Abstract: The main purpose of this dissertation is to establish a comparative analysis among several flat slabs, taking into account parameters such as the live load, the span, the material (normal or lightweight concrete) and whether or not post-tensioning is adopted.

In the analyzed model, each slab is composed of 9 panels, supported by columns. The study is organized into 4 groups. In group A, slabs with unbalanced spans are studied, which means that all 9 panels have the same dimensions in both directions. While the span in one direction is constant and equal to 8m, the span in the other direction varies every 2 meters from 10m to 20m. In the next group slabs show balanced spans, i.e. the dimension of the edge spans in each direction is 75% of the interior span, the latter being either 16m or 20m. Slabs in group C only have unbalanced, 16m long spans and are made of lightweight concrete, including classes 1.6, 1.8 and 2.0. All the above mentioned slabs are post-tensioned with a 1-way band beam system, unlike those in group D where the slabs are not post-tensioned and instead have drop panels in each column. The geometry of these slabs is equivalent to those in group C. For each group and each parameter presented, the live load ranged from 3 to 15 kN/m².

The main conclusions of this thesis are: slabs with unbalanced 16m long spans are about 11% more expensive than the balanced spans solutions; the current costs of lightweight concrete apparently prevents slabs composed of this material from being competitive against slabs with normal concrete; reinforced concrete slabs are about 51% more expensive than post-tensioned ones.

Keywords: Flat Slab, Band Beam, Post-Tensioned Concrete, Lightweight Aggregate Concrete
1. Introduction

1.1. General

The new architectural requirements regarding aesthetics and functionality of industrial and commercial buildings have been dictating an increase in the span. Because of this, flat slabs are becoming ever more a popular solution when it comes to designing those types of buildings. The goal of this paper is to understand how various factors such as the live load, the span, the material or even the presence of prestressing influence the design and finally the cost of the above mentioned slabs.

The requirement of longer spans and higher live loads naturally leads to the adoption of prestressing steel in order to keep the slabs thin and avoid implementing heavy drop panels. Consequently, that type of solution will be dealt with in more detail in this paper. Nevertheless, an economic and structural comparison between reinforced and prestressed concrete will be presented.

Another aspect studied in this paper is the adoption of lightweight aggregate concrete (LWAC) to replace the normal concrete used in the slabs. With a reduced self-weight, the adoption of this type of concrete could prove to be more economical than its counterpart, reducing the required reinforcement and prestressing steel needed to satisfy both ultimate and serviceability limit states.

1.2. Organization

The analyses are carried out on slabs composed of 9 panels, supported by columns.

The goal is to study the following aspects:

- To study the influence of span lengths and live load values on the materials quantities of structural concrete in industrial and commercial floors.
- To assess the difference between balanced spanned slabs over unbalanced ones (a balanced spanned slab being one in which the exterior spans have 75% the length of the interior ones)
- To evaluate the potential economical benefit of adopting LWAC, including classes 1.6, 1.8 and 2.0
- To assess the difference between reinforced concrete slabs and prestressed ones for such applications

The analyses are organized according to the following approach:
• **Group A** – slabs with unbalanced spans;
• **Group B** – slabs with balanced spans
• **Group C** – slabs made up of LWAC
• **Group D** – reinforced concrete slabs (with drop panels)

For each case presented in the above diagram, the live load ranged from 3 to 15 kN/m².

### 1.3. Structural System

The structural system adopted for post-tensioned and reinforced concrete slabs is shown in figures 1 and 2. The panels dimensions forced a solution of one-way band prestressed beams, for the post-tensioned solution. As for the reinforced concrete solution, rectangular drop panels under each support were adopted.

![Figure 1 - Structural system adopted for the prestressed solutions](image-url)
2. Materials

The materials adopted in the study are summarized in tables 1 and 2. Note that the costs of the materials are based on values provided by local suppliers.

