Numerical and Experimental Study of Parametric Rolling of a Container Ship in Waves

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Abstract

This study deals with the calibration and the validation of a theoretical method and its numerical solution to foresee the susceptibility of a vessel to the occurrence of parametric rolling. A frequency domain and a time domain code are used in conjunction for numerical calculations. This study aims to validate both codes by comparing them to the experimental results of scaled model testing of a C11 type container vessel.

In scope of this work, the forced oscillation tests are analysed and the parametric rolling experiments are compared to the results obtained using the numerical code. Furthermore, a user interface is developed to facilitate the interaction between the frequency domain and the time domain programs and to shorten the calculation times considerably.

In addition to validating the strip theory code in use, the analysis of the forced oscillation tests aim to assess the influence of taking instantaneous cross-sections into account on the calculation of added mass and damping for heave and pitch motions. As symmetric (upright) and the asymmetric (heeled) cross-sections are analysed, the study reveals the change in hydrodynamic coefficients due to the heeling of the vessel.

The parametric rolling experiments that have been conducted include head and following waves (regular and irregular) in two different loading conditions. This data is compared to the numerical results. Then, with the help of the user interface developed, complete numerical results are presented in polar diagrams.

Keywords

Parametric Rolling, Non-linear Ship Dynamics, Ship Stability in Waves, Strip Theory, Hydrodynamic Coefficients, Asymmetric Cross-Sections
Resumo

Estudo Experimental e Numérico do Balanço Paramétrico de um Navio Porta-Contentores em Ondas

Este estudo trata da calibração e validação de um método teórico e da sua solução numérica para prever a susceptibilidade de ocorrência de ressonância paramétrica num navio. Códigos de computador no domínio da frequência e do tempo são usados em conjunto para efectuar os cálculos numéricos necessários. Este estudo tem como objectivo validar os referidos códigos, comparando-os com os resultados experimentais realizados num modelo à escala de um navio porta-contentores do tipo C11.

No âmbito deste trabalho, testes de oscilação forçada são analisados e os ensaios de ressonância paramétrica são comparados com os resultados numéricos obtidos utilizando os códigos acima referidos. Além disso, é desenvolvido um interface de utilização para encurtar consideravelmente os tempos de cálculo.

O código da teoria das faixas utilizado para análise dos testes de oscilação forçada efectuados permitem ainda avaliar qual a influência de se considerar no cálculo da massa adicionada e do amortecimento em arfagem e cabeceio as variações instantâneas das secções transversais do navio. Esta análise diferenciada para secções transversais simétricas (navio direito) e assimétricas (navio adornado) possibilitam ainda identificar quais as variações impostas aos coeficientes hidrodinâmicos supra-citados devido à inclinação do navio.

Os ensaios experimentais de ressonância paramétrica em ondas regulares e irregulares realizados incluem ondas provenientes dos sectores à proa e popa para duas condições de carga do navio distintas. Este dados são posteriormente comparados com os resultados numéricos. Finalmente, utilizando o interface desenvolvido, é apresentado um conjunto alargado de resultados numéricos sob a forma de diagramas polares.

Palavras Chave

Ressonância paramétrica, Dinâmica de navios não-linear, Estabilidade de navios em ondas, Teoria das faixas, Coeficientes hidrodinâmicos, Secções transversais assimétricas.
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1.1 Ship Motions in a Seaway

Ships are built to carry materials or transfer people in a seaway. In order to achieve this goal, the vessel should meet certain requirements such as floating upright, possessing enough speed and being able to manoeuvre as required. The current knowledge on hydrostatics makes it relatively easy to design a vessel that will meet the requirements in calm water. Yet, ships rarely sail in calm water and mostly encounter more challenging conditions. Therefore, the motions and the stability of a ship in the seaway are severely affected by the waves.

The dynamic response given by the ship will differ for each particular vessel, and for each loading condition. Hence, apart from the basic stability calculations, it is important to design the vessels with a precaution against dynamic vulnerabilities. Changes to the ship hull form to reduce stability variations, reduction of the superstructure area or installation of roll-stabilization devices can all be considered at this point.

Often, these steps are not enough to ensure safety and it is important to carry out extensive studies of dynamic instabilities to provide the shipmaster with a decision support tool to prevent large amplitude motions in different encounter conditions. Considering the latest developments in computer technologies, the most convenient way to achieve this goal would be to install a real-time numerical simulation program onboard or present an output from a slower program that can be easily interpreted to help navigation in rough weather conditions.

1.2 Dynamic Instabilities in Waves

The effects of waves on the ship may include but are not limited to the motions themselves. There are also other dynamic effects that should be considered. For instance, spraying and green water on deck may be given as examples of problems that are not directly related to motions [1]. Specifically slamming, a condition where the pressures exerted on the ship by the water are very
high, is related to the motions and repeated slams will impair the ability of the personnel to work and may damage the vessel.

Along with these, there are other intact dynamic stability problems which are usually associated with large roll angles. The cause is generally the unfavourable tuning between the ship’s natural frequency and the wave encounter frequency. This mechanism, known as synchronous rolling, can be analysed under linear seakeeping theory. However, there are four other non-linear phenomena which may induce large roll angles that lead to cargo damage or capsize. The value of roll damping comes into play along with the consideration of the metacentric height as a dynamic variable rather than a static value. These four scenarios in which the non-linearities of the roll restoring moment play a leading role are:

- Parametrically excited roll
- Roll motion due to pure loss of stability
- Broaching-to
- Any given combination of these three scenarios

While all these phenomena are briefly explained in here, the focus of this study will be the understanding of parametrically excited roll. A ship sailing in longitudinal waves faces a change in transverse stability as the waterplane area increases on a wave through and decreases on a wave crest. When this oscillatory change occurs at approximately twice the natural roll frequency, the roll motion is amplified, possibly reaching to significant and unacceptably high amplitudes [2]. Figure1.1(b) and figure1.1(a) illustrate these changes of waterplane area on a wave crest and wave trough while the wave has a wavelength equal to the ship’s length. The changes lead to a decrease in the transverse righting arm when the wave crest is aligned with amidships and increase when the trough is aligned with amidships. Any attempt to describe this dynamic instability scenario should also consider at least the coupling effects between the heave, pitch and roll motions [3].

As in the parametrically excited roll case, the effect of having stability variations on longitudinal waves comes into play in roll motion due to pure loss of stability as well. In cases where the vessel encounters waves that place it in a position where it stays on a wave crest for long enough, the ship may experience a pure loss of stability due to the reduced righting arm caused by the crest. The situation is more likely occur if the vessel's speed is high and it is overtaking or following a wave. The length of the wave, along with its height should also be considered. A wavelength one to two times the ship's length will lead to greater change in waterplane area and therefore will reduce the righting arm to values that are most likely to induce instabilities.

Broaching-to occurs when the heading of the vessel changes so that the beam faces the waves and wind, and there’s a danger of capsizing. It is more commonly seen in small-craft or
fishing vessels. Usually broaching-to is accompanied by difficulties in steering. A breaking wave, approaching from the aft grips the stern and pushes the vessel sideways while the bow digs and holds its ground. The vessel may therefore attain large angles of heel, and further wave breaking upon the vessel may lead to capsize.

1.3 Motivation

Stability has been a concern for all floating structures that have ever been built. The first treaty that deals with this was passed in 1914 by the Safety of Life at Sea (SOLAS) convention. To further develop the code and to fill the gap of an international organization, the Inter-Governmental Maritime Consultative Organization (IMCO) was established in Geneva in 1948. The name was later changed to International Maritime Organization (IMO) in 1982. Although the convention has been updated to meet the necessities several times, dynamic instabilities have generally been overlooked. Only by the seventies, the German delegation has reported unexpected container losses in in longitudinal waves due to heeling angles around 40° to be considered by the IMO. Parametric roll resonance in head seas, specifically came into attention relatively recently [4]. Until now, American Bureau of Shipping (ABS) is the only classification society that has published a guideline on how to avoid parametric rolling in head seas [2] in 2004 (updated in 2008).

To address this issue, over the past nine years, Instituto Superior Técnico has been working on the development of a numerical model for the calculation of the movements of vessels in regular and irregular waves [3]. However, the code has not been fully validated, therefore it is necessary to conduct experiments and compare them to the results of the code. The experiments in question are the experiments from a scaled of a C11 type container ship that has been tested in the El Pardo towing in Madrid between the dates of 15 February 2009 and 13 March 2009. The work has been sponsored by the European Commission 6th framework project Hydralab III.
1.4 Objectives

This study aims to analyse and compare the data obtained from forced oscillation tests and parametric rolling tests to the numerical codes. To achieve this, a program that carries out the analysis of the forced oscillation tests will be developed and the results will be compared to the output of strip theory codes for upright and heeled vessel. Parametric rolling tests will then be compared to the time domain program results. Furthermore, in order to present numerical results for all headings and speeds, a method to create polar diagrams will be suggested. Additionally, the work targets to reduce the calculation times of the numerical program by developing an intermediate application that simplifies the process.

1.5 Outline

The work starts with the introductory chapter, which gives an outline and a brief description of the basic concepts of dynamic instabilities. Motivations are also presented. In order to carry out any further work in any field, it is a necessity to be aware of the latest advancements. Therefore, chapter 2 provides a review of the state of the art. The methods that are currently in use, their applications and limitations are discussed. Chapter 3 presents the theoretical background and the classical formulation of the hydrodynamic problem and its proposed solution. Linear seakeeping theory in regular and irregular waves is explained. Based on regular waves calculations, the superposition principle is introduced in order to reach a generalization for irregular waves scenario. The chapter concludes with a review of the non-linear seakeeping theory.

After the theory is covered, chapter 4 introduces the software that has been used to obtain the numerical results. Its functions as well as the usage are outlined. The input/output files and their structures are detailed. The user interface which has been developed in this study to automatically prepare the required files and iteratively run simulations is further explained in this chapter. The next chapter, chapter 5, describes the experimental work that has been conducted in the El Pardo towing tank, Madrid. The structure of the experiments are presented, along with the methodology.

The results obtained from the strip theory codes and the forced oscillation tests are compared in chapter 6. The hydrodynamic coefficients that are in question include the added mass and damping for pure heave and pitch motions. The comparison has been carried out for both the heeled vessel and the upright vessel. Chapter 7 then deals with the time domain code that is used for the prediction of parametric roll motion in regular and irregular waves. With the help of the user interface that was developed, polar diagrams are obtained to show results in all of the possible headings and speeds that the vessel is prone to suffer from parametric rolling. The work is concluded with chapter 8, presenting the general conclusion and discussing possible works.
2

State-of-the-Art

2.1 Introduction

The focus of chapter 2 is to review the state-of-art in large amplitude ship motions with a particular attention given to parametric rolling. The ideas behind two different numerical solutions, frequency domain and time domain, are presented along with the non-linearities leading to large amplitude motions. Chapter concludes with a brief discussion on possible methods for the prevention of parametric rolling.

2.2 Hydrodynamic Problem of Ship Responses in Waves

At the present, there are three different types of numerical methods to predict ship's responses in waves, which can be applied to different hydrodynamic problems and different types of vessels.

If the structure is very small compared to the encountered wavelength, then Morrison equation may be used. The assumption is that the force is composed of inertia and drag forces, and summed up linearly. On the other hand, in cases where the drag force is small compared to inertia, Froude-Krylov theory can be applied to small structures. The theory utilizes the incident wave pressure and pressure-area method. The third methodology is based on diffraction theory. It is applicable to structures which have a size comparable to the wavelength and where the structure alters the wave field in its vicinity, as in ships. In this case, the diffraction of the waves should be taken into account in the calculation of the forces.

This theory is known as the diffraction theory. A solution is possible for slender hull forms such as long hulled ships and barges. The Laplace equation is solved considering the boundary conditions, and the theory is termed the strip theory. Strip theory has been in use for ships for a relatively long time.

The motions caused by the waves are of concern even in the early design stage of ships.

\* The deviation in the direction of a wave at the edge of an obstacle in its path
Yet the complexity of the above mentioned methods and the environment in which the vessel operates have made experimental methods the only reliable method of prediction for a long time. The emergence of strip theory which simplified the three dimensional problem into manageable form has then introduced possibility of a quicker solution to the hydrodynamic calculations.

2.3 Frequency Domain Numerical Modelling of Ship Motions

The earliest theoretical works that can be mentioned on prediction of hydrodynamic loads in a seaway were carried out by Froude [5] and Krylov [6]. Although the theoretical research on wave-induced ship motions and hydrodynamic loads in a realistic seaway can be traced back to the work done by Froude and Krylov, the significant breakthrough was the strip theory developed by Korvin-Kroukovsky [7], which was the first motion theory suitable for numerical computations and had adequate accuracy for engineering applications. Unfortunately later, inconsistencies in the mathematics were found in this theory particularly, as it did not satisfy the Timman-Newman relationships [8]. Modified versions of strip theory have since been proposed, of which, the version developed by Salvesen, Tuck and Faltinsen [9] is mostly widely used in ship design. It provides satisfactory performance in the prediction of the motions of conventional ships as well as computational simplicity.

The traditional strip theories consider the vessel to be wall sided and the underwater geometry as not variable. This brings certain limitations such as not being able to predict the responses in severe seas as the changes in the underwater geometry become a considerable factor in large motion amplitudes. Time domain strip theory methods aim to overcome this shortcoming by using a time stepping scheme and calculation of the forces on the actual underwater part of the hull at each time step. Another limitation of conventional strip theory methods is their deviation from realistic results when frequency of oscillation is low or the ship speed is relatively high. These restrictions were lifted on slender-bodies by the “unified slender-body theory” of Newman [10]. The theory was further refined recently by Kashiwagi, et al. [11].

Apart from the strip theory based frequency domain solutions, three dimensional methods are also available. However, the numerical solution of the 3D panel methods largely depend on the computational power since the problem is too complex. The main change from strip theory methods is that this method can be applied to all structures even if the body cannot be considered slender. Two solutions are proposed, one depending on the Green function method by various authors and the other one considering the Rankine Panel Method initially by Dawson [12]. While the Rankine Panel Method reduces the complexity in calculation compared to Green function, its stability remains a concern.
2.4 Time Domain Numerical Modelling of Ship Motions

Time domain numerical methods focus on predicting the forces acting on the body and the movements at predefined time steps. At each given time step, the underwater part of the hull is calculated and the forces obtained using these instantaneous properties are transcribed into motions. It is an iterative process, where the output from the last time step becomes the input for the next step.

Standard strip theory is a method which considers small amplitude motions. The limitation comes from the consideration that the underwater geometry is not changing, which does not hold true for large amplitude motions. Certain improvements on this limitation have been suggested to overcome the problem and they include the calculation of hydrodynamic coefficients and forces based on the instantaneous conditions. In practical terms, it is important for this method to keep its simplicity in order to keep its advantage in terms of computational and modelling time. A fully time domain methodology has been adopted, first by Xia and Wang [13], then by Fonseca and Guedes Soares [14], and more recently by Pereira [15] with evaluation of convolution integrals over all previous time steps. Another methodology has been presented, first by Oakley, Paulling and Wood [16], then by Elsimiliawy and Miller [17] and Tao and Incelicik [18], and more recently by Fan and Wilson [19] and Ribeiro e Silva [3] where at each time step, the exact submerged part of each section is extracted, hydrodynamic coefficients and forces are then calculated based on this instantaneous body boundary. It has been shown that the change in hydrodynamic coefficients may amount to significant values due to large amplitude ship motions [20]. From an engineering point of view, this is a hybrid method, i.e. coupled in time and frequency domains where the equations of ship motions still keep a simple format.

It is also possible to consider a three dimensional time domain method. The frequency domain counterpart of these methods assume that the movements are sinusoidal and steady. Therefore it is not suitable to be used in transient stages of motion. Various authors have tried to overcome this problem by incorporating the Green function and this approach has proved to be more effective in time domain than frequency domain. This is due to the fact that it is easier to calculate the speed effects in time domain compared to frequency domain. Unlike strip theory methods, three dimensional methods are effective in calculating the large-amplitude motions.

When two dimensional methods and three dimensional methods are compared, the main advantage of 2D methods is found to be their simplicity. On the other hand, 3D methods turn out results that are closer to experimental data in expense of more computational power and more complex input data. In order to use a strip theory code, the input would be limited to the offsets of the vessel, while 3D methods would require a model of the hull including surfaces. In engineering, time is usually of greater concern and the reliability of strip theory methods is high enough to make them sufficient. Advances in computer technologies combined with improved coding would turn
the advantage to the 3D methods in a longer time scale but currently a method that would need
to present results in a relatively shorter time would need to depend on 2D strip theory methods.

2.5 Prediction of the Motions of Ships in a Seaway

In a realistic seaway, as the ship's underwater geometry changes, non-linearities in ship mo-
tions gain importance and they can be listed as:

- Variations of hydrostatic and Froude-Krylov forces
- Variation of roll damping due to ship's forward speed and maximum roll amplitude
- The ships directional stability due to the position in waves
- Large amplitude motions

These can be addressed by time-domain analysis. As discussed before, a fast 3D solution
is beyond the capabilities of the computers of this age and a 2D solution seems to be the most
appropriate way. Many other simplifications are needed but the non-linear terms in the equations
of motion would have to be included to take the movements and forces mentioned above into
account. For the purpose of this study, the parametric rolling is discussed more in detail.

Parametric Rolling in a Longitudinal Seaway

According to classical seakeeping theory, large roll responses are associated with beam seas
in cases which the ship's natural roll frequency coincides with the encounter frequency, leading
to dynamic amplification. However this is not the only case which results in large amplitude roll
motions. Certain non-linearities may also induce extremely pronounced rolling. The commonality
in these non-linearities is the change of roll restoring energy in time. Therefore this parameter
should be treated as a dynamic value instead of a constant value.

