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Study and Optimization of an Air Launched Nanosatellite Launcher

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Dissertação para obtenção do grau de mestre em

Engenharia Aeroespacial

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December 10, 2010

Acknowledgments

To my Professor, for helping lighten the burden.

To the cats, who have always said Meow.

To my family which encompasses three continents of awesome insanity.

To all my friends, of which I will forget some, but the A's , F, G, a surprising number of I's (or ı), L (never A), the M's (emigrated or otherwise), several P's, both R's, S, and last (but never the least lest she be offended) V.

To Her that even after all these years still gives me reasons to put up with her, play nice and go shopping (even for shoes).

Resumo

A utilização do espaço mantém-se uma proposta complexa e cara sendo limitada a um número muito restrito de operadores, uma vez que os satélites convencionais têm frequentemente grandes dimensões o que implica lançadores enormes e dispendiosos assim como estruturas dedicadas. Contudo, desenvolvimentos tecnológicos nos ramos da electrónica e miniaturização começam a alterar esta situação, visto começar a ser possível utilizar satélites muito mais pequenos para cumprir uma mesma missão, com óbvias vantagens em termos de custos tanto de construção, lançamento como de operação.

Uma classe de satélites de particular interesse é a dos nano-satélites, com massas entre 1 kg e 10 kg. Pelo seu reduzido tamanho e complexidade, é possível construi-los de forma barata vendo-se por isso um uso alargado nomeadamente por universidades, havendo ainda iniciativas a estimular o seu desenvolvimento como por exemplo as competições de cubesat.

Os lançadores, contudo, continuam virados para lançamentos de grandes satélites, o que tem sido um entrave ao uso destas tecnologias visto um satélite barato apenas o ser mediante a possibilidade de um lançamento de baixo custo.

Este trabalho explora a possibilidade de realizar um lançamento aéreo de um foguetão, utilizando um avião militar, capaz de colocar um nano-satélite em órbita. Uma ampla gama de factores foram considerados e estudados para obter um veículo final. Esses resultados podem servir de base para trabalhos futuros num lançador real.

Palavras chave: Nano-satélite, Lançamento aéreo, Foguete, Optimização, Baixo custo

Abstract

Space use remains an expensive and complicated endeavour mostly limited to a very restrict number of operators, as conventional satellites are frequently heavy and large devices, requiring massive and costly launchers as well as dedicated facilities for orbital placement. Technological developments in the field of electronics and miniaturization, however, are starting to change this; it is now possible to use a much smaller satellite to accomplish a given mission, with obvious advantages in terms of build, launch and operating costs.

A class of satellites of particular interest is the nanosatellites, with masses ranging between 1 kg and 10 kg. Due to their small size and reduced complexity it is possible to build them cheaply and thus they are seeing widespread use notably in universities, with initiatives such as the cubesat competition further stimulating their development.

Launcher systems, however, have remained geared towards heavy satellites which is proving to be a roadblock on the use of these technologies. A cheap satellite only remains a inexpensive endeavour with an equally cheap launching option.

The present work studies the feasibility of an air launch of a rocket using a military plane, capable of launching a nanosatellite into orbit. A wide variety of factors were considered and studied and a final vehicle obtained. The results may serve as a baseline to a future possible launcher.

Keywords: Nanosatellite, Air launch, Rocket, Optimization, Low cost

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Acronyms

Isp	Specific Impulse (s)
T/W	Thrust over Weight ratio

Variables

$a_{(x,y)}$	Acceleration, in the x or y direction (m/s^2)
A_t	Nozzle throat area (m^2)
A_e	Nozzle exhaust area (m^2)
A_{ratio}	Nozzle area ratio
A_{casing}	Exterior casing area (m^2)
$A_{nosecone}$	Exterior nosecone area (m^2)
A_{wet}	Complete exterior area (m^2)
C_D	Drag Coefficient
dt	Computational mesh size (s)
D	Drag (N)
E_M	Mass of the Earth (kg)
E_R	Radius of the Earth (m)
g	Gravitational acceleration (m/s^2)
g_0	Gravitational acceleration at the Earth's surface (m/s^2)
G	Gravitational constant ($m^3/Kg.s^2$)
h_{orb}	Orbital altitude goal (km)
h	Altitude (m)
h_b	Limit altitude of layer b (m)
k_{prop}	Propellant specific heat ratio
L	Structure length (m)
Le	Extra components length (m)
Ll	Linkage section length (m)

$L_{nozzle(1,2)}$	Nozzle length (stage 1,2) (m)
Lr_b	Temperature lapse rate, layer b (K/m)
m_{pay}	Payload mass (kg)
$m_{s(1,2)}$	Structure mass (stage 1,2) (kg)
$Mat_{mass(1,2)}$	Material mass (stage 1,2) (kg)
$Mat_{Vol(1,2)}$	Material volume (stage 1,2) (m^3)
Mat_{massg}	Pressurizing gas tank material mass (kg)
Mat_{Volg}	Pressurizing gas tank material volume (m^3)
Mair	Molar mass of air (kg/mole)
$pos_{(x,y)}$	Position, in the x or y direction (m)
P_{amb}	Ambient pressure (Pa)
P_{chamb}	Chamber pressure (Pa)
P_e	Exhaust pressure (Pa)
P_g	Pressurizing gas pressure (Pa)
P_p	Propellants pressure (Pa)
R_{prop}	Propellants gas constant (J/Kg.K)
Rd_e	Nozzle exit radius (m)
Rd_t	Nozzle throat radius (m)
Rd	Structure radius (m)
$Rd_{gas(1,2)}$	Pressurizing gas tank radius (stage 1,2) (m)
R_{air}	Air gas constant (J/Kg.K)
Re	Reynold’s number
S_f	Structural safety factor
t_s	Structure thickness (mm)
t_{gas}	Pressurizing gas tank thickness (mm)
t_1	Minimum required thickness, propellant tank pressure (mm)
t_2	Minimum required thickness, longitudinal stress (mm)
t_3	Minimum required thickness, transverse stress (mm)
T	Thrust (N)
T_0	Initial propellant temperature (K)

T_{chamb}	Chamber temperature (K)
T_e	Exhaust gas temperature (K)
T_p	Temperature (K)
T_b	Limit temperature of layer b (K)
$v_{(x,y)}$	Velocity, in the x or y direction (m/s)
v_e	Exhaust gases velocity (m/s)
V	Speed (m/s)
$W_{(L,T)}$	Weight, Longitudinal and transverse projections (N)

γ	Flight path angle with the horizontal (º)
λ	Structural finesse ratio [L/d]
$\lambda_{nosecone}$	Nosecone finesse ratio
ρ_{fuel}	Fuel density(kg/m^3)
$\rho_{oxidizer}$	Oxidizer density (kg/m^3)
ρ_0	Propellant initial density (kg/m^3)
ρ_t	Propellant density at nozzle throat (kg/m^3)
ρ_e	Exhaust gas density (kg/m^3)
ρ_{Mat}	Material density(kg/m^3)
ρ	Air density (kg/m^3)
ρ_b	Limit density of layer b (kg/m^3)
σ_{UT}	Ultimate strength (Pa)

1 Introduction

Rockets have been used by mankind for almost a thousand years, with records showing that by 1045 they were an integral part of Chinese military tactics (1). These were solid rockets, filled with gunpowder, used to bombard the enemies and were said to be heard from about 24 km away. Throughout the years technology evolved; rockets grew, their use spread and several efforts were made to improve them, including studies on staging, the introduction of metal casings by the end of the 18th century which allowed for higher thrust and led to a range of almost 2 km and replacement of guide vanes with fins. Afterwards, the greatest revolution was brought about by Robert H. Goddard who in March 16 1926 launched the first liquid fuel rocket, thus giving birth to modern rocketry.

The field was greatly developed with notable contributions from such people as Hermann Oberth, Konstantin Tsiolkovsky, and Wernher von Braun, amongst several others. Propelled by both the military and political climate of the second world war and the posterior cold war, rocketry evolved at an ever increasing pace. The nazi V2 rocket, developed during the war, may be considered as the first mass produced rocket using most of the systems found in modern rockets. After the war, and given their availability, intercontinental ballistic missiles were used as launchers for the first tests, with evolutions of these becoming the first generation of dedicated launchers such as the Soviet R-7 or the American Redstone. Growing performance requirements, especially due to the desire to reach the Moon, led to the need of larger rockets such as the Saturn V and the abortive Soviet N1. Nowadays the political climate is markedly less tense, which slowed both the arms and the space race, and consequently current rockets are mostly geared for the launching of commercial satellites – the Ariane V, the Soyuz family and the Delta and Atlas families.

1.1 Current Space missions

From the first satellite, Sputnik 1 launched on October 4 1957, there has been a remarkable technological evolution, yet not too much has changed in the panorama of space launches. With the development of satellite communications space rockets were geared towards the launch of these large and heavy devices with the market being almost completely dominated by the large organizations: the American Space Agency, the Russian Space Agency and, from 1973 onwards, the European Space Agency. This concentration of technology is perfectly justified by the characteristics of space development – the high costs involved, the connection with military development, the political nature of competition and the reduced market. It has been difficult to find options outside these three organizations and historically the prices for orbiting payloads have been very high, with eastern types having an advantage due to lower manpower and operation costs, and having some launchers even today directly supported by the military. The recent appearance of smaller companies offering new launcher systems, such as the Sea Launch or the Pegasus (detailed in section 2), has provided smaller operators with more options and may lead to a reduction in prices.

Satellite class (Low Earth Orbit)	Western Types	Eastern Types
Small – under 2000kg	18621	7074
Medium – 2 to 20 tons	11012	5307
Heavy – over 20 tons	9790	4291

Table 1-1: Average price per kg to Low Earth Orbit in dollars (2)

It is easy to see how launching even a small satellite, weighing a few hundred kilograms is an extremely expensive endeavour, limiting space access to a very restrict number of operators.

1.2 Developments in small Satellites

With the technological developments towards miniaturization in electronics, it is now possible to complete the same task with much smaller satellites, with projects using a 100 kg satellite to achieve (in 2003) what before required a 500 kg one (3). Since satellites are expensive to develop, build and launch, they are designed to last for the longest possible period of time. In the case of communications satellites, about half of their mass at launch is fuel, for correction manoeuvres throughout its whole lifetime. These developments allow for smaller satellites which will require less fuel and will contribute for a much more widespread access to space. In addition, initiatives such as the Cubesat projects are seeing universities building and launching their own satellites, further expanding space exploration.

Beyond the more common academic and monetary applications, there is a growing interest in this satellite class for other reasons, namely fast response and event observation. The ability to monitor events as they unfold is of great strategic importance allowing for much better and prompt decision making, in both military and humanitarian operations. There are two main ways to accomplish this goal: either by having a permanent constellation of satellites available (4), possible due to the low cost of each individual satellite, or by launching a satellite for a specific event. This last one requires a launcher with fast response capabilities, which is explored in the present work.

Another interesting use for nanosatellites is for space qualification of new technologies (5). After all the development and testing that takes place on the ground, until a component is considered safe and space qualified it is required to go through an even more extensive testing procedure, incurring significant costs. As such, the capability of testing the part in space using a lower cost satellite greatly cheapens and streamlines the process.

1.3 Launching issues

The use of this innovative type of satellites is yet to take off due to the difficulties found in launching them into space, as space launchers are tailored for large and heavy satellites. Smaller ones have to rely on “piggy back missions” going as secondary cargo. Indeed, this is hardly the perfect situation: it remains costly, with “Nasa’s Cubesat Launch Initiative” (an initiative allowing selected universities to orbit their satellites) having a clause stipulating a possible *“partial reimbursement of the launch and integration costs of up to $30,000 (thirty thousand dollars) per 1U Cube associated with their participation in the pilot project.”* (6), and, as a secondary mission, the operator has little power in the mission definition, both in terms of launch schedules and the orbital parameters.

1.4 Goal of this work

In general, the technological development of smaller satellites has not been accompanied by the development of efficient ways to launch them, leading to a situation where it is not possible to satisfy an ever growing demand. The objective of the present work is to study the feasibility of an airborne launch as a way to cheaply launch satellites that is simultaneously accessible to a wide range of users.

By making use of a widespread and available resource - aeroplanes - there are several factors that can possibly reduce the need for dedicated ground launch stations, simplifying launch procedures and likely reducing costs, while also providing some technical advantages to the launch itself. The ability to ignite the rocket at a significant altitude and, more importantly, with a considerable initial speed allows for reductions in the required rocket total mass, thus leading to a cheaper launch.

This work will analyse all the main parameters that most influence the performance of the system leading to a preliminary design of such a launcher of this type. Trade-off analysis will be performed to optimize the rocket to a reasonable degree.

2 Technological Overview

2.1 Launchers Analysis

Given the effort in time, means, and funds required for the development of a launcher, rockets are designed to have an extended lifetime, with small improvements being implemented along the service life. The space market is limited and its needs are assured by a small number of different rockets of different sizes, adapted for the main classes of possible missions. Three systems were chosen to provide an overview on the current technology.

