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Dynamic Analysis of the Masonry Arch
The Effect of the Extrados Filling

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Abstract: This paper presents an analysis on the behavior of the masonry arch concerning its stability. Initially it is addressed how the statics are comprehended, regarding the design methods to verify its stability and their effectiveness, from centuries ago to the modern day. A second part approaches Oppenheim's theory on the dynamic behavior of masonry. Oppenheim's results about the adequacy of the discrete element method to study masonry are confirmed and discussed. Furthermore, the dynamic effect of the extrados' filling is proved to be an advantage on the behavior of the structure for a semicircular arch. When studying its effects applied to an arch with an embrace of 157,5˚, no change in the behavior of the arch is acknowledged.

Key words: Masonry Arch, Filling of the Extrados, Line of Thrust, Dynamic Analysis, DEM

1. Introduction

The masonry arch and its behavior have been studied for long and the developments in understanding its behavior to static and dynamic loading have been drastically improved. However, verifying its stability so to guarantee the safety of the structure still presents obstacles. In the words of Kurrer (2008) “the arch, like the wheel, is impossible to date”. This curved structure is defined by its hiperstatic nature whose main advantage relies on the fact that under an uniform load, the majority of the tension installed is of compression. However, the collapse is possible through a plastic redistribution with the formation of three hinges, leaving the arch isostatic. The arch consists in a complex and peculiar structure, since it supports its weight and applied vertical loads creating compression load paths which arrive at the arch supports as horizontal or vertical impulses. So, altering the arch geometry means changing the values of these resulting impulses.

Masonry is one of the oldest building materials if not the oldest. Little has changed since the first days it is known to exist and, nowadays, on the age of concrete and metal, it remains as a smart and easy choice, due to the simplicity of its techniques and price. Masonry presents, however, a complex behavior because of its non-linear nature. The lower resistance of the existing joints makes it impossible to consider masonry as a material with an isotropic behavior.

Masonry is a composite material consisting of two very different components: the masonry units and a joining material (mortar). Due to its composite nature, the properties of masonry are strongly dependent upon the properties of its components. From the arrangement of the units to the characteristics of the mortar, all those factors may cause specific difficulties to the structural analysis of this kind of constructions (Sincraian, 2001).

In the masonry arch, three conditions regarding the behavior of the masonry are assumed: masonry has infinite compression strength; masonry has no tensile resistance; the sliding of the voussoirs is not considered. Even though the first two assumptions are not a fair representation of the reality, the third assumption has been confirmed through experimental results.

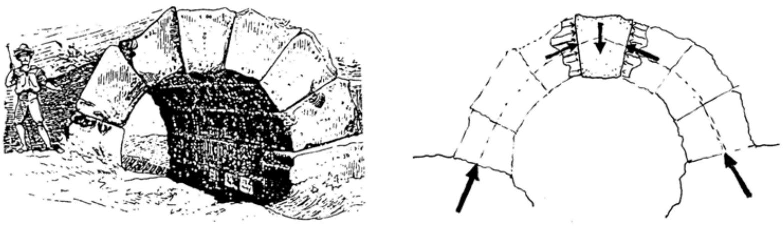


Image 1 – Etruscan arch (Huerta, 2001)

2. The Line of Thrust

Considering the works of Gago (2004), Kurrer (2008) or Nunes (2010), an exhaustive research on the masonry arch's development and on the knowledge of its behavior is presented. Gago (2004), after presenting the crucial historic findings of the arch's structural comprehension, focuses on the old methods used to execute historical buildings, some of them still present nowadays. Controlling the concepts behind these graphic and numerical techniques has improved our understanding of the masonry arch nature and how to correctly dimension it and therefore verify its stability. Furthermore, some of those methods serve as the basis for the new computational methods.

From all the important methods presented by Gago (2004), the thrust line is the one applied through all this project and exists nowadays in a simplified interactive software created by Block and Ochsendorf (2007) that allows the understanding of how the stability of the arch varies with its geometry. This parametric concept uses both the thrust line concept and the funicular polygon to draw the thrust lines to an arbitrary structure of our choice.

The locus of the centre of thrust forms a line, the "line of thrust." The path of this line depends, therefore, on the geometry of the arch, its loads and also on the family of plane joints considered. Since it was already stated that the masonry arch has no tensile strength, to respect this main property the line of thrust must be contained within the masonry arch. This concept derived from the knowledge of the behavior of the chain. The chain, on the other hand, has no compressive resistance but is able to sustain its loads forming the geometry that we all know. The masonry arch, being the opposite of the hanging chain, should present the same capability but upside down. This concept was presented by Hooke (1670) that concluded: "As hangs the flexible line, so but inverted will stand the rigid arch". So, if the arch has all its voussoirs of the same size, the line of thrust will have nearly the form of an inverted catenary (Huerta, 2001).