If a vessel travelling in longitudinal waves is considered, it can be demonstrated that when
the ship encounters a wave with a length that is approximately equal to the ship's length itself,
the righting arm increases in a wave trough and decreases with the wave crest aligned with the
midship. The result is a motion coupled with heave, pitch and roll and therefore the restoring
forces and moments should include relevant effects.

One of the first theories of parametric excitation came from Grim [21] in 1952 followed by
Kerwin [22] in 1955. This marks the early fifties as the time where studies start. Lately, it has
been Umeda et al. (1995) [23] and Hanamoto et al. (1996) [24] who has put in a comprehen-
sive study, containing proposals for criteria for pure loss of stability, broaching-to, surf riding and
parametric resonance in astern seas which was accepted and published by IMO [25]. In Umeda
and Hamamoto's case, the consideration was given to astern seas. However, head seas have
not attracted much attention until the a post-Panamax C11 class container-ship has encountered problems showing the practical importance of the phenomenon [26].

Specific importance to parametric roll in head seas was given by Burcher [4]. Later, it has been examined, using linear and non-linear theories by Ribeiro e Silva and Guedes Soares [27]. The linear model considered the Mathieu equation, and was not accurate enough to predict the motions under wave-induced parametric resonance conditions [3]. Therefore, a model that took the deck submergence and other non-linearities into consideration was proposed. Another single degree of freedom model was proposed by Bulian [28] based on quasi-static assumption for heave and pitch.

When the importance of coupling is considered in parametric roll, it can be said that the 1 degree of freedom (DOF) models cannot effectively deal with these effects. Therefore to overcome the shortcomings, a 5 DOF model was presented by Ribeiro e Silva, et al. [29]. Other authors such as Neves and Rodriguez [30] and Tondl et al. [31] have used 3 DOF approaches. These multi degree of freedom models consider the coupling between the motions to create a more realistic numerical model.

2.6 Prevention of Parametric Roll

In order for parametric roll to start, certain parameters, such as the roll damping and the auto-excitation forces, have to drop below or exceed threshold values respectively. Hence, in order to prevent the problem, one must consider either increasing the damping or reducing the moment created by the auto-excitation forces. Preventative measures such as active or passive roll stabilization devices aim to achieve this. An example to passive devices would be the “U” type tanks while active fins could be given as an example for the latter. The design of such devices is important as they should be over a certain “effectiveness threshold” to be worth the investment.

In cases that such investments are not seen feasible, a method would be to introduce a decision support system that would inform the shipmaster of foreseen danger. If the sea-state and the relevant vessel information, namely heading and speed, is known, it would be possible to avoid dangerous situations by using real-time calculation or pre-calculated data. Real-time calculation onboard the vessel would require appropriate hardware to predict the incoming waves and immense computational power thus it might not be practically applicable. On the other hand, pre-calculated data could be obtained and presented for vessels in form of polar diagrams for each sea state and loading condition. A shipmaster that is in possession of such diagrams would alter the route accordingly to avoid problematic combinations of the ship’s heading and speed.
3 Theoretical Background

3.1 Introduction

This section covers a brief review of the linear and the non-linear seakeeping theories, starting with the ideal potential flow hypothesis, proceeding with a brief explanation of the assumptions adopted to solve each of the hydrodynamic problems and ending with a presentation of the equations of motion for both cases.

As far as the linear theory is concerned the correct calculation of the motions is limited to small motions and a slender ship hull as they lead to small changes of the waterplane area which in turn makes linearisation possible. Also the initial theory considers sinusoidal waves, although the superposition principle extends the use to irregular waves by allowing to sum the sinusoidal functions. As solution is then obtained in frequency domain.

The small amplitude motion limitation makes it necessary to go one step further and to introduce the non-linear seakeeping theory and to solve the equations in time domain. However, the theory is very complex and therefore certain simplifications should be introduced to at least allow manageable calculation times. Therefore only the non-linear effects that are considered more important for practical engineering calculations are paid attention to. The Miller method, which has been used to estimate the roll damping in this work, has been discussed and the chapter ends with the presentation of the Mathieu equation which can be utilised to determine zones of instability under parametric rolling conditions.

3.2 Linear Seakeeping

The section presents the potential flow theory around a ship’s hull moving with a certain mean advance speed. The boundary value problem is solved to obtain the potential flow velocity around the ship considering the fluid as an ideal fluid (i.e. homogeneous, inviscid, incompressible and without surface tension). As the fluid is considered ideal, the flow field may therefore be rep-
represented by a scalar fluid velocity potential ($\Phi$) which complies with the continuity equation and boundary value problem within the fluid domain [32].

### 3.2.1 Coordinate Systems

In order to advance in the calculations a certain coordinate system should be decided on first. Three orthogonal and right handed coordinate systems are proposed. As illustrated in figure 3.1, the coordinate systems are:

\[ X_0 = (x_0, y_0, z_0) \implies \text{Fixed in Space} \]
\[ X = (x, y, z) \implies \text{Advances with the ship forward speed} \]
\[ X' = (x', y', z') \implies \text{Body Fixed} \]

Hence, all of the ship’s motions may be expressed in six degrees of freedom presented in the figure 3.2. The motions are given numbers following the $X$, $Y$, $Z$ coordinate convention presented in figure 3.1. The first three displacement coordinates ($\xi_1$, $\xi_2$, and $\xi_3$) represent the planar motions of surge sway and heave while ($\xi_4$, $\xi_5$, and $\xi_6$) represent rotational motions of roll, pitch and yaw along the $X$, $Y$ and $Z$ axes respectively.

### 3.2.2 Potential Flow

The potential flow hypothesis leads to a solution considering certain boundary values in order to linearise the potential flow problem. It is a set of simplifications over the real flow of a fluid.

Although the water density changes in the large scale of the ocean, around the ship the water density variations are very small, so they can be omitted to obtain an incompressible fluid in which the density stays the same. The fluid is therefore homogeneous as well. It may also be approximated that the fluid is inviscid, imposing no vorticity. This approach is justified based on the fact that water viscosity is quite small and the flow is kept irrotational over a large extend of the domain of the hydrodynamic problem that is considered here [3]. Furthermore, if the waves
are small, the surface tension would be the force acting on the ripples to level the water. As these
ripples are very small to affect the hydrodynamics of the ship and for larger waves the prominent
factor is replaced by gravity, the surface tension is of little practical importance for the engineer and
therefore may be neglected as well. Considering these limitations, certain formulations emerge:

• If the fluid is homogeneous and incompressible then the equation of conservation of mass
  reduces to the equation of continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(3.1)

where \(u\), \(v\) and \(w\) are fluid velocity vectors.

• If the fluid is inviscid then it is also irrotational (i.e. there is no vorticity, or it remains constant)
  and the fluid velocity vector may be represented by a scalar function, the velocity potential.
  In equation 3.2, \(\nabla\) stands for the gradient operator, \(\vec{V}\) is the velocity vector and \(\Phi\) is the
  velocity potential:

\[
\vec{V} = \nabla \Phi
\]

(3.2)

3.2.3 The Hydrodynamic Problem

Assuming that the fluid is inviscid and incompressible, the hydrodynamic problem is formulated
in terms of the potential flow theory. This means that the velocity vector of the fluid particles may
be represented by the gradient of a velocity potential as in equation 3.2. The fluid velocity potential
is defined as a scalar reducing the continuity equation to Laplace equation:
\[ \nabla^2 \Phi(\vec{x}_0, t) = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad (3.3) \]

Once the velocity potential is known, the determination of the fluid pressure may be carried out according to the Bernoulli equation:

\[ p(\vec{x}_0, t) = -\rho \left( \frac{\partial \Phi}{\partial t} + \frac{1}{2} |\nabla \phi|^2 + gz_0 \right) \quad (3.4) \]

In the equation 3.4, \( p(\vec{x}_0, t) \) represents the fluid pressure, \( \rho \) represents the fluid specific mass and \( g \) is the acceleration of gravity. Integration of the fluid pressures over the hull wetted surface results in the hydrodynamic forces acting on the ship.

A fully non-linear solution to the hydrodynamic problem is difficult and the linearisation of the problem is convenient. For this reason, sets of restrictions are applied to simplify the calculations. Different sets of restrictions lead to different seakeeping formulations (e.g. thin strip theory, slender body theory, strip theories, \( 2\frac{1}{2}D \) theory and panel methods). The boundary conditions are listed as:

- Slenderness of the hull
- Speed of the ship
- Amplitude of oscillation of the boundaries (free surface and hull)
- Frequency of oscillation of the boundaries

To linearise these boundary conditions, the velocity potential, \( \Phi(\vec{x}_0, t) \) has to be divided into two components, namely the steady flow (\( \bar{\Phi} \)) and the oscillatory flow (\( \tilde{\Phi} \)) and expressed as:

\[ \Phi(\vec{x}_0, t) = \Phi(x + Ut, y, z, t) = \bar{\Phi}(\vec{x}) + \tilde{\Phi}(\vec{x}, t) \quad (3.5) \]

The unsteady term (\( \tilde{\Phi}(\vec{x}, t) \)) is linearised using the small perturbations method and the boundary conditions are reduced to first order and expanded around the mean free surface using the Taylor expansion. The unsteady velocity potential is then decomposed further into incident (incoming) waves (\( \Phi^I \)), diffracted waves (\( \Phi^D \)) and radiated waves (\( \Phi^R \)):

\[ \tilde{\Phi} = \Phi^I + \Phi^D + \Phi^R \quad (3.6) \]

The radiation potential is then further decomposed into components related to each of the six oscillatory ship motions:

\[ \Phi^R = \sum_{j=1}^{6} \Phi_j^R, j = [1, 6] \quad (3.7) \]
After the linearisation and the application of the boundary condition and the Bernoulli equation, the equations for the exciting forces \( F^E \), radiation forces \( F^R \) and the hydrostatic forces \( F^H \) are obtained. The equations are then solved using an appropriate (e.g. strip theory) method.

### 3.2.4 Boundary Conditions

The solution for the Laplace equation presented with 3.3 can be obtained when the boundary conditions are defined. This section gives a brief description of the four boundary conditions imposed:

- Rigid-Body surface
- Free surface
- Sea bottom
- Radiation conditions at infinity

The rigid-body surface boundary condition ensures that the fluid does not penetrate the hull of the vessel and there are no void spaces between the fluid and the surface.

The free surface boundary condition states that the velocity potential must satisfy the kinematic and dynamic boundary conditions. The kinematic boundary condition implies that on this free surface, the vertical velocity of the fluid has to equal to the velocity of the very same surface, meaning that there’s no splashing of the water and all particles on the surface have the same velocity. The dynamic free surface boundary condition states that the fluid pressure at the sea surface is given by the dynamic equation (or Bernoulli’s equation) for irrotational flow.

The sea bottom boundary conditions states that the sea bed is at a very long distance from the free surface and therefore it is not affected by the motions. It is also defined as a deep water condition.

The radiation conditions at infinity simply state that the effect of the movements of the fluid at an infinite distance from the ship tends to zero, and therefore it may be taken as zero. It is a condition that is of higher importance to frequency domain methods.

### 3.2.5 Frequency Domain Solution of the Hydrodynamic Problem

The linear radiation and diffraction problems obtained can be solved by various theories including the strip theory. The incoming waves and the forced motions are assumed to be harmonic to simplify the calculations, therefore regular waves are in consideration. The switching from regular waves to irregular waves can be done through the superposition principle which will be explained later. The sections present a brief explanation of the forces involved.

Radiation forces result from the motion of a ship with forward speed. The force can be broken down into two components, one that is in phase with the acceleration of the motion and one in
phase with the velocity of the motion. For the acceleration component, the added mass should be included as well, presenting the concept of "equivalent mass". Added mass can be defined as the mass of the fluid that accelerates together with the body of the vessel due to the force in phase with the acceleration. The radiation force that is in phase with the velocity of the motion is related to the inviscid damping force that is proportional to the damping coefficient.

The hydrodynamic forces acting on the ship that is advancing with constant forward speed through a field of incident harmonic waves are called exciting forces. For the purpose of the calculations, the ship is considered to be restrained at its mean position without doing any oscillatory motion. These forces are further divided into two parts designated as Froude-Krylov forces and the diffraction forces. The Froude-Krylov forces are related to the incident wave field pressure and the diffraction forces are forces related to the perturbation of the incident wave field due to the presence of the ship itself.

Restoring forces result from the combination of the hydrostatic forces and the ship's own weight. For strip theory usage, certain simplifications are considered to linearise the values. The hydrostatic forces are calculated up to the still water surface \((z = 0)\), then angular motions are taken as small amplitude. Therefore, the waterplane of the vessel is subject to small changes. The ship's sides are vertical around the waterline leading to small changes of the waterplane area as well.

### 3.2.6 Equations of Motion

The equations of motion for the ships are based on Newton's second law which equates the mass times the acceleration of a body to the force acting upon it. In the case of a ship, the equilibrium is between the external forces (hydrodynamic) and the inertial forces associated to the ship's mass. Building on the assumption of small motions, the equation of motion is given as:

\[
\{ F^M_k \} = [M_{kj}] \{ \ddot{\xi}_j \}, \quad k, j = [1, 6] 
\]  

(3.8)

where the \([M_{kj}]\) term stands for the mass matrix and the \(\ddot{\xi}_j\) is the accelerations vector. The \(\{ F^M_k \}\) term is the summation of the radiation forces, exciting forces (which can be decomposed further into Froude-Krylov and diffraction forces) and the restoring forces. The final second order linear and homogeneous equations of motion are presented in following equations:

\[
\sum_{j=1}^{6} \{ (M_{kj} + A_{kj})\ddot{\xi}_j + B_{kj} \dot{\xi}_j + C_{kj} \xi_j \} = F^E_k 
\]  

(3.9)

Each of these equations is similar to any mass-spring-damper system equation. The \(F^E_k\) term represents the exciting forces on the right side of the equation and the damping, inertia and the restoring terms are on the left.
For the vessels with lateral symmetry, the system that consists of 6 coupled equations (equation 3.9) can be reduced to two uncoupled sets of equations of three equations each. The surge motion may be omitted in strip theory as the vessel is slender and the surge forces are small compared to others. While the heave and pitch motions are coupled between themselves, sway roll and yaw motions are coupled between each other.

The solution of a second order linear equation is assumed as harmonic:

\[ \xi_j(t) = \text{Re}\{\xi_j^A e^{i\omega t}\} = \xi_j^a \cos(\omega t - \theta_j) \] (3.10)

The equation presents \( \xi_j^A \) as the complex amplitude of motion, \( \xi_j^a \) as the real amplitude and \( \theta_j \) as the phase angle.

When the coupling effects are considered, the equations for the heave and pitch may be written as:

\[
[M + A_{33}(\omega)]\ddot{\xi}_3 + B_{33}(\omega)\dot{\xi}_3 + C_{33}\xi_3 + A_{35}(\omega)\ddot{\xi}_5 + B_{35}(\omega)\dot{\xi}_5 + C_{35}\xi_5 = F^E_3(t) \tag{3.11}
\]

\[
[I_{55} + A_{55}(\omega)]\ddot{\xi}_5 + B_{55}(\omega)\dot{\xi}_5 + C_{55}\xi_5 + A_{53}(\omega)\ddot{\xi}_3 + B_{53}(\omega)\dot{\xi}_3 + C_{53}\xi_3 = F^E_5(t) \tag{3.12}
\]

While equation 3.11 represents heave motion coupled with pitch, equation 3.12 represents the pitch motion including the coupling effects due to heave motion.

The other three modes, namely sway, roll and yaw are formulated with the following along with their respective couplings:

For sway:

\[
[M + A_{22}(\omega)]\ddot{\xi}_2 + B_{22}(\omega)\dot{\xi}_2 + A_{24}(\omega)\ddot{\xi}_4 + B_{24}(\omega)\dot{\xi}_4 + A_{26}(\omega)\ddot{\xi}_6 + B_{26}(\omega)\dot{\xi}_6 = F^E_2(t) \tag{3.13}
\]

For roll:

\[
[M + I_{44}(\omega)]\ddot{\xi}_4 + B_{44}(\omega)\dot{\xi}_4 + A_{42}(\omega)\ddot{\xi}_2 + B_{42}(\omega)\dot{\xi}_2 + A_{46}(\omega)\ddot{\xi}_6 + B_{46}(\omega)\dot{\xi}_6 = F^E_4(t) \tag{3.14}
\]

and for yaw:

\[
[M + I_{66}(\omega)]\ddot{\xi}_6 + B_{66}(\omega)\dot{\xi}_6 + A_{62}(\omega)\ddot{\xi}_2 + B_{62}(\omega)\dot{\xi}_2 + A_{64}(\omega)\ddot{\xi}_4 + B_{64}(\omega)\dot{\xi}_4 = F^E_6(t) \tag{3.15}
\]

For the coupling terms, the notation given as \( B_{ij} \) states the effect of the \( j \) mode on the \( i \) mode (i.e. \( B_{26} \) stands for damping in sway due to yaw).

The decoupling between the groups is a consequence of the symmetry of the ship’s hull forms about the xx axis. In certain dynamic instability scenarios these effects are important and should be considered.
3.2.7 Behaviour in Irregular Waves

It has been stated before that the equation 3.9 implies sinusoidal waves. On the contrary, ocean waves are highly irregular. Even so, they can be seen as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length, period or frequency and direction of propagation. This concept is known as the superposition principle and has been introduced first in hydrodynamics by St. Denis and Pierson [33]. The theory has its roots in electronics and communication field [34] and it states that the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually. A simple visual example using five components is presented in figure 3.3. As by definition a linear system is additive, the proposition holds true.