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Reinforcing Steel</th>
<th>Prestressing Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>C30/37</td>
<td>A500</td>
<td>A1670/1860</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>2,5</td>
<td>-</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0,2</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma ) (kN/m(^3))</td>
<td>25</td>
<td>78,5</td>
</tr>
<tr>
<td>( f_{ctm} ) (MPa)</td>
<td>2,9(^1)</td>
<td>-</td>
</tr>
<tr>
<td>( f_k ) (MPa)</td>
<td>30,0</td>
<td>500</td>
</tr>
<tr>
<td>( f_d )</td>
<td>20,0</td>
<td>435</td>
</tr>
<tr>
<td>( \epsilon_{ud} ) (‰)</td>
<td>3,5</td>
<td>10,0</td>
</tr>
<tr>
<td>( E ) (GPa)</td>
<td>33,0</td>
<td>200,0</td>
</tr>
<tr>
<td>Cost</td>
<td>100,0</td>
<td>0,9</td>
</tr>
<tr>
<td>(€/m(^3))</td>
<td>(€/Kg)</td>
<td>(€/Kg)</td>
</tr>
</tbody>
</table>

Table 1 - Main characteristics of the materials prescribed in this dissertation

\(^1\) The value of the flexural tension resistance of the concrete was calculated in each case according to article 3.1.8(1) of the EC2 – Part 1.1 by the formula: \( f_{ctm,fi} = \max \{ (1,6 - h/1000) f_{ctm}; f_{ctm} \} \)
## 3. Design

### 3.1. Post-tensioning

Due to the fact that slabs are elements with a very reduced thickness, a system of bonded flat ducts was adopted in order to maximize the tendons eccentricities. The geometry of these ducts is presented in table 3.

![Diagram of steel ducts](image)

<table>
<thead>
<tr>
<th>Steel Duct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
</tr>
<tr>
<td>h</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>b</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>s</td>
</tr>
</tbody>
</table>

(dimensions in cm)

![Table 3 - Geometry of the tendons](image)

Table 2 - Classes of the tested lightweight concrete

<table>
<thead>
<tr>
<th>LC30/33</th>
<th>1,6</th>
<th>1,8</th>
<th>2,0</th>
</tr>
</thead>
<tbody>
<tr>
<td>η₁</td>
<td>0,84</td>
<td>0,89</td>
<td>0,95</td>
</tr>
<tr>
<td>ηₑ</td>
<td>0,53</td>
<td>0,67</td>
<td>0,83</td>
</tr>
<tr>
<td>φ</td>
<td>1,32</td>
<td>1,67</td>
<td>2,07</td>
</tr>
<tr>
<td>ν</td>
<td></td>
<td></td>
<td>0,2</td>
</tr>
<tr>
<td>γ (kN/m³)</td>
<td>17,5</td>
<td>19,5</td>
<td>21,5</td>
</tr>
<tr>
<td>fₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑ euler</td>
<td>1,32</td>
<td>1,67</td>
<td>2,07</td>
</tr>
</tbody>
</table>
In order to take full advantage of the beneficial effects of post-tensioning (mainly maximum eccentricity, where the bending moments are higher), the following tendon layout was adopted in each case:

![Figure 3 - Tendon layout adopted](image)

The immediate and time dependent losses are summarized in the following table:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Immediate Losses</strong></td>
<td>15 %</td>
</tr>
<tr>
<td><strong>Time dependent Losses</strong></td>
<td>15 %</td>
</tr>
<tr>
<td><strong>Total Losses</strong></td>
<td>≈28 %</td>
</tr>
</tbody>
</table>

Table 4 - Prestressing losses

### 3.2. Serviceability Limit States

#### 3.2.1. Deflections

The criterion adopted in this thesis regarding maximum deflections in slabs in serviceability conditions is to limit them to \(\text{span}/500\) for the quasi-permanent loads. The deflections were calculated in the structural analysis software SAP2000™.

#### 3.2.2. Cracking

Since cracking in slabs leads to a substantial increase in their deformability (caused by the reduction of their rigidity), as well as a decrease in functionality and appearance, the criterion adopted was that **no slab would be cracked due to quasi-permanent loads** (except very locally in the region of the supports). This was accomplished by either increasing the prestress in tendons or the thickness of the slab. Additionally, in order to guarantee cracking control where tension is to be expected, minimum reinforcing steel was calculated in each slab according to the formula:

\[
A_{s,\text{min}}\sigma_s = k_c k_{ct,\text{eff}} A_{ct}
\]  

(1)
3.2.3. Vibrations

Due to the high levels of slenderness of the slabs in analysis, most of them with long spans and reduced thicknesses, it is important to make reference to the problem of vertical vibrations which may impair the slabs functionality. However, the task of accurately calculating the maximum vertical acceleration that a slab is expected to be subject to, is still a complex one, mainly because there is not yet a consensual definition of the appropriate load case, with which to assess those accelerations for buildings in general. That being said, the vulnerability of a slab regarding vertical accelerations can be roughly evaluated through the frequency associated to that vibration mode, as suggested in [2]. Because this thesis involves a general assessment of several slabs in different conditions, it is difficult to estimate what the minimum frequency of such a movement should be. Nevertheless, it is very likely that frequencies as low as 2 to 3 Hz can cause excessive vertical accelerations.