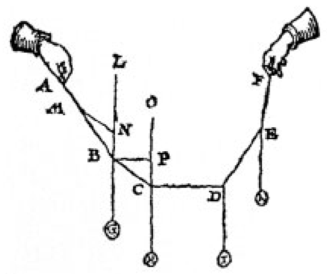


Image 2 – Catenary form. (Kurrer, 2008)

Gago (2004) and Rouxinol (2007), while presenting the structural behavior of the masonry arch, consider the effect of the filling of the extrados on their analysis. In a static analysis, it becomes clear that choosing any kind of method the filling will always increase the stability of the arch. This advantage can be proven through different effects.

Considering the filling of the extrados, and when comparing the simple arch with one that includes the filling in its analysis, the value of the minimal acceleration which starts the mechanism is smaller for the first one. The line of thrust verifies the arch’s stability when completely included in its thickness - the wider the thickness the more stable the arch. Even if the filling doesn’t present the same mechanical properties of the voussoirs, it allows the line of thrust to use it for its load path. Moreover, the filling increases the lateral resistance and stabilizes the supports of the arch by lowering the lateral impulses.

When induced an isolated concentrated load, its worst possible location for the arch’s stability seems to be around a third or fourth of the arch length. If the filling of the extrados is contemplated, the load is dispersed and the load ends up having a positive effect on the behavior of the arch (Gago, 2004).

Considering the semi circular arch (with an embrace of 180˚), Gago (2004) studies the geometry of the line of thrust bearing in mind how well it can adapt to the semicircular curve. It becomes clear that since the line of thrust derives from the catenary concept, its geometry approaches the parabolic expression and not the circular. So, it is understood that it can be difficult to draw a line of thrust within the thickness of a semicircular arch and therefore this type of arch is more instable.

3. Dynamic Analysis of Oppenheim

Once the behavior of the masonry arch in a static approach is comprehended, the next step, attending safety as the main concern nowadays, is to focus more and more on the dynamic analysis of the arch and more importantly of the masonry. A significant part of the worldwide existing construction consists on old masonry buildings made of stone or brick blocks. Because they also constitute the old urban center of numerous cities, especially in Europe (many of them are relevant architectural and cultural heritage), and because they are representative of an earlier building tradition, there is a strong need of preservation. These structures are very vulnerable to seismic events, which have often caused massive damage or, even worse, their destruction (Sincraian, 2001).

DeJong (2009), in his study of the dynamic response of the masonry arch, inquires about the difficulty of the determination of the safety of masonry structures in seismic regions. He explains with three different arguments. The first relates to the absence of modern engineering design on the old constructions that have instead resulted from empirical expertise. So, masonry assessment methods are naturally far behind those for modern steel and concrete structures.

Secondly, the long existence of many masonry structures implies several unknown variables. In most cases, geometry is difficult to determine because construction drawings do not exist and environmental factors have caused material degradation, support displacements, and damage. Thirdly, the basic nature of masonry still remains a complex problem regarding its modeling. Although the Finite Element Method (FEM) represents the most widespread structural analysis, it is a method tailored toward continuous structures, while, as mentioned, masonry is discontinuous by nature. So, the best method to model masonry is the discrete element method (DEM) presented further.

Following the work of Housner (1963) on the dynamic behavior of the block, Oppenheim (1992) studies the dynamic behavior of a masonry arch considering an arch with a center-line radius R_A=10m, a thickness t_A=1,5 m, seven voussoirs and an angle of embrace β_A=157,5. This author developed an analytical model that describes the arch as being a rigid body of four-link mechanism and predicts failure of the arch in response to a given base impulse excitation.

To first address this problem, Oppenheim seeks the equation of motion of the arch which can be derived using Hamilton's principle and Lagrange's equation:

$$\frac{\partial}{\partial\theta}\left(\frac{\partial T}{\partial\theta}\right)-\frac{\partial T}{\partial\theta}+\frac{\partial V}{\partial\theta}=Q$$

$$M(\theta)ma^3\ddot{\theta}+L\,(\theta)ma^2\dot{\theta}+F(\theta)ma^2g=P(\theta)ma^2\ddot{x}_g$$

$$4370*\ddot{\theta}+59000*(\dot{\theta})^2-867=239\ddot{x}_g$$

This expression is obtained considering the rotations of the voussoirs as dependent to each other and therefore the problem results in a single degree of freedom.