To explain the process, a simple case of the wave pattern observed at a fixed point \((x = 0, y = 0)\) and neglecting the directions of the waves may be components can be considered. This signifies that all wave components are travelling in the same direction resulting in a long crested irregular wave. According to the superposition theory, this irregular wave will consist of a large number of regular components, each having the equation:

\[
\zeta_i(t) = \zeta_i \cos(-\omega_i t + \epsilon_i)
\]  (3.16)

Where \(\zeta_i\) is a component amplitude corresponding to wave frequency \(\omega_i\) and \(\epsilon_i\) is the random phase angle. The total wave system is then assumed to be a summation of many (theoretically infinite number) of independent components:

\[
\zeta(t) = \sum_i \zeta_i \cos(-\omega_i t + \epsilon_i)
\]  (3.17)

This form also makes it possible to define these wave components in terms of a variance spectrum \(S(\omega)\) which is not discussed further. As a reminder, it is important to note that the
process works both ways. Random waves may be created by summing up a high number of components or they may be broken down into components to be analysed further.

3.3 Non-Linear Ship Motions

The linear formulation of the seakeeping theory based on the strip theory, considers motions of small amplitude and harmonic waves. The inviscid hydrodynamic forces are decomposed into several independent components (i.e. radiation, excitation, restoring) and the forces are considered to be linear. For this linearity to be valid, the amplitude of the motions should be small and the ship's sides should be considered vertical above the waterline. Given the case, the dependency of the hydrodynamic coefficients is reduced to the vessel speed and the encounter frequency. The frequency domain solution along with spectral techniques is adequate to grasp the idea of a vessel's dynamic behaviour in waves.

On the other hand, when the intent is to study the responses and non-harmonic non-linear (large amplitude) motions of the vessel in more severe seas, the frequency domain solution provides neither the necessary accuracy nor the formulation is able to capture the physics of some important phenomena. In this situation some non-linearities gain importance over others:

- **Non-linear excitation**: When the motions are considerably large, the underwater shape of the hull is subject to large changes and introduces the necessity to include the wetted surface variations ($S_W$) in the calculations. Moreover, it is not possible anymore to linearise the boundary condition about the mean wetted surface.

- **Non-linear effect in the potential flow**: The waves of high slope cannot be linearised. They induce large amplitude oscillatory motions making it impossible consider certain boundary conditions related to linear models.

- **Viscous Effects**: Viscous effects gain significant importance in the asymmetrical sway, roll and yaw movements due to the separation of the flow and the growth of the boundary layer.

- **Shipping of Water on deck**: When the water reaches the deck as a result of large amplitude motions, the shipping of water may considerably change the behaviour of the vessel. Therefore in some cases of small vessels it should be considered as relevant.

In order to incorporate these non-linearities, the computer code used in this work, ShipAtSea, takes a quasi-static approach to calculate the non-linear restoring coefficients in heave, roll and pitch motions in waves taking the instantaneous submerged hull into consideration [3]. This is due to the dependency of hydrodynamic components mainly on the overall submerged hull form while the quasi-static hydrostatic component is strongly dependent upon the wave passage and hull-shape [27].
While it would be possible to calculate the hydrodynamic coefficients (added mass and damping) at each time step, this would lead to the complication of the program and increase the calculation time. Therefore if such an application is necessary is subject to question, and will be addressed later in this work.

### 3.3.1 Equations of Motion

The equations of motion relate the exciting forces and the inertial forces with the mass of the ship. In the non-linear time domain case, the equations are solved with a numerical procedure for each time step. The non-linear equations of motion in time domain are given by:

\[
\sum_{j=1}^{6} \{(M_{kj}(t) + A_{kj}(\omega, t))\ddot{\xi}_j + B_{kj}(\omega, t)\dot{\xi}_j + C_{kj}(t)\xi_j\} = F_{Ik}(t) + F_{Dk}(t) + F_{Pk}(t) + F_{Rk}(t) + F_{Wk}(t) + F_{DkW}(t), \quad k = [1, 6]
\]  

(3.18)

The \( F_{Ik}(t) \) and the \( F_{Dk}(t) \) are respectively Froude-Krylov forces and the diffraction forces as in the frequency domain case. The \( F_{Pk}(t) \) is force created by the propeller, \( F_{Rk}(t) \) is the force due to the rudder, \( F_{Wk}(t) \) is the force exerted by the wind and finally \( F_{DkW}(t) \) term stands for the force induced due to the shipping of water on deck. The mathematical model in this work does not consider the forces exerted by the propeller, the rudder, the wind and the water on deck.

### 3.3.2 Effects of Surge Motion on Hydrostatic and Froude-Krylov Forces

In some conditions the surge motion has a significant effect on the exciting forces. Due to the changes in the applied pressure of the incoming wave, this should be also taken into consideration when calculating Froude-Krylov forces. Approximation methods can be used if the three dimensional velocities of the wave are unknown or the formulation is based on a 2D method such as the strip theory.

Hutchison and Bringloe [35] have suggested the following method as an improvement over the method suggested by Belenky and Sevastianov [36]:

\[
F_{1k}^T(t) = -\rho \nabla_0 \zeta_0 \omega^2 K(k_0) \cos(\beta) \cos(w t)
\]

(3.19)

Where \( k_0 \) is the wave number and \( K(k_0) \) is given by:

\[
K(k_0) = \frac{\sin \left( \frac{k_0 L}{2} \cos(\beta) \right)}{\frac{k_0 L}{2} \cos(\beta)}
\]

(3.20)

The calculation of the Froude-Krylov forces in longitudinal irregular waves is made considering the spectral peak frequency and the significant wave height.
3.3.3 Viscous Effects on the Transverse Plane

The viscous effects in sway, roll and yaw are considerable. Yet it is very difficult to model the viscous effects in sway and yaw. The damping in roll motion is worthy of particular notice, and should be estimated as accurately as possible to allow better estimation of the non-linear phenomena. Most methods that are utilized to calculate the damping include an iterative process. The maximum amplitude of the response depends on the estimated maximum amplitude of roll, therefore the process needs to be repeated if the estimated and the obtained roll angles are far from a pre-defined margin. This problem is solved in this work through the user interface which is explained further in chapter 4. The roll damping estimation method that was used is the Miller’s method [37]. The method presents the total damping in roll ($\varsigma$ in equation 3.21) as an addition of two components: the linear damping term ($\varsigma_1$) and the quadratic damping term ($\varsigma_2$):

$$\varsigma = \varsigma_1 + \varsigma_2 \sqrt{\xi d}$$  \hspace{1cm} (3.21)

The linear term ($\varsigma_1$) is given by:

$$\varsigma_1 = C_U 0.00085 \frac{L_{pp}}{B} \sqrt{\frac{L_{pp}}{G M_T}} \left[ \left( \frac{F_n}{C_b} \right) + \left( \frac{F_n}{C_b} \right)^2 + 2 \left( \frac{F_n}{C_b} \right)^3 \right]$$  \hspace{1cm} (3.22)

and the non-linear term is given by:

$$\varsigma_2 = 19.25 \left( A_{bk} \sqrt{\frac{L_{BK}}{b_{bk}}} + 0.0024L_{pp}B \right) \frac{b_{BK}^3}{L_{pp}B^3 T C_b}$$  \hspace{1cm} (3.23)

The variables used for calculation are as follows:

- $A_{bk} = l_{bk} b_{bk}$, one sided area of bilge keel (m$^2$)
- $l_{bk}$, length of bilge keel (m)
- $b_{bk}$, beam of bilge keel (m)
- $G M_T$, initial transverse metacentric height (m)
- $C_U$ is the correction factor for the speed effect. While it can be considered as unity, for slender ships such as frigates, $C_U$ is given by:

$$C_U = 4.85 - 3.00 \sqrt{G M_T}$$  \hspace{1cm} (3.24)

3.3.4 Simulation of Parametric Rolling in Longitudinal Waves

If the hull is considered symmetric and the waves longitudinal, it can be clearly seen that there is no wave exciting forces for the roll motion. In other words ($F^I_1(t) = F^P_1(t) = 0$) holds true. For this reason, the mathematical model given by the equation 3.14 reduces to:
\[(I_{44} + A_{44})\ddot{\xi}_4 + B_{44}(\xi_4, \dot{\xi}_4) + \Delta GZ(\xi_3, \xi_4, \xi_5, \zeta, \lambda, x_G) = 0\]  
(3.25)

which presents the damping and the restoring coefficient as non-linear terms. In the equation 3.25, \(\Delta\) is the weight of the ship, \(GZ(\xi_3, \xi_4, \xi_5, \zeta, \lambda, x_G)\) is the instantaneous righting arm where \(x_G\) is the relative position between the ship and the waves.

The non-linear \(GZ\) term can be linearised for some basic numerical simulation models [38], but the process is omitted here. The final linearised righting arm is:

\[GZ(\xi_4, t) = GM_{(wave)} \times \xi_4\]  
(3.26)

It is possible to write the equation of roll in a simpler form if the previous linearisations of the damping and restoring coefficients are considered:

\[\ddot{\xi}_4 + b_{44eq}\dot{\xi}_4 + \omega_{44}^2[1 + a_0 + a_1\cos(\omega t)]\xi_4 = 0\]  
(3.27)

where:

\[a_0 = \frac{GM_{trough} + GM_{crest}}{2GM_{(sw)}} - 1\]  
(3.28)

\[a_1 = \frac{GM_{crest} - GM_{trough}}{2GM_{(sw)}}\]  
(3.29)

where \(GM_{(sw)}\) is the GM in still water. In equation 3.27 further definitions are presented by:

\[\omega_{44}^2 = \frac{\Delta GM_{(sw)}}{I_{44} + A_{44}}\]  
(3.30)

\[b_{44} = \frac{B_{44eq}(\xi_4, \dot{\xi}_4)}{I_{44} + A_{44}}\]  
(3.31)

where equation 3.30 formulates the natural roll frequency squared and equation 3.31 provides the linearised damping coefficient.

It is possible to reduce the equation 3.27 to the Mathieu equation. The equation reveals important aspects of the ship motions as for certain encounter frequencies (\(\omega_e\)) it becomes unstable. Physically this means that a motion that is of increasing amplitude, up to a certain limit value, would be obtained by:

\[\frac{d^2\xi_4}{d\tau^2} + b_{44eq}\frac{d\xi_4}{d\tau} + (\delta + \varepsilon\cos(\tau))\xi_4 = 0\]  
(3.32)

Where:

\[\delta = \left(\omega_{44}/\omega\right)^2 \quad \text{and} \quad \varepsilon = a_1\left(\omega_{44}/\omega\right)^2\]  
(3.33)
Figure 3.4: The Ince-Strutt Diagram: The stability (unshaded) and instability (shaded) zones are demonstrated.

The calculations of the limits are beyond the scope of this study but the results corresponding to different \( \varepsilon \) and \( \delta \) values then can be visualized as stable and unstable zones in an Ince-Strutt diagram of which an example is presented in figure 3.4. The shaded regions in the figure demonstrate where instabilities may occur while the unshaded regions demonstrate stability (safe) regions. The border lines in the figure are considered unstable and are added to the instability zone.
4

The Computer Code

4.1 Introduction

In chapter 3, a brief review of the theoretical background of linear and non-linear seakeeping theories was presented. The frequency domain solution has been based on the strip theory. The time domain solution for the non-linear theory has been presented using the hydrodynamic coefficients as an input from the frequency domain calculations. The simplified governing equation of parametric rolling in the form of a Mathieu equation had also been presented.

The focus of chapter 4 is the description of the computer programs that are used to perform these calculations. Two different programs, a frequency domain code and a time domain code were developed by Instituto Superior Técnico in the recent years [38]. The frequency domain program in question is a strip theory based computer code that calculates the hydrodynamic coefficients and the initial exciting forces for the time domain code. The time domain code uses that data to simulate six degrees of freedom ship motions and to estimate the parametric rolling. These two programs are briefly explained along with their input and the output files. In this work, a third program was developed to facilitate the usage of these two and to speed up the calculation process. It includes but is not limited to a user interface which provides a graphical form to enter the required input. Then the program carries out all of the intermediate steps required by the other two codes and presents a complete output along with the graphics. This interface will be put under specific focus and will be explained in detail.

4.2 The Frequency Domain Program

As stated before in the chapter, the time domain code depends on the output from the frequency domain solution. Therefore before obtaining the motions in time domain, it is necessary to run the frequency domain executables to obtain the hydrodynamic coefficients.

The frequency domain program uses the “close-fit” method of Frank [39] for the calculations.
Usually around 21 stations are enough to describe the geometry, making use of smaller stepping between the strips at places where the geometry changes quickly or abruptly. The maximum points that can be utilized for each strip is 128 while 10 points is usually enough. For general strip theory use, the vessel needs to be modelled up to the waterline as the sides are accepted as vertical from then on. For the calculation of the hydrodynamic coefficients in heeled positions, the vessel has to be modelled completely.

The input file required for the program is named “fddata.dat”. It provides definitions of the geometry along with certain structural and geometrical characteristics of the vessel such as mass, moments of inertia, metacentric height of the vessel and longitudinal and vertical center of gravity. At the end of the file, there are the variables defining the encounter conditions (i.e. wave frequency and the speed of the vessel).

The executables needed for a complete calculation of frequency domain results can be presented as follows (order of execution is to be preserved to get correct results):

- **ADDAMP.EXE**: The executable is for calculating the added mass and damping coefficients for the given encounter conditions utilizing the Frank’s Close-fit method and the strip theory.

- **FEDIFRA.EXE**: The executable is for calculating the excitation forces (i.e. Froude-Krylov and Diffraction) for the given encounter conditions utilizing the Frank’s Close-fit method and the strip theory.

- **SOLVE.EXE**: The last executable of the frequency domain calculations solves the equations of motion to obtain the motions.

The flowchart explaining the processes of the frequency-domain numerical calculations is presented in figure 4.1.

### 4.3 The Time Domain Program - ShipAtSea

The time domain program, ShipAtSea, calculates the motions in six degrees of freedom (6DOF) utilizing the output of the frequency domain calculations. The restoring forces are nonlinear, therefore they are calculated at each time step. The values depend on the incoming wave, the geometry of the vessel and the position of the wave relative to the vessel. The obtained results are used to solve the equations of motion for each time step, using numerical integration methods of Runge-Kutta 4th order differential equations. The resulting motions are then presented to the end-user as an output. The flowchart regarding ShipAtSea is presented in figure 4.2.

ShipAtSea makes use of 5 different flags to decide on the calculation methods. These variables are “boolean-type” and are given in the input file as “0” or “1”. The flags are explained further below:
Figure 4.1: Frequency Domain Program Flowchart

- **Flag 1**: When set to “0”, hydrodynamic coefficients, hydrostatic coefficients and the diffraction forces are calculated around the instantaneous free-surface instead of average free surface.

- **Flag 2**: When set to “1”, the Froude-Krylov forces are calculated instantaneously, regardless of flag 1.

- **Flag 3 & Flag 4**: When both of these flags are set to “1”, the program calculates non-linear forces for asymmetric transverse sections along with the added mass and damping at each integration step.

- **Flag 5**: This flag is used for roll stabilization devices such as passive U tanks or active fins.

For the time domain calculations, certain input files are needed. Apart from “SHIPATSEA.IN”, which is specific to the time domain program, the other input files are prepared through processing of the frequency domain output files. All input files are explained further below:

- **{geometry file name.GF}**: This is the geometry file input required for ShipAtSea to run. It is a format used by hydrostatics programs such as Autohydro™ and GHS™.

- **Excitforce.dat**: This is an array of values representing the exciting forces in all six modes.
• **MaDmpStf.dat**: The file includes the Added Mass, Damping and Stiffness matrices in all six modes.

• **ShipAtSea.IN**: ShipAtSea specific input file. Provides the settings and values for the simulations.

Apart from these files, in case of random waves, three more files are included, one as an executable and one as an input file and finally the output file which is then read by ShipAtSea:

• **RandomWaves.IN**: The Random Waves input file includes data such as the spectrum type, number of wave frequencies and other spectrum specific data.

• **RandomWaves.EXE**: The executable uses the input file to generate random waves by applying the decomposition principle on generating harmonic components with randomly generated phase angles.

• **RandomWaves.OUT**: The output file consisting of the harmonic wave components.
The output of the program is “ShipAtSea.OUT” which is the ship’s motions, calculated in all six modes along with other data such as the wave elevation, instantaneous restoring coefficients and the excitation forces.

4.4 The ShipAtSea User Interface (GUI)

The results of the time domain ship motions come as the joint work between the frequency domain and the time domain programs. For each different simulation condition (e.g. speed, heading, wave frequency, different roll damping estimation methods) data should first be prepared to obtain the frequency domain output which is to be then processed as time domain input. The following action would be to prepare the time domain specific input and to obtain the end results. The final preparation would be to include graphics in the output, making it easier to interpret the results.

This process requires the user to be knowledgeable about the input and output file structures and also requires considerable time. In cases that larger-scale simulations are required (e.g. polar diagrams or any other “survey-type” results) the total number of simulations would be overwhelming. To overcome this difficulty, a user interface which lets the user visually decide what calculations are required without being too concerned about the structures of the files was designed. The flowchart regarding the program is presented in figures 4.3 and 4.4. The interface also handles consecutive operations, running “ranges of simulations”. Therefore, to get the results of a range such as the speed from 1 kts to 10 kts, it is only necessary to define the end points and the stepping increment in between. While the main aim of the ShipAtSea user interface is to save time on consecutive operation and make easier input and output possible, it has other features such as the “iterative-damping” option. These features will be outlined in this section.

