ABSTRACT

The work described in the present thesis is a short review of the most common wave energy converter operating nowadays all around the world. Furthermore, a discussion of the hydrodynamic and dynamic performance of one device targeted to harness ocean wave energy by rolling motion is presented. To perform the study, the coastal zone of Leixões, Portugal, is chosen; and main hull characteristics are modified aiming to match the fundamental period of the operational zone. Frequency domain transfer functions are obtained, viscous roll damping corrections are introduced, and the power take off influence is also taken into account. Moreover, analysis based on irregular seas states is made aiming to estimate the responses in a real sea state. The results show that the designed geometrical shapes guarantees amplitudes in roll higher than 3º for the most common zero crossing period and the observed significant wave height. Further, the analysis of the mooring system reveals that less heavy materials for the mooring line reduces the vertical loads on the fairlead and the anchor point. In addition, subsurface floaters may be added to the mooring system in order to further reduce the load, especially the vertical load at the fairlead.

NOMENCLATURE

- $D$: Moulded depth
- $B$: Moulded breadth
- $T$: Draught
- $L_{pp}$: Length between perpendiculars
- $\Delta$: Displacement
- $VCG$: Vertical centre of gravity
- $Cb$: Block coefficient
- $\beta$: Heading angle
- $GMt$: Transversal metacentric height
- $A_{ij}$: Added mass coefficients
- $B_{ij}$: Damping coefficients
- $C_{ij}$: Hydrostatic restoring coefficients
- $F_k$: Complex exciting forces
- $\rho$: Water density

1. INTRODUCTION

Renewable energy such as: solar and wind energy have been extensively studied during last years. However, one energy source which has remained relatively untapped to date is ocean wave energy. Ocean wave energy has several advantages over other forms of renewable energy since waves are: more constant and predictable, and with higher energy densities enabling devices to extract more power from smaller volumes at lower costs, thus reducing visual impact [1]. WECs have to fulfill requirements as: survivability, serviceability and practical installation. Hence, WECs can be designed to survive 1 to 50 year storms or to move to the ‘fail safe’ avoiding extreme loading. Other requirements are: (a) good power capture over the range of most commonly incident wave frequencies, considering the ‘phase or complex conjugate control’ [2], and (b) ensure harmony between device and PTO to achieve maximum efficiency.

Most of WECs harness ocean wave energy while simultaneously generating waves, thus, their hydrodynamic problem is a combination of diffraction and radiation problems [2]. Then, WAMIT and other approaches are used to study WECs. WAMIT, was employed by [3] and [4] to study WECs operating isolated and in array. Moreover, devices as the Wave dragon [5] and the WavePlane convert energy by capturing the water volume of overtopped waves [6]. They are studied by a time-dependent mild-slope equation and the Boussinesq model. A mild-slope equation was used by [7] to analyze the Wave Dragon.

1.1 OVERVIEW OF OCEAN WAVE ENERGY DEVICES

Ocean wave energy has been organized in different classifications and they are: according to distance from shore and relative to technological type.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Sub-classification</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from shore</td>
<td>Near shore</td>
<td>WaveRider, Oyster</td>
</tr>
<tr>
<td>Technologies type</td>
<td>Oscillating water column</td>
<td>Aquaside, FlexPower</td>
</tr>
<tr>
<td></td>
<td>Oscillating wave surge converter</td>
<td>WaveRider, Oyster</td>
</tr>
<tr>
<td></td>
<td>Foil booster</td>
<td>PowerBuoy, PatoSeagot</td>
</tr>
<tr>
<td></td>
<td>Pressure differential</td>
<td>CETO</td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Classification according to distance from shoreline are: shoreline, near shore and offshore WECs. On the other hand according to technological type devices can be divided as: attenuators, overtopping, oscillating water column, oscillating wave surge converters, point absorbers and pressure differential. Table 1 presents a classification of the most common devices.

2. THEORY

The response of a vessel in a seaway is a complicated phenomenon involving the interactions between the vessel dynamics and several distinct hydrodynamic forces. All responses are nonlinear to some extent but in many cases when nonlinearities are small a linear theory will yield good predictions. In the case of WEC, ignoring interactions between mooring systems and power cables, the floating device during operations can be assumed as free-floating bodies. Therefore, rigid body oscillatory motions are solved in the frequency domain to obtain the six degrees of freedom. This is calculated by solving a set of coupled linear differential equations, which represent the equilibrium:

$$\sum_{j=1}^{6} \left( M_{kj} \eta_j + A_{kj} \eta_j + B_{kj} \dot{\eta}_j + C_{kj} \eta_j \right) = F_k e^{i\omega t},$$