Considering that the minimum ground acceleration - necessary to transform the arch into a mechanism and hence to initiate rocking motion - may be obtained through equivalent static analysis, for this arch in particular, as=0,37*g. Ground motions which never exceed this level would cause the arch to move as a rigid body with the ground so this value can be used as a safety limit that distinguishes collapse from equilibrium. These two domains of behavior don't consider the duration and magnitude of the impulse, so DeJong (2009) adds two more possible regions in his study of the arch. DeJong (2009) admits that the arch can experience four general responses to the applied impulse. For an impulse with a large duration and magnitude, the arch fails immediately during the first half cycle of motion. This region is labeled "Mode II collapse". For smaller impulses, the arch fails during the second half cycle of motion and the resulting region is labeled "Mode I collapse". Even smaller impulses imply the arch to rock back and forth until it reaches stability. This region is labeled "Recovery". Finally, for impulses of lower magnitude, as it was mentioned, the rocking mechanism is not initiated. This region is labeled "No hinging".

Oppenheim (1992) also studied how the geometry of the arch contributes to the stability of the structure. He first acknowledged a scale effect since the arch radius is present in the differential equations of motion. So, arches with the same proportions (t_A/R_A , β_A) but different sizes will have different dynamic responses.

Also, the thickness of the arch plays a major role on the results of the arch under a dynamic analysis. As seen in a static approach, if the stability of the arch is obtained from the inclusion of the line of thrust in its thickness, the wider the thickness the more stable the arch is.

Once the arch reaches the minimum acceleration, that starts the mechanism, was obtained, Oppenheim (1992) also concluded that in the case of the semicircular arch this almost doesn't require an acceleration to start the mechanism that will eventually lead it to collapse. Considering the number of voussoirs, more voussoirs would make the shift in mechanisms less drastic, and therefore the spacing of the failure curves would be more even. That said, increasing the number of voussoirs increases the resolution of the possible initial hinge locations which are found using equivalent static analysis.

4. Discrete Element Modeling

Discrete element (DE) methods derive from a perspective where the material is viewed as an assembly of distinct bodies, the masonry units, interacting along their boundaries. The distinction between finite and discrete elements codes has become somewhat blurred by the evolution of these methods as they borrowed features from each other. For example, blocks in DE models need no longer to be assumed rigid, but may have internal FE meshes. On the other hand, FE models are being used to represent discontinuities at smaller scales, for example, using joint elements to partition bricks into small polygonal particles, to study fracturing (Lemos, 2007).

Lemos (2007) presents three different aims for the application of DE models. The first objective consists in qualitative identification of possible deformation and failure modes. A second objective relates to the usage of DE models in the interpretation of experimental test results or damage observed in the field. Finally, their use as a structural analysis tool in engineering practice is becoming more widespread.

DE models pretend essentially to represent masonry as an assembly of component blocks or particles in mechanical interaction. Admitting the shape of the elements, Lemos (2007) presents two different models of the elements: block DE models, composed of sets of polygonal or polyhedral bodies, which are the most widely used; and secondly particle DE models, based on circular or spherical particles, aimed at a representation of the materials at a finer scale, extensively studied in the work of Rouxinol (2007).

The main characteristics of the DE models consists in the following (Itasca, 2004; Rouxinol, 2007; Sincraian, 2001; and Lemos, 2007):

1. These models allow the assumption that blocks are rigid and the system deformability is concentrated in the joints, allowing, however, deformable block formulations;

2. Interaction between blocks is represented by sets of point contacts, or sets of edge-to-edge contacts, although there is no attempt to obtain a continuous stress distribution throughout the contact surface;

3. Their most particular characteristic consists on fully allowing separation between blocks, and that the analysis may carry on even in the large displacement regime. Most codes perform contact detection and update tasks automatically;

4. Finally, these methods employ time-stepping algorithms, either in a real-time scale or as a numerical device to solve quasi-static problems.

5. Case Studies

This work considers the studies of Oppenheim (1992) on the masonry arch and pretends, using the discrete method formulation UDEC, to study three case scenarios:

- confirming the minimum acceleration values obtained by Oppenheim (1992) for both the arch with an embrace of 157,5◦ and the semicircular arch;
- verifying the effect of the filling of the extrados on the behavior of the arch when submitted to a dynamic analysis considering again both arches;
- confirming this effect of the filling of the extrados, comparing the values of the collapse of a simple arch, an arch considering only the weight of the filling and finally an arch that models the filling as discrete elements.