The interface was programmed using Visual Basic Applications for MS Excel. While other programming languages may be faster in operation, this method is highly adaptable. File reading operations are considerably easier as well, considering the complexity of the structures dealt with. MS Excel’s cell logic makes it easy to find and replace each data. It is an highly detailed program in itself and with the total size being around 3000 lines.

4.4.1 Initiation of the Interface

After the initiation, the interface presents two tabs for the run modes and two other tabs for the settings. The run modes are classified as “single run” and “survey-run” modes. The setting tabs consist of the “options” tab and “damping options” tab. The “options tab” has mostly debug-type settings. “Damping options” include required input for various roll-damping methods. The extra “information” tab is self-explanatory. Each of these tabs is presented and explained further below.
Figure 4.3: ShipAtSea User interface flowchart, single run module
Figure 4.4: ShipAtSea User interface flowchart, survey run module
4.4.2 Single Run Mode

The single run mode is designed to stop the interface from handling the initial input and output files of the frequency domain time domain programs. Necessary conversation for the intermediate files are still carried out. In cases that the user would like to try and use his own prepared files, and just let the interface run the executables and prepare the output, the use of this mode is appropriate. No initial input file gets changed during the process, the frequency domain program is run, the output is converted into input for ShipAtSea and ShipAtSea is run. The results are prepared with graphics (if requested) as an MS Excel file. The main purpose of this mode is to allow the user flexibility to try his exact settings without converting input types between the time and frequency domain programs. The single run mode is presented in figure 4.6. In cases that the waves that are to be tested are irregular, there is an extra input box introduced, which will be explained further in detail, later on in the text.
4.4.3 Survey Mode

The use of survey mode of the interface (see figure 4.7) is intended for “range-operations”. It can also be used to perform a single simulation at any given speed, heading, frequency and/or wave height, letting the interface introduce the necessary changes on the files. Unlike the single mode, all input files get overwritten with the requested data in all cases.

There are 4 ranges that the user can survey:

- $\lambda/L_{pp}$: Defines the wavelength over the length of the vessel, therefore the frequency of the wave
- Speed: The speed range in knots
- Heading: The range of headings in degrees
- Wave Amplitude: Wave amplitude† range in meters.

The data for the survey is entered in the form of a matrix. Each range has three values ($X$, $Y$, $Z$) where $X$ represents the start, $Y$ is the end point and $Z$ is the increment. The order of the start and end points do not matter (e.g. $X = 180$, $Y = 0$, $Z = 5$ for heading angles yields the same results with $X = 0$, $Y = 180$, $Z = 5$). At the end of the simulations, the output files are moved to a folder and a report file is prepared for easy interpretation of the complete simulation results. The report file is explained in detail later in the chapter.

4.4.4 Options

Figure 4.8 represents the options tab. The options tab is more geared towards debug-type options. It has two subsections, an informative section and an optional section. The informative section is to control if the values of the ship are read correctly by the program. The tab also

†Please note that the range is the amplitude, NOT the wave height
introduces the possibility of checking the values used for the vessel without going through the input files themselves bypassing the need for knowledge of the file structure. In order to do that control, the “Show Vessel Info” button is used.

The “$\lambda/L$” informative button calculates the corresponding wave frequency for any given wavelength to ship’s length ratio.

The optional section has two settings. They are self-explanatory:

- **I Will Modify Frequency Domain Output Files**: This option stops and waits for the user to introduce any changes deemed required on the ShipAtSea input files before running the executable. It is useful, in cases where the user would like to change a certain value of the hydrodynamic matrix or the exciting forces for any reason, after the frequency domain program has run.†

- **Do not draw graphs**: If a very large number of simulations are requested from the program, the total calculation time accumulates to a high number depending on the simulation type (regular/irregular waves). Therefore a time-saving option is needed to optimize the total runtime*. This check-box forces the interface to skip the graphics that are prepared at the end of each simulation, shortening the simulation time.

### 4.4.5 Damping Options

The correct estimation of the roll damping is of crucial importance in estimation of non-linear dynamic instabilities such as parametric rolling. With the aim of providing better input for the time domain simulations program ShipAtSea, the user interface is capable of handling different damping estimation methods. The tab is presented in figure 4.9.

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†A word of caution is that this setting will make the simulations stop and wait for user input after each single simulation if run in conjunction with the survey mode.

*on average, drawing of graphs takes approximately 0.5 to 1 second of time per simulation
The “method” section of the the options tab includes three damping estimation methods and two optional boxes. While Miller’s method is completely implemented, the “Energy methods” would require a tuning for each vessel which may be done through the code. The roll damping estimation methods are:

- **Miller Method**: The method is explained in chapter 3, section 3.3.3.
- **Energy Method**: Energy method using only the linear term
- **Energy Method**: Energy method using linear + quadratic terms

As noted, the energy methods have to be specifically calibrated for each vessel. This may be done through the code and their implementation into the program is not fully completed. Currently for this project, the Miller’s damping method is in use. Furthermore, depending on the type of the estimation method, there are other settings that can be used in conjunction with the method to calibrate the method further.

- **Use Iterative Damping Estimation**: Most of the damping estimation methods depend on an estimated maximum roll angle value. It is not always possible to get the right estimation of this parameter in the first try, making it possible to approach the correct value only in a few tries. This “iterative damping estimation problem” is solved by the use of this option. The maximum roll angle obtained is checked against the maximum roll angle estimated and, if it is within a pre-defined precision, is accepted as correct. If the values are out of the required precision, the simulation is repeated until a correct value is reached.

- **Use Damping Matrix**: Although convenient, the “iterative damping estimation method” is time consuming. If a matrix with the known values of maximum roll angles for the specific test conditions (heading and speeds) calculation time could be reduced by reading the values directly instead of taking the long route and retesting. Also if irregular waves are considered,
the roll value is ever-changing depending on the incoming wave profile and its continuity. In such cases the values can be read through files prepared for sinusoidal waves of similar conditions, leading to a better estimation. The “best-match” method is used (i.e. the closest heading and speed are matched to the available data).

The use of “iterative damping estimation” creates the necessity for further input from the user. These settings are classified under Iteration Settings which are:

- **Maximum iterations**: The maximum number of iterations that will be carried out before deciding on a final estimated maximum roll angle for the damping estimation method. In the case presented in figure 4.9 the iterations will be repeated a maximum of 5 times and if no result is obtained within the given precision, the last value will be accepted as correct.

- **Precision**: This is the precision expected from the program while comparing the estimated and the obtained maximum roll angles. Figure 4.9 has the value defined as five, therefore if the estimated maximum roll angle is 28 and the obtained is 32, the program accepts the values and goes onto the next simulation.

Variables is the last section in the damping options:

- **Phi**: The estimated maximum roll angle. This value may be presented as a variable if iterative damping estimation is set or a constant if the option is not taken. If the damping matrix is in use, the value will be overwritten with whatever is read from the matrix.

- **Lbk**: “Length of the Bilge Keels” for Miller’s Method calculations. Obsolete if the method is different.

- **Bbk**: “Beam of the Bilge Keels” for Miller’s Method calculations. Obsolete if the method is different.

- **Multiplier**: This is an extra setting, which allows the user to take a percentage of the damping calculated by the method chosen. The value obtained from the method is multiplied by this number before being used as an input for ShipAtSea. If a certain damping value is known beforehand, this adjustment might serve as fine-tuning to the required value in changing encounter conditions.

### 4.4.6 Information

The tab presents “programmer information” for both time-domain numerical code ShipAtSea and the user interface. Also contains compatibility data. The screen is presented in figure 4.10.
4.4.7 Irregular Waves

ShipAtSea is capable of simulating irregular waves by applying the superposition principle. Therefore the user interface should be adapted to preparing the related input/output files. In single run and multiple run tabs, an extra input box is added automatically (see figure 4.11) in cases that the simulation type is defined as “1” (irregular) by the sea flag in the initial input files of the time domain program (ShipAtSea.IN). The number of runs that are to be repeated for each encounter condition is defined. The naming of the output files also change accordingly and that is to be detailed when output files are described.

For irregular waves calculations three other files (executable, input and output) are in use:

- waves.in: The input file has definitions for the number of sinusoidal components and the spectrum specific data.
- RandomWaves.exe: The executable file to generate random waves.
- waves.out: The output file that’s read by the time domain simulations.
4.4.8 Interface Related Files

There are two extra interface related files. While one of the files is prepared automatically if it is not present the other file is required only if a certain setting (damping matrix) is utilized.

- *dampinginput.dat*: To not enter the same settings repeatedly, it is convenient to use additional files to backup settings. This file backs up the last damping settings used, and is created automatically if it is not present.

- *DampingMatrice.dat*: The file can be prepared by any program and will have to be present in the folder to be used for *use damping matrix* option. The structure consists of a three column matrix in the form of:

  $\begin{bmatrix}
    \text{Heading}_1 & \text{Speed}_1 & \text{Max. Roll Angle}_1 \\
    \text{Heading}_2 & \text{Speed}_2 & \text{Max. Roll Angle}_2 \\
    \vdots & \vdots & \vdots \\
    \text{Heading}_n & \text{Speed}_n & \text{Max. Roll Angle}_n
  \end{bmatrix}$

4.4.9 Interface Output

After all of the settings are defined and the program starts, ShipAtSea interface goes to the informative mode and keeps the user updated on which simulation is being run, and under which settings. An example is presented in figure 4.12. The informative screen displays which executable is being run at that moment in the first line. The frequency, heading, speed and wave height being simulated are displayed in the second line and the damping settings in the third line. There is an option to save and quit if it is desired to stop the simulations which finishes the last simulation being carried out and classifies the output files as will be explained next.

The output obtained from ShipAtSea is prepared by the user interface as an MS Excel output. An extra report file that sums up all the simulation results and the settings used to obtain them is prepared. The files are then moved into a folder that's named according to the date and the survey information. As this moving of the files may take a very long time when the number of simulations are very high, the file counting screen is shown to prevent the user from thinking that
there is a problem with the process (see figure 4.13). When the process is completed, the user is informed as in figure 4.14.

While the interface works in the “temp” folder created under its root, the final naming of the folder is different. The first 13 characters of the folder name is the date in which the simulations are completed, presented as (without the curly brackets):

- {YEAR}{MONTH}{DAY}_ {HOUR}{MINUTE}_ {Simulation Data}

The “Simulation Data” is named according to the following convention:

- {Name of the vessel}_ {The heading range}_ {The frequency range}_ {The speed range}

As an example, the folder name:

- 20101208_2318_C11_150to140deg_1.0to1.0LmbOL_0to16kts

would represent simulations that were run for the vessel “C11”, in 2010, December 8th at 23:18h. The simulations were run from 150 to 140 degrees of heading, at wave length to ship’s length ratio 1 and at a speed range of 0 to 16 knots.

The regular and irregular simulations are also named differently to include required information.

For the regular waves, each single output file is named according to the following convention:

- {Vessel name}_ {Frequency}_ {Heading}_ {Wave length}_ {Wave height}
An example is:

- C11_Frq0.383_Hd180_8.0kts_Lw420_Hw6

Which represents the simulations for the vessel C11, at a wave frequency of 0.383, heading of 180 degrees, wave length of 420 degrees and a wave height of 6 meters.

Irregular waves are named slightly differently:

- { Vessel name }_{ Frequency }_{ Heading }_{ Zero up-crossing period }_{ Significant wave height }_{ The number of time-space realizations }

It may be exemplified by:

- C11_Frq0.485_Hd140_0.5kts_Tz12.96_Hs8_Run(s)_5

Which represents the simulations for the vessel C11, at a wave frequency of 0.485, heading of 140 degrees, zero up-crossing period of 12.96 seconds, a significant wave height of 8 meters and the file contains 5 different realizations of space and time.

The folder also includes a report file which lists all the simulations carried out and details the results along with the settings that have been used to obtain them. The report files start with the word “Rep.”. The naming of the report file is very similar to each file name:

- Rep_C11_180to0deg_1.0to1.6LbdOL_0to10kts_0317_1600.xls

The report file above represents the simulations for the vessel C11, from 180° to 0° of heading, from 1 to 1.6 ship’s length ratios, and from 0 to 10 knots. The simulations had been completed at March (month 03) 17, at 16 hours. The report file is not only important as a report, but may also be used for creating other input data such as the one used for the polar plots. Additionally, it is not possible to open each file and check the results if the number of simulations reach numbers that are double, triple or even quadruple digits.

The headers that will be found in the report file will be:

<table>
<thead>
<tr>
<th>File Name</th>
<th>Roll Angle [Deg]</th>
<th>Freq [Rad/sec]</th>
<th>Heading [Deg]</th>
<th>Speed [Kts]</th>
<th>( \lambda/L_{0p} )</th>
<th>VCG [m]</th>
<th>Damping Coeff.</th>
<th>Wave Amp. Phi used</th>
<th>Date</th>
</tr>
</thead>
</table>

40
5

Experimental Work

5.1 Introduction

The preceding sections have introduced the theoretical background, the numerical procedures and the programs used to evaluate the non-linear instabilities of ships. Any mathematical model that is used to estimate ship motions requires validation of its results to demonstrate reliability in future conditions. With this aim, a series of experiments were conducted to assess the instantaneous hydrodynamic coefficients and the parametric rolling characteristics of a C11 class container vessel model. The tests have been carried out in CEHIPAR towing tank, Madrid. The experimental programme consisted of captive model tests, free-decay tests, forced oscillation tests at various heel angles and parametric rolling tests. While the experimental programme had been prepared and managed by the author’s supervisors, the author had the opportunity to follow the experimental work and to analyse the data which are described in the following chapters. This chapter deals with the presentation of the vessel and these tests.

5.2 The Scope of the Experimental Work

Parametric rolling is induced by time varying roll-restoring moment in waves and estimating these instantaneous values accurately is of paramount importance for the modelling and simulation of the phenomenon [40]. It is a widely accepted fact that, to explain the wave effect on roll-restoring moment, the Froude–Krylov assumption can be used. Therefore, in order to gain a better insight of critical conditions where parametric rolling is likely to occur, a preliminary testing of the adopted device was conducted “to measure the wave effect on the roll-restoring moment” following the procedure referenced from Hashimoto et al. [41].

In order to measure the value mentioned above, a dynamometer was used with a heaving rod and a hinged mechanism to allow free heave and pitch motions on the model while it had been transversely and longitudinally kept aligned into the incoming wave trains. For this reason,
Figure 5.1: Schematic diagram of the experimental apparatus to measure heave and pitch hydrodynamic coefficients.

Surge, sway and yaw motions were fixed and magnitudes of their forces were captured by the dynamometer which was installed at the same height as the height of the propelled point.

As mentioned before, for the estimation of parametric rolling the availability of an adequate motion calculation routine capable of predicting the time domain large amplitude responses under resonant conditions is important. With larger motions, at each time step the underwater section changes and the velocity potential has to satisfy the new boundary conditions. Therefore, added mass, damping and the excitation forces are time-dependent, producing non-linearity. It is also important to assess the extent of this change, which has been aimed by carrying out forced oscillation test on the inclined vessel.

To obtain the hydrodynamic coefficients that are used in the equations of motion, the use of a strip theory based calculation routine is highly favoured. These routines are easily accessible routines and require relatively small computational effort for practical engineering applications at the initial design stage. Yet, when being used for purposes such as estimation of parametric rolling, the theory is pushed far beyond its generally accepted limits in particular respect to its restriction on the assumption that the motions are small. Hence, not only the free surface boundary condition is linearised about the undisturbed water surface, but also the body boundary condition is expanded about the mean position. Therefore, the question that was asked for the experiments that are described further in the chapter is whether a good correlation in experimental and computational results for the heave and pitch hydrodynamic coefficients (added mass and damping) will be obtained when compared with the strip theory calculations of the frequency domain program. The data required for the evaluation of the radiation forces was acquired by forcing the model to execute harmonic heave and pitch motions while being towed along the tank at a critical speed and several heeled positions. The analysis of the time series result in the estimates of linear added mass and damping for each of the segments of the model.

The model is segmented in order to conduct the forced oscillation tests. Separate data for
each segment is measured for the motions and moments. The segmenting of the model along with the dimensions in millimetres is presented in figure 5.1. The scaled model consists of several pieces separated by cuts at certain stations. The gaps between the pieces are sealed by means of a flexible rubber strip stuck to the hull in the inner part. The pieces are held together by the help of a longitudinal backbone of continuous metal beam structure. The beam is fitted with strain gauges at the stations to measure vertical forces and moments. The forced oscillations were produced by two vertical linear actuators (see figure 5.2). Each actuator consists of a worm screw driven by step motors. The aft actuator is clamped to the towing carriage so that it's always kept vertical. The forward actuator’s connection to the carriage is made through a hinge allowing small rotation angles in the longitudinal vertical plane, therefore the model always pitches about its natural position.

5.3 The Selected Vessel

For the purpose of this study, a ship that is well documented as prone to parametric rolling [26] had been selected. A graphical representation of the hull form and the characteristics of the vessel are presented in figure 5.3 and table 5.1 respectively. The vessel itself is worthy of particular attention because it is the same type of ship that has lost 1/3 of its deck containers and damaged another 1/3 on the way in a severe storm in October 1998 [26]. At times the vessel reached yaw angles of 20 degrees port and starboard sides, making course keeping almost impossible. Main engine overspeed trips and shaft vibrations together with pounding reflected significant pitch amplitudes. Port and starboard rolling as much as 35 to 40 degrees was reported in simultaneously with extreme pitching. The event has attracted wide and renewed attention due
Table 5.1: Main particulars of the C11 type container vessel

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars ($L_{pp}$)</td>
<td>262.00</td>
<td>m</td>
</tr>
<tr>
<td>Depth at main deck ($D$)</td>
<td>24.40</td>
<td>m</td>
</tr>
<tr>
<td>Breadth at design waterline ($B_{DWL}$)</td>
<td>40.00</td>
<td>m</td>
</tr>
<tr>
<td>Displacement at design waterline ($\n_{DWL}$)</td>
<td>76056</td>
<td>t</td>
</tr>
<tr>
<td>Block coefficient ($C_B$)</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>Draught at amidships ($T_{DWL}$)</td>
<td>12.34</td>
<td>m</td>
</tr>
<tr>
<td>Transverse metacentric height in still water ($GM_T$)</td>
<td>1.973</td>
<td>m</td>
</tr>
<tr>
<td>Natural roll period, linearised in waves ($T_{44}$)</td>
<td>22.78</td>
<td>s</td>
</tr>
<tr>
<td>Maximum speed ($U$)</td>
<td>20</td>
<td>knots</td>
</tr>
</tbody>
</table>

Figure 5.3: C11 Hull form

to the evidence of parametric rolling in head seas.

