Optimization of the reinforcement configuration of an OPENCELL® type flat panel

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ABSTRACT

The purpose of this thesis is to perform a structural optimization of a flat plate having a sandwich configuration. This task is developed computationally through the interface between an optimization algorithm and the finite element method, with the goal of maximizing the stiffness of the structure.

In particular an OPENCELL® configuration, a new way of building sandwich panels, is studied and the optimization process performed involves the search for the optimum reinforcements arrangement in terms of elastic energy.

The innovation of this work lies in the selection of the design variables, chosen to adapt well to an OPENCELL® panel, and in the fact that the study is implemented by considering the panel composed as regular repetition of a basic cell whose configuration does not vary across the sandwich. In this manner the panel could be built in a more automatic way both under a computational and an industrial points of view, reducing considerably manufacturing times and costs.

The results for the optimal configurations are presented for different loads and boundary conditions, considering single load and multiload optimization cases.

Keywords: Structural static analysis, Sandwich structures, OPENCELL®, Finite Element Method, Optimization algorithm, Reissner-Mindlin plate theory, Matlab®, ANSYS®.

1. INTRODUCTION

Sandwich structures are a relatively new class of composite plates introduced and further developed during the second half of the nineteenth century, that found a really extensive use in the aerospace field. In fact, they present a high specific strength and stiffness, meaning that their mechanical resistance and stiffness are higher than standard plates with the same mass, and for these reasons many components of aircrafts and spacecrafts have been built using that configuration.

A sandwich structure is an advanced laminate configuration composed by two thin external plates, named skins, spaced by a sandwich core that works as reinforcement and increases the stiffness: in this way, in fact, the area moment of inertia of the plate’s section results bigger, being proportional to the cube of the height of the section [1, 2,3].

These advantages, combined with a high impact resistance and with a good thermal insulation, make sandwiches widely used in engineering and an interesting subject for a structural optimization.

Among the several kinds of sandwiches existing, the so called OPENCELL® configuration has been chosen.

The innovation introduced by this arrangement consists in the fact that no material is added to create the core of the panel, because it is obtained from the external plates themselves: from one of them some reinforcements are obtained by cutting and bending, and then attached to the opposite plate by welding or gluing. Typically only one skin is cut in order to leave
the other intact and able to withstand pressures or concentrated loads, that are then transferred to the whole panel. Figure 1 presents the most common OPENCELL® configurations, whilst figure 2 shows an OPENCELL® with a shell arrangement.

![Figure 1: Different kinds of reinforcement](http://www.ply.pt/)

![Figure 2: Shell OPENCELL®](http://www.ply.pt/)

The main advantage of an OPENCELL® structure is the reduction of weight compared to a conventional sandwich because no material is added, and for this reason they can be reasonably considered a good target on which implement a computational optimization.

2. OBJECTIVES AND EXPECTED RESULTS

The objectives of this thesis are:

1. developing a computational model able to perform an optimization of an OPENCELL® plate;
2. creating an interface between the engineering tools Matlab® and ANSYS® in order to implement such optimization;
3. finding appropriate design variables in order to best model the OPENCELL® reinforcements chosen;
4. establish parametric models in terms of design variables, loads and boundary conditions, in order to be able to study different kind of problems;
5. performing optimization studies with the tool created.

The expected result of this work is the optimal reinforcement configuration that maximizes the stiffness of the structure for particular loads and boundary conditions.

3. THEORETICAL REVIEW

3.1 Reissner-Mindlin plate

A plate is a solid body bounded by two parallel planes whose lateral dimensions are large compared to the distance between them. The Reissner-Mindlin theory can be considered an extension of the Kirchhoff-Love theory [2, 4], because it takes into account shear deformations along the thickness, that are instead neglected by their predecessors. Since in the finite element analysis performed in this thesis those effects are considered, the theory on which it is based is the one developed by Reissner-Mindlin [5].

This theory is based on the assumption that lines originally normal to the midsurface of the plate remain straight but not necessarily perpendicular to it after deformation.

Moreover, stresses in the thickness direction are negligible, $\sigma_{zz} = 0$, condition known as plane stress, and the transversal displacement of the plate is small compared to its thickness.