In order to confirm the results obtained by Oppenheim (1992) in his work, two arches are modeled with the characteristics as follows

Table 1 - Oppenheim's arches geometry definitions

	Radius (*m*)	Thickness (*m*)	Embrace	Voussoirs
Arch 1	10	1,5	157,5◦	7
Arch 2	10	1,5	180◦	7

Both arches are submitted to two different analyses – static and dynamic. The first one consists on submitting the arch to horizontal loads that consider the arch's mass and an acceleration that will vary so to find the minimum that cause the mechanism. A second analysis applies the same different values of acceleration, this time to the bases on which the arch rests.

The results obtained are presented in the following table

Table 1 – Static and dynamic analyses results of the Oppenheim's arches

	Static Analysis	Dynamic Analysis		Oppenheim's values
		*F=m*a*	*as*	
Arco 1	0,30**g*	0,29**g*	0,14**g*	0,37**g*
Arco 2	0,14**g*	0,12**g*	0,06**g*	0,16**g*

The results obtained by Oppenheim (1992) were of 0,37*g for the first arch and around 0,15*g for the second one. The values obtained in the static analysis show a similitude to the values this paper was aiming for. In the dynamic analysis, however, the results are far beyond the values predicted.

Although it might seem that both analyses would return the same results, in fact this parallel is only possible in rigid systems, not the case with the arches studied. Although they are modeled as rigid, the system isn't perfectly rigid, which provokes an amplification of the results causing the static analysis to return higher values.

We may now conclude that it is perceptible how the semicircular arch is less stable than the arch nº 1 (with an embrace of 157,5˚). The difference between them implies that the arch nº1 with a slight decrease of the angle of embrace has double the stability of the arch nº2.

A second case study involves the analysis of the study of the effect of the filling of the extrados. At this point, the filling is modeled through its weight. So, for both arches, it is considered that the voussoir and the filling have the weights 21 kN/m^3 and 22 kN/m^3 respectively and the areas of both elements are calculated so to obtain the new density of each voussoir when considering the filling.

The same two analyses are applied to both arches and the results are as presented

Table 3 - Results of the analyses for Arch 1

Arch 1 (157,5º)		
Static Analysis	With filling	0,29**g*
	Without filling	0,29**g*
Dynamic Analysis	With filling	0,17**g*
	Without filling	0,14**g*

Table 4 - Results of the analyses for Arch 2

Arch 2 (180º)		
Static Analysis	With filling	0,17**g*
	Without filling	0,12**g*
Dynamic Analysis	With filling	0,12**g*
	Without filling	0,06**g*

Again the results of both static and dynamic analysis are significantly different due to the reasons explained before.

What is interesting is to observe that in the case of the first arch, the existence of the filling has no relevant contribution to the stability of the arch contrarily to what happens when regarding the effects of the filling considering the line of thrust.

In the case of the second arch, being a more unstable arch, due to its angle of embrace, the effects of the filling are obvious and represent a positive consequence on the stability of the structure. This difference of the effect between arches can be related to geometrically non linear effects and to the fact that being a more unstable structure, it is also more vulnerable to any improvement or change in general. So, as it was already known in a static approach, the filling of the extrados, for unstable arches like the semicircular one, represents an advantage to its stability.

Finally, in the last case scenario, it is compared the results obtained considering three arches: one simple, one with the weight of the filling and finally modeling the third arch with discrete elements.

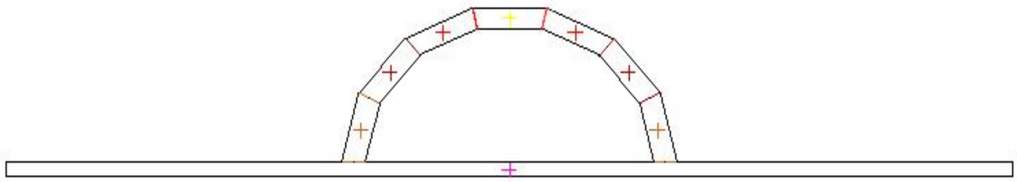


Image 4 – Semi-circular arch considering the extrados filling weight

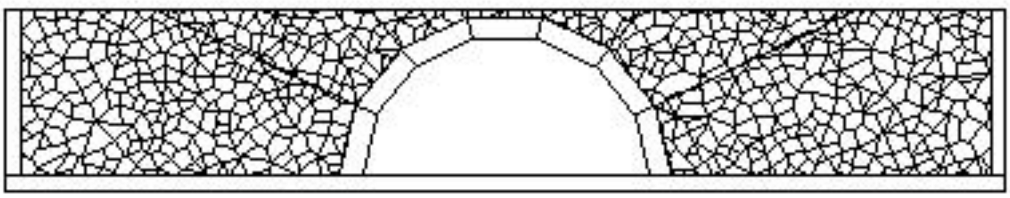


Image 5 - Semi-circular arch considering the extrados filling weight through discrete elements

The arch considered is a semicircular arch with the same characteristics of the arch before presented, apart from having the filling of the extrados confined between the arch and two rigid walls. These walls were distanced two times the radius of the arch from the bottom voussoirs, so to prevent the walls from having a resistant effect on the arch.