2.1.1 Pegasus

Introduced in 1989, the Pegasus launch system is a three stage aerial launch vehicle geared towards smaller payloads (maximum weight around 450 kg) also having the relevance of being the first privately developed launch system (7). The system is carried aloof up to almost 12 km by a specially converted Lockheed L1011 and released at 0.82 Mach, followed by a 5 second coast phase until the first stage ignites. The three stages are all single engine solid propellant rockets, the first stage includes a delta wing that generates lift during the initial part of the flight, with the second and third stages being ordinary rocket sections. By being totally mobile, it gives operators an unprecedented degree of flexibility in mission planning. It is possible to define the orbital inclination angle right at launch, saving a costly plane-change manoeuvre in space, weather problems are negligible as ignition takes place over cloud coverage and time windows are much more readily available as the plane can move into position. Pegasus flights are currently valued at 11 million dollars (or 24500 dollars per Kg) amongst the most expensive per mass unit, yet the low overall price when compared with larger delivery systems combined with the flexibility of being the main mission has offered an opportunity for smaller operators to orbit their payloads. This has been the system's greatest contribution.

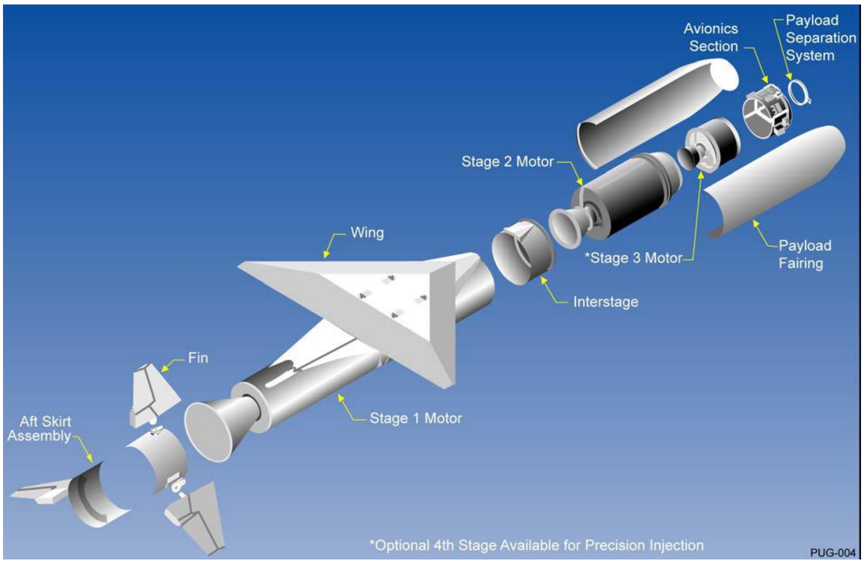

Figure 2-1: Pegasus Layout (7)

Total Mass	23130 Kg
Total Propellant Mass	19709 Kg
Length	16.9 m
Diameter	1.27 m
Payload	450 Kg
Mission price	$11M

Table 2-1: Pegasus Main Characteristics (7)

While an order of magnitude larger than the case developed in this work, both in dimensions and carrying capacity, the Pegasus project is proof of both the suitability of airborne launches as way to increase access to space and the capability of private ventures to achieve that goal. Unlike the subject of this work, the Pegasus makes use of wings, leading to a much heavier structure. Their need comes mostly from the heavy weight of the system coupled with a low manoeuvrability carrier plane, requiring a long horizontal initial flight phase.

2.1.2 Ariane 5

With the go ahead given in 1986 and the first launch in 1996 the Arianne 5 is Europe's flagship launcher, composed of two parallel solid rocket boosters strapped to a two staged linear rocket (8). It is a completely conventional rocket launched from Kourou Space Centre with a maximum payload of 20 tons to LEO, the solid boosters providing the majority of thrust for the initial lift-off. Designed to be human-rated from the onset, the original engines used a mix of monomethylhydrazine (MMH) and nitrogen tetroxide (N_2O_4) later converted to Liquid Hydrogen and Oxygen for better performance. After some initial mishaps the system has proven itself to be extremely capable, flexible and reliable. Several nosecones are available, allowing for different volumes for different payloads, and it is possible to fit up to three individual satellites thanks to the use of specialized mounts. Current prices are estimated to be in the vicinity of 180 million dollars per launch, or around 10000 dollars per kg (2).

Total Mass	777 tons
Total Propellant Mass	399 tons
Length	46 – 52 m
Diameter	5.4 m
Payload	20 ton
Mission price	$180 M

Table 2-2: Ariane V main characteristics (8)

The vehicle has a completely different size and set of capabilities when compared to the case pursued in this work, but being the market dominant force it is worth to review.

Figure 2-2: Ariane 5 Layout (8)

2.1.3 Sea Launch

The Sea Launch is a consortium that was created in 1995 by four companies from the United States (Boeing), Russia (Energia), Norway (Aker Solutions) and Ukraine (PO Yuzhmash) to operate commercial space launches (9). The system has three main elements:

- Rocket – an especially modified Zenit-3SL
- Sea Launch Commander – a sea vessel that operates as final rocket assembly factory and mission control
- Ocean Odyssey – a mobile launch platform

Unlike other conventional launchers, this system is completely mobile, with the rocket parts being built in several countries and then moved to California for integration before being loaded into the Sea Launch Commander for ferry to launch destination. The rocket is then transferred to the Ocean Odyssey platform where a completely automated launch takes place. This mobility allow the launches to always be performed at the maximum efficiency location be it the equator for geostationary launches (it is estimated to provide an increase of 15% in mass compared to a cape Canaveral operated rocket) or a particular inclination. By launching from the middle of the ocean, there are also savings in costs (from using a simpler launch procedure and reduced insurance premiums due to lower risks) and increase in safety as it is possible to avoid largely populated areas during the critical phases of the ascent. The system had some problems: besides 3 unsuccessful missions out of 30 (Ariane V has 4 out of 53) there have been some significant problems unrelated to engineering issues. There have been some political questions raised regarding the export of technology, arising from the international nature of the venture, and the company has filed for bankruptcy once, being now owned in 85% by the Russian partner Energya.

Total Mass	462.2 tons
Length	59.6 m
Diameter	3.9 m
Payload	15.876 ton
Mission price	$85 M

Table 2-3: Zenit 3SL main characteristics (9)

Figure 2-3: Sea Launch System (9)

2.2 Air Launch Studies

There has been some research that, to a varying degree, explores the possibility of the air launch, focusing on some particular aspects of the problem.

2.2.1 MDO study of an airborne system

The issue of an air launched rocket for nanosatellites was used as a case study for a Multidisciplinary Optimization (MDO) analysis (10). The analysis was started with a number of parameters set: the use of a 3-stage rocket, with solid propellant second and third stages; the mission objective was defined to be a circular orbit at 700km of altitude with at least a 7.5kg payload; the use of an F4 Phantom as the carrier; and a release altitude of 12000 meters. The remaining parameters were left open (within acceptable intervals) as part of the optimization algorithm with the only worth mentioning being the first stage propellant, with provisions for either a solid rocket stage or a hybrid engine, and the launch velocity, 0.8 or 1.5 Mach. While the main focus of the paper is the MDO analysis per se and not the engineering problem itself, the results are relevant for the current work.

From the four possible combinations, subsonic launch solid engine, subsonic launch hybrid engine, supersonic launch solid engine and supersonic launch hybrid engine the most effective one was found to be the supersonic with the hybrid engine. The launch speed conclusions are directly applicable, as the same issue is present. The engine selection is an indication of the value of using more efficient liquid fuel solutions over solid propellants. We did not apply these conclusions directly in the present work. Regarding the launch speed, the use of more recent and efficient planes as carriers, discussed further ahead, allowed us to use a higher launch speed of around Mach 1.8. Similarly, the considerations about the use of a higher efficiency fuel were taken one step further with the selection of a fully liquid solution, as shown in the relevant section. Furthermore, the work developed by the authors was towards a numerically optimized solution, while our objective is to provide an in depth analysis of the parameters' interactions.

2.2.2 Launcher considerations

There have been some studies focusing on the launch and initial moments of the rocket where three main issues were addressed (11): initial angle, aerodynamic surfaces and control, and carrier influence. Regarding the initial angle, it was concluded that the larger the value (from 0º to 40º) the more efficient the launch. Not only did it make it possible to get a higher mass into orbit but, by having a lower maximum dynamic pressure, a higher fraction of the total mass was useful payload instead of structure. Regarding the aerodynamic surfaces, it is mentioned that the stabilizers reduce the performance of the vehicle, and thus are only used to control the rocket between separation and ignition with the rest of the control being provided by vectored thrust. A solution using wings for the first stage was also tested, with results showing a reduction in useful payload of almost 25%. Finally an analysis was performed where a rocket designed to be launched form the Eurofighter Typhoon was launched from a Mig-25, which has a much higher performance in terms of both speed and ceiling. The results clearly show the advantages of this solution as the Mig-25 was able to launch almost 150 kg more, or an extra 50%.

Several important lessons can be learned from this. Regarding the launch angle, the results are somewhat inconclusive, as several factors interact with the initial angle. The control and surface analysis' conclusions are directly applicable in the present case, validating the selection made to use solely thrust vectoring and to forego the use of wings. Finally, the mother ship tests show the value of using the highest performance vehicle available. In comparison, the present work aims to analyze the relation and influence of the carrier in conjunction with the remaining parameters, in order to improve the whole system.

2.2.3 Launch System Study

Using the Boeing AirLaunch vehicle as base, the study of a coplanar air launch with a gravity turn trajectory was addressed in (12). The importance of being able to move the rocket into position is discussed, analyzing how it allows for greater flexibility by removing the constraint posed by launch windows, and how it allows for better performance by launching the payload with the final orbital inclination as opposed to using an orbital plane change manoeuvre. As in other analyses, the trajectory chosen for the atmospheric part of the flight is a gravity turn, a type of trajectory that, by definition, ensures zero angle of attack, which avoids possible structural complications. This work shows the interest and feasibility of a more flexible system - provided by the mobile first stage - and validates the use of a gravity turn trajectory for atmospheric flight, both important issues in the present work.

3 Parameter analysis

3.1 Overview

In the present chapter the factors that influence the performance of the system are identified, as well as the influence they have on each other, in a bid to find a way to optimize the design of a rocket. A discussion of each parameter is carried out and selected baseline values justified. The calculations performed to obtain the results are also shown and explained.

The factors considered relevant were divided in three groups, according to the type of influence they have on the launch: Carrier plane, with influence in the initial conditions, Rocket, influencing the capabilities and Trajectory, defining the stresses and some performance requirements. An outline of the main variables in the study, indicating any relations between factors that are of special interest, is presented here for reference of the more detailed analysis:

Carrier plane – The mother ship defines most of the initial conditions of the launch, which subsequently influence the whole flight and performance. The main parameters and their relations can be found in Table 3-1.:

Factor	Parameter influence	Dependencies
Carrier Capacity	Defines suitability of the carrier for the rocket	Independent
Release Altitude	Determines initial potential energy and the Drag	Independent
Initial Speed	Determines initial kinetic energy	Independent
Initial Angle	–	Depends on mass flow rate
Drop Mechanism	How the rocket is attached to the carrier	Depends on speed regime

Table 3-1: Main parameters of the Carrier plane

Rocket – The main focus of this work is on all the parameters representing the various sub-systems of the rocket, and how they interact. These can be found in Table 3-2, organised by sub-system.

Configuration	-	Independent
	Propulsion	
Propellant selection	Defines practically everything in the propulsion system	Independent
Nozzle Area ratio	Defines the nozzle efficiency	Dependent on initial altitude
Mass flow rate	Defines thrust	Dependent on initial angle
Pressure system	Defines structural weight	Independent
	Aerodynamics	
Nosecone finesse ratio	Defines nosecone drag	Independent
Body finesse ratio	Defines body drag	Dependent on structural calculations
	Structure	
Material selection	Defines the structural weight	Dependent on maximum stress
Body finesse ratio	Defines body mass	Dependent on aerodynamic calculations

Table 3-2: Main parameters of the rocket

Trajectory – the trajectory determines the stresses supported by the rocket as well as the required burn patterns.

3.2 Carrier Plane

Being an aerial launched vehicle, the mother ship selection takes a central role in defining the full capabilities of the rocket. As it will be seen the use of a high performance fighter aircraft is the best option. The role of the carrier is to impart to the rocket the maximum possible energy, which can be achieved by going as high and as fast as possible, in all aspects acting as a first stage of the launcher assembly. This increased efficiency can have two outcomes: it either leads to a lighter and simpler rocket, reducing costs, or it allows the same rocket to carry a heavier payload.

Another relevant issue is the ferrying of the rocket. To ensure safety of operation, as well as optimal launch conditions, the carrier is expected to transport the rocket a certain distance from the base of operations (the delivery of the rocket to the operator is a non issue, as it may be taken in a container by a conventional cargo plane). A lower operational radius means either a lower flexibility, requiring a stricter choice of launch locations, or the use of tanker aeroplanes. It should also be noted that this ferry takes some time, which may have other consequences.

Due to all this, a trade-off analysis must be performed, taking into consideration the various limitations each carrier vehicle has.

3.2.1 Study Variables

The main variables regarding the carrier are:

- Physical constraints – Two issues must be taken into consideration regarding the physical capabilities of the carrier: maximum weight possible and size constraint (length and height). These constraints do not directly affect the performance of the vehicle, they serve as limits to the aeroplane classes' suitability.
- Altitude – The release altitude represents little in itself seeing as the best that can be expected from a conventional plane will be between 12 km and 15 km, under 10% of the total orbital altitude (expected to be in the 200-700 km range) – to which corresponds under 5% of the total potential energy variation required to reach orbit, and an even smaller fraction of the total orbital energy. However, the change in atmospheric conditions with altitude play a major role in the overall performance of the vehicle as air density (and consequently aerodynamic drag) drops exponentially with altitude by a factor of 5 at 12 km, 10 at 16 km, and 20 at 20 km, greatly reducing drag losses.
- Speed – The objective of the vehicle is to impart a certain speed to a given payload, at a desired altitude. The speed imparted by the mother ship represents a direct save in the total required fuel (and thus weight). Considerations had to be taken regarding the launch flight regime: subsonic or supersonic. While the second represents a much greater save, as not only is the rocket travelling faster but it will not have to expend fuel to beat the transonic drag,

the supersonic release of the vehicle presents some complex engineering issues. Thus, both options were considered, focusing primarily on the feasibility of the supersonic release.