Based on the previous hypothesis and considering a time independent problem, the Reissner-Mindlin displacements field assumes the following form:

$$
\begin{align*}
    u(x, y, z) &= u_0(x, y) - z\theta_x \\
    v(x, y, z) &= v_0(x, y) - z\theta_y \\
    w(x, y, z) &= w(x, y)
\end{align*}
$$

where $\theta_x$ and $\theta_y$ are the rotations of the transverse normal about the $y$ and $x$ axes respectively.
Considering the stress-strain law of an isotropic material, and the static equilibrium of an infinitesimal plate element of area \( dx \cdot dy \) and thickness \( t \) on which a transverse load \( q_x \) is applied, the equilibrium equations as a function of the transversal displacement \( w \) and rotations \( \theta_x, \theta_y \) become

\[
\frac{Et}{2(1+\nu)}\left(-\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\right) = -q_x
\]

\[
-\frac{E}{2(1+\nu)}\left(\frac{\partial^2 \theta_y}{\partial y \partial x} + \frac{\partial^2 \theta_y}{\partial y \partial x} - \frac{\partial^2 \theta_y}{\partial y \partial x} + \frac{\partial^2 \theta_y}{\partial y \partial x} + \frac{\partial^2 \theta_y}{\partial y \partial x}\right)
\]

\[
\frac{E}{2(1+\nu)}\left(-\frac{\partial \theta_y}{\partial y} + \frac{\partial w}{\partial y}\right)
\]

3.2 Finite Element Method

The finite element method is a numerical method widely used in engineering to solve problems that involve complicated physics, geometry and boundary conditions. Dividing a given domain into a set of subdomains, called elements, a complicated function defined over the whole space is decomposed into simpler ones, defined in the subdomains. In this way, the resolution of the problem becomes much easier in terms of computational effort, although it involves some numerical errors.

Considering the simplification due to plate assumptions, a static analysis FEM assumes the form

\[
[K][d] = [F]
\]

where \([K] = \int_A \int_{-t/2}^{t/2} [B]^T[D][B] \, dz \, dS\) is the stiffness matrix of an element, \([F]\) is the external load vector, and \([d]\) contains the unknowns of the problem, namely the transversal displacement and the rotations expressed in equations 1 [4].

3.3 Optimum design

The extreme competition that nowadays has taken root in all the economical and industrial fields forces companies to look for new ways of designing their products. Optimum design refers to those processes that try to find the best solution of a problem, responding to the requirements in a more efficient way.

A general optimization process consists in the search for a set of design variables

\[
x = (x_1, x_2, \ldots, x_n)
\]

that, subject to some constraints of the type

\[
h_j(x) = h_j(x_1, x_2, \ldots, x_n) = 0 \quad j = 1, \ldots, m_e
\]

\[
g_i(x) = g_i(x_1, x_2, \ldots, x_n) \leq 0 \quad i = m_e + 1, \ldots, m
\]

minimizes (or maximizes) a certain objective function

\[
f(x) = f(x_1, x_2, \ldots, x_n)
\]

The optimization method used in this thesis is a gradient based mathematical programming method, from the Matlab® optimization toolbox. It is a class of optimization processes whose search procedure consists of creating a sequence of approximate solutions until a termination criteria is reached, and they need to evaluate the gradient of the objective function at each iteration [7].

4. FORMULATION OF THE PROBLEM

The plate to optimize is a square with a fixed side length \( L \), number of holes on a row \( N \), thickness \( t \) and isotropic material properties \( E \) and \( \nu \). These constants, saved on a .txt file read by ANSYS®, are parameterizable in order to allow the user to change the geometry of the plate and the material properties. In table 1 the values assigned to these parameters in the optimizations performed in this work are shown.

<table>
<thead>
<tr>
<th>ANSYS® constants</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>1 m</td>
</tr>
<tr>
<td>( t )</td>
<td>5 mm</td>
</tr>
<tr>
<td>( E )</td>
<td>72.4 GPa</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>( N )</td>
<td>5</td>
</tr>
</tbody>
</table>
The reinforcements chosen to model the core of the sandwich are square holes present on one skin, from which four identical triangular fins are obtained by cutting and bending, and finally welded to the opposite skin as shown in figure 3.