where $k, j = 1, \ldots, 6$

These excitation forces $F_k$ and ship motions $\eta_j$ can be conveniently represented on a right handed Cartesian coordinate system, $X = (x, y, z)$, fixed with respect to the mean position of the vessel and origin in the plane of the undisturbed free surface. As shown in Figure 1, the translatory displacements in the $x, y, z$ directions are respectively surge $\eta_1$, sway $\eta_2$, and heave $\eta_3$, while the rotational displacements about the same axis are respectively roll $\eta_4$, pitch $\eta_5$, and yaw $\eta_6$. Subscripts indicate forces in the $k$-direction due to motions in the $j$-mode. $M_{kj}$ are the components of the mass matrix for the ship, $A_{kj}$ and $B_{kj}$ are the added mass and damping coefficients, $C_{kj}$ are the hydrostatic restoring coefficients, and $F_k$ are the complex amplitudes of the exciting forces.

![Figure 1: Coordinate system and six modes of motion, and definition of the vessel heading angle.](image)

This investigation uses WAMIT software to calculate the potential flow hydrodynamic coefficients and harmonic wave exciting forces. The equations of motion are solved in the frequency domain to obtain response amplitude operator (RAO) for the floating structure.

WAMIT is based on the linear and second-order potential theory for analyzing floating or submerged bodies, in the presence of ocean waves. The panel method is used to solve for the velocity potential and fluid pressure on the submerged surfaces of the bodies. WAMIT evaluates separate solutions for the diffraction and the radiation problems for each of the prescribed modes of motion. Then the relevant hydrodynamic parameters including added-mass and damping coefficients, exciting forces, RAOs, the pressure and fluid velocity, and the mean drift forces and moments. More details can be found in WAMIT user manual [8], while the theoretical formulation for derived responses can be found in [9].

3. A WEC BASED ON ENERGY CAPTURE BY ROLL MOTION

The study of the hydrodynamics and dynamics for a floating WEC harnessing ocean wave energy by rolling motion is presented. The WEC is assumed to operate in the coastal zone of Leixões, Portugal. To maximize energy capture hullforms have to accomplish:

- The resonance period should meet the most common period in the Portuguese coastal area of Leixões, which is around 5-9 [s] (74.33 % of the registered waves). For the purpose of the study a period of $T \approx 7.5$ [s] has been chosen in order to aid to a better understanding of the main hull parameters.
- The C.G. should be placed to the centre of the waterplane determined by the waterline (hereafter waterplane), this because major space for the PTO and major facility of installation on board. Moreover, this will simplify the moments applied to the PTO since it is a rotational converter and not translational.
- The need of a small metacentric height (Gmt), in order to have less restoring moments, contributing to the incensement of the roll angle at periods even different than the resonance period.
- In addition, it is also important to consider the heave performance in order to avoid higher acceleration in the vertical direction, this considering the device when will be tow to the operational area.

3.1 Influence of the Hullform in Roll Motion

Roll motion is related to hull parameters by: (a) the waterplane area, (b) hullform, (c) the vertical centre buoyancy and (d) the vertical centre of gravity. However, the analysis is only carried out for items a, b, and c, while the centre of gravity is hold at the water plane area.
To match the resonance period it can also be done by adding a mass sprig system well-known as latching control. However, the present work pretends to find the enhancement hullform to further work apply control. To understand the behavior of the hullform in roll motion, different geometries were analyzed.

From the analysis, it is observed that the elliptical shape has smaller percentage of the added moment of inertia followed by the triangular and the rectangular shape (see Table 2). The advantage of this smaller added moment of inertia can be seen when comparison between the elliptical and the triangular geometry are made. In this case, the smaller amount of added mass experienced by the elliptical shape explains why the elliptical design has smaller resonance period even when both have approximately the same GMt. When comparing to the rectangular design the advantage of the elliptical geometry is more evident, since the rectangular shape needs smaller GMt values and larger displacements to match the same resonance period and this can only be achievable by moving the centre of gravity or increasing the displacement which is impractical ($C_r = 0.96$). Therefore, the future discussion is focused on the triangular and the elliptical shape.

In Fig. 3, roll motions are plotted for the elliptical and the triangular design. It can be observed that resonance roll period for the triangular shape is 7.75 [s] with amplitude of 5.7° [deg/m] and the elliptical shape experience a resonance period of 7.5 [s] with roll amplitude of 12.5° [deg/m]. It also seen that the roll amplitudes for the elliptical is bigger than experience for the triangular geometry for a wave period ranging from 4 [s] till 11 [s] for quartering bow and beam seas. This variation can be understood from the exciting moment applied to the floating structure. This is shown in Figure 2.