- Dock and release – The docking mechanism defines how the weight of the structure stresses it and thus leads to further constraints for the structural design. The release method is also a factor.

3.2.2 Support Rail and Release

We start by analyzing the rocket support system, as it directly influences the flight regime. The support rail has the function of securing the rocket during the flight to the launch location onboard the carrier. Care must be taken to allow a sufficient number of aircraft (the larger the better) to be compatible with the system. With the ever growing movement towards military commonality this should not be a problem and will not be further discussed in this work.

Two systems are commonly used for the release of payloads and that may be used with the rocket. The first is a drop system, where the rocket would be released from the carrier with ignition only happening after the rocket is at a distance from the plane. This not only excludes supersonic release but also requires the rocket to have some kind of stabilization (usually with aerodynamic surfaces) to maintain control without thrust. The second method relies on a rail attachment to which the rocket is fixed for transport with the rocket being ignited while still connected, for launch. This system allows for a supersonic launch, care only required to ensure that the aircraft is not damaged. For all this, the Rail option has been selected.

3.2.3 Aeroplane Classes

As a way to provide a framework for the analysis, the aeroplanes considered were split into three separate categories, each with its own particular characteristics:

- High Performance Fighter Aircraft (such as F4, F16, Mig 29) – the most widespread of all, offering the best performance and flexibility but lower weight capacity
- Large Bomber Aircraft (such as Tupolev 160, B1, B52) – exceptional carrying capacity and good performance, but limited availability as only the United States and Russia currently operate this kind of craft
- Patrol Craft (Nimrod, P3 – Orion) – can be considered to be in the middle in terms of performance and availability.

Civilian craft were not considered since they fail to provide better performance than the ones selected and their use would require extensive conversion, contrary to the spirit of the objective. Each class of carrier has different characteristics, thus changing the design constraints. The parameters selected as being relevant were performance, both in maximum velocity and altitude, carrying capacity and availability, as low number of operators reduces the reach of the project. Since Large Bombers are an extremely scarce resource, available only to a small number of countries which also possess the other two classes, we decided not to pursue them as a viable alternative.

3.2.3.1 Fighter Jets

The most widespread jet planes available, they are also the ones presenting the best performance both in terms of speed and attainable altitude. The main drawback is their small size, which limits the takeoff weight and maximum size of the rocket as well as the maximum mission range for launch - that may be a significant problem depending on the launch Country's geography. A selection of the most relevant aircraft (chosen for performance and availability) including both western and eastern types can be observed in Table 3-3.

Aeroplane	Service Ceiling	Top Speed	Range	Payload	Operators
Mirage 3 (5)	17000 (18000) m	2,350 km/h	2400 [4000] km	4000 [400] kg	6
F4	12190 (18300) m	2,370 km/h	1360 km	8480 kg	7
F5	15800 m	1700 km/h	1405 km	3200 kg	22
F16	18000+ m	2,410 km/h	1100 km	2700 kg	25
F18	12190 (15000) m	1,915 km/h	1074 km	6215 kg	8
Mig 21	19000 m	2,125 km/h	1580 km	1000 kg	18
Mig 29	18013 m	2400 km/h	1400 km	3500 kg	24

Table 3-3: Fighter Jet Examples (13)

From the data acquired, and accommodating the largest amount of aircraft, the limits are presented in Table 3-4, representing a wide variety of suitable carriers while maintaining flexibility for the rocket design. As can be seen in Table 3-3, the selected values are lower than the maximum allowed for the majority of the planes considered, namely speed and ceiling. This is because the top speed of an aircraft is reached at an altitude inferior to the service ceiling and also, lower values allow a performance margin to accommodate the necessary manoeuvres.

Service Ceiling	Top Speed	Range	Payload
12 km	2000 km/h	1000 km	2500 Kg

Table 3-4: Fighter Jet Mission Limits

3.2.3.2 Patrol Planes

Patrol planes are designed to remain aloft for large periods of time surveying (usually marine) territory armed to deal with expectable threats. Compared to Jet Fighters, these vehicles are of much lower performance, both in absolute speed and service ceiling, making up for it with a much greater load carrying capacity and mission range. See Table 3-5 to review data.

Aeroplane	Service Ceiling	Top Speed	Range	Payload	Operators	Cruise Missile
P3 - Orion	10400 m	750 km/h	8944 km	9000 kg	18	yes
Nimrod	12800 m	918 km/h	11119 km	9980 kg	1	yes
Atlantique 2	9145 m	648 km/h	9075 km	3,500 kg	5	no (exocet)
C-295M	7620 m	480 km/h	1,333 (4300) km	9250 (4550) kg	11	no

Table 3-5: Patrol Plane Examples (13)

The selected mission profile is an average of the available craft:

Service Ceiling	Top Speed	Range	Payload
10000 m	700 km/h	5000 km	4000 kg

Table 3-6: Patrol Plane Mission Limits

In comparison to the previous class of carrier planes, the initial conditions are much less favourable, providing a smaller fuel save, and the initial angle will be much lower requiring a different approach for the whole problem especially in terms of aerodynamic support. On the other hand a Patrol Plane is able to carry a much larger and heavier rocket and does not presents the issue of a supersonic launch.

3.2.4 Plane Manoeuvre

The complete definition of the carrier flight path and losses is out of the scope of this work. The final stages of the flight are considered to be a wide loop with a low climb rate, and without loss of forward speed. This simplified approach of ignoring losses in the initial stages of flights has been used in literature such as in (11) and should be addressed in future works.

3.2.5 Initial Angle ($\gamma 0$)

It was determined that for a certain initial angle a minimum value of thrust is needed, leading to a minimum mass flow ratio $\dot{m}$. However an increase in $\dot{m}$ increases the structural weight and therefore leads to worse results. A trade-off analysis was performed to optimize the solution. Preliminary calculations show that there is an increase in efficiency with a more vertical launch, up to a maximum determined by the thrust characteristics, which is in agreement with the results of (11). Given these interactions, the calculations will be presented in section 3.4.2.6. (Propulsion).

3.2.6 Selection

All things considered, we decided to use a Jet Fighter as the carrier plane. The final analyses will show that the maximum mass of the rocket will be less than 1 ton, well within the maximum limits acceptable for the selected jet fighters, justifying that selection, with the Patrol Craft's greatest advantage becoming irrelevant. On the other hand, the much higher performance offered by the fighters (maximum altitude, top speed) allows for a more efficient launch leading to a lighter and cheaper rocket – as intended.

As a result of the analysis and discussions above, the selected initial point can be found in Table 3-7 where the Initial Angle value (measured with the horizontal) is also presented for convenience, although the reasoning for its selection is still to be presented (section 3.4.2.6).

Initial Speed	550 m/s
Initial Altitude	12 km
Initial Angle	62.5º

Table 3-7: Initial Point

3.3 Trajectory

The definition of a possible trajectory and the implication of the possible trade-offs between the parameters of the problem is central to the issue of putting a payload in a selected orbit. Given that this represents a complex optimization procedure which would go beyond the scope of this work, only a simplified analysis is to be performed.

3.3.1 Orbital requirements

For the payload to be inserted into orbit, it must reach the desired altitude with a certain speed. The selected Cartesian approximation proves itself invaluable, as it allows us to fully separate the vertical (altitude) and horizontal (velocity) components. In this very particular case, it is also possible to analyze the motion in both axes in a completely independent way, which is extendable to the velocity vectors. We have thus performed an energetic interpretation of the vertical movement, by calculating the kinetic energy that is associated with the vertical velocity:

$$\Delta Potential\ Energy_{Orbit} = m_{pay}\left(\frac{G * E_M}{E_R} - \frac{G * E_M}{E_R + h_{orb}}\right) \qquad [3.1]$$

$$Final\ Mechanical\ Energy_{vertical} = \Delta Potential\ Energy_{Orbit} \qquad [3.2]$$

$$Mechanical\ Energy_{vertical} = Kinetic\ Energy_{vert} + Potential\ Energy \qquad [3.3]$$

$$Kinetic\ Energy_{vert} = \frac{1}{2} m_{pay}\, v_y{}^2 \qquad [3.4]$$

$$Potential\ Energy = m_{pay}\left(\frac{G * E_M}{E_R} - \frac{G * E_M}{E_R + pos_y}\right) \qquad [3.5]$$

With v_y and pos_y relative to particular moments in time. The orbital velocity obtained by

$$v_{orb} = \sqrt{G * \frac{E_M}{E_R + h_{orb}}} \qquad [3.6]$$

And the required velocity change is simply the difference between the final velocity and the velocity at the start of the burn

$$\Delta v = v_{orb} - v_{x0} \qquad [3.7]$$

3.3.2 Constraints

Much depends on the trajectory, which has to efficiently fulfil the following criteria. As to minimize fuel mass, it has to attain the right balance between spending time at a lower altitude, for a more energetically efficient burn, or higher where drag is much reduced. Unlike in a land based launch, where the required thrust is mostly determined by takeoff weight, in the air launch it is defined by the trajectory, where it is possible to have the Thrust over Weight ratio $T/W < 1$. Finally, the trajectory also determines the loads the Rocket will be subjected to; care must be taken to ensure that the force is kept as axially aligned as possible, reducing the angle of attack to the bare minimum.

3.3.3 Gravity turn

A possible solution to this problem is to have the vehicle perform a so called gravity turn trajectory during atmospheric flight. This trajectory consists in maintaining the thrust vector aligned with the velocity vector at all times thus ensuring zero angle of attack, giving the rocket an initial angle which is increased by action of the gravity force (14). As the atmospheric flight cannot be directly solved due to the present of drag forces, it was necessary to iteratively calculate the solution. The equations that govern the gravity turn are (14) (15):

$$Impulse = T - D \qquad [3.8]$$

$$g = \frac{G * E_M}{\left(E_R + pos_y\right)^2} \qquad [3.9]$$

$$a_x = \frac{Impulse * \cos(\gamma)}{mass} \qquad [3.10]$$

$$a_y = \frac{Impulse * \sin(\gamma)}{mass} - g \qquad [3.11]$$

$$\gamma = \frac{v_y}{v_x} \qquad [3.12]$$

Those that can't be explicitly solved required the application of numerical methods, and we chose a very simple first degree backwards finite differences scheme for them

$$v_{x,y}(i) = a_{x,y}(i) * dt + v_{x,y}(i-1) \qquad [3.13]$$

$$pos_{x,y}(i) = pos_{x,y}(i-1) + v_{x,y}(i-1) * dt \qquad [3.14]$$

3.3.4 Trajectory calculation

The selected ascent trajectory is split into three parts, determined using the cartesian approximation:

- The first burn will give the rocket enough potential energy to reach the target altitude. As it starts with the release from the carrier plane, in atmosphere, it will be a gravity turn type of trajectory that will be maintained during the whole burn.
- In between burns there will be a small coasting phase (20 seconds) during which separation occurs, and the rocket manoeuvres itself for the second burn.
- We considered the second burn as being horizontal, accelerating the payload up to orbital velocity, as the orbital altitude has been guaranteed by the vertical speed acquired during the first burn.

As the second burn is horizontal as opposed to a Gravity turn type of trajectory, the accelerations are calculated from eqs [3.15] and [3.16] with the rest of the iterative system of equations remaining the same.

$$a_x = \frac{Impulse}{mass} \quad [3.15]$$

$$a_y = -g \quad [3.16]$$

This provides a realistic if not completely real path for the rocket, a close approximation to what usually takes place in a launch. It has the advantage of being simple since the horizontal impulse (second burn) does not interfere with the vertical speed provided to increase potential energy, due to the Cartesian approximation used. It has never been our goal to calculate a completely accurate and optimized solution for the trajectory problem, since it would require too much time both in the analysis of the problem and in the iterative process to obtain each solution but it is sufficiently good for the goal intended.

3.4 Rocket

In this section all of the discussions relating to the rocket system itself will be held, analyzing the various components and factors that must be integrated to obtain a good performance.

3.4.1 Preliminary topics

3.4.1.1 Rocket outline

The rocket is in principle one of the simplest flying devices in terms of general layout and theoretical functioning. It consists of a nosecone, usually housing the payload located at the top, followed by a cylindrical segment containing the propellants and other necessary components and ending in the engine nozzles. Since it is designed to also operate in the vacuum of space it carries both the fuel and the oxidizer, either together (in solid propellants or monopropellant options) or separately. They are combined and ignited in the combustion chamber and consequently expanded through the nozzle, the thrust being generated by reaction of the accelerated exiting gases combined with pressure forces.

The basic functioning of a rocket can be easily expressed by Tsialkovsky's Rocket equation (14)

$$\Delta v = v_e * \ln\left(\frac{m_{pay} + m_s + m_{propellant}}{m_{pay} + m_s}\right) \quad [3.17]$$

$$v_e = Isp * g_0 \quad [3.18]$$

This equation provides an exact solution in vacuum conditions, with correction for non perfect conditions usually represented by extra Δv terms added to the left side of the equation, to take into account effects such as drag losses or gravity losses.