**Figure 3: Plate geometry**

The dimensions and orientations do not vary from hole to hole over the panel, which means that in terms of computational modelling and, most of all, in terms of industrial fabrication it involves much less effort since the entire plate can be considered as the regular repetition of a basic cell composed by two plates, one with a single hole and linked to the other through the four fins obtained.

Then the optimization process could be seen as the search for the basic cell’s arrangement that best suits particular measure of the whole panel performance, leaving a certain generality to the study.

The cell’s geometry is shown in figure 4, where the hole dimension $b$ and orientation $\theta$, together with the cell dimension $a = \frac{L}{N}$, are represented. In this figure, the fins form an angle of 90° with the plane of the plate.

**Figure 4: Basic cell geometry**

The structural optimization performed regards the maximization of the stiffness of the plate when it is subject to particular loads and boundary conditions, that is equivalent to the minimization of the total elastic energy of the plate, and therefore of the elastic work of the external load: $f(\mathbf{x}) = \{d\}^T \{F\}$. Then, the total elastic energy of the panel is the objective function of the optimization.

The objective function depends on some design variables that should have a considerable influence on its value. In particular, it has been noticed that the elastic energy is strongly dependent on the hole dimension $b$ and, to a lesser extent, on the hole orientation $\theta$. Moreover, the industrial manufacturing of such a panel would be easier in terms of time and machinery use because it involves only few operations that can be made with not really complicated tools, in particular cutting and bending. Therefore, these parameters have been chosen as design variables.

As final study and starting point for a possible future work, also the inclination $\gamma$ of the triangular fins acting as reinforcements is considered as design variable and, in this case, once the hole dimension $b$ and then the orientation $\theta$ are kept fixed in order to have only two design variables in each optimization that, otherwise, would be computationally heavier and slower. Angle $\gamma$ is shown in figure 5, that represents the basic cell with inclined tips from two different points of view.

**Figure 5: Basic cell with inclined tips i) General view    ii) Side view**

There are some limitations on the values that the design variables can assume in the optimization.

Except of the simple bounds that delineate the upper and lower values of the variables in the feasible
domain, there is a nonlinear constraint relating $b$ and $\theta$, that forces the hole to stay within the boundaries of the basic cell and not to invade the adjacent ones. This relation is

$$b \cdot \{ \cos (\theta) + \sin (\theta) \} \leq \beta \cdot a$$

(7)

where $\beta$ is a coefficient greater than zero and less than one that ensures a reasonable clearance between the hole and the cell side. It was chosen to be $\beta = 0.8$.

On the other hand, a different optimization has also been performed without the constraint 7, but imposing the upper bound of $b$ equal to

$$b_{up} = \frac{\beta \cdot a}{\sqrt{2}}$$

(8)

that is the maximum value that it can assume when $\theta = 45^\circ$. This is done to assign more importance to the orientation $\theta$ in the optimization results: in a bending loaded plate, otherwise, it may assume the trivial value $0^\circ$ in order to allow the maximum hole dimension, because it reflects directly the panel height, that is the configuration presenting the greatest bending stiffness.

5. COMPUTATIONAL MODEL

5.1 Optimization process

The optimization is performed through the `fmincon` algorithm of the Matlab® Optimization toolbox. It is classifiable as a gradient based method, and it “it attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate” [6].

The objective function that `fmincon` evaluates at each iteration is a Matlab® file that calculates the total elastic energy of the panel by running ANSYS® through a DOS command, respecting the nonlinear constraint and the bounds explained in section 4. At this regard, a certain importance is assumed by the choice of the options supported by `fmincon`: in particular, the maximum number of iterations and functions evaluations allowable are set equal to 70 and 200 respectively, a termination tolerance of $10^{-12}$ is assigned to the variables $x$ and to the objective function. Moreover, the minimum and maximum changes in variables for the evaluation of the finite difference gradient are defined, but it is really important to choose them carefully. In fact, it is not possible to set different increments for $b$, $\theta$ and $\gamma$, therefore their values need to be scaled in the optimization in order to assign them the same influence on the objective function, otherwise the search direction would not be precise.