These three arches were submitted to an historical earthquake which occurred in Portugal on the year of 1969. The intensity of the earthquake is varied through λ so it becomes clear the arches' resistance. The choice of the earthquake used in this model had, like the choice of the arch geometry, no intention beyond permitting a representative example.

The results for the forces of collapse are plotted in the next table and the behavior is visible in images 6 and 7

Table 5 – Collapse values for the three modeled arches

Collapse values (λ) of the arch	
Simple Arch	10,5
Arch considering the weight of the filling	17,75
Arch with the filling as discrete elements	27,5

As previously observed, the filling proves to have a positive effect on the arch stability. However, it becomes clear that through the modeling of the weight it is impossible to consider the lateral stiffness in the arch behavior. So, to provide a proper modeling, filling has to be represented through independent elements.

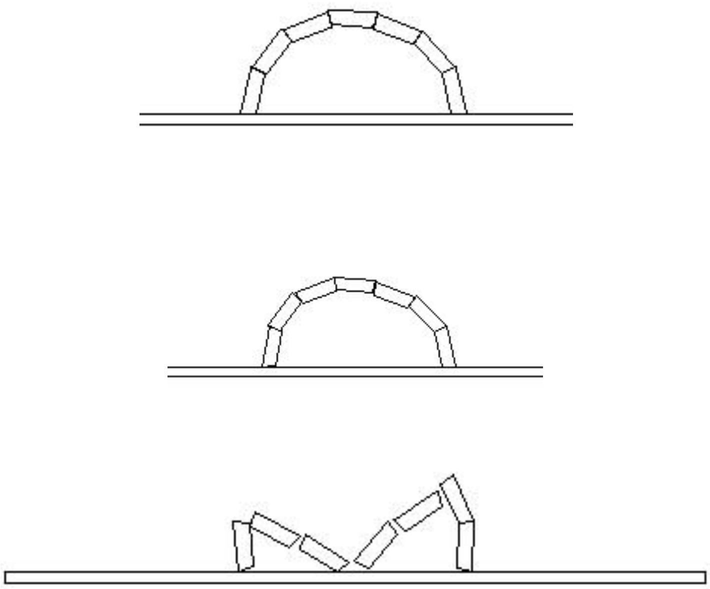
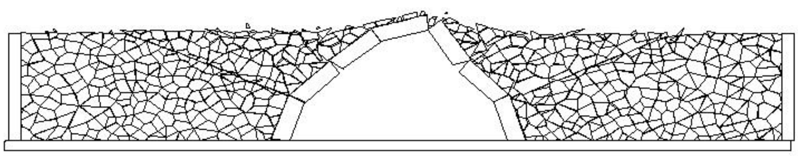


Image 6 – Collapse of the semi-circular arch considering the extrados filling through it's weight



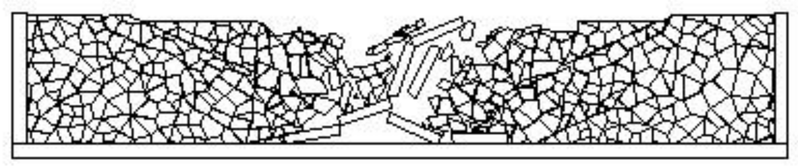


Image 7 – Collapse of the semi-circular arch considering the extrados filling with discrete elements

6. Conclusions

In this paper it is presented the importance of comprehending the behavior of the masonry arch to allow a correct intervention in historical buildings or to guarantee the safety of the structure. It is presented one of the oldest methods to design the masonry arch – the line of thrust – and used to understand the static effect of the filling of the extrados.

Following the work of Oppenheim (1992), the minimum accelerations necessary to form the mechanism for arches of different geometries are searched and the resulting values confirm the ones defined by Oppenheim (1992).

The filling of the extrados is modeled considering its weight on each voussoir and although it shows no relevant effect on the arch with an embrace of 157,5º, in the semicircular arch the raise of its stability is obvious.

Finally the effect of the filling is confirmed with a second analysis, and the two possible models of the assumption of the filling are compared. The modeling of the filling through discrete elements shows an obvious advantage towards the simpler one that only considers the filling's weight, since only the first models the lateral stiffness of the arch.

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