The model experiments have been conducted at Canal de Experiências Hidrodinâmicas de El Pardo (CEHIPAR), Madrid. The tank has a length of 150 meters, width of 30 meters and a depth of 5 meters. A detailed sketch of the towing tank is presented in figure 5.5. The tank utilizes a flap type wave maker and an overhead towing carriage (see figure 5.4).

5.4 Experiments

To fully evaluate and compare numerical results, three groups of tests were carried out in CEHIPAR. The tests consisted of calm water tests and ship model basin tests. The changes in the setup of the model (load conditions) or different wave frequencies require further calibration and lengthens test times. For this reason the load conditions were minimized. Although it is possible to test oblique waves, time and equipment limitations state that the tests should be considered in two major parts; head and following waves. Slightly oblique waves were tested as well ($10^\circ$, $20^\circ$, $160^\circ$ and $170^\circ$). In order to avoid recalibration of wave periods in oblique waves, the desired encounter frequency was obtained not by changing the wave frequency and keeping the model speed the same, but by keeping the same wave frequency and changing the model speed to return the same encounter frequency. The conditions that are found to be the most critical (i.e. most prone to parametric rolling) were identified beforehand by means of numerical simulations with ShipAtSea reducing the number of required tests. The three initial groups of tests are presented...
They mainly consist of the initial setup, captive model tests, and the forced oscillation tests. Detailed information on the experiments are presented in tables 5.3 to 5.7.

While the activities were being planned by the author’s supervisors, attention had been paid to keep the time allocated to a maximum of 16 runs per day including all activities. The scale of the model has been selected as 1/65.

The first part of the tests focused on the initial setup and parametric rolling tests in head waves. The model had to be set up and the instrumentation and data acquisition equipment had to be calibrated for the correct reporting of parameters such as the wave height, heading, moments and forces. The model had to be tested for floatation and stability in the calm water tank to assess the
draught and inclination.

Parametric roll is very dependent on the wave encounter conditions which depend on the wavelength and the wave height. Hence, it is of utmost importance that the waves generated for the tests are correctly defined. For that reason, the initial setup includes generating waves of different frequencies and amplitudes to test out the required wavelength range.

Another prominent parameter in encounter frequency is speed. This parameter has to be calibrated as well by making sure that the carriage moves the model at the speed that is deemed critical (i.e. 7.97 knots). Therefore the carriage has also been tested for consistency.

Having the equipment and the model ready, parametric rolling in regular head waves were tested under different encounter conditions varying the wavelength to ship’s length ratio while the loading conditions were kept the same. Three different wave heights and four different wavelength to ship’s length ratios were considered for this section. Apart from regular head waves, polychromatic head and oblique waves were examined using different number of wave components ranging from 3 to 11. Group 1 activities also included the assessment of transverse metacentric height and the natural period ($T_n$) by means of inclining experiments and free-roll decay tests.

Once the critical conditions for the regular waves are identified, the vessel was then tested for equivalent conditions in irregular waves having an appropriate wave spectrum consisting of the highest possible number of harmonic components. As encounter conditions constantly change when irregular waves are considered, the next batch of tests consisted of testing different wave realizations in space and time. Irregular waves experiments were repeated not only for head seas, but also for slightly oblique seas of 170° and 160°. Providing that the critical advance speed, wavelength and wave amplitude were well identified in regular and irregular waves, second and third group of experiments were carried out as captive model tests and forced oscillation tests.

The second and third groups of experiments were related to the analysis of the hydrodynamic coefficients. The roll-restoring moment was obtained by towing the model in different heel angles and speeds along the basin. The added mass and damping coefficients were obtained through forced oscillation tests which were repeated at different heeled positions and oscillation periods. The aim was to get the necessary data to validate the numerical results that will be obtained from two different strip theory codes assessing symmetric and asymmetric cross-sections. Different amplitudes of motion also allowed the assessment of non-linearities in the coefficients. Two different speeds were also tested allowing the assessment of speed effects. Although the forced oscillation tests were conducted in calm water conditions with wavemakers stopped it was impor-
### Table 5.3: Part I, Sets 1 to 3 – Parametric Rolling in Head Waves

<table>
<thead>
<tr>
<th>Set No &amp; Settings</th>
<th>Description of First Group of Activities - Hydrostatic Parametric Rolling Model Tests in Head Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set 1</strong></td>
<td><strong>Regular Head Waves</strong></td>
</tr>
<tr>
<td></td>
<td>- Initial set up of the scaled model and all the required mounting and calibration of instrumentation and acquisition data equipment.</td>
</tr>
<tr>
<td></td>
<td>- The scaled model is tested to assess floatation &amp; stability in the calm water tank. Therefore, draught measurements and inclining experiments are carried out at the design load condition.</td>
</tr>
<tr>
<td></td>
<td>- Waves of different frequencies and amplitudes are generated, and wave probes are positioned in the working station of the seakeeping model basin to measure wavelengths: first equal to 0.8 $L_{bp}$ of the model and then increasing up to 1.4 $L_{bp}$, passing by the condition where the wavelength is exactly equal the $L_{bp}$.</td>
</tr>
<tr>
<td></td>
<td>- Investigation of the conditions in which the parametric rolling occurs in regular head waves ($\beta = 180^\circ$), varying the ships speed and wave frequency (corresponding to a variable wavelength: first equal to 0.8 $L_{bp}$ of the model and then increasing up to 1.4 $L_{bp}$, passing by the condition where the wavelength is exactly equal the $L_{bp}$) and different wave amplitudes as well.</td>
</tr>
<tr>
<td></td>
<td>- Assessment of the transverse metacentric height ($GM$), the natural period ($T_n$) and traces of free-roll decay curves in still water and in head waves close to critical wave conditions (i.e. same wavelength and wave height experimentally determined and then one up and one down variations of these parameters).</td>
</tr>
<tr>
<td><strong>Set 2</strong></td>
<td><strong>Polychromatic Head and Oblique Waves</strong></td>
</tr>
<tr>
<td></td>
<td>- Investigation of the conditions in which parametric rolling occurs in N harmonic wave components in head and oblique waves ($\beta = 0^\circ, 10^\circ, 20^\circ$) and (3, 5, ..., 11 harmonic wave components), considering the equivalent wave heights and mean wave encounter frequencies recorded during the most critical test of parametric rolling in regular head waves (1 harmonic wave component).</td>
</tr>
<tr>
<td><strong>Set 3</strong></td>
<td><strong>Irregular Head and Oblique Waves</strong></td>
</tr>
<tr>
<td></td>
<td>- Investigation of the conditions in which parametric rolling occurs in irregular head and oblique waves, with forward advance speed, considering the same significant wave heights and modal wave encounter frequencies recorded during the most critical test of parametric rolling in regular head waves.</td>
</tr>
<tr>
<td></td>
<td>- Repetition of the tests with the same spectral energies but with different wave realizations in space and time.</td>
</tr>
</tbody>
</table>
Table 5.4: Part I, Set 4 – Analysis of Roll Restoring Moment

<table>
<thead>
<tr>
<th>Set No &amp; Settings</th>
<th>Description of Second Group of Activities Hydrostatics Stability and Captive Model Tests in Head Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 4</td>
<td>Captive Model Tests in Head Waves, Evaluation of Roll-Restoring Moment</td>
</tr>
<tr>
<td></td>
<td>• GM = 1.973m</td>
</tr>
<tr>
<td></td>
<td>• After initial testing of the measurement device for the roll-restoring moment (dynamometer), captive model tests in head waves are conducted for the scaled model at 4 advance speeds ($U = [0, 4, 8, 12]$ Kts), three heeled positions ($\Phi = [180^\circ, 170^\circ, 160^\circ]$) and 7 ($\frac{L}{\lambda_{bp}} = [0, \frac{1}{2}, \frac{2}{3}, 1, 1\frac{1}{2}, 2]$) wavelength to ships length ratios.</td>
</tr>
</tbody>
</table>

Table 5.5: Part I, Set 5 – Analysis of Hydrodynamic Coefficients (Added Mass and Damping)

<table>
<thead>
<tr>
<th>Set No &amp; Settings</th>
<th>Description of Third Group of Activities Forced Oscillation Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 5</td>
<td>Forced Oscillation Tests for the Analysis of Damping and Added Mass</td>
</tr>
<tr>
<td></td>
<td>• After initial testing of the measurement devices (load cells, strain gauges and linear potentiometers), forced oscillation tests will be conducted for the segmented model at critical advance speed of 8 knots, 3 heeled positions ($\Phi = [0^\circ, 5^\circ, 10^\circ]$), 7 oscillation periods ($T = [T_1, ..., T_7]$ sec), and 3 amplitudes for heave and pitch motions ($X_1, X_2, X_3$) either in meters or degrees.</td>
</tr>
</tbody>
</table>

tant to consider the wall effects caused by the sides of the basin and take precaution to avoid the problem.

The critical conditions regarding the head seas and the following seas were different. Therefore, the model had to be recalibrated with a new metacentric height before starting second part of the tests. Furthermore, natural period ($T_n$) and free roll-decay curves were also assessed in this load condition. After the recalibration, parametric rolling in regular following waves, polychromatic following and oblique waves (with 3 to 11 components) and irregular following and oblique waves were conducted. Unlike head waves, the tested wave heights for following waves were reduced to two (6 and 8 meters) and Only zero speed was tested. The number of wavelength to ship’s length ratios were kept the same. The irregular waves experiments were repeated with different time-space realizations and same spectral energies. The experiments end with the second group of activities: captive model tests for the evaluation of the roll restoring moment. There is no third group of tests in part two, as calm water and forward speed were the requirements for those tests.
Table 5.6: Part II, Sets 6 to 8 – Parametric Rolling in Following Waves

<table>
<thead>
<tr>
<th>Set No &amp; Settings</th>
<th>Description of First Group of Activities - Hydrostatic Parametric Rolling Model Tests in Following Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set 6</strong></td>
<td><strong>Regular Following Waves</strong></td>
</tr>
<tr>
<td>GM = 0.978m</td>
<td>Initial set up of the scaled model and all the required mounting and calibration of instrumentation and acquisition data equipment.</td>
</tr>
<tr>
<td>$\beta = 0^\circ$</td>
<td>The scaled model is tested to assess floatation &amp; stability in the calm water tank. Therefore, draught measurements and inclining experiments are carried out at the design load condition.</td>
</tr>
<tr>
<td>$\lambda/L_{bp} = [0.8, 1.0, 1.2, 1.4]$</td>
<td>Investigation of the conditions in which the parametric rolling occurs in regular head waves ($\beta = 0^\circ$), varying the ships speed and wave frequency (corresponding to a variable wavelength: first equal to 0.8 $L_{bp}$ of the model and then increasing up to 1.4 $L_{bp}$, passing by the condition where the wavelength is exactly equal the $L_{bp}$) and different wave amplitudes as well.</td>
</tr>
<tr>
<td>$H_w = [4.0, 8.0]$</td>
<td>Assessment of the transverse metacentric height ($GM$), the natural period ($T_n$) and traces of free-roll decay curves in still water and in head waves close to critical wave conditions (i.e. same wavelength and wave height experimentally determined and then one up and one down variations of these parameters).</td>
</tr>
<tr>
<td>Decay Tests with and without waves ($U = [0, ..., 20]$kts)</td>
<td>Measurements of the wave profile along the length of the model should be taken at different speeds in still water conditions.</td>
</tr>
</tbody>
</table>

| **Set 7** | **Polychromatic Following and Oblique Waves** |
| GM = 0.987m | Investigation of the conditions in which parametric rolling occurs in N harmonic wave components in head and oblique waves ($\beta = 0^\circ$, $10^\circ$, $20^\circ$) and (3, 5, ..., 11 harmonic wave components), considering the equivalent wave heights and mean wave encounter frequencies recorded during the most critical test of parametric rolling in regular head waves (1 harmonic wave component). |
| $\beta = [0^\circ, 10^\circ, 20^\circ]$ | |
| $\lambda/L_{bp} = [1.2]$ | |

<p>| <strong>Set 8</strong> | <strong>Irregular Following and Oblique Waves</strong> |
| GM = 0.987m | Investigation of the conditions in which parametric rolling occurs in irregular following and oblique waves, with zero speed, considering the same significant wave heights and modal wave encounter frequencies recorded during the most critical test of parametric rolling in regular following waves. |
| $\beta = [0^\circ, 10^\circ, 20^\circ]$ | Repetition of the tests with the same spectral energies but with different wave realizations in space and time. |</p>
<table>
<thead>
<tr>
<th>Set No &amp; Settings</th>
<th>Description of Second Group of Activities Hydrostatics Stability and Captive Model Tests in Following Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 9</td>
<td>Captive Model Tests in Following Waves, Evaluation of Roll-Restoring Moment</td>
</tr>
<tr>
<td></td>
<td>• After initial testing of the measurement device for the roll-restoring moment (dynamometer), captive model tests in following waves are conducted for the scaled model at 3 advance speeds (U = [0, 4, 8] \text{ Kts}), three heeled positions (\Phi = [0^\circ, 5^\circ, 10^\circ]) and 7 (\frac{\lambda}{L_{bp}} = [0, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 1, 2]) wavelength to ships length ratios.</td>
</tr>
</tbody>
</table>
6

Validation of the Hydrodynamic Coefficients Prediction

6.1 Introduction

The complete structure of the tests was introduced in chapter 5. This section deals with the analysis of the results. Particularly two sets are under focus in this study. First, set 5 is dealt with, which includes the data processing of forced oscillation tests and the analysis of damping and added mass for heave and pitch motions. Then in the next chapter, the results obtained from the parametric rolling experiments of the sets 1 and 6, namely the regular head and following waves are compared with the numerical results. For the ease of reading, the respective tables of tests are repeated before their related sections.

The forced oscillations tests were presented in chapter 5 in table 5.5. The tests consist of forcing the model to execute harmonic heave and pitch motions at a predefined frequency and amplitude. While one part of tests consisted of oscillations at no forward speed, the other part has been performed at 7.97 knots of forward speed along the towing tank. During the motions, the forces acting on each segment (see figure 5.1) and the total force acting on the model were measured.

The analysis of these tests leads to obtaining the added mass and damping coefficients in their relative modes, which are of very high importance as the dynamic stability of the vessel largely depends on the hydrodynamic coefficients (see equation 3.18). Hence, it is very important to validate the numerical method that has been used to predict the phenomenon. The validation of the hydrodynamic coefficients for pure heave and pitch is the subject of this chapter, although cross-coupling effects due to heeling should also be investigated in the future.

Apart from the upright position, the same set of experiments were repeated with $5^\circ$ heeled position and $10^\circ$ heeled position of the vessel. These heeled angle results are also compared between themselves using 3D graphics representing frequencies.
As CEHIPAR provides the measurements from the tests in ship scale there was no further need for conversion of measurements. Nevertheless, for the reader’s information the conversion factors are given in table 6.1.

<table>
<thead>
<tr>
<th>Table 6.1: Scale Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
</tr>
<tr>
<td>Vertical Motions</td>
</tr>
<tr>
<td>Vertical Forces</td>
</tr>
<tr>
<td>Shear Forces</td>
</tr>
<tr>
<td>Bending Moments</td>
</tr>
<tr>
<td>Speed</td>
</tr>
</tbody>
</table>

6.2 Forced Oscillation Analysis Methodology

For the analysis of the time records, a MATLAB program, named “Forced Oscillation Tests Analysis (FOTA)” was developed and used. The data files presenting the results have their own format and the routines that were used to read those input files were provided by CEHIPAR. The results in this section are for pure heave and the pitch motions (i.e. the hydrodynamic coefficients $A_{33}, B_{33}, A_{55}, B_{55}$).

Before the analysis of the time series, whether the data measured needs to be filtered further should be discussed. It should be noted that all software filtering techniques cause an artificial delay as in figure 6.1, changing the phase angles of the signals. The amount of delay is a balance between the cut-off frequency and the amount of filtering (i.e. as the signal is filtered more, the delay increases). After an initial test carried out to assess if the signal can be analysed without software filtering, the end decision was to omit software filtering as the signals were clean enough. Therefore any artificially introduced delay is avoided in this study.

In order to obtain the sinusoidal that represents the time series, MATLAB’s curve-fitting algorithm was chosen as it is efficient and fast. The algorithm utilised to obtain the first harmonics of the signals was satisfactory and the quality check was made through visual inspection. Figure 6.2 illustrates the quality of the harmonic analysis technique applied. There was a consistently high coherence in the experimentally obtained data for all the cases tested. In rare cases that the curve-fitting does not perform as expected, the solution was to analyse another time-space in the same test record, with preferably 1-2 seconds of shifting.