The “active-set” option of `fmincon` is chosen for the optimization process, because it can take large steps, adding speed to the procedure, and the algorithm is effective on problems with non-smooth objective function and constraints [6]. In fact, the elastic energy function is slightly non-smooth due to the limitations of the FEM, in particular to the dependence of the mesh created by ANSYS® at each step on the design variables values.

5.2 Static analysis

The static analysis required to calculate the elastic energy value of the plate is performed by the engineering tool ANSYS® that, launched by Matlab® at each step, reads an APDL code containing the FE commands in batch mode, that is an automatic execution of the program without any user intervention.

The design variables $b$, $\theta$ and $\gamma$ (only in the last analysis) are defined in a .txt file written by Matlab® and read by ANSYS® at each iteration: this is the key of the interface between the two tools.

The FE element that best suits the plate configuration of the sandwich panel is the type SHELL, and the most indicated for the ANSYS® static analysis of flat plates are SHELL63 and SHELL93 [8].

It has been opted for SHELL93 element, that presents eight nodes instead of four, and consequently the FE analysis would be more precise, because a bigger number of nodes implies a numerical solution closer to the exact one. Although this makes the
computational process slower, it was preferred to try to reduce numerical errors.

Once that the input parameters of table 1 and the design variables are imported in ANSYS®, the geometry creation starts with the definition of the keypoints of the basic cell. The design variables $b$, $\theta$ and $\gamma$ appear explicitly in this phase, and the hole is specified completely in their terms. Afterwards lines and areas are created.

The connections between the reinforcements and the inferior plate have been modeled as lines instead of points, in order to reduce stress concentration in their vicinity and to be closer to reality.

In the optimization performed a triangular mesh is chosen because, besides a better quality of the solution due to a greater number of nodes, a parametric study showed that, in that case, the elastic energy as a function of $b$ and especially of $\theta$ is smoother.

Moreover, it is possible to choose the degree of refinement of the mesh through two parameters, $m$ and $p$, present in the APDL file and modifiable by the user: greater their values, more refined the mesh.

The optimizations are performed with a not too refined mesh and once a solution is reached, a more refined static analysis with the obtained optimal design variables is performed in order to obtain a higher quality of the FE solution. In figure 6 the load and boundary conditions of the shear loaded panel (cases C, E) is shown.

6. RESULTS

In the following table the cases studied in the optimizations are presented, for different boundary conditions, loads and constraints on the design variables.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Boundary condition*</th>
<th>Design variables</th>
<th>Load**</th>
<th>Optimization constraint***</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C-C-C-C</td>
<td>$b \theta$</td>
<td>Uniform pressure</td>
<td>(7)</td>
</tr>
<tr>
<td>B</td>
<td>C-C-C-C</td>
<td>$b \theta$</td>
<td>Uniform pressure</td>
<td>(8)</td>
</tr>
<tr>
<td>C</td>
<td>C-F-F-F</td>
<td>$b \theta$</td>
<td>Shear forces</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Combination of A and C</td>
<td>$b \theta$</td>
<td>Combination of A and C</td>
<td>(7)</td>
</tr>
<tr>
<td>E</td>
<td>Combination of A and C</td>
<td>$b \theta \gamma$</td>
<td>1-3)Uniform pressure</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td>2-4) C-F-F-F</td>
<td>$b \theta \gamma$</td>
<td>2-4)Shear forces</td>
<td></td>
</tr>
</tbody>
</table>

The optimizations are performed with a not too refined mesh and once a solution is reached, a more refined static analysis with the obtained optimal design variables is performed in order to obtain a higher quality of the FE solution. In figure 6 the load and boundary conditions of the shear loaded panel (cases C, E) is shown.

![Figure 6: Panel with a clamped side and shear forces](image)

Moreover, because of a possible dependence of the optimal panel configuration on the choice of the starting values used by fmincon, different optimizations are performed for different ones, and here only the best solution in terms of elastic energy will be presented.

