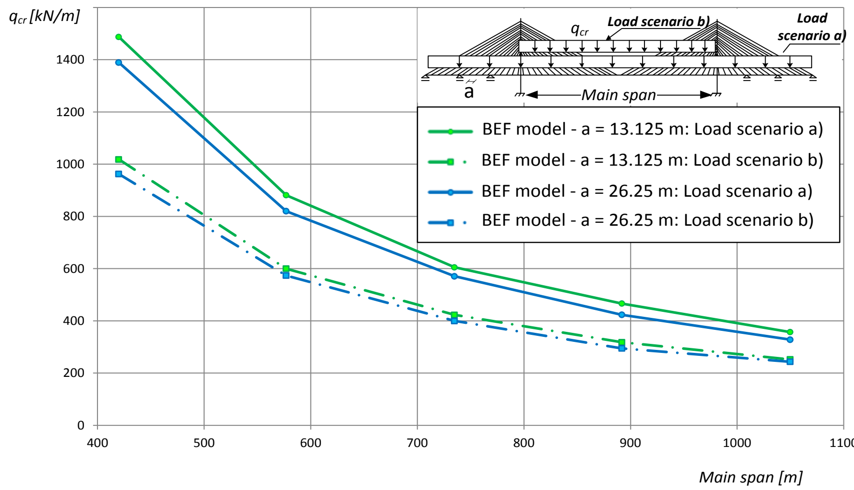

Fig. 10 *Buckling loads for different stays spacing*

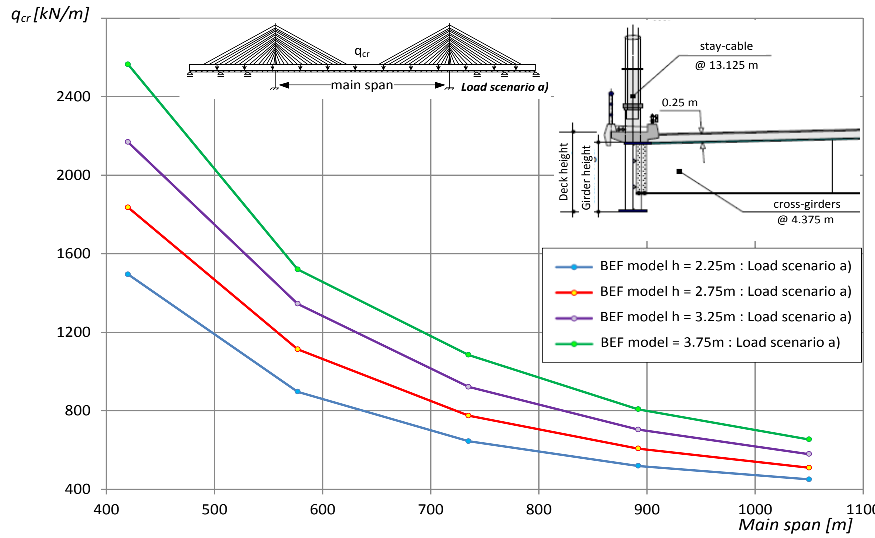

Fig. 11 *Buckling loads for different girder heights*

5.5 Bending stiffness of the deck influence

The bending stiffness of the deck affects the critical load as shown in *eq. 5* This equation and the results shown in Fig 11 make it possible to conclude that the greater the bending stiffness of the deck is, the greater it's critical load is as well. This increase of the bending stiffness of the deck is obtained taking into account higher girders (*Table 3*).

Table 3 *Bending stiffness of the deck considered*

girder height (m)	*EI [kN/m²]*
2.25	6.487E+07
2.75	9.901E+07
3.25	1.417E+08
3.75	1.936E+08

5.6 Towers geometry influence

The geometry of the tower's cross section used in the initial model has the shape of an *H*. The use of a nonlinear analysis, performed with *SAP2000*, on tridimensional models with *A* and inverted *Y* shapes leads to the conclusion that the geometry of the tower's cross section does not seem to affect the critical load of the deck, since the analysis' results are similar for all types of geometry [7].