Table 2: Main hull parameters and hydrodynamic coefficients for the triangular, elliptical and rectangular geometries

| Shape     | L [m] | B [m] | T [m] | G [Tons] | LE | BT | Tb | Ibx | Iyy | Izz | EMt | GMT | GMt | Roll | T [s] | EM | Add |
|-----------|-------|-------|-------|----------|-----|-----|----|-----|-----|-----|-----|-----|------|-----|-----|-----|
| Triangular | 12.00 | 6.00  | 2.00  | 43.68    | 2.00| 3.0 | 0.30| 50.1| 0.81| 0.00| 2.40| 4.80| 4.80| 0.96| 1.24 | 0.44| 5.7 | 7.75 | 4.32 | 12.18% |
| Elliptic  | 12.00 | 6.00  | 2.00  | 52.29    | 2.00| 3.0 | 0.64| 127.2| 0.83| 0.00| 2.40| 4.80| 4.80| 0.58| 1.38 | 0.45| 12.5| 7.50 | 197.28 | 7.91% |
| Rectangular | 12.00 | 6.00 | 2.00 | 157.95 | 2.00| 3.0 | 0.96 | 219.0| 0.88| 0.00| 2.40| 4.80| 4.80 | 0.26 | 1.56 | 0.59 | 19.7 | 7.00 | 245.84 | 20.00% |

Then to maximize energy production and match the resonance period of Leixões, the floating devices should math the following requirements:

- A relative cross sectional area closer to the waterline in order to increase the exciting moment.
- A small second moment of area and therefore a small hull displacement in order to keep the same resonance period with smaller GMt values, which will guaranty bigger roll motions for wave frequencies different than the resonance.
- The elliptical shape seems to be a best choice since the roll motion is bigger compared to the triangular shape. The small percentage of added moment of inertia permits to match the resonance period at smaller GMt values, and the bigger cross sectional area, closer to the waterplane, increases the exciting moment. Its slenderness also important since it decreases heave motion avoiding vertical movements and therefore improving the operability of the device (see Fig. 3).

**Figure 2:** Exciting moment on the vessel hull

**Figure 3:** Heave and roll motion for the triangular and the elliptical geometries in following, beam and bow quartering waves, respectively.
3.2. Hullform Analysis Including the PTO System

Considering what has been stated before, the next step is to modify the hull shape enhancing its performance for roll motion. In this section, modelling of the PTO and its influences in roll motion is considered for a floating structure carrying multiple and a single PTO.

Many WECs have been proposed and a remarkable example is the work of [3] where a device harnessing wave energy by pitch motion, is presented. This motivates to study a device harnessing ocean energy by roll motion since it experiences bigger amplitudes than the pitch motion.

To accomplish this work, the physical model of an apparatus for gyroscope propulsion is considered (see US patent No US 6,705,174 B2). Thus according to its physical model; the PTO can be addressed similar to the mechanical principle of the well know gyroscopic stabilizer. Which according to [10] this type of establishers are very effective and can reduce around to 60% to 80% of the roll motion but were left for reduction of roll motion due to large spare requirements.

The moment, $M_z$, comes from the conservation of angular moment and is given by:

$$M_z = I_z \omega \theta$$

Where $I_z$ is the mass moment of inertia of the spinning element about the axis of spin, $\omega$ is the angular velocity of spin, and $\theta$ the velocity of rolling motion.

Figure 4 plots the RAO for roll motion for a given hull geometry with and without the influence of the PTO. It can be observed that a significant reduction of the roll amplitude when applying the PTO.

![Figure 4: Roll motion in bow quartering and beam waves, with the PTO, considering $I_z \omega = 2.0$ k and 30k $\text{Nms}/\text{rad}$](image)

Table 3: Main hull parameters and hydrodynamic coefficients of OPTMAL01 geometry

<table>
<thead>
<tr>
<th>Shape</th>
<th>$L$ [m]</th>
<th>$B$ [m]</th>
<th>$T$ [m]</th>
<th>$\Delta$ [tons]</th>
<th>$I_{G}$</th>
<th>$E_{G}$</th>
<th>$c_{b}$</th>
<th>$I_{xx}$</th>
<th>$V_{cg}$</th>
<th>$I_{yy}$</th>
<th>$I_{zz}$</th>
<th>$I_{xx}$</th>
<th>$B_{xx}$</th>
<th>$B_{yy}$</th>
<th>$B_{zz}$</th>
<th>$B_{x}$</th>
<th>$B_{y}$</th>
<th>$B_{z}$</th>
<th>$E_{zz}$</th>
<th>$E_{yy}$</th>
<th>$E_{xx}$</th>
<th>$E_{xy}$</th>
<th>$E_{xz}$</th>
<th>$E_{yz}$</th>
<th>$E_{xyz}$</th>
<th>$E_{xxxx}$</th>
<th>$E_{yyyy}$</th>
<th>$E_{zzzz}$</th>
</tr>
</thead>
</table>
| OPTMAL01 | 12.00 | 4.50 | 1.80 | 52.13 | 2.62 | 2.5 | 0.54 | 54.6 | -0.72 | 0.00 | 1.80 | 4.00 | 4.80 | 1.05 | 3.33 | 16.0 | 11.6 | 6.75 | 103.29 | 3.18%