* C-C-C-C: plate clamped on four sides; C-F-F-F: plate clamped on one side and free on the others
** Uniform pressure of 5000 Pa applied on the skin without holes; Shear forces of 20 N each applied on the nodes of the top part of side A (figure 6)
*** The values in brackets correspond to equations numbers 7 (nonlinear constraint) and 8 (side constraint)
6.1 Case A

The first case to be presented is the panel clamped on the four sides, with an uniform pressure applied on the plate without holes, and considering the nonlinear constraint 7 on the design variables. Table 3 shows the optimization results, presenting in particular the optimal values of the design variables $b_{\text{opt}}$ and $\theta_{\text{opt}}$, the minimum value of the elastic energy reached, the number of iterations and function evaluations accomplished, and the Matlab® algorithm stopping criterion. The upper and lower bounds of the variables in these optimization are: $b_{\text{low}} = 0.06 \text{ m}$, $b_{\text{up}} = 0.16 \text{ m}$, $\theta_{\text{low}} = 0^\circ$, $\theta_{\text{up}} = 90^\circ$.

Table 3: Case A results

<table>
<thead>
<tr>
<th>$b_{\text{opt}}$ [m]</th>
<th>$\theta_{\text{opt}}$ [rad(deg)]</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0 (0)</td>
<td>0.08817</td>
<td>2</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

The stopping criterion (exitflag=1) is the best expectable, and indicates the convergence of the algorithm.

Since this is classifiable as a bending case, the optimization process tries to maximize the stiffness of the structure by reaching the greatest possible height of the panel, reflected by $b_{\text{opt}} = 0.16 \text{ m}$, that coincides with the upper bound of $b$. As a result, and because of the presence of the nonlinear constraint 7, the maximum value of $b$ is achievable only by setting the hole orientation $\theta$ to $0^\circ$ (or $90^\circ$). In figure 7, the displacement distribution is represented for a more refined static analysis, whereas figure 8 shows the optimal configuration of the basic cell for this load condition.

Figure 7: Case A displacements distribution: i) top view   ii) bottom view

6.2 Case B

This case differs from the previous one because of the elimination of the nonlinear constraints 7 on the design variables, that is substituted by a different upper bound on the hole dimension $b_{\text{up}}$. Consequently, the upper and lower bounds of the variables in these optimizations become: $b_{\text{low}} = 0.06 \text{ m}$, $b_{\text{up}} = 0.11314 \text{ m}$, $\theta_{\text{low}} = 0^\circ$, $\theta_{\text{up}} = 90^\circ$. Table 4 shows the optimization results.

Table 4: Case B results

<table>
<thead>
<tr>
<th>$b_{\text{opt}}$ [m]</th>
<th>$\theta_{\text{opt}}$ [rad(deg)]</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1131</td>
<td>0.7845 (44.9)</td>
<td>0.14443</td>
<td>11</td>
<td>64</td>
<td>5</td>
</tr>
</tbody>
</table>

The stopping criterion (exitflag=5) means that the optimality criteria are not completely satisfied. The solution, in this case, presents an angle of approximately $45^\circ$ and the maximum hole dimension allowed, $b_{\text{up}}$, in fact it is another case of bending loaded plate. Therefore the influence of the hole rotation is shown and the value of $45^\circ$ reflects the symmetry of the problem.

Anyway, the optimal value of the elastic energy is about 40% greater than the corresponding value of case A, because the height of the panel is smaller and it has a more important effect on the elastic energy than the hole orientation. The displacement distribution on the panel with a more refined mesh can be seen in figure 9, and in figure 10 the optimal arrangement of the basic cell is displayed.

* Exitflags: 1 = First-order optimality measure less than options; 0 = Number of iterations exceeded; 4 = Magnitude of the search directions less than options; 5 = Magnitude of directional derivative less than options
6.2 Case C

The third case analyzed concerns the effects of a shear load, in fact tangential forces are applied on the top part of the free side of the panel. The upper and lower bounds of the variables are the same as case A: \( b_{\text{low}} = 0.06 \text{ m}, \ b_{\text{up}} = 0.16 \text{ m}, \ \theta_{\text{low}} = 0^\circ, \ \theta_{\text{up}} = 90^\circ. \)

<table>
<thead>
<tr>
<th>( b_{\text{opt}} ) [m]</th>
<th>( \theta_{\text{opt}} ) [rad(deg)]</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.7784</td>
<td>4.45216*</td>
<td>8</td>
<td>47</td>
<td>5</td>
</tr>
</tbody>
</table>

The optimal value of \( b \) coincides with the lower bound \( b_{\text{low}} \) imposed in the Matlab\textsuperscript{®} algorithm. This is physically explainable by recognizing that when the hole dimension is smaller, the surface of the plate with holes is bigger, and it offers a greater resistance to the shear flux. In figure 11, the displacement distribution is represented for a more refined static analysis, whereas figure 12 shows the considerable twist of the plate that tends to rotate around the y-axis.