However some authors [8] refer that towers with *A* or inverted *Y* shapes provide an improved global stability to the structure.

5.7 Towers bending stiffness influence

Some types of cable-stay bridges, like the multiple span cable-stay bridges, require towers with greater longitudinal bending stiffness. A nonlinear analysis performed with *SAP2000* on the 420 m model shows that a higher value on the tower's stiffness leads to the increase on the critical load of the deck (*Fig. 12*). However, in case the towers are too flexible (with about an eight times smaller *EI*), they lose their stability first.

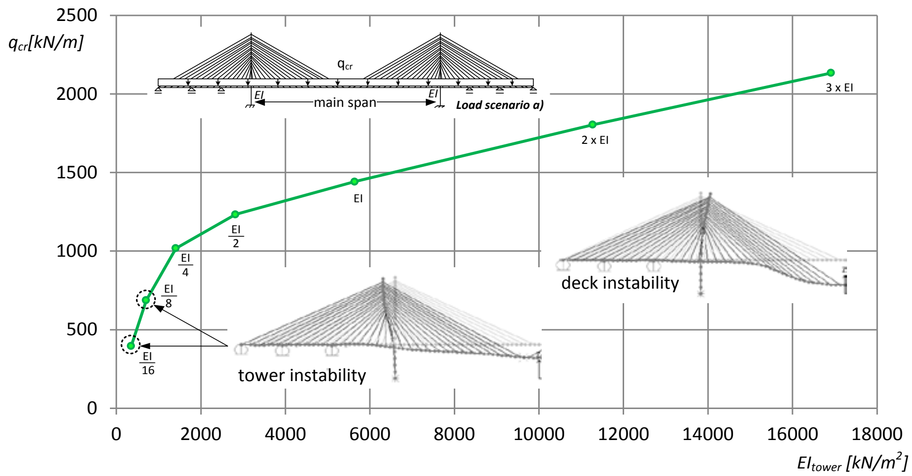

Fig. 12 *Buckling loads for different bending stiffness of the tower*

5.8 Connection between deck and towers influence

On the initial model, the deck is only supported by the stays and intermediate and final piers. The inclusion of a simple support between the deck and the towers is analyzed through a nonlinear analysis performed with *SAP2000*. The results show very similar critical loads in both cases, with or without the supports' inclusion. Consequently, the support of the deck on the towers does not affect its stability.

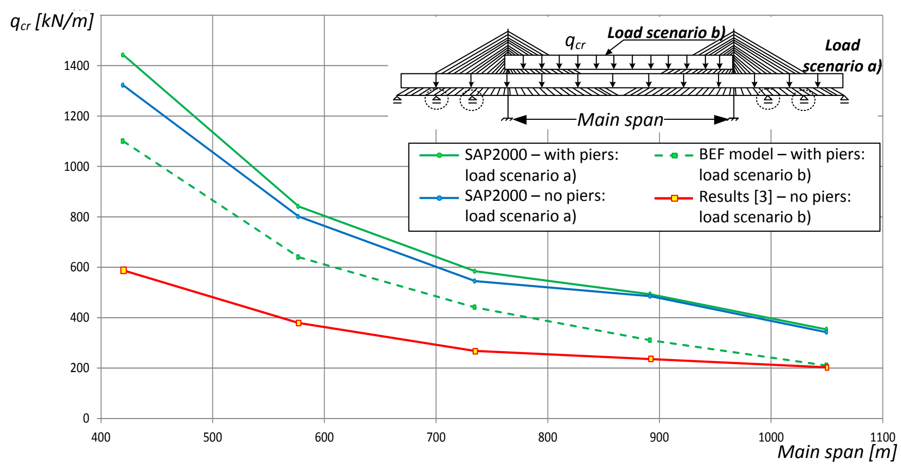

Fig. 13 *Buckling loads when intermediate piers on the lateral spans are considered or not*

5.9 Intermediate piers influence

Under service conditions, the intermediate piers at the lateral spans of cable-stayed bridges improve the deck's stiffness and stability [4]. In order to examine the impact of those piers on the deck's overall stability, the two intermediate piers of the case study (*Fig. 2*) were fully eliminated. *Fig. 13* shows the result of a nonlinear analysis performed with *SAP2000*.