For the towing tank personnel, it is more convenient to deliver more than one test in a single file as in figure 6.3. This way, in one run, different amplitudes of oscillations can be tested to save time. Yet, this creates the necessity to separate a time record into parts, and also to remove the transient stages where the amplitude of the motion is increasing but has not reached its steady value. The analysis subroutine includes the code to do such trimming of the records as well.
Figure 6.1: Artificial delay caused by software filtering

Figure 6.2: Curve-fitting Algorithm
6.2.1 Forces acting on the model

As explained in chapter 5, the model is connected to two vertical actuators. The heave and pitch motions are obtained by forcing the model to move at certain oscillation periods by those actuators positioned at the aft and forward sections of the model. Then the resulting forces and the motions are measured by means of load cells located at the actuators. The test data provides the total forces as $F_{fore}$ and $F_{aft}$. The summation of these forces total to the force acting on the model:

$$F_{\text{total}} = F_{fore} + F_{aft} \quad (6.1)$$

As the distances of the heaving rods ($x_{fore}$ and $x_{aft}$) are known, the moment forcing the model to pitch can simply be calculated by:

$$M_{fore} = x_{fore} \cdot F_{fore} \quad (6.2)$$

$$M_{fore} = x_{aft} \cdot F_{aft} \quad (6.3)$$

$$M_{\text{pitch}} = -M_{fore} + M_{aft} \quad (6.4)$$

These values form the total forces and moments acting on the model and they can be separated further into three components: The hydrodynamic, hydrostatic and the inertial forces. The hydrodynamic coefficients relate only to the hydrodynamic forces, therefore it is necessary to strip the total force off hydrostatic and inertial components before proceeding with the calculations of the hydrodynamic coefficients.
Inertial forces could be measured by forcing the model to oscillate in open air by hanging it from the actuators. The resulting forces and moments would be purely inertial. These forces may then be subtracted from from the total force, leaving only forces related to the vessel being in the water. If this data is not available, the same force can be assessed numerically as it was done in this work.

Another prominent force that acts on the vessel is the buoyancy force. This restoring force is easy to calculate and should be subtracted from the total force obtained as well. Considering that there is no forward speed, what’s left will be the hydrodynamic force, to which the hydrodynamic coefficients are related to. In instances where the speed is high \(Fn > 0.25\), there will be an extra dynamic lift on the vessel and its amount will depend on the geometry. In those cases, the lift should be taken into account as well.

The total forces acting on the body may be presented in components as:

\[
F_t = F_R + F_I + F_D
\]

where:

- \(F_t\); total force
- \(F_R\); the restoring force
- \(F_I\); the inertial force
- \(F_D\); the hydrodynamic force

The restoring forces depend on the immersion of the vessel in the water. They are related to the underwater geometry and the amount of change in the restoring forces with the movement of the vessel is related to the waterplane area. There are different possibilities in handling this force. A fully-linear approach may be taken, taking the draught of the vessel as a fixed value making the waterplane area of the vessel a constant. This would simplify the calculations.

Realistically, it would be necessary to apply the draught and the wave profile to the vessel and obtain the corresponding water plane area. While possible, this would require constant calculation at each time step for the given vertical displacement leading to a more complex calculation.

A third approach would be to take draught as a variable and change the waterplane area instantaneously with heave disregarding the wave profile. For heave forced oscillation tests, there are no incoming waves and the heave along the length of the vessel is the same. Therefore this method would be applicable easily to further the precision of the results. Initially for this work, the waterplane area corresponding to the instantaneous draught was calculated through a pre-defined polynomial for the vessel. Later on, by comparing the instantaneous values, it was clear that the change in the waterplane was negligible and a fixed value assumption was causing no deviation from realistic results. Finally, for the heave motion, restoring forces were defined as:
where the variables are:

• $F_R$, the restoring force

• $A_{wp}$, the waterplane area

• $g$, the acceleration of gravity

• $A$, amplitude of the motion

and for the pitch motion:

$$F_R = \Delta g GM_L$$  \hspace{1cm} (6.7)$$

where the variables are:

• $F_R$, the restoring force

• $\Delta$, the mass of the vessel

• $g$, the acceleration of gravity

• $GM_L$, longitudinal metacentric height

The inertial force can be calculated through Newton's second law, $F = m \ddot{z}$. As the motion is oscillatory, the acceleration can easily be expressed:

$$z = A \sin(\omega t)$$  \hspace{1cm} (6.8)$$

$$\dot{z} = A \omega \cos(\omega t)$$  \hspace{1cm} (6.9)$$

$$\ddot{z} = -A (\omega^2) \sin(\omega t)$$  \hspace{1cm} (6.10)$$

As indicated by the sign the inertial forces oppose the motion. Integrated into $F = m \ddot{z}$ the resulting equation is:

$$F_I = A m \omega_o^2$$  \hspace{1cm} (6.11)$$

The variables in the equation are:

• $F_I$, the inertial force

• $m$, the mass of the vessel

• $\omega_o^2$, the oscillation frequency squared
• $A$, the amplitude of the motion

After the forces acting on the body are defined, it is important to note that the forces and the motions will not be in phase. There is always a phase lag between the forces and the motions which is calculated by:

$$ (\varphi) = (\varphi)_{\text{force}} - (\varphi)_{\text{motion}} $$

(6.12)

The phase lags were calculated as an end result of the curve-fitting. A positive phase angle means a lead of the force signal relative to the motion while a negative value represents that the motion is leading.

6.2.2 Analysis of Dynamic Forces and Moments

When all the components of the total force are identified, the removal of those forces leave only the hydrodynamic force. The added mass and damping are then given by:

$$ A_{ij} = \frac{F_D \cos(\varphi)}{A \omega_o^2} $$

(6.13)

$$ B_{ij} = -\frac{F_D \sin(\varphi)}{A \omega_o} $$

(6.14)

The indexes $i$ and $j$ denote the mode of the movement and the hydrodynamic coefficient in that mode, or the coupling. For instance, $A_{55}$ and $B_{55}$ are added mass and damping in pitch respectively. $A_{45}$ would be heave added mass due to pitch motion.

6.3 Experiments

The experiments analysed in this chapter consist of heave and pitch forced oscillation tests. The model was tested at different speeds, amplitudes of motion, periods and heel angles. The results were compared to the numerical results obtained from the strip theory codes developed at IST and explained in chapter 4. Two strip theory codes were used, one to assess the upright position and the other one for the asymmetric cross sections.

Heave motion experiments are presented in table 6.2 and pitch motion experiments is summarised in table 6.3. It should be noted that any given test is repeated for all of the other values presented in the table. Therefore if speed is picked as 0 knots, there will be 3 heel angles, 6 oscillation periods and 3 amplitudes of heave amounting to a total of 54 tests. The total number of tests summarized in the table are 108. The same holds true for pitch.
Table 6.2: Summarized table of the forced oscillation tests (Heave)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>speeds (kts)</td>
<td>0 / 7.97</td>
</tr>
<tr>
<td>heel angles (deg)</td>
<td>0 / 5 / 10</td>
</tr>
<tr>
<td>amplitudes of heave (m)</td>
<td>0.4 / 0.8 / 1.2</td>
</tr>
<tr>
<td>oscillation periods (s)</td>
<td>9.2 / 11.2 / 12.9 / 15.8 / 17.1 / 18.3</td>
</tr>
</tbody>
</table>

Table 6.3: Summarized table of the forced oscillation tests (Pitch)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>speeds (kts)</td>
<td>0 / 7.97</td>
</tr>
<tr>
<td>heel angles (deg)</td>
<td>0 / 5 / 10</td>
</tr>
<tr>
<td>amplitudes of pitch (deg)</td>
<td>1 / 2 / 3</td>
</tr>
<tr>
<td>oscillation periods (s)</td>
<td>9.2 / 11.2 / 12.9 / 15.8 / 17.1 / 18.3</td>
</tr>
</tbody>
</table>

### 6.4 Forced Oscillation Analysis Results

The experiments that were presented in the previous section covered a wide range of conditions of different speeds, heel angles and periods. By taking different periods under consideration, the strip theory responses at different frequencies were examined. Speed effects give consideration to the low Froude number assumption, and heel angles focus on the wall-sidedness of the vessel and how the hydrodynamic coefficients change. It is important to clarify a few points before presenting the results.

The figures presented below show the geometry of three sections taken from different places along the length of the ship. By looking at figures 6.4(a) to 6.4(c), it is visible that the vessel itself is not a wall sided vessel apart from the midship section. There is a “parallel body” but its extension is rather short. The forward section of the ship is “V-shaped” while the aft section is far from a 90° angle with the calm water surface. This way, strip theory is pushed to its limits at the end stations considering the wall-sidedness assumption.

The speed for the tests was 7.97 knots corresponding to $F_n = 0.081$, which can be considered a low Froude number. Therefore, the vessel is not bound to suffer from high hydrodynamic lift.
related problems seen in higher speeds and this is compliant with the theory requirements.

The last topic of interest would be experimental errors themselves. Forced oscillations require
a complete sinusoidal motion. While it is expected to have a transient stage before the motion
reaches its peak value, it is then expected to have steady time series. As with every experiment
there are times, albeit very rarely, that this does not hold true. In such cases there are difficulties
identifying which part of the time record should be analysed for more realistic analysis. These kind
of tests end up giving extremely deviating values and are more likely to occur in low amplitudes
of motion. Usually it is not the motion that is showing large fluctuations but the force. Figures
6.5(a) and 6.5(b) illustrate such a case. While the heave motion shows an steady amplitude, the
force amplitude is fluctuating significantly and the curve fitting is averaging the force which leads
to erroneous damping or added mass values. These conditions were rare, and those values were
not eliminated from the results.

After these considerations, it is possible to examine the results in a clearer light. Generally,
heave and pitch results should be looked at separately. However there is one common point in
both motions; in both experiments better agreement with numerical calculations in higher frequen-
cies is expected. This is related to the high frequency assumption of strip theory.

6.4.1 Interpretation of the results

In the figures 6.6(a) to 6.11(d), complete results for heave and pitch motions are presented.
The experimental results are represented by points where each shape represents a different am-
plitude of motion. The numerical calculations are represented by a line. The X axis shows the
periods of oscillation and the Y axis shows the actual value of the hydrodynamic coefficient.

It is worthy of notice that there are two codes used in this comparison. While the first code
calculates the upright numerical values the second code carries out calculations for the heeled
vessel. As giving zero degrees of heel (i.e. defining the vessel as upright) to the second code
would result in the same values as the first code, only one line defines the numerical results.

The heave motion results are presented in figures 6.6(a) to 6.8(d). When the heave added mass is examined, it seen as being in closer agreement with the theory at higher frequencies. Non-linear effects are visible as the results from different amplitudes of tests tend to “spread-out” especially in lower frequencies (for a clear example, see figure 6.6(c)). If the effect of speed is considered, speeding up to 7.97 knots does seem to introduce higher deviations from the experimental results, especially in higher frequencies. In general, it is possible to state that the values are slightly underestimated apart from the 10° (figures 6.8(a) and 6.8(c)), where the correlation is very good. This finding may be interesting to assess the performance of strip theory in heeled sections.

The damping in heave motion, $B_{33}$, is estimated well by the numerical code when there is no speed, yet there is a tendency to underestimate it with speed. The non-linearities are highly visible also for heave damping. Regardless of speed, the values are underestimated for the 10 degrees of heel (figures 6.8(b) and 6.8(d)). Generally the curvature of the damping results follow the average of the experimental results. In here, the damping results are not extremely affected by the oscillation period, going beyond the general strip theory assumption of high frequency motion.

The pitch added inertia and damping results are presented in figures 6.9(a) to 6.11(d). When the added inertia is paid close attention to, it is seen that the experimental values are represented
Figure 6.7: Added mass and damping coefficients for heave, 5° of heel

Figure 6.8: Added mass and damping coefficients for heave, 10° of heel
Figure 6.9: Added inertia and damping coefficients for pitch, 0° of heel

Figure 6.10: Added inertia and damping coefficients for pitch, 5° of heel
Figure 6.11: Added inertia and damping coefficients for pitch, $10^\circ$ of heel

very well numerically at no forward speed, but are slightly underestimated when forward speed is present. The results seem to show less non-linear effects due to different amplitudes of motion. That may be associated to the fact that there is less change of underwater geometry in pitching compared to heaving for this vessel. Again as the heave damping, the period of motion does not seem to play an important role in the estimation of pitch added inertia.

The damping is overestimated in all cases with this trend diminishing in larger heel angles and high frequencies. The curvature of the line that represents the numerical damping values follow the experimental results but over accentuating the shape. The best results are seen for the 11.2 seconds period and then for both ends of the curve (towards 9.2 seconds and towards 18.3 seconds) they get further away from experimental values. The biggest overestimation occurs at the longest periods of motion, agreeing with the high frequency assumption of strip theory in this case. The non-linear effects are not very noticeable and the experimental results are very close for each amplitude of motion.

### 6.4.2 Analysis of Experimental Heeled Tests

The previous section has presented the comparison of hydrodynamic coefficients obtained from a strip theory code, to the experimental results. All results were presented separately. While that approach is enough for understanding the performance of strip theory, further probing is re-
quired to fully understand the effect of heel angles on the results of the forced oscillation analysis. For this reason the values obtained from the 0° of heel tests should be compared with 5° and 10° tests. This work aims to address that by making use of three dimensional figures, where the X axis represents the heel angles, Y axis the periods of oscillation and the Z axis the normalized values of the hydrodynamic coefficient itself. This way, the curvature along the heel angle axis would show the changes of hydrodynamic coefficients due to heel angles.

It is important to minimize the effect of all other variables for the added mass and damping values in order to enhance the visibility of deviation only due to the heel angles. This simplification is done by taking an average of the results obtained from all amplitudes of tests for a given period (i.e. the results for 0.4m, 0.8m and 1.2m are averaged for each period of heave and the results for 1° 2° and 3° are averaged for each period of pitch.) Therefore, it is important to note that the results presented here are the results obtained from the experiments. No effort is taken towards eliminating significantly deviating values while calculating the average. The averaging of values disregards the non-linear effects if any. Furthermore, the added mass and damping values were normalized by dividing each value to the maximum value of each frequency to emphasize the heel angle related differences only and increase readability. The complete results are presented in figures 6.12(a) to 6.13(d).

When parametric roll is considered, wave lengths equal to or larger than ship’s length ratios are commonly seen to excite the ship into high amplitudes of roll. As this study focuses on parametric rolling, it is more important to see what happens in longer waves, therefore longer periods of oscillation. For this reason, the Y axis values showing the longest periods are turned towards the reader in the Y axis \( T'(s) = 18.3 \). Shorter periods are also important but closer attention is paid to longer periods. A last note should be the color bars representing the change. They are not standardized, meaning that each figure has its color bar defined with different values. This should be noted while the figures are examined.

When the heave added mass results (figures 6.12(a) and 6.12(c)) are analysed, a somewhat unexpected outcome can be seen. Without any data at hand, one might have expected a general tendency to increase or decrease in the hydrodynamic coefficient as the vessel heels towards one side. This seems to be the case for high frequencies with although difference between the upright position and the heeled angle is very small (around 5%). However, for lower frequencies the results show an increase in added mass towards 5 degrees and then a sharp decrease, even down below the upright position values at 10 degrees of heel. This drop becomes more prominent when the vessel is in motion (i.e. at 7.97 knots). The added mass depends on the underwater geometry and the pressure distribution, therefore this is a specific case for this vessel (C11 container ship). This statement holds true for the other coefficients as well.

The heave damping the results do not follow the same trend. From figures 6.12(b) and 6.12(d)) it is seen that the values are increasing as the vessel is heeled to the sides. The change is the
around the same with or without speed, especially for higher frequencies. As the frequency gets closer to 18.2 seconds, figure 6.12(b) shows less change compared to 6.12(d) (for higher speed, the increase is around 20% and for the zero speed it’s around 30%). This may be attributed to experimental measurements as the rest of the frequencies turn in closer results between themselves. Furthermore, unlike added masses, a more even increase is visible regardless of the frequency of oscillation.

The pitch added inertia (figures 6.13(a) and 6.13(c)) show a different trend compared to the added mass results. Generally there is a decrease in the coefficient as the vessel is heeled to the side. The common point is that the changes are higher in longer frequencies of motion. Usually there is a drop reaching 20% at longer periods of motion. For higher frequencies of motion, the change does not even reach 10%, and stays around 5%. Figure 6.13(c) represents a sharp drop at 11.2 seconds period for the 10 degree heel angle. This may more be attributed to experimental measurements rather than a tendency as the rest of the figure follows very smoothly.

Pitch damping (figures 6.13(b) and 6.13(b)), on the other hand show the opposite behaviour by
increasing as the heel angle increases. The changes are significant regardless of the oscillation period, and reach up to 10%. It is difficult to say whether the speed contributes significantly to the change, as the results in general seem to fluctuate rather largely.

6.5 Remarks on the Analysis

If all results are reconsidered, it may be stated that the strip theory code used gives equally good results regardless of the heel angle. This may suggest that it can be considered as a candidate in the estimation of the hydrodynamic coefficients of asymmetric cross-sections. The pitch damping is an exception to the other coefficients as it is generally overestimated in longer frequencies.

As a final note on the change of hydrodynamic coefficients due to heeling, even if the changes may seem small in percentage, it is important to remember that the hydrodynamic matrix is composed of a large number of components. Adding up even 5 percent of change in each value
may lead to a big change in ship motions when the entire matrix is considered. More prominent differences in the behaviour of the vessel may be seen, especially in lower frequencies where each coefficient is subject to bigger shifts. It may therefore be necessary to consider an iterative calculation of hydrodynamic coefficients for the simulation of non-linear phenomena.