Figure 5: Heave and roll motion for OPTMAL01 geometry in following, beam and bow quartering waves, respectively, and a profile view of elliptical geometry
3.2.1. Hullform Analysis for Three Spheres

The hullform designed here is namely OPTMAL01. This is designed with a waterplane area as a result of modification of the elliptical shape (less breadth at the fore and aft) with length of 12[m], beam 4.5[m] and draught 1.8[m]. Main hull parameters and the hydrodynamic coefficients are presented in Table 3, while a perspective view is show in Fig. 5.

Moreover, a rounded shapes (with nearly elliptical water plane areas) has been chose for the design. This since the contribution of viscous damping and other damping sources to the total roll damping are less, therefore their effects are less influents in rolling motion and consequently also increases roll amplitudes. Other advantage of the rounded shape is the smaller values of the block coefficient adding to match the required resonance period at smaller GMt.

OPTMAl01 has been analyzed with WAMIT considering an external damping of 30k [N-m-s] for the three spherical PTOs. Viscous damping factor of 0.07 has been taken from the experimental work of [11]. RAOs are presented in Fig. 5 for heave, pitch and roll motion.

Table 3 presents the hydrodynamic coefficients for the proposed design. It is seen that it experiences smaller amount of added moment of inertia, and this is even less than the obtained for the elliptical geometry (see Table 3 and 2). The reason of this hydrodynamic behaviour is mainly due to the rounded shape of OPTMAL01.

From Fig. 5, it can be seen that the RAO for heave motions is less than 1 for all wave periods. Hence, the device will not experience higher vertical motion. This is convenient when the device will be towed and since it also has to be moored, therefore, simplifying the stationkeeping requirements and design for the WEC.

In Figure 5 also plots for the RAO for the pitch motion is presented. This shows that the device experiences bigger pitch amplitude which could limit the operation of the device, however this does not represent bigger problems since the resonance for pitch is around 4 [s] which is normally associated to small waves amplitudes.

In the case of roll the proposed geometry presents a broader shape for bow quartering and beam waves. This can be explained due to its relative smaller GMt achieved for this geometry which a result of the smaller $I_{xx}$ and the smaller vessel displacement, consequently decreasing the GMt. However, any change of GMt vary the resonance period, but it does not happen for OPTMAL01 due to the rounded cross sections which has small added moment of inertial permitting to hold the resonance period at even smaller values of GMt (see Table 3).

| Shape | $L$ [m] | $B$ [m] | $T$ [m] | $D$ [m] | $L/B$ | $B/T$ | $cb$ | $V_{cb}$ | $V_{0.05}$ | $V_{0.06}$ | $V_{0.07}$ | $V_{0.08}$ | $V_{0.09}$ | $V_{0.1}$ | $V_{0.15}$ | $V_{0.2}$ | $V_{0.25}$ |
|-------|---------|---------|---------|---------|-------|-------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| UFO02 | 6.00    | 5.60    | 1.80    | 15.24   | 1.87  | 2.9   | 0.42  | 13.7    | 0.63    | 0.63    | 0.63    | 0.63    | 0.63    | 0.63    | 0.63    | 0.63    | 0.63    |
| sec. sphere D4 | 3.90 | 5.90 | 1.80 | 11.22 | 1.80 | 2.2 | 0.50 | 12.3 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| sec. sphere D4 | 3.90 | 5.90 | 1.80 | 11.22 | 1.80 | 2.2 | 0.50 | 12.3 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |

Table 4: Main hull and hydrodynamic parameters UFO02 and the sphere

[Figure 6: Heave and roll motion for UFO02 and the sphere geometries in following, beam and bow quartering waves, respectively, and a profile view of geometries.]
3.2.2. Hullform Analysis for One Sphere

Configuration and the hull shape for a floating device designed to carry only one PTO is analyzed. Table 4 presents the main hull parameters and the evaluated roll resonance period. The present discussion is made for proposed geometries namely UFO02 and spherical. Both geometries are designed to have the resonance period lying in the bandwidth of 5-9 [s].