3.4.1.2 Configuration

Since the performance of the rocket is extremely dependent of the structural mass, as can be understood from the equation [3.17], methods have been used to reduce it as much as possible, first by using the lightest components available, and then by the use of rocket staging. By separating the rocket into several stages it is possible to achieve a much higher performance, as the heaviest components required for the initial stages are shed and the final flight carried on by a smaller vehicle with much smaller parasitic mass (14). In the present case, the carrier plane may be considered a first stage and thus the final number of stages was studied to verify the best configuration.

There are then 4 possible configurations for the rocket: single stage rocket, linearly staged rocket, single stage rocket with boosters, linearly staged rocket with boosters. The option of using boosters, usually high powered solid fuel motors fired during ascension from the ground (a situation that does not take place in an air launch), would create extra constraints in terms of interaction with the carrier plane (both in terms of available space and attachment systems). Because of that it was decided not to pursue that option. To test the remaining options we have analysed the case, shown in Figure 3-1, where two rockets, one with one stage and the other with two, fulfilled the same mission. This is just a qualitative analysis to assess the impact of the number of stages in this problem with the values (both initial conditions and final results) having little meaning by themselves and only the comparison between stages is relevant.

As can be seen from Figure 3-1, the staged option is much more efficient since it is able to fulfil the same mission with a much lower initial mass. While there could be more to gain by using an extra number of stages, it would be a much smaller difference (as there are reduced returns with increasing number of stages) and it would increase costs and complexity. As the two stage rocket's dimensions are well within the maximum carrying capacity available, this was the selected staging option since it offers a good trade-off between simplicity and performance.

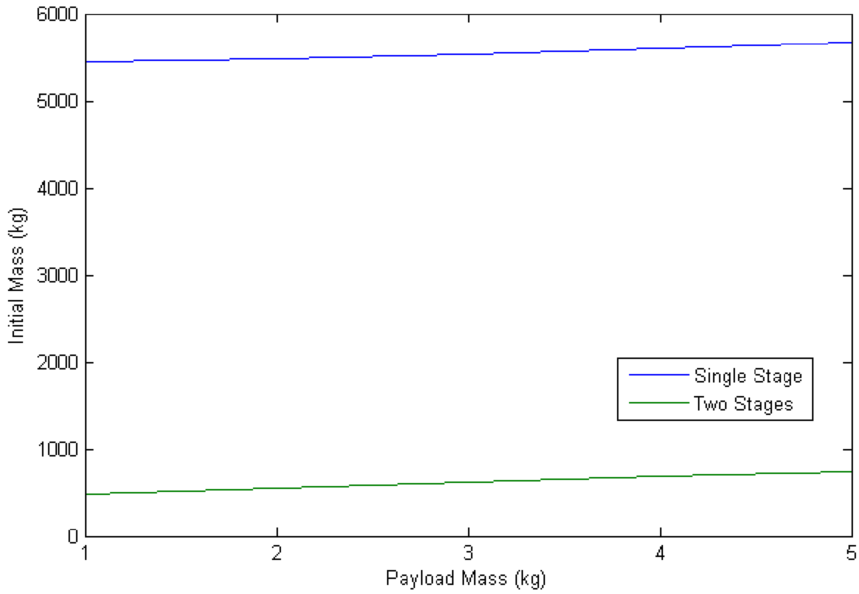

Figure 3-1: Staging effect on initial mass required

The stages' division follows the trajectory plan, the first stage fulfils the first burn (the gravity turn phase) and the second stage the second burn (horizontal acceleration). This separation allows the system to perform each trajectory step with a single burn, without requiring the complication of a restart able engine. Also, as each engine only fulfils one flight regime (gravity turn or horizontal boost) it can be optimized for that. Finally, during the coast stage between burns, the first stage is dropped, together with the inter stage linkage section and the nosecone, to minimize mass. As this happens in space, the nosecone is no longer necessary.

3.4.2 Propulsion

3.4.2.1 Propellant options

The propellant selection is one of the most important aspects of the whole vehicle, as it directly influences thrust, size of the rocket, efficiency, weight, and others. There are currently three main types of propellants for this kind of vehicle, each with differing characteristics. The methodology used in (16) was used in the whole section, to maintain consistency.

- Solid Propellants –used since ancient times they are the simplest to design and operate. They consist of a solid mixture of fuel and oxidizer, called the grain, cast with an appropriate shape including an indentation for the burn chamber that controls the burn rate. The main advantages from their use are the simplicity of having fuel and oxidizer in a single solid mixture without requiring pumps and reservoirs, the low maintenance required (for this reason being chosen for ICBM's (16) and high density allowing a smaller rocket for a same weight. On the other hand, they have the lowest efficiency, with maximum specific impulse Isp of about 285 s; the whole structure case works as a burn chamber and thus needs reinforcement, becoming heavier; it is the most prone to accidents from impact, or from grain defects; and there is no thrust control, after ignition the burn pattern defined by the grain is followed until burnout.

- Liquid Propellants – these consist of a wide range of different substances stored in liquid form in appropriate tanks. They have several advantages: a wide variety of choices allows for precise fitting to the mission requirements; the availability of thrust control (usually within a range) including shutdown and re-ignition (in appropriately designed engines); they have the highest efficiency of chemical rockets, with usual values of Isp between 300 s and 450 s. The main disadvantages are a smaller density (depending on the propellant chosen); the added weight of both the pressurization system, be it a gas tank or a turbo pump and the valves and plumbing; depending on the selection of propellant, there may be added constraints regarding use or storage, some are cryogenic requiring special plumbing, pumps and operating procedures; many have high toxicity.
- Hybrid Propellants – these stand in the middle, usually consist of a solid fuel with a liquid propellant. Comparatively to Solid Propellants they present a higher efficiency and some burn control, while compared to a Liquid solution they present a higher density leading to a smaller rocket and are mechanically simpler.

3.4.2.2 Propellant Type Selection

We selected a Liquid Propellant. While it provides an added degree of complexity, both in preliminary calculations and in further design developments, the advantages of the liquid system such as the possibility of controlling the thrust pattern and greater safety in operation where considered in this selection. However, the main reason was the superior efficiency of the Propellant: being this an air launch, the maximum payload is extremely limited unlike a ground rocket which can theoretically always grow. As such, this added efficiency allows for a lesser amount of propellant corresponding to a lighter rocket. It is subsequently necessary to select the specific type of Liquid Propellants of which there are two main categories, with several subcategories. They can be split according to the number of substances used:

- Monopropellants – they are single substance propellants, which are either used cold (usually by means of a catalyst to enable the release of energy) or are capable of burning alone. They have low efficiency but a great degree of simplicity in their use.
- Bipropellants – they work by mixing a fuel and oxidizer which are then ignited in a thrust chamber. More complex, they are nevertheless much more efficient than even hot monopropellants.
- Tripropellants – these are usually bipropellants with an extra chemical added to improve efficiency. While they are the most efficient chemical fuels, they are extremely complex, have never been used in a standard vehicle, and most of the additives are extremely toxic.

And according to storage method:

- Earth Storable – these are chemicals that are liquid at room temperatures.
- Cryogenic – these need to be stored at very low temperatures to remain liquid. While they are the best mixtures available (Liquid Hydrogen and Oxygen) they require heavy and well designed storage and isolation systems, and due to boil off cannot be kept in storage for long.

3.4.2.3 Propellant Selection

Two of the premises of the present work are that the rocket will be carried a given distance before ignition and that it should be possible to launch it at very short notice. The use of a cryogenic propellant requires a lot of preparation time, as the whole system has to be flushed of any contaminant beforehand, and it is also subject to boil-off, which is the propellant loss through escape valves to avoid excessive pressure as it evaporates. It must also be topped off very close to the launch moment, which prevents the rocket to be kept ready to use. All these factors make cryogenic propellants highly unsuitable for use in this type of rocket. Consequently, an Earth Storable Solution has been selected. Regarding the composition, we selected a Bipropellant since it would not make sense to have the added complexity of a Liquid system (vs. Solid) and then use a Monopropellant with low efficient, and the Tripropellant solution was deemed too complex and untested.

The most relevant parameters for the propellant are a high density, preferably a low Chamber Temperature and a high Isp. The usual options for oxidizer are oxygen, fluorine, nitrogen tetroxide, red fuming nitric acid and hydrogen peroxide. The use of oxygen requires a cryogenic solution, which was deemed unacceptable, and both Fluorine and Red fuming nitric acid are extremely toxic, further complicating ground operations. Given that, liquid nitrogen tetroxide (N_2O_4) was selected as oxidizer for which there are two suitable choices of fuel: an equal mix of unsymmetrical dimethylhydrazine (UDMH) and hydrazine, and Rp-1.

Oxidizer	Fuel	Mixture ratio (by mass)	Density (kg/m^3)	Chamber Temperature (K)	Molar mass (kg/mol)	k
Nitrogen Tetroxide	UDM + Hydrazine	2	1.21e3	3372	22.6	1.24
	RP-1	3.4	1.23e3	3293	24.1	1.23

Table 3-8: Fuels using nitrogen tetroxide as oxidizer (16)

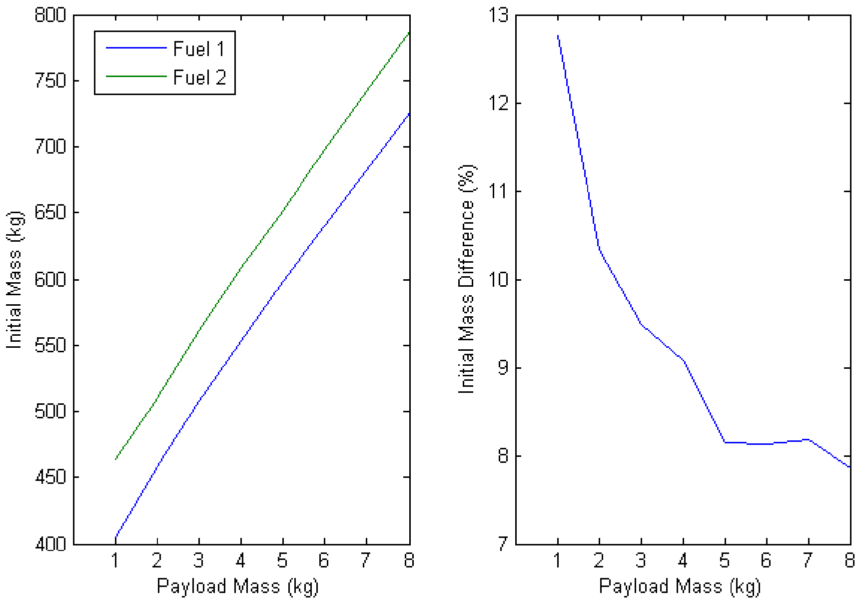

Figure 3-2: Fuel comparison (fuel 1 – UDMH and Hydrazine, fuel 2 – RP-1)

This selection goes beyond merely engineering or performance factors. The first option has a much higher propellant density, both from a higher fuel density as well as a more oxidizer rich mixture (because the oxidizer is denser than either fuel) which in turn allows for a smaller vehicle, correspondingly lighter. However, it is also an extremely toxic fuel, requiring special handling and care especially in case of leakage. RP-1, on the other hand, leads to a less efficient rocket but a safer one, with the added advantage of being cheaper.

The fuel selected was Rp-1. It is more readily available, cheaper and, most importantly, much safer to handle, and seeing as the performance penalty in relation to the other option is not significant.

3.4.2.4 Propellant Parameters

The parameters selected as standard, picking the average value when an interval was available (such as density) are depicted in Table 3-9.

Fuel Density	915 Kg/m^3
Oxidizer Density	1413.5 Kg/m^3
Oxidizer to Fuel Mass Ratio	3.4
Propellant Molar Mass	24.1 Kg/mol
Specific heat ratio (k)	1.23
Chamber Temperature	3290 K
Chamber Pressure	6894757.29 Pa

Table 3-9: Propellant Parameters (16)

3.4.2.5 Nozzle

The main driving parameter of the nozzle is the design altitude at which it operates at maximum efficiency, which defines the area ratio between the nozzle throat and exit and consequently the length and mass. As the rocket operates both in space and atmosphere, a balance has to be found between a smaller area ratio for low altitude operation and a large area ratio for vacuum all the while trying to minimize mass and physical dimensions. Regarding the optimization, an analysis must be performed with the area ratio going from the optimal value at launch altitude to the optimal value for vacuum with the aim of finding the best trade-off point. In this, the air launch proves extremely advantageous, as pressure variations are much higher at low altitude due to the exponential decay with altitude of the atmospheric density, as can be seen from the Figure 3-4, with the launch altitude of 12 km shown.