The range of frequencies of the slabs studied in this thesis is presented in the following graphs. Note that the graphs depict the variance with the span of the slab (x axis) as well as with the live load.
Graph 1 - Frequencies associated with the vertical movement of the different types of slabs

3.3. Ultimate Limit States

3.3.1. Bending

Figure 4 illustrates the resistance mechanism of a slab when subject to bending and compression, as well as the equations from which the area of ordinary reinforcement is calculated from.
\[ F_c = 0.85 \times f_{cd} \times 0.8x \times b \]  
\[ \Delta F_p = A_p \times (f_{pyd} - \sigma_p) \]  
\[ F_s = A_s \times f_{yd} \]  

Note:

| Bonded: | \( \sigma_p = \frac{P_{co}}{A_p} \) |
| Unbonded: | \( \sigma_p = f_{pyd} \) |

Equilibrium of moments:
\[ \sum m_{As} = 0 \iff F_c \times b_c - \Delta F_p \times b_p = m_{std} \implies x \]  
Equilibrium of forces:
\[ \sum F = 0 \iff F_c - \Delta F_p - F_s = P_{co} \implies A_s \]  

Figure 4 - Resistance mechanism of a slab subject to bending and compression

Prior to the calculation of \( A_s \), it is necessary to guarantee that the tension in the reinforcement and prestressing steel reach \( f_{yd} \) and \( f_{pyd} \), respectively.

3.3.2. Punching Shear

The punching shear is a main concern in what regards to the design of flat slabs, particularly the ones with such long spans. However, this issue is not approached in this paper, since this verification would only lead to adopting drop panels in the slab, or local punching shear reinforcement, a small concern once the main goal of this thesis is to make a comparative analysis of the different solutions mentioned in the introductory chapter.

4. Comparative analyses

4.1. Balanced Spans vs. Unbalanced Spans

As the following graphs show, unbalanced spans require in average higher thicknesses than balanced spans. This is mainly caused by the fact that unbalanced spans are more flexible, thus requiring more prestress in the exterior panels than in the interior ones. However, this additional prestress would generate high compression stresses in the concrete, leading to poor serviceability behavior. In order to decrease those stresses to tolerable values, an increase in the thickness of the slab is required. As
depicted in graph 2, there is an increase of about 9% in the average thickness of the unbalanced span slab when compared to the balanced one.

Because of the fact that increasing the thickness of the slab is more efficient than solely increasing the quantity of prestressing steel, there is not a significant difference between the quantity of prestress in balanced or unbalanced span slabs. In fact, for high live loads, a balanced span slab requires more prestress than an unbalanced one.

As shown in graph 4, the difference in cost between a balanced and an unbalanced slab is in average 11%. Bear in mind however that this difference is closely related to the relative area of the balanced and unbalanced spans in the total area. Consequently, the bigger the total area, the lesser the difference in cost between those two types of slabs. This way, in a real commercial or industrial structure where the total area of slab is expected to be higher than the one studied in this paper, the difference between these types of slabs is likely to be less than 11%.
4.2. Normal Concrete vs. Lightweight Concrete

A slab composed of lightweight concrete requires in general lower band thicknesses, as shown in the graphs below. However, in order to maintain an adequate serviceability behavior (low deformation and cracking), the thickness of the slab between bands needs to be greater than the one in normal concrete, especially for heavy live loads, resulting in higher mean thicknesses of the slabs composed of LWAC.

The main benefit when adopting LWAC is shown in the following graphs:
These graphs show that when adopting LWAC there is a significant decrease in the quantity of prestressing steel and, more important, in the reinforcement steel.

All things considered, as graph 6 shows, in spite of the reduction in the quantities of steel, the high price of LWAC makes these solutions up to 20% more expensive than those in normal concrete.

Further analyses of how the price of LWAC influences the total cost of the structure showed that for the LWAC solutions to be as expensive as the normal concrete ones, their prices had to be reduced as the following table shows:

<table>
<thead>
<tr>
<th>Class</th>
<th>Actual</th>
<th>Break-Even</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>180</td>
<td>122</td>
<td>↓ 32%</td>
</tr>
<tr>
<td>1.8</td>
<td>166</td>
<td>118</td>
<td>↓ 29%</td>
</tr>
<tr>
<td>2.0</td>
<td>131</td>
<td>111</td>
<td>↓ 15%</td>
</tr>
</tbody>
</table>

Table 5 - Cost of the different classes of LWAC (LC30/33) to match the cost of normal concrete slabs

The cost reduction is significant in every class, especially for classes 1.6 and 1.8.