For load scenario a) this change does not seems to affect the critical load value of the deck. However, for load scenario b, and according to *Pedro* [3,5], when no piers are considered, there is a decrease in the critical load of nearly 50%. For longer spans, the impact of intermediate piers on buckling loads is not so important.

6 Conclusions

The beam on elastic foundation (BEF) model, and the Klein's simplified method provided similar results to a nonlinear analysis. Therefore, these two types of methodologies are capable of analyze the global stability of the deck. The modification on both models taking in account the effect of the towers displacement, on the foundation's stiffness of the deck, enabled the analysis when only the main span was loaded. For a load scenario when only half of the deck's main span was loaded, another modification, considering a simplified distribution of the compressive force and foundation's stiffness, provided results in agreement with other researches.

With the parametric study presented, it was possible to identify the influence of: 1) the deck load pattern, 2) the stays arrangement, 3) the towers height an stays spacing, 4) the stiffness of the deck, 5) the towers geometry and bending stiffness, 6) the connection between deck and tower and 7) the intermediate piers. It is shown that the deck stability is sensitive to some of these parameters, but in any case modern long-span cable-stayed bridges are more likely to be governed by strength than by deck instability [3]. These conclusions are in agreement with those by *Taylor* [8] and *Pedro* [3].

REFERENCES

1 Adeli, Y.; Zhang, J. − « Fully Nonlinear Analysis of Composite Girder: Cable-Stayed Bridges » − Computers & Structures – Vol. 54, Nº2, pg. 267-277. Elsevier Science, 1995.

2 Klein, J-F. – « Ponts Haubanés: Comportement et Stabilité des Tablier Minces » - Ecole Polytechnique Fédéral de Lausanne, These Nº 833. Lausanne, Avril 1990.

3 Pedro, J. J. O. − « Pontes Atirantadas Mistas: Estudo do Comportamento Estrutural » − Tese de Doutoramento, Instituto Superior Técnico. Lisboa, Julho 2007.

4 Pedro, J. J. O. − « Pontes de Tirantes – Concepção, Dimensionamento e Construção » − Instituto Superior Técnico. Lisboa, 2010.

5 Pedro, J. J. O.; Reis, A. − « Stability of Composite Cable-Stayed Bridges » − EUROSTEEL 2011. Hungary, August 2011.

6 Reis, A.; Camotim, D. – « Estabilidade Estrutural » . Editora McGraw-Hill de Portugal, 2001.

7 Shu, H-S.; Wang, Y-C. − « Stability Analysis of Box-Girder Cable-Stayed Bridges » − ASCE - Journal of Bridge Engineering, January 2001.

8 Taylor, P. R. − « What are the limiting criteria governing the maximum span of composite cable-stayed bridges? » − Composite Bridges – State of the Art in Technology and Analysis. Proceeding of the 3rd International Meeting. Madrid, January 2001.

9 Taylor, P.; Kaneko, S.; Bergman, D. – « Buckling stability and secondary stress effects in the deck girders of cable-stayed bridges » – International Conference AIPC-FIP, Proceedings - Vol. 2, pg. 223-230. Deauville, October 1994.

10 Timoshenko, S.; Gere, J. − « Theory of elastic stability » − pg. 94-113, 2nd edition, McGraw-Hill Book Company, 1961.

11 Timoshenko, S. − « Resistência dos Materiais » − pg. 326-328, 1ª edição, Ao Livro Técnico S.A.. Rio de Janeiro, 1969.

12 Wang, Y-C. − « Number of Cable Effects on Buckling Analysis of Cable-Stayed Bridges » − ASCE - Journal of Bridge Engineering, November 1999.