To analysis the design UFO02 and Sphere, their hull form has been analyzed with WAMIT considering damping of 10k [N-m-s] due to the spherical PTOs system has been included and a viscous damping factor of 0.009 has been chosen. The viscous factor used here, has been toke from the experimental work of [11]. Figure 6 shows the two shapes to be compared. It also is important to notice that the spherical shapes namely here is not the whole semi-spherical, instead is a part of it, this is the reason of why the waterplane area has a diameter of 3.98[m] and not 4 [m] which is the diameter of the spherical geometric (see Table 4).

From Figure 6, it can be seen that for UFO02 the RAOS for heave motions is less than 1 for all wave periods which is useful since the device is suppose to be moored, thus avoiding higher loading on the mooring system . In the case of the sphere, it experiences amplification at its resonance period which can result in bigger loading at wave frequency for the mooring system.

Figure 6 also plots the RAO for pitch motion for both designs. Amplitudes developed by the UFO02 design are bigger than the spherical form and also their respectively resonance period are different. However, pitch amplitudes presented by UFO02 are subjected to smaller wave periods and consequently smaller wave amplitudes than the experienced by the spherical design.

Roll motion are plotted in Fig. 5 and a broader shape is presented for both designs which are approximately the same for bow quartering and beam waves. The similarity of in roll can be understood when looking to Table 4 where the resonance period, the added mass and the potential damping are found similar for both. Finally from the analysis, of these designs to carry one PTO, it seems that the UFO02 shape is better than the spherical shape.

3.2.3. Motions Responses in Irregular Seas

The present analysis estimates the responses of the floating device in a given irregular sea. This analysis is made considering the same wave spectrum approaching to the device from incident wave angles form 180º to 0 º. However, the discussion is only restricted to incident angles form 120º to 30º, which represent the area of operability of the device. This since it is designed to operate near shore. Vertical acceleration and other longitudinal responses are not presented due to small responses experienced in quartering and beam seas.

Table 5 shows the chosen couple wave period and height for the analysis. The chosen values represent the most probable of occurrences for area 16, they were retrieved from the global wave statistics. Hereafter denominations of sea state refer to table 5.

Table 5: Sea states most probable of occurrence in the area 16, Portugal.

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Sea state 2</th>
<th>Sea state 3</th>
<th>Sea state 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>T (s)</td>
<td>H (m)</td>
<td>T (s)</td>
</tr>
<tr>
<td>1</td>
<td>0.2, 7.5</td>
<td>0.6, 7.5</td>
<td>1.6, 8.2</td>
</tr>
<tr>
<td>2</td>
<td>0.1, 7.5</td>
<td>0.9, 7.5</td>
<td>1.7, 8.5</td>
</tr>
<tr>
<td>3</td>
<td>0.2, 7.5</td>
<td>0.8, 7.5</td>
<td>1.8, 8.5</td>
</tr>
<tr>
<td>4</td>
<td>0.2, 7.5</td>
<td>0.9, 7.5</td>
<td>1.9, 8.5</td>
</tr>
<tr>
<td>5</td>
<td>0.3, 7.5</td>
<td>1.0, 7.5</td>
<td>2.0, 8.5</td>
</tr>
<tr>
<td>6</td>
<td>0.3, 7.5</td>
<td>1.1, 7.5</td>
<td>2.1, 8.5</td>
</tr>
<tr>
<td>7</td>
<td>0.4, 7.5</td>
<td>1.2, 7.5</td>
<td>2.2, 8.5</td>
</tr>
<tr>
<td>8</td>
<td>0.4, 7.5</td>
<td>1.3, 7.5</td>
<td>2.3, 8.5</td>
</tr>
<tr>
<td>9</td>
<td>0.5, 7.5</td>
<td>1.4, 7.5</td>
<td>2.4, 8.5</td>
</tr>
<tr>
<td>10</td>
<td>0.5, 7.5</td>
<td>1.5, 7.5</td>
<td>2.5, 8.5</td>
</tr>
</tbody>
</table>

Figures 7 present the results of the roll angle and the lateral acceleration for the OTPMAL01, UFO02 and the sphere geometries. In the polar plots the radio represents the sea state (according to Table 5), while angles describe the angle of incidence of the waves (see Fig. 1). As an example the response at sea state 1 is plotted in the polar diagram by the radii 0 to 1, which is related to significant wave heights $H_z = 0.1 – 0.5$ and zero crossing wave period of $T_z = 7.5$ respectively.