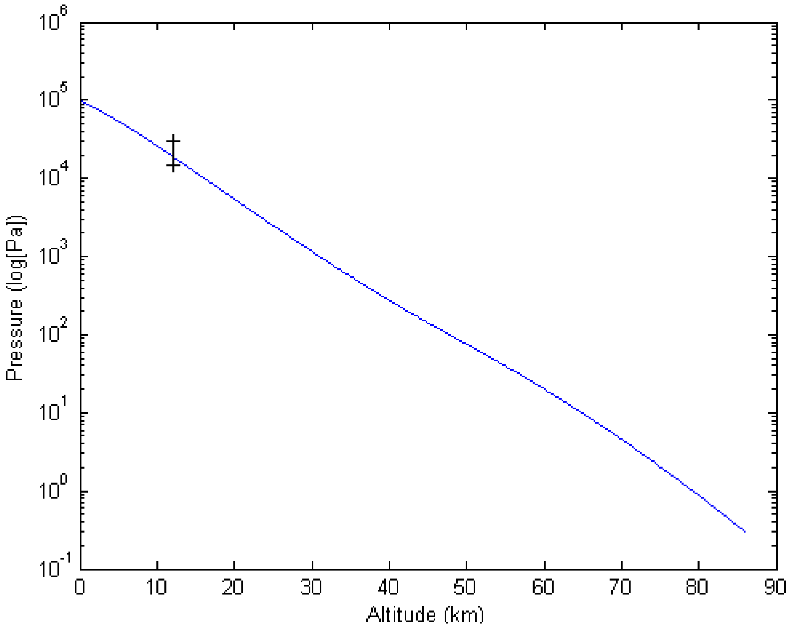

Figure 3-3: Pressure variation with altitude

For each area ratio we calculated the thrust efficiency at each altitude (with Isp representing the efficiency) as well as the physical dimensions, shown in figure 3-4. We considered a perfect isentropic expansion through the nozzle, using the corresponding equations together with the adequate geometric relations to obtain all the nozzle data (16):

$$\rho_0 = \frac{P_{chamb}}{R_{prop} * T_{chamb}} \quad [3.19]$$

$$\rho_t = \frac{\rho_0}{\left(\frac{k_{prop}+1}{2}\right)^{\frac{1}{k_{prop}-1}}} \quad [3.20]$$

$$T_t = 2 * \frac{T_{chamb}}{k_{prop}+1} \quad [3.21]$$

$$A_t = \frac{\dot{m}}{\rho_t * \sqrt{k_{prop} * T_t * R_{prop}}} \quad [3.22]$$

$$Rd_t = \sqrt{\frac{A_t}{\pi}} \quad [3.23]$$

$$v_e = \sqrt{2 * \frac{k_{prop}}{k_{prop} - 1} * T_{chamb} * R_{prop} * \left(1 - \frac{P_e}{P_{chamb}}\right)^{\frac{k_{prop} - 1}{k_{prop}}}} \quad [3.24]$$

$$\rho_e = \frac{\rho_0}{\left(\frac{P_{chamb}}{P_e}\right)^{\frac{1}{k_{prop}}}} \quad [3.25]$$

$$T_e = \frac{T_{chamb}}{\left(\frac{P_{chamb}}{P_e}\right)^{\frac{k_{prop} - 1}{k_{prop}}}} \quad [3.26]$$

$$A_e = \frac{\dot{m}}{\rho_e * v_e} \quad [3.27]$$

$$Rd_e = \sqrt{\frac{A_e}{\pi}} \quad [3.28]$$

$$A_{ratio} = \frac{A_e}{A_t} \quad [3.29]$$

$$L_{nozzle} = 0.8 * \frac{Rd_e - Rd_t}{\tan\left(15 * \frac{\pi}{180}\right)} \quad [3.30]$$

$$mass_{nozzle} = \pi * \rho_{nozzle} * \left(\begin{array}{c} 2Rd_t * L_{nozzle} + \tan\left(15 * \frac{\pi}{180}\right) * L_{nozzle}^2 * t_{nozzle} \\ + L_{nozzle} * t_{nozzle}^2 \end{array} \right) \quad [3.31]$$

$$T = \dot{m} * v_e + A_e(P_e - P_{amb}) \quad [3.32]$$

$$Isp = \frac{T}{\dot{m} * g_0} \quad [3.33]$$

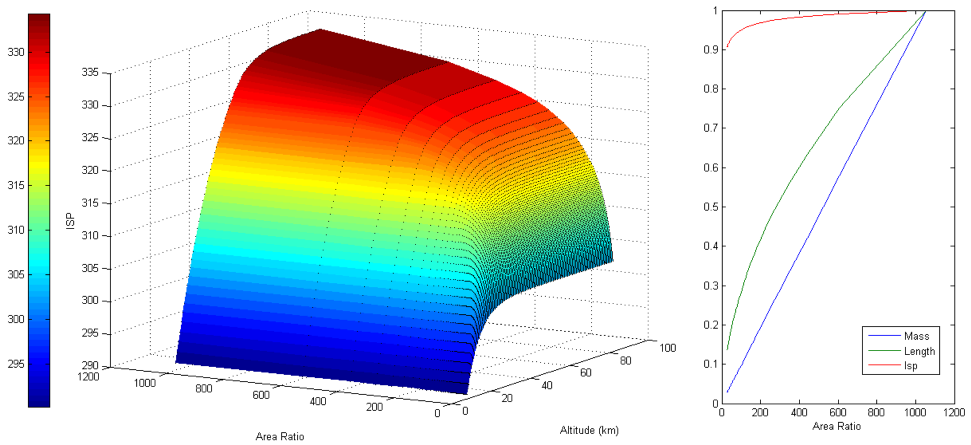

Figure 3-4: Area ratio effect on nozzle

As it is more efficient, we used a bell nozzle. Since the physical characteristics depend on the propellant mass ratio, a dimensionless plot was obtained. Several conclusions can be drawn from Figure 3-4; first of all, due to the air launch, the Isp of the nozzle does not change significantly whether optimized for the launch altitude or space. The vacuum Isp has a value of 303 s when optimized for launch altitude and 334 s for space (under 10% variation), with both cases showing a value of 290s at the launch altitude. It is also easy to see, from the right plot of Figure 3-4, that the physical dimensions of the nozzle have a much more extreme scaling. From both these premises we decided for a compromise value between the two limits that tries to optimize the problem. After some numerical experiments whose results are shown in Figures 3-5 and 3-6, the second showing a more precise analysis, the area ratio was set at 80. The smoother layout seen in figure 3-5 arises from this, as the interpolation points are wider apart leading to a softer evolution of the curve. Since the Isp does not change much with altitude and that most of the thrust action takes place in space, we used the approximation of the Isp being constant and with the vacuum value.

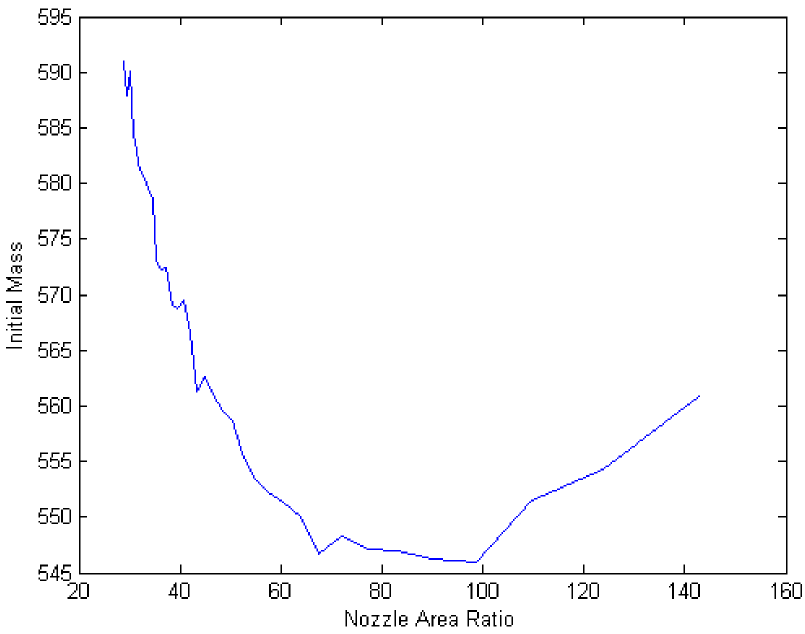

Figure 3-5: Nozzle Area Ratio Optimization #1

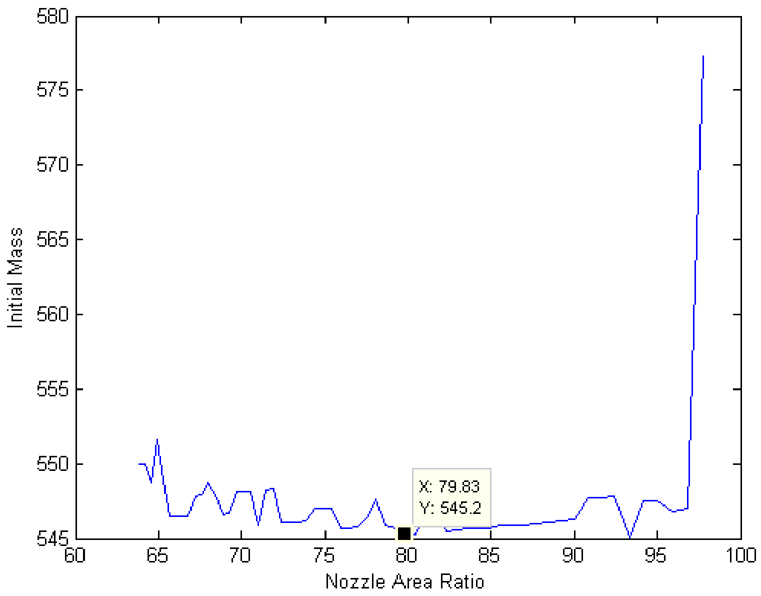

Figure 3-6: Nozzle Area Ratio Optimization #2

3.4.2.6 Mass Flow and Initial Angle

As already mentioned in 3.1, the minimum allowable propellant mass flow is dependent of the initial angle (measured with the horizontal) with contradicting effects – lowering the initial angle lowers the initial mass but requires a higher propellant mass flow while lowering the propellant mass flow lowers the initial mass but requires a higher initial angle. Both parameters were tested together for what was considered the minimum mission the system should be capable of delivering (1 kg and an orbit of 300 km of altitude) and the maximum mission (10 kg at 700 km altitude orbit):

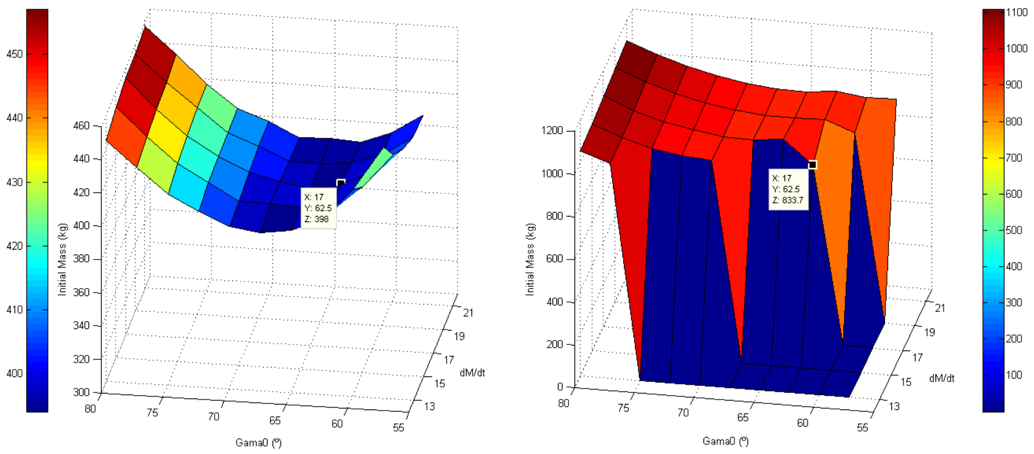

Figure 3-7: Initial Mass evolution with Propellant Mass Flow with Initial Angle

	1 kg at 300km					10 kgs at 700 km				
º \ ṁ	13	15	17	19	21	13	15	17	19	21
80	444,43	450,72	453,78	453,94	457,01	1053,59	1062,43	1080,79	1097,08	1110,09
77,5	429,35	433,64	436,25	438,12	440,87	1004,42	1009,57	1021,73	1040,97	1054,97
75	414,62	418,57	421,36	424,34	424,10		970,47	981,95	1000,00	1013,65
72,5	405,35	408,82	410,13	410,09	410,73		949,53	954,05	966,14	979,26
70	397,23	398,52	401,85	403,22	404,46		938,70	933,59	944,17	954,32
67,5	394,48	398,09	397,02	395,54	395,49			929,31	929,00	939,82
65	397,22	396,62	393,89	394,98	396,06			943,89	929,89	937,42
62,5	404,89	398,25	397,95	396,59	395,12			833,74	933,46	949,19
60	424,15	411,13	405,70	402,48	401,61				880,07	932,13
57,5	450,39	432,00	421,31	414,16	413,31					937,73

Table 3-10: Initial Mass evolution with Propellant Mass Flow with Initial Angle

It is possible to achieve the minimum mission's orbit using a higher angle and lower mass flow but to ensure compatibility with the full spectrum of expectable missions, only the values also allowing for the heavier one were considered. Analysing Figure 3-7 it can be seen that the maximum efficiency point, where the initial mass required to fulfil the mission is lower, is similar for both cases. There are some slightly more efficient value combinations for the minimum mission yet the difference is almost insignificant (under 1%) and the advantages for the maximum mission are considerable, the second best value being over 5% larger. After this initial test to find the general location of the maximum efficiency point, a more precise one with a finer mesh was performed, to more accurately locate it.