This is due to the fact that the external transversal force is not applied on the shear centre of the section, but only on one skin, producing therefore a torque.

* Exitflags: 1 = First-order optimality measure less than options; 0 = Number of iterations exceeded; 4 = Magnitude of the search directions less than options; 5 = Magnitude of directional derivative less than options

6.3 Case D: Multi-objective optimization

Once that the optimal configurations for bending and shear cases have been found individually, it is a natural consequence to study them applied together in the same optimization in order to see the influence of both cases on the final values of the design variables.

In fact, in any practical application these kind of loads act simultaneously and it would be more useful to optimize the plate for a general load condition.

Therefore in this part a multi-objective optimization is analyzed, that is so called because in the same step two separate static analysis are performed. From each of them, the corresponding value of the elastic energy is calculated and the final objective function will be their weighted average.

The boundary conditions must be the same for both loads: the C-F-F-F boundary condition is chosen.
Moreover, since the objective function depends on the single energy values, care is taken in order that they have at least the same order of magnitude, otherwise the final solution would depend exclusively on the dominant load.

Calling $E_1$ and $E_2$ the elastic energy values of bending and shear plates respectively, the new objective function becomes:

$$ f(b, \theta) = \alpha_1 E_1(b, \theta) + \alpha_2 E_2(b, \theta) \quad (9) $$

where $\alpha_1$ and $\alpha_2$ are weights whose sum is equal to one, and that are modifiable in order to assign more importance to one load than to the other.

In the following table, the results of the optimizations performed changing the weights values are presented; the nonlinear constraint of equation 7 is taken into account, whereas the upper and lower bounds of the variables are the same as case A: $b_{\text{low}} = 0.06 \, \text{m}$, $b_{\text{up}} = 0.16 \, \text{m}$, $\theta_{\text{low}} = 0^\circ$, $\theta_{\text{up}} = 90^\circ$.

### Table 5: Multi-optimization results

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$b_{\text{opt}}$ [m]</th>
<th>$\theta_{\text{opt}}$ [rad(degree)]</th>
<th>Elastic energy [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.113138</td>
<td>0.7806 (44.7)</td>
<td>4.238560</td>
</tr>
<tr>
<td>D2</td>
<td>0.7</td>
<td>0.3</td>
<td>0.113138</td>
<td>0.7809 (44.7)</td>
<td>4.240919</td>
</tr>
<tr>
<td>D3</td>
<td>0.3</td>
<td>0.7</td>
<td>0.106669</td>
<td>0.7865 (45)</td>
<td>4.205339</td>
</tr>
<tr>
<td>D4</td>
<td>0.9</td>
<td>0.1</td>
<td>0.144570</td>
<td>1.5487 (88.8)</td>
<td>3.644372</td>
</tr>
<tr>
<td>D5</td>
<td>0.1</td>
<td>0.9</td>
<td>0.074551</td>
<td>0.7822 (44.8)</td>
<td>3.731640</td>
</tr>
</tbody>
</table>

As visible, increasing the weight coefficient of one load respect to the other makes the solution closer to the optimal configuration of the plate subject to that load.

This is physically reasonable since, as one should expect, the optimal configuration of the plate subject to a combination of those loads is an average of the individual ones, giving more weight to the load that presents the highest coefficient in equation 5.

### 6.4 Case E

In these optimizations, the design variables become $b$ and $\gamma$ in the first two cases, $\theta$ and $\gamma$ in the last two. The loads and boundary conditions are the same as cases A and C.

Actually, under a geometrical point of view, the modelling of the fins inclination resulted to be easier if expressed in terms of the ratio between the two segments $\overline{AB}'$ and $\overline{AB}$ shown in figure 14, instead of $\gamma$.