The result shown that OPTMAL01 and UFO02 experiences approximately the same responses while sphere presents a slightly improvement. It can be seen as well that devices operate between 3º and 5º for sea states 3 [$H_z = 1.6 – 2.5; T_z = 7.5$], between 8º and 12º for sea state 4 [$H_z = 2.6 – 3.5; T_z = 8.5$], which are the most probable of occurrence for Area 16. The importance of this analysis relays on how much the device will roll while operating in an irregular seas, this since the device is target to generate energy by the roll motion. Thus, from the analysis made for the three geometries presented, it is highly probable that proposed hullforms including the effect of the PTO will roll at amplitudes higher than 3º at the design operational area.

Figure 7 also plots lateral acceleration felt onboard (at the forest part of the hull device). Devices experiences different responses. The spherical shape presents higher values for this criterion, followed by OPTMAL01 and then by UFO02. However, the acceleration for all shapes does not represent any hazard for devices, since the value of the lateral acceleration is relative small, half of the acceleration gravity (0.5g).

The small variation in the GMt and therefore the similarity in the resonance period, permits for all devices to experiences similar responses in irregular seas. Therefore, no significant differences can be seen in the
performance of these hull shapes. However, the lateral
acceleration can be used as a selecting criterion when we
look for the best configuration and hull shape form. Also,
the ratio cost and energy production can be used.

Figure 7: Motion in roll and lateral accelerations calculated at the bow for the geometries OPTMAL01, UFO02 and
sphere. The first three polar plots belongs to roll motion while the remaining to lateral acceleration

3.3. Mooring Line Analysis

Mooring lines are subject to environmental forces such as: wind, current, tidal and ocean waves. Wind as well
current forces might be divided into mean and fluctuation
forces, and vortex induced vibrations. While wave forces
can be divided into components as: The 1st order forces
at wave frequencies (WF); 2nd order forces and higher
order forces. 2nd order forces comprises mean wave drift
forces, forces at sum frequencies (HF) and forces at
lower frequencies (LF), while higher order forces
comprises wetted surface effects, ringing and viscous
(non-potential).

Those environmental forces are considered according to
the analysis proposed. Generally, the analysis of mooring
cables is divided in two stages: during pretension and in
service. When the cable is addressed during pretension a
static analysis can be performed neglecting the
environmental factors (such as current and wind when
the mooring cable is considered a heavy cable). While
when analysis is made in service, different approach can
be used to solve the dynamic and hydrodynamic
problem.

Mooring lines can be built of all chain, wire rope,
chain/wire combination or synthetic mooring lines.
Other components can be used as a clump weight, spring
buoy. Clump weights improve the performance and
reduce costs. They provide concentrated weight at the
point close to the seabed thus replacing a portion of chain
and increasing the restoring force of the mooring leg.
While spring buoys can be applied to reduce weight of
mooring lines that must be supported by the vessel hull,
reduce effects of line dynamics in deep water and reduce
vessel offset for a given line size an pretension

Anchor systems are available in a variety, some of the
most common are: drag embedment anchors, pile
anchors (driven, jetted, drilled and grouted), caisson
foundations (suction anchors), gravity anchors and
propeller embedment anchors

Figure 8: Coordinate systems and forces action on the cable [12]
The forces acting on the cable segments (see Fig. 8) in water are: the tension \(-\mathbf{T}\ddot{\mathbf{u}}\) and \((T + dT)(\mathbf{\dot{u}} + d\mathbf{\dot{u}})\); effective weight; \(-G_u d\mathbf{\dot{u}}\); tangential drag force; \(\mathbf{F} = F dl \mathbf{\ddot{u}}\); normal drag force; \(\mathbf{H} = H dl \mathbf{\ddot{v}}\) and the binormal drag force; \(\mathbf{Q} = Q dl \mathbf{\ddot{\omega}}\). where \(T\) is the tension along the cable, \(G_u\) is the effective weight per unit length in water, \(F\), \(H\), and \(Q\) are the tangential, normal and binormal drag forces caused by ocean currents per unit length, respectively. The equilibrium of a cable suspended in water results in the following differential equations:

\[
\frac{dT}{dl} = G_u \sin \alpha - F \quad (3)
\]

\[
\frac{d\theta}{dl} = \frac{H}{T \cos \alpha} \quad (4)
\]

\[
\frac{d\alpha}{dl} = \frac{1}{T} (G_u \cos \alpha - Q) \quad (5)
\]

The tangential, normal and binormal drag forces per unit length in water can be acquired through Morison’s equation [13]. Moreover the constitutive relation of the mooring cable is assumed to be continuous and extensible with a linear elastic stress–strain relationship given by Hooke’s Law:

\[
T = E A \varepsilon = E A \left(\frac{dl}{l_0} - 1\right) \quad (6)
\]

where \(E\) is the Young’s Modulus, \(A\) the cross-sectional area of the unstrained cable, \(\varepsilon\) is the unit elongation and \(l_0\) is the unstrained cable length. Under axial tension the unit effective weights of cable in water, \(G_u\) and the cross-sectional area \(A\), must be modified as \(G_u = G_{u0} / A_0 / (1 + \varepsilon)\) \(A = A_0 / (1 + \varepsilon)\).