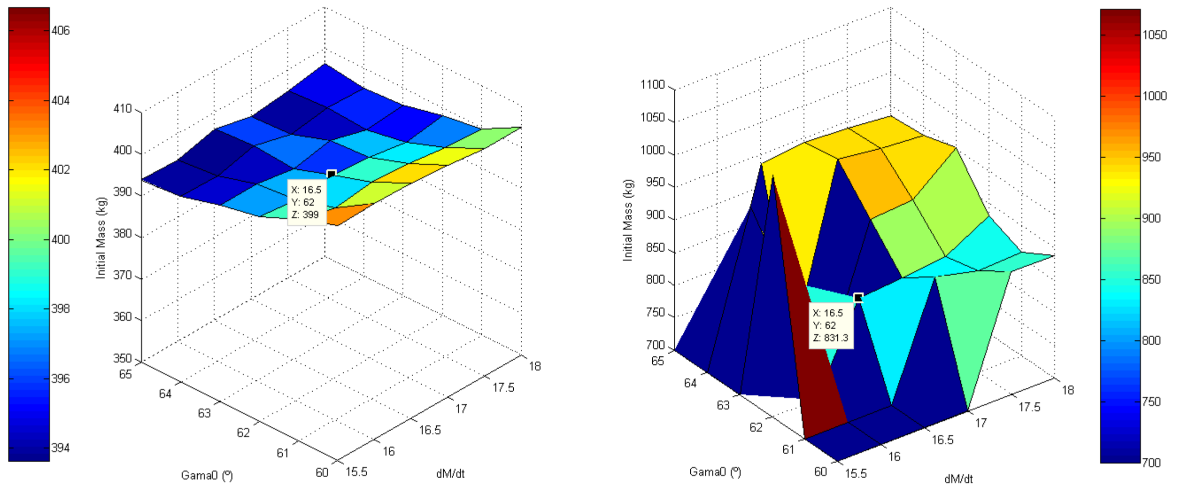

Figure 3-8:Initial Mass evolution with Propellant Mass Flow with Initial Angle, second analysis

On the plot on the right, showing the maximum mission, the non solutions were shown as 700 kg of initial mass instead of 0, to increase readability.

	1 kg at 300km						10 km at 700 km					
º \ ṁ	15,5	16	16,5	17	17,5	18	15,5	16	16,5	17	17,5	18
65	393,87	393,64	395,90	393,89	394,97	396,46			936,56	943,89	940,14	933,52
64	394,53	394,41	396,69	394,37	395,58	395,19		929,45		951,68	942,52	937,67
63	397,22	397,36	395,80	397,63	395,14	395,95		850,11		892,93	897,55	951,41
62	399,22	398,10	398,98	398,29	396,77	398,02	1071,71		831,27	837,29	843,44	880,53
61	403,20	401,04	402,11	401,55	400,29	399,71				872,55	855,60	858,40
60	406,42	406,68	406,50	405,70	404,71	404,85					888,45	887,21

Table 3-11:Initial Mass evolution with Propellant Mass Flow with Initial Angle, second analysis

In Figure 3-8 the new analysis is shown, with the same mission objectives and with the study variables (angle and mass flow) closer to what was found to be the highest efficiency point in Figure 3-7. As before, priority was given to the maximum mission. Comparing to the previous analysis, it is actually seen that it provides a better solution, and thus will be used in the final model. While a slightly smaller value of mass flow (16.5 kg/s vs 17 kg/s) shows slight improvements, it is much more sensitive to the initial angle value (as can be seen by the lack of solutions for the maximum mission), and was thus not selected. Anyway, the difference in efficiency relatively to the selected value is small.

The mass flow ratio of the second stage was also analysed with results showing that the efficiency point is dependent on the mission, expressed by the (relatively) high horizontal error for lower values. Since the objectives emphasize versatility, the selected value was 0.5 kg/s as it fulfils the most demanding mission. Figure 3-10 shows the dimensionless initial mass (initial mass divided by the maximum value) and the horizontal error on the second plot, both versus propellant mass flow ratio.

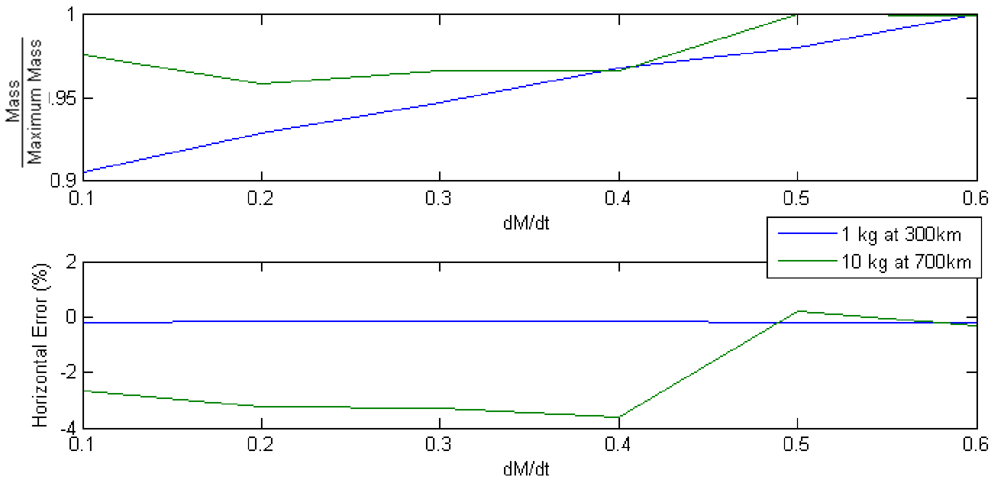

Figure 3-9: Second Stage Propellant Mass Flow

As the mass flow ratio is constant during each stage burn, and as we approximated the Isp as the vacuum Isp, the thrust is also constant during the burn.

3.4.2.7 Pressure system

To pressurize the propellants there are two main types of systems, the use of a turbo pump assembly or of a tank of pressurized gas. The first option requires the selection of a turbo pump cycle from the several available, each with varying degrees of efficiency and complexity, and offers the best propulsive performance. On the other hand, a Bipropellant turbo pump is an intricate machine, increasing price and possibly failure rates. The mass of this type of system is almost constant for a given thrust and does not change with fuel mass, so no judgement of the matter can be held beforehand. The second solution is to have a tank of an inert gas (usually helium) connected to the fuel and oxidizer tanks. While it is a simpler solution, the gas tank is relatively heavy due to the required pressure, and the propellant tanks have to also be kept at a higher pressure and thus need reinforcement. The selected option was the pressurized gas tank. While the mass gain cannot be discussed without an in depth analysis, the difference in complexity was the reason for the selection.

3.4.3 Aerodynamics

3.4.3.1 Model

Aerodynamic calculations are amongst the most computationally intensive, usually requiring CFD codes to provide accurate solutions and making the whole problem extremely time consuming. As a way to reduce the numerical workload we used a simplified aerodynamic model (17), which is an analytical model that was verified to be accurate by comparison with standard codes such as Missile Datcom and Aeroprediction 98 for Mach numbers between 1.5 and 6. As the rocket is launched with a rail system, meaning power is always available, it was possible to design the rocket with no aerodynamic control surfaces (something in accordance with the findings in (11)), and as such only the sections pertaining to longitudinal force were considered. The model's calculations used are now presented. The cone coefficients are determined by

$$C_{Dnc} = \left(0.083 + \frac{0.096}{M^2}\right)(5.73\,\theta)^{1.96} \qquad [3.34]$$

(with θ *being half the cone angle in radians*)

$$C_{Dnp} = D_{Dnc}\left[\frac{0.08(15.5 + M)}{3 + M}\right] \qquad [3.35]$$

With [3.34] corresponding to a perfect cone and [3.35] to a parabola. We adopted the latter since it provides a lower drag. This is applicable for a nosecone having a finesse ratio over 2.5, with higher values presenting lower drag (from a lower value of θ) but a higher weight due to a larger cone. A trade-off analysis was performed to obtain the best value and it is shown in Figure 3-10. where the y axis is a dimensionless measure of the required initial mass. It is observed that the decreasing drag is the predominant factor for the initial values with the increase of mass finally becoming more important later on. We selected the optimal value of 4.6 for the nosecone finesse ratio.

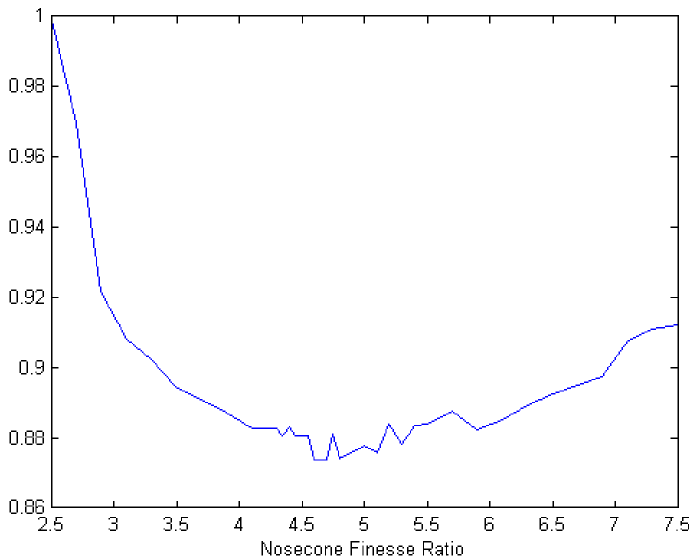

Figure 3-10: nosecone finesse ratio optimization

Viscous drag is the main component of supersonic drag, calculated by

$$C_{Dv} = \begin{cases} \dfrac{1.328}{\sqrt{Re}}, & Re < 10^6 \\ \dfrac{0.427}{(\log_{10} R - 0.407)^{2.64}}, & Re > 10^6 \end{cases} \qquad [3.36]$$

$$Re = \frac{V * L}{\nu} \qquad [3.37]$$

$$\nu = \mu/\rho \quad [3.38]$$

And finally, the base drag from

$$C_{Db} = 0.3129e^{-0.387M} \quad [3.39]$$

The final drag coefficient is the sum of all the separate contributions

$$C_D = C_{Dnp} + C_{Dv} + C_{Db} \quad [3.40]$$

$$D = C_D 0.5\rho v^2 A_{wet} \quad [3.41]$$

$$A_{wet} = A_{nosecone} + A_{casing} \quad [3.42]$$

$$A_{nosecone} = \pi * \frac{\frac{L}{\lambda^2}}{12 * \lambda_{nosecone}{}^2} * \left((\lambda^2_{nosecone})^{1.5} - \frac{1}{8}\right) \quad [3.43]$$

$$A_{casing} = \pi * \frac{L^2}{\lambda} \quad [3.44]$$

3.4.3.2 Optimization

The shape of the rocket is completely conventional with a nosecone on the top and a cylinder behind, with the only parameter that may be optimized being the finesse ratio of the cylinder section which has to be chosen balancing drag and structural mass. This study will be performed in the structural section.

3.4.3.3 Atmospheric Model

To obtain the atmospheric data necessary for the calculation of the Drag forces, we used the 1976 version of the U.S. Standard Atmosphere model (18). This model divides the atmosphere into layers, calculating the data using barometric formulas, depending on whether Lr_b is 0 or not, and is presented here.

Layer [*b*]	Altitude [h_b] (m)	Static pressure [P_b] (pa)	Density [ρ_b] (Kg/m3)	Temperature [T_b] (K)	Temperature lapse rate [Lr_b] (K/m)
0	0	101325	1.2250	288.15	-0.0065
1	11,000	22632.1	0.36391	216.65	0.0
2	20,000	5474.89	0.08803	216.65	0.001
3	32,000	868.019	0.0132	228.65	0.0028
4	47,000	110.906	0.00143	270.65	0.0
5	51,000	66.9389	0.0006	270.65	-0.0028
6	71,000	3.95642	0.000064	214.65	-0.002

Table 3-12: 1976 U.S. Standard Atmosphere model data (18)

For Lr_b=0

$$P = P_b * e^{\left(-g_0 * M_{air} * \frac{h-hb}{R_{air} * Tb}\right)} \quad [3.45]$$

$$\rho = \rho_b * e^{\left(-g_0 * M_{air} * \frac{h-hb}{R_{air} * Tb}\right)} \quad [3.46]$$

$$T_p = T_b \quad [3.47]$$

And for $Lr_b \neq 0$

$$P = P_b * \left(\frac{T_b}{T_b + Lr_b * (h - h_b)}\right)^{\frac{g_0 * M_{air}}{R_{air} * Lr_b}} \quad [3.48]$$

$$\rho = \rho_b * \left(\frac{T_b + Lr_b * (h - h_b)}{T_b}\right)^{-\left(\frac{g_0 * M_{air}}{R_{air} * L_b}\right) - 1} \quad [3.49]$$

$$T_p = T_b + (h - h_b) * Lr_b \quad [3.50]$$

Using for the air constants the values: R_{air}=8.31432 J/Kg.K and M_{air}=0.0289644 kg/mole. Finally, the dynamic viscosity was calculated using Sutherland's formula for air:

$$\mu = 18.27 * 10^{-6} * \frac{291.15 + 120}{T + 120} * \left(\frac{T_p}{291.15}\right)^{1.5} \quad [3.51]$$

3.4.4 Structure

3.4.4.1 General Description

The structure component of the rocket has two main tasks: to withstand the structural stresses and to cover all the vehicle's systems. It must be strong and stiff enough to accomplish the first while at the same time remaining light enough not to hinder the vehicle's performance. The main components are described below and shown in a simple sketch in Figure 3-11:

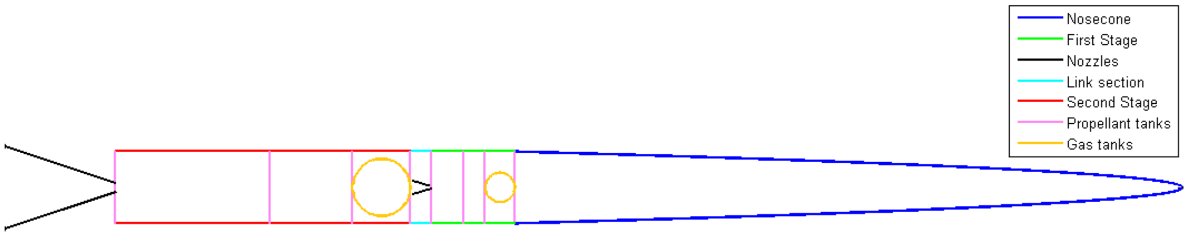

Figure 3-11: Rocket Components

General

- Casing – the outer shell of the vehicle, shaped like a cylinder. It encloses all the subsystems and supports the loads
- Electronics – including the guidance and communication systems.
- Nosecone – forming the top of the rocket, it houses the payload and is shaped as a parabola to minimize aerodynamic drag.