Hybrid methods may also be considered, meaning the matrices, for each frequency, may be calculated for different heel angles such as 5°, 10° and 15° instead of each angle and then the closest value to the obtained heel angle in the iterative process might be taken for the next iteration. This approach would save considerable calculation time while allowing higher precision in the estimation of large amplitude ship motions. However, it would be difficult – if not impossible – to do the calculations this way as there is a coupling between the motions and for a given heave amplitude at any moment the same pitch would not be obtained. In this case each coefficient would have to be checked separately.
7 Prediction and Validation of Parametric Roll

7.1 Introduction

The previous chapter has dealt with the validation of numerically obtained hydrodynamic coefficients by comparing them with the experimental results. The main idea behind any numerical procedure should be to put it into use to serve a certain purpose. In case of parametric rolling, the numerical estimation of the motions experienced by the vessel can help the masters of the ship on how to handle the ship in certain unfavourable conditions. As dynamic instabilities are conditions that may lead to loss of cargo, damage to the vessel, injuries to the crew and even to the loss of the vessel due to capsizing, proactive and preventative measures are of very high importance. There would be certain different ways to achieve this goal: The propagation of the waves and the corresponding induced movements of the ship can be calculated real-time onboard, having the measurements of the sea state. Or eventually they may be pre-calculated for the conditions the vessel will be in. Real-time calculation requires considerable computational power. The requirements may be exemplified by Edgar et al. [42] as having 500 seconds of sampling for a prediction of 100 seconds, with the wave heights being measured from one nautical mile distance from the vessel.

Therefore, ignoring the non-stationarity of parametric rolling in waves, a more simplified pre-calculation routine currently seems to be a viable option. The results from numerical simulations with non-linear time domain computer codes can present a solution for the risk assessment of parametric rolling. The downside of such a method is that simulations for a very large number of conditions would have to be carried out in advance considering the variability of environmental factors and load conditions. The advantage would be the accessibility of the method as the polar plots may be prepared at any computer without the need of a device onboard and without real-time sea state measurements.
The pre-calculated simulations have to be presented to the deciding authority of the ship’s course in a form that is easily understandable. For this purpose, polar plots bring together a form that is both easy to comprehend and prepared off-board. One thing should be noted, as the polar plot is a two-dimensional medium, it would have to define a limit for the intensity of the motion and classify it as dangerous. In other words, it is forced to introduce a severity criterion by itself. This shortcoming can be limited by the use of three dimensional polar plots but they would reduce the readability. The best option is to present a colour coded two-dimensional polar plot which introduces the amplitude of the motion in colour codes and the heading/speed of the vessel in radial and polar axes.

The aim of this chapter is to prepare such graphics for the container vessel C11 that has been under discussion. First, the chapter deals with the experimental results obtained by testing the vessel in different encounter conditions as presented in chapter 5. Then the results are compared with the numerical results of ShipAtSea. After the validation of the results, polar plots for different conditions are presented.

7.2 Parametric Rolling Regular Waves

In the experiments, both regular and irregular waves were tested. In this study they will be dealt with separately, starting with regular waves. The regular waves experiments at CEHIPAR were conducted at different wave heights, wavelengths, loading conditions and speeds. The speed was defined as the most critical one, 7.97 knots for head waves. For following waves, the vessel had no forward speed (i.e. 0 knots). As the experiments are very time consuming, the load conditions that were tested were limited to two. In the first load condition, the GM was set equal to 1.973 m, and the second load condition was defined with the GM value lowered to 0.978 meters. They have been used for head waves and following waves respectively.

For both head waves and following waves, the wave length to ship’s length ratios considered were $\lambda/L_{pp} = [0.8, 1.0, 1.2, 1.4]$. The wave heights considered were $H_w = [4.0, 8.0, 10.0]$ for head waves while 10 meters of wave height was omitted in following waves. The table below summarizes all of the parametric roll experiments for regular waves:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Head Waves/Load Condition 1</th>
<th>Following Waves/Load Condition 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (knots)</td>
<td>7.97</td>
<td>0</td>
</tr>
<tr>
<td>GM (m)</td>
<td>1.973</td>
<td>0.987</td>
</tr>
<tr>
<td>Wave Height (m)</td>
<td>6 / 8 / 10</td>
<td>6 / 8</td>
</tr>
<tr>
<td>$\lambda/L_{pp}$</td>
<td>[0.8, 1.0, 1.2, 1.4]</td>
<td>[0.8, 1.0, 1.2, 1.4]</td>
</tr>
</tbody>
</table>

Below, the results obtained from the experiments are presented along with the corresponding simulation results. Using the code and the interface that was explained in detail in chapter 4, the
same settings used for the experiments were repeated numerically. The maximum roll angles were compared.

Two different tables are presented for the results, 7.2 for head waves and 7.3 following waves. Before discussing the results, it is important to note that while the code has performed better in head waves, in following waves parametric rolling was not encountered numerically. In order to replicate the results of the following waves experiments to the highest extent, an adjustment to the numerical transverse metacentric height ($GM_t$) was required. Therefore the numerically ($GM_t$) was taken as 3.124 meters instead of the experimental value of 0.987 meters for following waves. As a reminder, $\lambda/L_{pp}$ represents the wavelength to ship’s length ratio in the following tables:

Table 7.2: Experimental and Numerical Results

<table>
<thead>
<tr>
<th>$\lambda/L_{pp}$</th>
<th>$H_w$ [m]</th>
<th>Roll (Num) [$^\circ$]</th>
<th>Roll (Exp) [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>6</td>
<td>1.26</td>
<td>32.15</td>
</tr>
<tr>
<td>0.8</td>
<td>8</td>
<td>1.35</td>
<td>33.43</td>
</tr>
<tr>
<td>0.8</td>
<td>10</td>
<td>1.48</td>
<td>35.72</td>
</tr>
<tr>
<td>1.0</td>
<td>6</td>
<td>19.68</td>
<td>23.88</td>
</tr>
<tr>
<td>1.0</td>
<td>8</td>
<td>34.71</td>
<td>25.32</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
<td>1.32</td>
<td>27.27</td>
</tr>
<tr>
<td>1.2</td>
<td>6</td>
<td>2.04</td>
<td>1.3</td>
</tr>
<tr>
<td>1.2</td>
<td>8</td>
<td>1.45</td>
<td>1.91</td>
</tr>
<tr>
<td>1.2</td>
<td>10</td>
<td>1.27</td>
<td>1.83</td>
</tr>
<tr>
<td>1.4</td>
<td>6</td>
<td>1.36</td>
<td>0.53</td>
</tr>
<tr>
<td>1.4</td>
<td>8</td>
<td>1.29</td>
<td>0.84</td>
</tr>
<tr>
<td>1.4</td>
<td>10</td>
<td>1.32</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 7.2 represents the comparative results for head waves. The results require further explanation. To begin with, there are no conditions in which parametric rolling has been detected numerically but has not been encountered experimentally. This suggests that the model is not prone to overestimating the encounter conditions that may lead to roll and suggesting unrealistic cases of parametric roll. $\lambda/L_{bp} = [1.2, 1.4]$ corresponds to these ratios.

For $\lambda/L_{pp} = 1$ and $\lambda/L_{pp} = 0.8$, there is a different scenario. For $\lambda/L_{pp} = 0.8$ parametric rolling has been captured experimentally, reaching roll angles well over 30 degrees. The motion is larger than the case where the wavelength is equal to the ship’s length (around 25 degrees). This motion was not captured by the mathematical model. On the other hand, $\lambda/L_{pp} = 1$ was captured and the results were close to experimental values apart from the experiment where the wave height is 10 meters. However, if numerically, $\lambda/L_{pp} = 0.95$ is tested instead unity, the mathematical model also detects parametric rolling and the amplitude reported is 49 degrees. The reason for this behaviour is discussed further in the chapter while discussing the effect of wave height on parametric rolling.

The complete following waves results are given in table 7.3. In this condition, all cases led to parametric rolling regardless of the wavelength to ship’s length ratios and wave heigh. This motion was not captured by the numerical model. However, if the vertical centre of gravity of the model
was lowered to 16.25 meters (GM becomes 3.142) the model was able to predict the roll. As the transverse metacentric height relates to the restoring forces and the stiffness, this suggests that there were changes in stiffness that were not captured by the numerical model. Using the altered metacentric height, the roll motion at $\lambda/L_{pp} = 1.0$ was captured, with the 8 meter wave height presenting a result very close to experimental. Interestingly, while $\lambda/L_{pp} = 1.2$ was missed by the code, $\lambda/L_{pp} = 1.4$ was captured again for the 8 meter wave height. Consequently, the effect of wave height becomes prominent also in following waves along with head waves. This will be discussed next.

Table 7.3: Experimental and Numerical Results – Numerical GM: 3.142

<table>
<thead>
<tr>
<th>$\lambda/L_{bp}$</th>
<th>$H_w$ [m]</th>
<th>Roll (Num)[°]</th>
<th>Roll (Exp)[°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>6</td>
<td>1.16</td>
<td>35.78</td>
</tr>
<tr>
<td>0.8</td>
<td>8</td>
<td>1.42</td>
<td>35.79</td>
</tr>
<tr>
<td>1.0</td>
<td>6</td>
<td>16.34</td>
<td>35.79</td>
</tr>
<tr>
<td>1.0</td>
<td>8</td>
<td>43.91</td>
<td>38.19</td>
</tr>
<tr>
<td>1.2</td>
<td>6</td>
<td>1.15</td>
<td>31.53</td>
</tr>
<tr>
<td>1.2</td>
<td>8</td>
<td>2.38</td>
<td>34.06</td>
</tr>
<tr>
<td>1.4</td>
<td>6</td>
<td>37.12</td>
<td>25.96</td>
</tr>
<tr>
<td>1.4</td>
<td>8</td>
<td>1.44</td>
<td>26.34</td>
</tr>
</tbody>
</table>

7.2.1 Effect of Wave Height and Speed on Parametric Rolling

As mentioned in the section above, wave height plays an important role in parametric rolling. Another prominent factor that closely relates to the encounter conditions is the speed. Both of these variables will be examined closely. Figures 7.1(b) and 7.1(a) present the effect of changing the wave height and speed. Before taking a closer look at the figures, it should be noted that the results were prepared using the iterative damping estimation of ShipAtSea and a tuned version of the Miller’s method to get closer damping values compared to the experiments. As stated before, Miller’s roll damping estimation is a function of roll angle and speed. In the iterative estimation calculations of the interface, there is a certain precision value, and if the roll value used for the estimation of the roll damping is not within the tolerance levels of the roll obtained, the simulation is repeated with the obtained roll as the roll damping estimation input.

There are encounter conditions where parametric roll is very sensitive to the roll damping value. While the damping is always a major factor, according to the simulations, for certain conditions parametric roll reaches very high angles when the damping goes below a threshold value and stops completely when the damping rises above that value. Therefore in the following figures, as the estimated roll angle changes with the iterative estimation, there are certain “jumps” in the figures (i.e. between 6.5 to 7 meters of wave heights and 7 to 8 knots of speed). As the roll angle that was encountered in the simulations get to higher values, the user interface reacts by using a higher maximum roll angle for damping estimation. This lowers the obtained roll angle for the next
iteration. If it drops below the pre-defined precision, the estimation is lowered as well, this time raising the roll angle. This is the reason for the non-systematic change in the results presented.

**Effect of Wave Height**

If extreme cases of wave height are considered, very small waves may not excite the ship into parametric rolling. However, this does not mean that as the height increases the roll angle will keep increasing. Other factors such as deck immersion and slamming will come into play, and will change the behaviour of the vessel. Therefore what is expected to be seen, is an increase in roll angles up to a certain wave height and then a decrease. How sudden this decrease will be is an important question.

In order to see this, the experiments had been repeated with three different wave heights: 6, 8 and 10 meters. Experimentally, the vessel attains roll angles with negligible differences in between. It is very probable that if the wave height was increased even more, there would be a cut-off wave height. In the experiments very high roll angles obtained through high waves have created physical problems leading to damage to the model, therefore this exact value has not been obtained. Both experimentally and numerically, results are presented here for $\lambda/L_{pp} = 1$ and the heading of 180 degrees where the results are more in agreement.

Because it is not as time consuming as the experiments, to get a more detailed idea of the effect of wave height, more points were calculated numerically compared to the experiments. For each 0.1 meters of wave amplitude, another simulation was run. As the total number of simulations for this case is 52, only the results that have lead to roll angles larger than 19 degrees are presented in table 7.4. The wave height of 8.8 meters is included on purpose although there is no parametric rolling, in order to define where the motion stops.

The “phi used” column lists the estimated roll angle used to calculate Miller's roll damping. The phi values also illustrates the jumps in the figure more clearly if the wave heights of 6.4, 6.6, 6.8
Table 7.4: List of wave heights in which the obtained roll angle was greater than 20°

<table>
<thead>
<tr>
<th>Roll Angle [Deg]</th>
<th>Wave Amplitude</th>
<th>Phi Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.68</td>
<td>6.00</td>
<td>19.68</td>
</tr>
<tr>
<td>21.14</td>
<td>6.20</td>
<td>20.08</td>
</tr>
<tr>
<td>23.4</td>
<td>6.40</td>
<td>20.08</td>
</tr>
<tr>
<td>19.12</td>
<td>6.60</td>
<td>45.23</td>
</tr>
<tr>
<td>45.89</td>
<td>6.80</td>
<td>20.37</td>
</tr>
<tr>
<td>22.29</td>
<td>7.00</td>
<td>46.14</td>
</tr>
<tr>
<td>42.33</td>
<td>7.20</td>
<td>46.14</td>
</tr>
<tr>
<td>36.81</td>
<td>7.40</td>
<td>46.14</td>
</tr>
<tr>
<td>45.6</td>
<td>7.60</td>
<td>36.09</td>
</tr>
<tr>
<td>45.75</td>
<td>7.80</td>
<td>36.09</td>
</tr>
<tr>
<td>34.71</td>
<td>8.00</td>
<td>36.09</td>
</tr>
<tr>
<td>35.82</td>
<td>8.20</td>
<td>36.09</td>
</tr>
<tr>
<td>34.51</td>
<td>8.40</td>
<td>36.09</td>
</tr>
<tr>
<td>57.36</td>
<td>8.60</td>
<td>5</td>
</tr>
<tr>
<td>1.26</td>
<td>8.80</td>
<td>57.36</td>
</tr>
</tbody>
</table>

and 7.0 are examined more closely. The damping adjustments are explained in more detail in the effect of speed, along with the precision of the estimation. The requested precision is 10 degrees in both cases. This results table is obtained simply as a portion of the report that is provided by the user interface that has been explained previously in the work.

The figure 7.1(a) shows a numerical cut-off wave height at around 8.6 meters. As explained before if the experiments were continued with higher waves, there would have been an experimental cut-off as well. In order to understand why numerically the motion is not detected it should be considered that with waves of higher amplitudes dynamic effects such as slamming and the coupling between the symmetric and antisymmetric modes may come into play. As this motion changes the natural motion of the vessel and slows it down, there is a possibility that it also changes the natural roll frequency. The current mathematical model does not consider this. It is possible that this may be the reason why at large amplitude waves the parametric roll fades out numerically while the experiments still report parametric rolling. In an attempt to prove this point, the wavelength that is slightly different was numerically tested. As mentioned above in the chapter, if this ratio is taken as 0.95 instead of unity – corresponds to the wave frequency of 0.498 instead of 0.485 to numerically state how small the change is – roll angles of 49 degrees can be obtained for the 5 meter wave height.

Effect of Speed

The effect of speed on parametric rolling is only examined numerically. The reason for this is that the experiments require a considerable amount of resources and therefore they were only carried out using the critical speed which led to the highest roll angles. Although there are no experimental results presented, the heading is kept the same (180°, head waves), wave height was fixed at 4 meters and \( \lambda/L_{pp} = 1 \) was preferred for consistency. The data consists of 32
speeds, ranging from 0 to 15 knots in steps of 0.5 knots.

The most prominent result that can be seen from figure 7.1(b) is that parametric rolling starts rather suddenly and slowly deteriorates with speed. At 6 knots, sharply, the vessel starts to roll reaching dangerously large amplitudes. As it keeps speeding up, the value keeps increasing up to the point where the vessel has attained 11 knots, and then starts to decrease slowly, waning out at 15 knots. Iterative damping interferes with the increase at around 8 knots. Then it follows along the results and does not leave room for sudden jumps. This is related to the precision value defined, which in this case was taken as 10 degrees. However, there are certain settings that make keeping the precision impossible, such as 6 knots. If the damping is lowered below a certain threshold, the program will obtain high roll angles, and if those roll angles are tried for the next iteration, rolling will stop completely leading to another iteration with a rather small roll angle. This will continue until the maximum number of iterations defined is reached. The results for 5.5 to 15 knots of speed are presented in table 7.5.

Unfortunately it has not been possible to confirm this finding experimentally due to the reason mentioned above. However, if this result is considered to be correct, it may be advisable for the ships at sea to slow down in case of parametric rolling, if possible, as speeding up seems to result in a solution that is much slower.

### 7.2.2 Comparison of Regular Waves Time Series

To achieve a better understanding of the data being analysed, it is not sufficient to see only the maximum roll angles, but also, the time series representing the simulations should be com-
pared. For this reason, the test where the numerical and experimental results are matched closely were put under further focus. As seen before in tables 7.2 and 7.3, the best agreement with the numerical results is in load condition 1, and where where $\lambda/L_{pp} = 1$ with a wave height of 4 meters. Speed is equal to 7.97 knots. The time series show the wave elevation, heave, roll and pitch. A characteristic of parametric rolling is having the heave and pitch oscillation periods twice the period of roll which should be taken into consideration when the figures are examined. The experimental time series is given in figure 7.2 and the numerical time series is given by figure 7.3.