Finally, if the LWAC prices were the same as the ones for normal concrete, the economy of this solution would not surpass 7% of the total cost for normal concrete. This clearly shows that even if the price of LWAC were not an issue, its adoption in order to obtain more economical solutions is not significant.
4.3. Post-tensioned Concrete vs. Reinforced Concrete

It is widely known that for spans as big as the ones analyzed in this paper, reinforced concrete solutions are never economically competitive when compared to post-tensioned concrete. The reasons for such a fact are clearly demonstrated in the following graphs, starting with the overall thickness of the slab:

![Slab thickness at the support](image1)

![Average thickness of the slab](image2)

Table 6 - Thicknesses of reinforced vs. prestressed concrete slabs

The thickness of slabs in reinforced concrete (RC) is always higher than the one in prestressed concrete (PC), especially for higher live loads. In average, the thickness of RC slabs can be as much as 50% higher than PC slabs.

While the increase in concrete is significant, the main difference between RC and PC lies on the quantity of reinforced concrete. Even though RC solutions don’t have the cost of prestress, its absence leads to a whopping increase of 160% in the quantity of ordinary reinforcement.

![Ordinary Reinforcement Index](image3)

Table 7 - Reinforcement steel of reinforced vs. prestressed concrete slabs
All the above considerations lead to the conclusion that a reinforced concrete balanced 16m long span slab is in average 52% more expensive than one in prestressed concrete. This clearly shows that there is no match for prestressed concrete for this range of spans.

Table 8 - Average cost of reinforced vs. prestressed concrete slabs
5. Conclusions and further developments

The main conclusions of this paper are summarized as follows:

- The total cost of either unbalanced or balanced span slabs is not directly proportional to the span. In fact, a span increase of 100% (10m to 20m) leads to an increase in the total cost of about 85% only.

- An unbalanced span slab is in average 11% more expensive than a balanced one. This increase is mainly due to the higher quantity of ordinary reinforcement that the first type of slabs requires, due to the fact that thicker bands demand more minimum reinforcement.

- The current high prices of lightweight concrete in Portugal discards any economical benefit by adopting such a material. Unless its price reduces by about 30%, LWAC slabs will still remain a more expensive solution, when compared to normal concrete. Moreover, a further analysis has led to the conclusion that even if LWAC had the same price as normal concrete, the maximum reduction in cost would be about 7% (for LWAC of class 1.8), which does not inspire a great deal of potential reduction in the global cost of the solution. This material should only continue to serve its original purpose: weight reduction.

- For a slab with balanced 16m long spans, a solution in reinforced concrete will be about 40% more expensive that one with prestressed concrete, apart from the fact that it will be a solution of poorer quality. This price increase is due to the fact that the absence of prestress leads to a rise of about 161% in the cost of ordinary reinforcement. Moreover, a reinforced concrete solution requires about 50% more concrete, due to the size of the drop panels that must be adopted in a slab this long. It is also important to mention the poorer quality of such a solution: on the one hand, the maximum deflection permitted had to be restricted to span/250, since enforcing span/500 would lead to economically unfeasible solutions (as an example, requiring a total deflection of span/500 in a balanced 16m long span with a live load of 3 kN/m² would imply a thickness of 1,10m over the columns and 0,40m at mid span!); on the other hand, and for the same reasons, it is almost impossible to guarantee that the slab will not crack over the quasi-permanent loads. This cracking leads to a significant increase of the time dependent deformation, as well as making the slab more vulnerable to weather conditions.
Future developments of this paper might include:

- In-depth analysis of more cases of balanced span slabs;
- Analyze more classes of LWAC;
- Extend the study of reinforced concrete slabs to try to find the span for which both reinforced and prestressed concrete solutions cost the same;
- Compare the results of this paper to composite concrete and steel solutions;
- Analyze the design difference between adopting bonded or unbonded prestressing steel

6. Acknowledgements

I want to express once again my deepest gratitude to Professor João Almeida for his kindness, support and permanent availability, which greatly helped me to complete this thesis. He has played a big part in motivating me in the start of my professional career and I see him as a role model. I was very fortunate to have him as my tutor.

I would also like to thank Dr. António Cabrita for his prompt and eager availability to review my English grammar and vocabulary. A person with his kindness is very rare.

7. References