Propulsion

- Nozzle – at the bottom of the rocket, expands the combustion gases and directs them correctly.
- Propellant tanks – of cylindrical shape, they are integrated into the casing. They contain the pressurized fuel and oxidizer.
- Pressure gas tank – a spherical tank (to maximize strength) that contains the gas required to maintain propellant flow.
- Supplementary mass – accounts for such things as plumbing, actuators, thrust chambers, cooling...

Methods for calculating each of these were devised (shown in section 5.6) thus obtaining the final structural mass of the vehicle.

3.4.4.2 Structural Stresses

The stresses applied on the vehicle were divided into three classes, according to type/source.

- Axial tensions – both lengthwise and perpendicular.
    - Longitudinal – the forces applied are Thrust and Drag completely aligned with the longitudinal axis, and a component of the vehicle Weight, creating a compression stress.
    - Transverse – there are two situations: during transportation onboard the carrier the force applied is the total weight; during flight, and as the velocity and thrust vectors are aligned for the whole trajectory, it is solely a component of the vehicle Weight.

These forces generate the flight loads, completely supported by the case. They are calculated by:

$$Force_{Long} = T + D + W_L \quad [3.52]$$

$$Force_{Trans} = W_T \quad [3.53]$$

$$W_L = mass\left[a_x \cos(\gamma) + a_y \cos(\frac{\pi}{2} - \gamma)\right] \quad [3.54]$$

$$W_T = mass\left[a_x \sin(\gamma) + a_y \sin(\frac{\pi}{2} - \gamma)\right] \quad [3.55]$$

- Manoeuvring torques – generated during attitude changes.

As there is only one significant manoeuvre during the flight, and it occurs during a coasting phase (thus with less applied forces), it was defined by approximation that the manoeuvre is performed softly enough that the stresses generated are lower than those during normal operation and can thus be ignored from the analysis.

- Propellant pressure

The propellant tanks are integrated into the casing, meaning that it is also subjected to the gas pressure. Since the pressurizing gas has a higher pressure, an independent spherical tank was used, its mass calculated separately. The propellant tanks are both pressurized at 0.5 MPa and the gas tank at 6MPa, values chosen as a little inferior than conventionally used in (16) to reduce tank stress.

3.4.4.3 Finesse ratio optimization

As discussed in section 3.4 the structural finesse ratio has influence in both the structural weight and aerodynamic drag, a balance having to be found that optimizes the trade-offs. Plotting the evolution of both factors, in a dimensionless form in Figure 3-12:

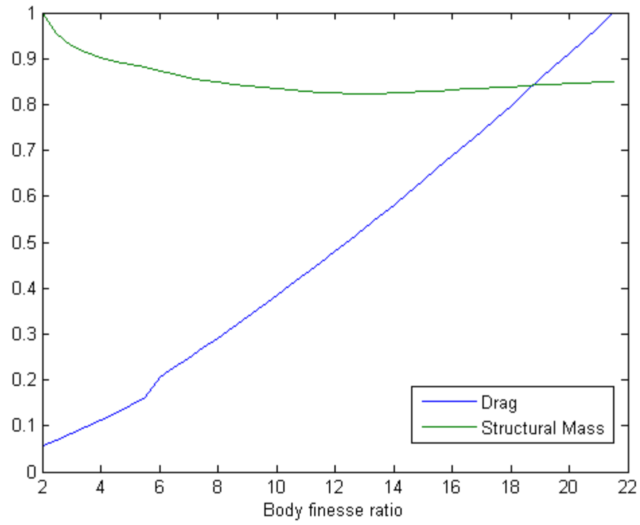

Figure 3-12: Drag and Mass evolution with Finesse Ratio

It is seen that while the vehicle's structural mass has a minimum, the drag force increases constantly with the finesse ratio and as such further analysis is required. However, it is safe to assume from the above data that the optimum value will be lower than the structural mass minimum value, which occurs at a body finesse ratio value around 13. For this analysis three different missions were tested, with the initial mass for each finesse ratio value calculated.

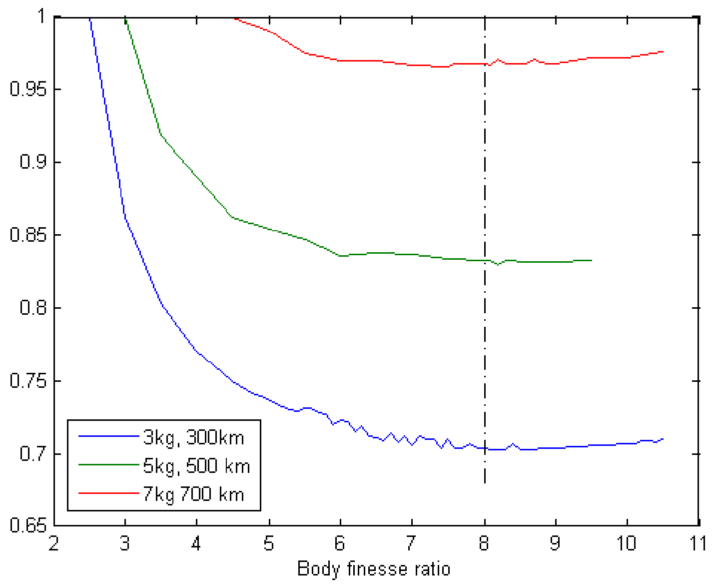

Figure 3-13: Initial mass evolution with body finesse ratio

From Figure 3-13 we see that the initial rocket mass drops steeply with the increase of the finesse ratio, reaching a plateau around a value of 6.5 (depending on the mission) and then starts to gently rise. We chose the value of 8, as it provides a low value across all the missions.

3.4.4.4 Structural layout

The interior layout has been simplified as much as possible. After obtaining the propellant mass, calculations were made to obtain the corresponding oxidizer and fuel masses and respective volumes, increased by 10% as extra volume to accommodate evaporation (16). Using the previously decided structural finesse ratio and the just calculated volume required by the propulsion systems, the structural radius is derived, again increased by 10% to account for plumbing and other systems not explicitly detailed. The rest of the data is calculated from geometrical constraints. The only differences in these calculations between having one or two stages are the existence of two separate spherical gas tanks and the upper stage nozzle, which increases the length by a fixed amount.

The casing layout was kept simple, with the cylindrical outer shell and the tanks simulated by three circular ends inside the shell having the same thickness. This leads to an over dimensioning of the structure, since the tank deposits aren't required to support the same kind of loads, giving the results a certain extra degree of safety. Equations [3.56] through [3.70] show these calculations. Starting with the volume of the propellants and the pressurizing gas (for each stage) (16):

$$Volume_{oxidizer} = \frac{1}{\rho_{oxidizer}} propellant\ ratio * \left(\frac{mass_{propellant}}{propellant\ \ ratio + 1}\right) * 1.10 \qquad [3.56]$$

$$Volume_{fuel} = \frac{1}{\rho_{fuel}} \left(\frac{mass_{propellant}}{propellant\ ratio + 1}\right) * 1.10 \qquad [3.57]$$

$$Volume = Volume_{oxidizer} + Volume_{fuel} \qquad [3.58]$$

$$mass_{gas} = P_p * Volume * \frac{k_{gas}}{R_{gas} * T_0 * \left(1 - \frac{P_p}{P_g}\right)} * 1.25 \quad [3.59]$$

$$Volume_{gas} = mass_{gas} * R_{gas} * \frac{T_0}{p_g} \quad [3.60]$$

$$Rd_{gas} = \frac{Volume_{gas}}{\left(\frac{4}{3} * \pi\right)^{\frac{1}{3}}} \quad [3.61]$$

The structural dimensions may be calculated, for a two stages using equations [3.62] – [3.64] and for a single stage [3.65] and [3.66]

$$L_e = 2 * Rd_{gas\,1} + 2 * Rd_{gas\,2} + L_l \quad [3.62]$$

$$L_l = 1.10 * L_{nozzle\ 2} \quad [3.63]$$

$$Volume_{total} = Volume_1 + Volume_2 \quad [3.64]$$

$$L_e = 2 * Rd_{gas} \quad [3.65]$$

$$Volume_{total} = Volume \quad [3.66]$$

The final dimensions are obtained from eqs. [3.67] and [3.68] and for two stages, the length of each stage is calculated as shown in [3.69] and [3.70].

$$Rd = 1.10 * 0.5 * \left[\frac{4}{\pi} * \frac{Volum\ e_{total}}{\lambda - L_e}\right]^{\frac{1}{3}} \quad [3.67]$$

$$L = 2 * \lambda * Rd \quad [3.68]$$

$$L_1 = \frac{L - L_e - L_l}{1 + \frac{Volume_2}{Volume_1}} + 2 * Rd_{gas\,1} \quad [3.69]$$

$$L_2 = L - L_1 - L_l \quad [3.70]$$

It should be noted that the length calculated is for the cylindrical section only, not taking into consideration the nosecone or the nozzles (except for the linkage section connecting the first and second stage).

3.4.4.5 Nosecone

As explained in the Aerodynamics section, the shape of the nosecone is that of a paraboloid with a given *L/d* relation (4.6, for the optimal solution) and defines the available volume for payloads, primarily as a function of structural diameter. Since this capacity is an important, if not defining, characteristic of the vehicle we deemed it relevant to obtain an estimate of its value. As the usable volume depends on the shape of the payload, three approximations are considered:

- Single volume
- Two volumes, the second having half the radius of the first
- Three volumes, the second having three quarters the radius of the first, the third half the radius of the first.

The results shown are divided by the maximum volume point for a single volume with the radius ranging between [0,1]. As such, obtaining the available volume for a different configuration or size is just a matter of multiplying by the corresponding factor.

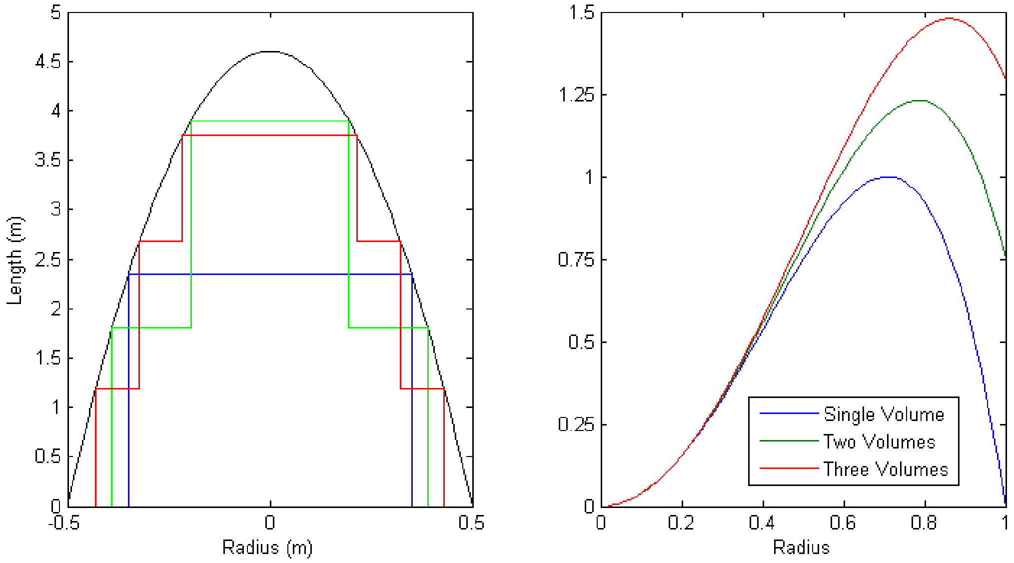

Figure 3-14: Nosecone Volume

Besides the maximum volume, it is also relevant to be able to obtain the relation between full nosecone length and the length of each section, for each volume combination. The values are shown in Table 3-12, presenting the maximum length for each volume as a percentage of the total nosecone length.

Layout	Volume	Nose Length %
Single Volume	Single Volume	51 %
Two Volumes	First Volume	39.2 %
	Second Volume	84.8 %
Three Volumes	First Volume	26 %
	Second Volume	58.4 %
	Third Volume	81.5 %

Table 3-13: Nosecone volume Length

3.4.4.6 Materials

The structural algorithm leaves the material selection open, meaning calculations are performed for all the materials chosen as acceptable for the project and then the lightest solution selected. The material list chosen is presented in Table 3-13, as well as the most relevant characteristics.