![Figure 14: Geometry of the basic cell with inclined fins](image)

$$ \lambda = \frac{\overline{AB}'}{\overline{AB}} \quad (10) $$

Referring to figure 14 and to equation 10, a simple calculation led to the following relation between $\lambda$ and $\gamma$.

$$ \gamma = \sin^{-1} \left( \frac{\overline{BB}'}{\overline{BC}} \right) = \sin^{-1} \left( \frac{1-\lambda}{\lambda} \right) \quad (11) $$

**E1 – Bending loaded plate ($b$, $\gamma$):** considering $b_{\text{low}} = 0.4$, $b_{\text{up}} = 1.2$, $b_{\text{low}} = 0.06 \, \text{m}$, $b_{\text{up}} = 0.16 \, \text{m}$, the optimization results are shown in table 6.

### Table 6: Case E1 results

<table>
<thead>
<tr>
<th>$b_{\text{opt}}$ [m]</th>
<th>$\lambda_{\text{opt}}$ [-]</th>
<th>$\theta_{\text{opt}}$ [rad(degree)]</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>1.1938</td>
<td>(-13.2)</td>
<td>0.084031</td>
<td>8</td>
<td>50</td>
<td>5</td>
</tr>
</tbody>
</table>

**E2 – Shear loaded plate ($b$, $\gamma$):** the optimization results are shown in table 7.

### Table 7: Case E2 results

<table>
<thead>
<tr>
<th>$b_{\text{opt}}$ [m]</th>
<th>$\lambda_{\text{opt}}$ [-]</th>
<th>$\theta_{\text{opt}}$ [rad(degree)]</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.4(46.1)</td>
<td>0.040993</td>
<td>4</td>
<td>12</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
E3 – Bending loaded plate \((θ, γ)\): considering \(θ_{low} = 0°\), \(θ_{up} = 90°\) \(λ_{low} = 0.4\) and \(λ_{up} = 1.2\), the optimization results are shown in table 8.

<table>
<thead>
<tr>
<th>(θ_{opt} ) rad</th>
<th>(γ_{opt} ) deg</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7710( (44.2^\circ) )</td>
<td>1.2( (-14^\circ) )</td>
<td>0.174764</td>
<td>8</td>
<td>88</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 8: Case E3 results

E4 – Shear loaded plate \((θ, γ)\): the optimization results are shown in table 9.

<table>
<thead>
<tr>
<th>(θ_{opt} ) rad</th>
<th>(γ_{opt} ) deg</th>
<th>Elastic energy [J]</th>
<th>Iterations</th>
<th>Function evaluations</th>
<th>Exitflag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7852( (45^\circ) )</td>
<td>0.4( (46.1^\circ) )</td>
<td>0.050990</td>
<td>16</td>
<td>70</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 9: Case E4 results

In figure 15, the displacements distributions of cases E are shown, whereas their optimal basic cell configurations are presented in figure 16.

Figure 15: Displacement distributions [m]: i)E1 ii)E2 iii) E3 iv) E4

Figure 15: Optimal basic cell configurations: i)E1 ii)E2 iii) E3 iv) E4

7. CONCLUSIONS

The main conclusions that were possible to obtain from this work are:

- The optimal configuration of the panel withstanding an orthogonal uniform pressure, namely the bending case, consists of maximizing its height and therefore the holes dimension. When \(γ\) is used as design variable, its optimal value implies inclined fins, and consequently a lower panel height.
- The optimal configuration of the panel withstanding the shear load consists of minimizing the hole dimension (therefore the panel height), and setting the hole orientation to \(45^\circ\). When \(γ\) is used as design variable, it assumes the optimal value of \(46^\circ\), that coincides with its lower bound;
- When the bending and the shear loads are applied together in the multi-objective optimization, that considers \(b\) and \(θ\) as design variables, the optimization results lie between the optimal values obtained for the individual cases: in particular, changing the weights assigned to each loads makes the solution closer to one case than to the other.

REFERENCES


* Exitflags: 1 = First-order optimality measure less than options; 0 = Number of iterations exceeded; 4 = Magnitude of the search directions less than options; 5 = Magnitude of directional derivative less than options