### 3.3.1. Numerical Model

Herein the mooring line is analyzed during pretension to determine its influence and to select the best suitable for the WEC, this from a static point of view. Discussion of designs, by means of: one or two systems, different mooring radius and cables properties, and the use of buoys for water depth of 50 [m].

When pretension analysis is consider, a static approach is chosen to study the mooring problem. In this process, the mooring cable is discretized into \(n\) segments with the same \(dy = D / n\), where \(D\) is the anchoring radius in the horizontal direction and \(n\) is the segment division number of the mooring cable.

The end point of each segment is numbered by the index \(i\), which runs from 1 at beginning of the system till \(n+1\) at the end of the system. The coordinate of the anchor point \((x_1, z_1)\) is assumed to be \((0,0)\). Here, the following parameters should be known prior to calculation: design pretension load \(T_{x_{11}}\); water depth \(H\); anchor point; anchoring radius \(D\); cable properties, including Young’s modulus \(E\), cross-sectional area \(A_0\), unit effective weight of cable in water \(G_{u0}\), and buoyancy force and position, if buoy is applied. It can be known that the coordinate of the top point \((x_{n+1}, z_{n+1})\) is equal to \((D, H)\). This analysis considers heavy mooring cables therefore drag forces are neglect. The iterative equations are:

\[
T_i = T_{x_{i+1}} - dli G_{ui} \sin \alpha_i \quad (7)
\]

\[
\alpha_i = \alpha_{i+1} - dli \left(G_u \cos \alpha_i / T_i\right) \quad (8)
\]

\[
G_{ui} dl_i = G_{ui} dl_i \quad (9)
\]

\[
e_i = Ti (A_0 / E_i) \quad (10)
\]

The cable is discretized with the same \(dy = D / n\), so the length of strained segment is:

\[
dl_i = dy / \cos \left(\alpha_i + \alpha_{i+1} / 2\right) \quad (11)
\]

Other option to obtain the unstrained segment is from Equation. 10:

\[
dl_{i0} = dl_i / (1 + e_i) \quad (12)
\]

The geometry equations of the strained cable are:

\[
x_i = x_{i+1} - dl_i \cos \alpha_i
\]

\[
z_i = z_{i+1} - dl_i \sin \alpha_i \quad (13)
\]

With the given coordinate \((x_{n+1}, z_{n+1})\), the iterative procedure can be initiated with an assumed elevation angle at the top point \(\alpha_{n+1}\) as the \(\arctan(H / D)\). Fig. 9 shows that in each iterative step, the calculated coordinate of the anchor point \((x_1, z_1)\) should be compared with the real anchor point \((0,0)\) to see if the error \(E_i = (x_1^2 + z_1^2)^{1/2}\) is greater than \(\delta_1\), where \(\delta_1\) is a specified small quantity. If \(E_i > \delta_1\), \(\alpha_{n+1}\) needs to be actualized and the running process star again until \(E_i \leq \delta_1\). When to cables are used the analysis is run till tension at the joint present a difference less than 10%, this save larger computational time.
3.3.2. Analysis of Mooring Line Configurations

The analysis is divided in eight combinations. Table 6 presents design configuration chosen for mooring line. The studies are namely mooring 1 till mooring 8 and only mooring 2 is design with one system. The study also considers two types of material, a chain with $G_{w} = 2.86 \times 10^{6}\text{[kN/m]}$, $A_{E} = 12.997 \times 10^{6}\text{[kN]}$ and $BS = 1.5 \times 10^{4}\text{[kN]}$; and steel wire: $G_{w} = 0.814 \times 10^{6}\text{[kN/m]}$, $A_{E} = 1.15 \times 10^{6}\text{[kN]}$ and $BS = 1.6 \times 10^{4}\text{[kN]}$, where $G_{w}$ is the weight per unit length in water $A_{E}$ the stiffness and $BS$ the breaking strength. To study the influence when some parameters are vary, the design of the mooring lines are grouped, the classification can be seen in table 7.

Table 6: Mooring lines characteristics for each design.