A comparison of these figures shows that roll motion develops to its maximum amplitude faster experimentally. Before going on any further with the discussion, it should be noted that the numerical roll amplitude ($34.7^\circ$) is larger than the experimental ($25.3^\circ$). Numerically this value is reached around 600 seconds, and experimentally around 300 seconds. If the initial roll amplitude for the simulations were defined to be higher, the development of parametric rolling would be quicker, but the amplitude of initial roll is experimentally very small as well. The initial values of sinkage and trim are calculated by the user interface, using the instantaneous wave elevation at the time $t = 0$ seconds. The required data is obtained from the frequency domain code results. Therefore those values are not arbitrary, and are effective in the results.

When the roll motion is checked against heave and pitch, both numerically and experimentally it is visible that the roll period is twice the pitch/heave period, confirming the expected values. When the other amplitudes are compared, the amplitude of pitch motion seems to be in agreement between the simulations and experiments, while the heave motion gets underestimated by the numerical code. This may be a factor in slower development of parametric roll motion as less heave motion means less change in the underwater geometry of the vessel and therefore less change in the transverse metacentric height.

### 7.3 Parametric Rolling in Irregular waves

It is impossible for a vessel to encounter a pure sinusoidal wave in a realistic condition. The waves that will be encountered will be irregular waves which can be subdivided into a large number of sinusoidal components using the superposition principle. For this reason, it is important to see how the model behaves in irregular waves. The results will also demonstrate the practical importance of parametric rolling. To experimentally assess the susceptibility of the C11 class container vessel, the series of experiments have included also irregular wave configurations. The zero-upcrossing period was defined as $T_z = 12.95$ during the tests. Both loading conditions that have been mentioned before have been tested, using different speeds. The head waves load condition was tested not only in head waves but also in slightly oblique waves by changing the speed of the vessel. The reason to change the speed was to keep the encounter frequency constant, without having the towing tank personnel readjust and recalibrate the wavemakers and
Figure 7.2: Experimental time series, \( \lambda/L_{pp} = 1 \), Wave Height 4 meters, Speed = 7.97 knots
Figure 7.3: Numerical time series, $\lambda/L_{pp} = 1$, Wave Height 4 meters, Speed = 7.97 knots
save time. For the following waves, no such calculation was carried out and the vessel has been tested at zero speed. The results obtained from these tests are presented in table 7.6.

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>Speed [kts]</th>
<th>Heading [°]</th>
<th>Roll Angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.97</td>
<td>180</td>
<td>6.4</td>
</tr>
<tr>
<td>1</td>
<td>8.16</td>
<td>170</td>
<td>17.1</td>
</tr>
<tr>
<td>1</td>
<td>8.36</td>
<td>160</td>
<td>7.3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>25.3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>10</td>
<td>28.3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>20</td>
<td>28.5</td>
</tr>
</tbody>
</table>

Examining the table, the practical importance of parametric rolling is eminent. Four out of six tests have shown parametric rolling. While in head waves have been less problematic, and the maximum roll angle has been 17 degrees, in all following waves experiments, the obtained roll angle has been larger than 20 degrees.

In order to better understand how parametric rolling has developed in these cases, and the danger involved, it is necessary to take a closer look at the time records as well. Apart from the maximum roll angle obtained, it is important to see how quickly parametric rolling has developed in irregular waves, how long has the vessel been in resonance, and how long did it take for the motion to fade away. For a vessel, experiencing roll for 10 seconds will not yield the same severe results with continuous roll motion lasting several minutes. It has been mentioned before that model testing is very time and resource consuming. For this reason, and for the fact that the results are interesting to be compared in between themselves as well, time records of all four experimental tests that have led larger roll motions are presented with figures 7.4 to 7.7.

The head waves time series (figure 7.4) show parametric rolling at around 17 degrees, but the time is rather short. Around 1.5 minutes. Then the motion stops very quickly. The development is slow but also with small amplitude motions, which may go unnoticed for the crew to leave enough time for them to react. Figure 7.5 shows a similar series to figure 7.4, this time in following waves. Again there is not much time for the crew to react. Figures 7.6 and 7.7 prove to be much more interesting. Before commenting on those experiments, it is important to note that the vessel is not moving in following waves. Therefore, theoretically the time allowed to carry out these tests is infinite, as the vessel never reaches the end of the towing tank but is stationary with 0 speed. For this reason, it may be argued that if it was possible to wait longer in head waves with higher speeds, there may have been more severe roll angles. Considering that, following waves figures 7.6 and 7.7 show time development of parametric rolling not only once, but twice in the same test run. In both cases the amplitude of roll motion reaches more than 20 degrees.

For irregular waves it is very difficult to repeat the same profiles and same settings to examine the capabilities of the mathematical code. Therefore, a test was chosen as an example to present. It was explained before that the interface is capable of running the program that generates random
Figure 7.4: Irregular waves, experimental results, heading $170^\circ$, 8.16 knots
Figure 7.5: Irregular waves, experimental results, heading 0°, 0 knots
Figure 7.6: Irregular waves, experimental results, heading $10^\circ$, 0 knots
Figure 7.7: Irregular waves, experimental results, heading $20^\circ$, 0 knots
waves and repeating the same settings with different time and space realizations. To obtain the irregular seas results here and in the following section which presents polar diagrams, 5 different time-space realizations were tried to obtain the maximum angle of roll that is to be reported. In all cases, the zero up-crossing period is 12.95 seconds. The vessel speed is the same. The only variable that changes is the actual incoming waves. For this reason, a complete set consisting of 5 runs is presented. While only one run is presented along with heave and pitch motions and the wave elevation, for the other four runs only the roll motions are given. A numerical set with 170° of heading and 8 knots of speed was chosen as the values are very close to the experimental values that lead to parametric rolling under the same conditions. The detailed results of these numerical simulations are as follows:

Table 7.7: Irregular waves simulations, 170°8 knots

<table>
<thead>
<tr>
<th>Run number</th>
<th>Roll angle [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.8</td>
</tr>
<tr>
<td>2</td>
<td>22.7</td>
</tr>
<tr>
<td>3</td>
<td>19.7</td>
</tr>
<tr>
<td>4</td>
<td>01.2</td>
</tr>
<tr>
<td>5</td>
<td>22.1</td>
</tr>
</tbody>
</table>

As illustrated in the table, almost all runs have lead to parametric rolling except for one. While the maximum amplitudes of motion have been similar in all cases, the behaviour was different and shows varies between runs. In run 1 (figure 7.4), the motion develops gradually similar to a sinusoidal wave simulation. While run two develops later, run three develops and stops. Run four shows no signs of parametric rolling and run 5 is again similar to run 1.

As the wave profiles are not identical, it is very difficult to compare the numerical and experimental results fairly. It can be stated that generally, the motion develops more slowly in numerical simulations compared to experiments. Therefore, as the starting times are short, it seems to be more probable to be taken by surprise in a realistic seaway. On the other hand, the numerical roll durations are longer as well. Experimentally, roll has lasted at most around six minutes (see figures 7.6 and 7.7). It is not possible to check and compare all of the numerical results in an attempt to get an average of roll motion durations, as the total number of irregular simulations obtained in this study is more than 14000. However, the examples presented here in figures 7.8 and 7.9 show that much longer times around 10 minutes are obtained twice, probably revealing a general tendency.

7.4 Polar Plots

The main idea behind having this mathematical model is to predict dangerous situations where parametric rolling occurs and make it possible to avoid them. As stated before in the chapter, this work aims to see if it is possible to provide guidance to the shipmaster through pre-calculated
Figure 7.8: Irregular waves simulation: heading 170°, 8.0 knots, Run 1
Figure 7.9: Irregular waves simulations: Time series of roll motion, heading $180^\circ$, 8.0 knots, Runs 2 to 5
polar plots. The diagrams presented here were prepared using a MATLAB code that has been developed specifically for this purpose.

In order to fully understand the data that is required to obtain the diagrams, the resolution of the data should be mentioned beforehand. In the previous section, it was shown that parametric rolling is very sensitive to changes in speed. For this reason, the speeds that have been tested ranged from 0 to 15 knots, in steps of 0.5 knots representing speeds of [0, 0.5, 1, 1.5, ..., 14, 14.5, 15] knots. Headings were also assessed in detail from 0 to 180° in 2 degrees of increment (e.g. 180°, 178°...). For this reason the total number of simulations that are used for each polar plot adds up to 2821, representing 91 headings and 21 speeds. Each of these simulations calculate a time space of 1200 seconds. This case holds true for regular seas. Furthermore, iterative damping was used to calculate the roll damping making the actual number of unrecorded simulations much higher. The iterative damping estimation tries each condition a maximum of 5 times, to obtain a precision of 10 degrees. The results that were obtained without satisfying these roll damping requirements were discarded.

For irregular seas, the number of simulations required are considerably higher. The headings and speed that were used were the same with the regular waves. However, the wave profiles generated for the simulations would be different for same speed and heading on different tries due to the irregularity. For this reason, to overcome the probability of missing out certain cases that lead to roll motion, each speed and heading combination was simulated using five different irregular wave profiles. In the irregular polar plot that will be presented, only the result that represents the maximum of these five runs was taken into consideration. Therefore the total number of simulations for the irregular waves polar plots are 14105 of which 2821 of the worst case results (i.e. cases that have led to largest amplitudes of roll) are presented.

Given this very high number of simulations, how important the programmed interface is very clearly shown. It impossible to carry out this extremely high number of simulations if the process required constant interaction from an engineer. Instead, a computer was set beforehand and completed the calculations automatically, running 24 hours for days or weeks until all required solutions were obtained. This raises another question of how this large volume of data is turned into polar plots, as it is equally impossible to go through 14000 results, pick the highest degrees of roll and plot them. What eases the task is the reports file of the interface. As it automatically reports a list of all simulations carried out, speeds, headings, and the maximum roll angles obtained, there is already a matrix that represents a detailed list of simulation results and their settings.

Further programming was required to convert this outcome to input for MATLAB and get the polar plots as end results. The program and the MATLAB code are not explained in detail but the process is as follows: The output is obtained from the reports file of the interface. Then this output is automatically sorted into the matrix format required by the MATLAB program that prepares the polar plots. Polar plots are than made transparent to allow visibility of the axis which is a
specifically important as the polar plotting routine of MATLAB does not present a good solution in 3D surfaces. If the polar plots were given as 2D diagrams, in two colors, it would be necessary to define an arbitrary value where parametric roll develops (for example mark the 20 degree margin only). For this reason the polar plots were color coded, showing the intensity of the motion along with the heading and speed. In the polar diagrams (figures 7.10, 7.12 and 7.11) the radial axis represents the speed of the vessel and the angular coordinates represent the headings. Each polar plot has its own color bar, representing colors for the corresponding roll angles.

For regular waves, the wave height was fixed at 4 meters. Both load conditions that have been previously presented in table 7.1 have been simulated. The first polar diagram in (figure 7.10) represents the load condition with the metacentric height of 1.97 meters. head waves and oblique waves turn in large amplitudes of roll in this case. Similar to the simulations that were presented before in the chapter, it starts rapidly at approximately 7 knots and slowly wanes until the speed of 15 knots is reached. At approximately 12 knots the amplitude is already below 20 degrees. This plot confirms, for all headings, the previous speed survey results that suggest a sudden start and a gradual decline. The second polar diagram (figure 7.11) represents a load condition where the transverse metacentric height equals 0.987 meters. According to these results, there is a more concentrated zone where parametric rolling is encountered. The same loading condition seems to be dangerous on low speeds up to 5 knots regardless of the direction of the wave. The only
way to stop parametric rolling in this case would be to speed up beyond 5 knots into head waves reaching to 7 - 8 knots.

The polar plot for the irregular waves tested are presented in figure 7.12. The number of simulations required to obtain such a plot are very large. Each simulation also takes longer times, as they require a more sophisticated calculation process relating to not only one but more wave components. The spectrum that was utilized in this work was a Jonswap spectrum, and the number of wave components were 41. The frequencies of the waves range from 0.2 rad/sec to 2 rad/sec. Significant wave height is the same with the regular waves (4 meters.) The zero up-crossing period is 12.95 seconds where the wavelength to ship’s length ratio would equal to unity if the waves were regular. As the required time for each simulation increases and five repetitions were carried out using the interface for each speed and heading, this polar plot represent mirrored results to save on calculation time. Results from 0 to 180 degrees were obtained and mirrored to the 180 - 360 degrees side of the plot. Ideally, as the 180 - 360 side of the plot would also be tested and therefore they would not be symmetric as in this case. From the polar plot it is easy to say that the process is not stationary. If this case showed a resemblance to the regular seas results (figure 7.10) for the same loading condition, it might have been possible to compare them and use these regular waves results as a guidance. The way that the results are unrelated makes this idea obsolete. However, if the plot is examined closely, the vessel can be noted as generally
being prone to parametric rolling. Constantly, there are cases where the roll angles reach 20 degrees and more. They seem to be more prominent in speeds up to 10 knots. This polar plots suggests that the difficulty in relating these results to the irregular seas results means that onboard calculations for the estimation of parametric rolling using the measured instantaneous wave profiles should be given priority. Although this approach leads to a more expensive solution as it requires more computational power and extra hardware, switching from regular waves to irregular waves does not seem to be a viable option according to these results. However, if a numerical model of the vessel is simulated in irregular waves and these are the results that are obtained, it may be used to classify the vessel as prone to parametric rolling and the crew may at least be informed to take precaution.
8 Conclusions

8.1 Conclusions

The main focus of this study has been on parametric rolling and the aim was the validation of a numerical method that has been developed at Instituto Superior Técnico in the recent years. To achieve this, the experimental results from the forced oscillation tests and parametric rolling test of a C11 type container ship were compared to the numerical calculations. The computer program utilized here takes a modular approach and there are two different parts to the code which work in conjunction. Each part was paid specific attention to. To facilitate their usage and to make uninterrupted iterative calculations possible, a third program which includes a user interface was developed to prepare the necessary in between files. The output is also prepared by the program as MS Excel output with graphics.

To summarize its functions, the interface introduces the necessary changes to the frequency domain program input files, runs them, gets the output, prepares the time domain input files, gets the time domain program output files and prepares them in a presentable format. Additionally, it organizes the output files and produces a report file listing all the simulations and the results along with their settings. The intermediate program also handles roll damping methods and is capable of doing iterative estimations with the given precision and maximum tries. Apart from these functions, the development of such program has made obtaining polar plots possible. This is due to the fact that range surveys (e.g. a survey of all speed from 0 knots to 10 knots in steps of 0.1 knots) can now be carried out without interruption and a very high number of simulations are needed for polar plots (for irregular plots may reach up to 15000 simulations). This part of the work is not limited to this thesis and has been used in other projects as well. Future work was also made easier.

In order to validate the time domain and the frequency domain codes, the results of the analysis from the model testing experiments that were carried out at the CEHIPAR seakeeping basin, Madrid were compared with the numerical results. The results of the forced oscillation tests reveal...
that the frequency domain code generally does a good job of estimating pure heave and pitch hydrodynamic coefficients (added mass/inertia and damping). The correlation between experiments and numerical results were not heavily affected by the heeling of the vessel and therefore shows that strip theory has been equally reliable for heel angles up to 10 degrees for this vessel. While the forced oscillation tests were conducted, three different amplitudes were selected. Therefore the non-linearities were also analysed. The findings were different for heave and pitch and heave motion showed more non-linearity. When changes of hydrodynamic coefficients due to the heeling of the vessel has been examined closely, at longer periods of oscillation they were found to be much more significant amounting to 20% at times. In this regard, calculation of the hydrodynamic coefficients in time domain have been suggested.

The performance of the time domain code has been assessed by comparison to the parametric rolling tests. To get a full understanding, different wavelengths, wave heights, speeds and loading conditions were compared. The numerical code has performed better in head waves when compared to following waves. For the latter, certain adjustments to the stiffness of the vessel were necessary to detect parametric rolling. For the wave height of 5 meters, parametric rolling was not detected at head waves. This particular case has been further evaluated by examining the effects of wave height and it has been found that there may be a synchronization breakdown due to a change in natural frequency at high waves. In order to get a better understanding of the vessel’s behaviour numerically and experimentally, time series of parametric rolling motion had been presented for numerical and experimental results. It has been seen that generally parametric rolling develops quicker experimentally in comparison to numerical simulations. Further investigation on irregular waves has been carried out and time series were presented as well. Both experimentally and numerically parametric rolling has been observed for irregular waves making it of practical concern for the crew and the vessel owner.

To see the complete numerical results for various headings and speeds, polar diagrams have been developed for the container vessel in two different load conditions. A third polar plot was prepared for irregular waves. The method to obtain these polar diagrams relies heavily on the user interface and the method was discussed further. With the comparison of regular waves and irregular waves polar plots, it was seen that the phenomenon is non-stationary and a clear connection between regular and irregular waves is not present. However the irregular polar plot has presented a general idea that this vessel is prone to parametric rolling as roll angles larger than 20 degrees were quite common.

### 8.2 Future work

The occurrence of parametric rolling is largely dependent on the hull form and the encounter conditions. While the focus has been on container ships, other hull forms such as fishing vessels
are not completely excluded from encountering this phenomenon. The nature of their work im-
poses severe danger for the fisherman onboard as well, if large roll angles are to be encountered. 
This study may be extended onto different hull forms to see how the code fares with them to widen 
its area of application.

The asymmetric cross-section experiments have revealed the change in added masses and 
damping coefficients of heave and pitch due to the heeling of the vessel. The calculation of these 
coefficients may be incorporated on the time domain code to take the differences into account 
and to develop the code further. Also, continuing the analysis of the forced oscillation experiments 
would allow the calculation of coupling effects for the pitch into heave added mass and damping 
coefficients as well as the heave-roll coupling effects for asymmetric cross-sections.
References