<table>
<thead>
<tr>
<th>Item</th>
<th>$g$</th>
<th>$r$</th>
<th>$h$</th>
<th>$D$</th>
<th>$A_{E}$</th>
<th>$BS$</th>
<th>$1$</th>
<th>$2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mooring 1</td>
<td>2</td>
<td>50</td>
<td>50</td>
<td>45</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 2</td>
<td>1</td>
<td>50</td>
<td>50</td>
<td>45</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 3</td>
<td>2</td>
<td>50</td>
<td>40</td>
<td>65</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 4</td>
<td>2</td>
<td>50</td>
<td>30</td>
<td>60</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 5</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>60</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 6</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>40</td>
<td>175</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Mooring 7</td>
<td>3</td>
<td>30</td>
<td>30</td>
<td>45</td>
<td>120</td>
<td>Wire</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Mooring 8</td>
<td>3</td>
<td>30</td>
<td>20</td>
<td>45</td>
<td>120</td>
<td>Wire</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

From Table 6, it is observed that requirements of buoyancy increases as the mooring radio decreases. In group 2 (chain material for both system 2) When steel wire is used a considerable reduction of the buoyancy is found for mooring 6 and 7 in group 4, and this can be confirm when comparison between mooring 6 and 7 in group 4 is made. The results for mooring 1 and mooring 5 present that when the buoy is located and deeper positions (for mooring 5) less buoyancy is needed.

Table 7: Mooring line tensions and characteristics

<table>
<thead>
<tr>
<th>Item</th>
<th>$T_{1}$ (kN)</th>
<th>$T_{2}$ (kN)</th>
<th>$T_{3}$ (kN)</th>
<th>$T_{4}$ (kN)</th>
<th>$T_{5}$ (kN)</th>
<th>$T_{6}$ (kN)</th>
<th>$T_{7}$ (kN)</th>
<th>$T_{8}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>2</td>
<td>50</td>
<td>50</td>
<td>45</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 2</td>
<td>1</td>
<td>50</td>
<td>50</td>
<td>45</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 3</td>
<td>2</td>
<td>50</td>
<td>40</td>
<td>65</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 4</td>
<td>2</td>
<td>50</td>
<td>30</td>
<td>60</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 5</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>60</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 6</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>40</td>
<td>Chain</td>
<td>10</td>
<td>5</td>
<td>Chain</td>
</tr>
<tr>
<td>Group 7</td>
<td>3</td>
<td>30</td>
<td>30</td>
<td>45</td>
<td>Wire</td>
<td>10</td>
<td>10</td>
<td>Wire</td>
</tr>
<tr>
<td>Group 8</td>
<td>3</td>
<td>30</td>
<td>20</td>
<td>45</td>
<td>Wire</td>
<td>10</td>
<td>10</td>
<td>Wire</td>
</tr>
</tbody>
</table>

Finally form the analysis made in group 1, needs of including a buoy is pointed out this since considerable reduction of the vertical load is observed and the importance of considering materials with less weight since it decreases loads from the mooring into WECs. Less heavy materials are also needed due to the

Figure 9: Two dimensional profile view of mooring
reduction of the net buoyancy required (see Table 6). It is important to point out that there might be an extra loading on the mooring due to the buoy at motion at wave frequencies, thus the depth at which the buoy will be placed should be studied.

4. CONCLUSIONS

A large variety of WECs are found in the literature, which is classified in different form. However, the classification of WECs according to the technological type appear to be one of the most useful as it permits to compare a broad range of devices at a certain stage.

From the analysis made for a WEC harnessing ocean energy by roll motion the following were found: the resonance period and the amplitudes of roll motions are dependent of the metacentric height, added moment of inertia and the structural inertia of the floating structure. The analysis shows that waterplane with a geometrical form closer to an ellipse gives less second restoring moment and consequently maximizes rolling motion thus energy extraction.

Considering all stated above, three geometries were proposed and analyzed with WAMIT taking into account the effects of the PTO. RAOs were obtained and no significant variation was observed in roll motion. Furthermore, an analyzed in irregular seas were perform, and from results roll motion higher than 3° for most of the commonly observed significant wave heights and zero crossing wave periods were found. Moreover smaller accelerations were observed which is important for the performance of the device.

The analysis shows the importance of the main hull characteristics for a WEC aiming to harness ocean by oscillation motions. The geometrical characteristics of the floating device determine the behaviour of the structure at sea and consequently delimit the efficiency of the device in terms of energy extraction and conversion. It is also important to mention that the device must be designed for a range of periods and not only for the resonance period in order to enhance its overall performance. Hence, it is required to count with small GMT values, since it helps to decrease the restoring moment and consequently maximizes rolling motion thus energy extraction.

In addition, a set of proposed mooring designs were studied. The analysis reveals the relevance of using less heavy materials for the mooring cable and the importance of buoyancy of to reduce vertical loads at the fairlead. Moreover, significant reduction of the vertical load at the fairlead was found when varying the mooring radio.

5. REFERENCES