Material	Ultimate Strength (MPa)	Density (Kg/m^3)	Minimum thickness (mm)
E glass reinforced plastic	1030	1940	1
Kevlar 49 reinforced plastic	1310	1390	1
Titanium alloy	1240	4600	0.25
Steel alloy	2000	7840	0.25
Aluminium alloy	455	2790	0.25

Table 3-14: Material properties

The minimum thicknesses were chosen from the availability of metal sheets that thin, for the alloys, and to ensure good orthotropy and characteristics, for the composites. These are calculated as shown in eqs. [3.71] – [3.74], each associated by one type of stress (propellant tank pressure, longitudinal stress and transverse stress respectively).

$$t_1 = P_p * S_f * \frac{Rd}{\sigma_{Ut}} \quad [3.71]$$

$$t_2 = \sqrt{\frac{Force_{Long} * S_f}{\sigma_{Ut} * \pi} + Rd^2} - Rd \quad [3.72]$$

$$t_3 = \frac{Force_{Trans} * S_f}{\sigma_{Ut} * 2 * L} \quad [3.73]$$

$$t_s = larger\ of\ (t_1, t_2, t_3) \quad [3.74]$$

We found the best material to be the Titanium alloy, the thickness varying according to the size of the rocket (due to differing missions) as can be observed in Figure 3-15 where the structural mass of the first stage (divided by the lightest option) for each material is shown. The masses are calculated as seen in eqs [3.75] – [3.79]. The numerical subscript found in some terms represents the stage (when applicable).

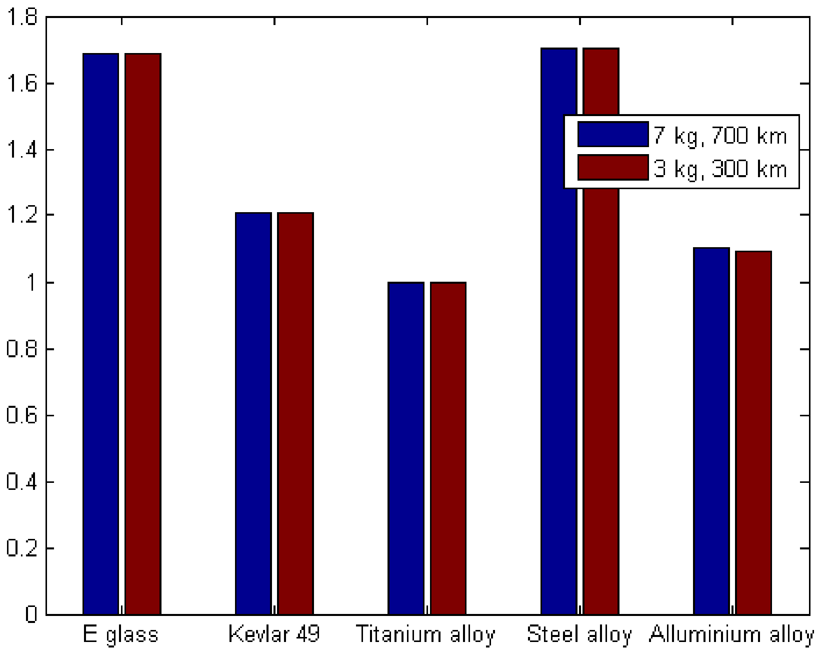

Figure 3-15: Structure mass according to material

$$Mat_{Vol\,(1,2)} = \pi * [(Rd + t_s)^2 - Rd^2] * L_{(1,2)} + 3 * \pi * Rd^2 * t_s \quad [3.75]$$

$$Mat_{mass\,(1,2)} = Mat_{Vol\,(1,2)} * \rho_{Mat} \quad [3.76]$$

$$Mass_{nosecone} = [\pi * (Rd + t_s)^3 * 2.5 - \pi * Rd^3 * 2.5] * \rho_{Mat} \quad [3.77]$$

$$Mat_{Vol_g\,(1,2)} = 4 * \pi * \left(Rd_{gas\,(1,2)} + t_{gas}\right)^2 - Rd_{gas\,(1,2)}^2 \quad [3.78]$$

$$Mat_{mass\,s_g} = Mat_{Vol_g} * \rho_{Mat} \quad [3.79]$$

The nozzles and burn chamber have to be constructed in a material capable of withstanding the high temperature these components are exposed to without significant loss of strength. We selected a Niobium C130 alloy, similar to the one used in the Space Shuttle OMS system's nozzles. Shown in Table 3-14 are the most relevant characteristics (19).

Density	8850 kg/m^3
Melting Point	2350 +-50 ºC
Temperature (K)	Ultimate Tensile Strength (Mpa)
300	396.45
810 – 1144	275.79
1366	193.05
1644	82.73

Table 3-15: C130 alloy characteristics

3.4.4.7 Electronics and guidance

It is somewhat complicated to estimate the mass required for the electronics of the system. Searching for cubesat components, we have found communications sets weighting from 80 to 200 g. We thus estimated a total mass of 300 g for all the required electronics.

3.5 Software

We present here a quick outline of the software designed to perform the calculations, a more in depth guide can be found in the appendix. The software was developed in Matlab in a modular form with all the calculations for the structure, aerodynamics, propulsion and trajectory being performed separately and then integrated. The program works with a loop. In each iteration the results from the previous are used as input for the various modules (an initial guess for the first) resulting in the required initial mass, until convergence is achieved. Due to the nature of the problem requiring the use of a propagator, it was not possible to efficiently use parallel calculating for the whole program, but it was used when possible to accelerate calculations. The time for each run was dependent on the objectives (a larger rocket had a higher flight time and thus more calculations) and also on computer fluctuations and in table 3-16 we show some values obtained from the 209 analyses required to obtain the data displayed on figure 4-1.

Shortest Run	226.2 s
Average Run	422.3s
Longest Run	1995.6 s

Table 3-16: Analyses Time

3.5.1 Numerical Convergence

As a verification of the numerical convergence of the code, a quick analysis was performed to test the effect of increasing the number of points by reducing the width of the computational mesh.

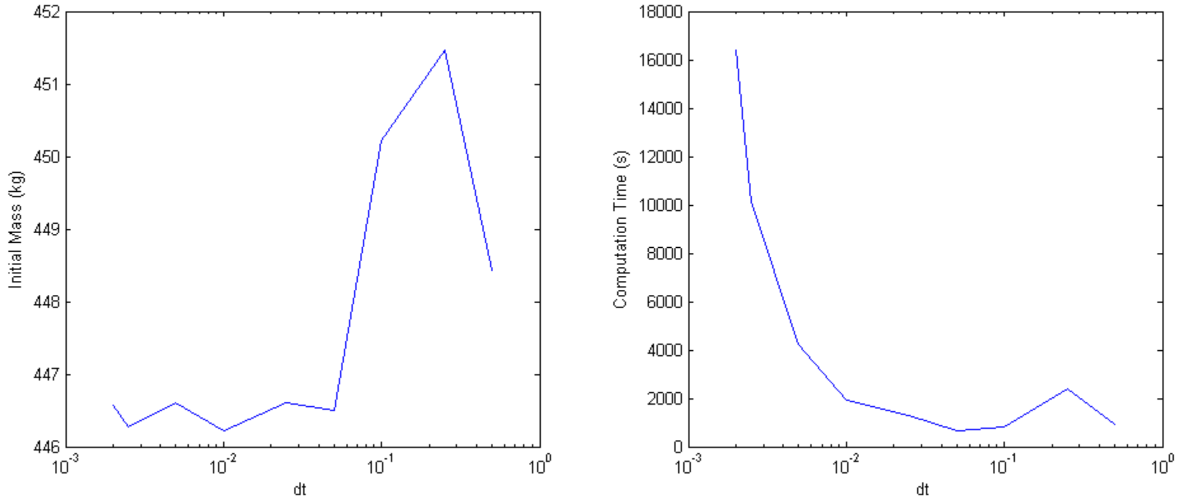

Figure 3-16: Convergence analysis

What can be observed in Figure 3-16 is that the reduction of the mesh after a threshold brings little change in the results, while increasing the computational time taken exponentially. Thus, the value of d*t* used was $2.5*10^{-2}$ as a compromise solution.

4 Design

With all the main design variables well defined one of the goals of the present work is complete, the study of the parameters that define an air launched nanosatellite launcher. We now will present the rocket as an integrated system.

4.1 Mission Layout

The vehicle's size is dependent of the desired mission. As a generic case, the selected mission was a circular orbit at a given altitude with a satellite of a certain mass. Extrapolation to elliptical orbits is easy to determine. The mission variables are:

- Payload mass: 1 to 10 kg, the limits for the nanosatellite class.
- Orbital Altitude: 250 to 700 km, low Earth orbits. The lower limit is able to support the typical lifetime of many missions while the upper limit is still admissible for the class.

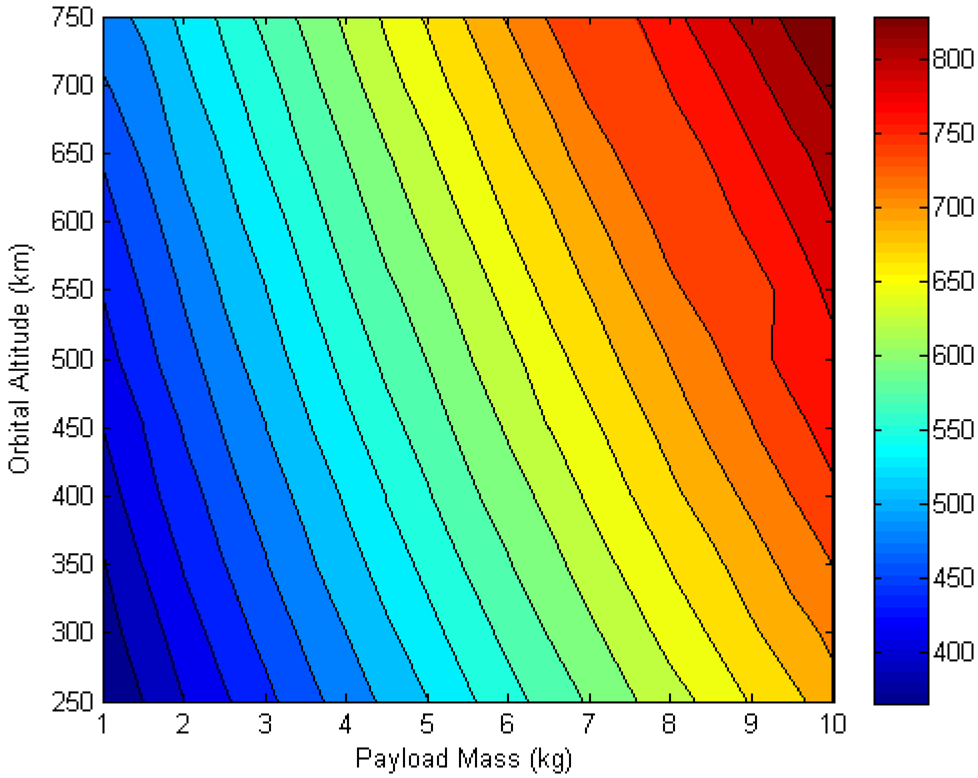

Figure 4-1: Initial mass evolution with mission parameters

The results are shown in Figure 4-1. The colours correspond to the initial mass of the rocket (the values are shown in the sidebar). There is a very clearly defined and regular evolution of mass with the desired final mission. It can also be seen that the desired payload mass has a bigger influence on the rocket size, shown by the clear bias of the curves' slope has towards the payload mass axis.

4.2 Design Points

Having obtained a "pareto front" representing the optimal rocket for a given mission, we can choose a design point and perform an in depth analysis of a single rocket. In the interest of flexibility and providing a range of solutions, it was decided to analyze three significantly different rockets, each optimized for a different mission: 3kg at 300 km altitude (Rocket 1), 5 kg at 500 km altitude (Rocket 2) and 7 kg at 700 km altitude (Rocket 3). These three rockets include one in the lower limits of the range, one average and one that is almost in the top limit for the desired missions, we can consider these sets of tests representative of the full range of missions. The most relevant data of each rocket is presented in Table 4-1.

	Rocket 1	Rocket 2	Rocket 3
Initial mass (minus payload)	443.61 kg	578.00 kg	706.78 kg
First stage fuel mass	331.5 kg	464.53 kg	595.43 kg
First stage structural mass	34.94 kg	42.00 kg	48.53 kg
Second stage fuel mass	69.28 kg	63.60 kg	55.21 kg
Second stage structural mass	7.895 kg	7.868 kg	7.608 kg
Structural mass	42.83 kg	49.87 kg	56.14 kg
Length	3.480 m	3.823 m	4.103 m
Structural radius	0.218 m	0.239 m	0.256 m
Single volume payload capacity	0.0744 m^3	0.1013 m^3	0.1253 m^3
Nosecone Length	2.001 m	2.198 m	2.359 m

Table 4-1: Design points

The relative size of the stages change with the mission: the first stage's mass increases with a more demanding mission, representing the required energy to raise a higher mass to a higher altitude, while the second stage becomes lighter for the more extreme mission, which is due both to a lower orbital velocity at a higher orbital altitude as well as a higher velocity at the end of the gravity turn manoeuvre. The initial mass always increases with the more demanding the mission (heavier load / higher altitude), which leads to a larger rocket.

4.2.1 Inverse Calculation

Having chosen those three representative rockets it was considered useful to analyze the full range of capabilities of each one, that is, the spectrum of missions each is capable of fulfilling. The calculations were performed by assigning a target altitude and calculating the allowable payload mass, with the results presented in Figure 4-2.

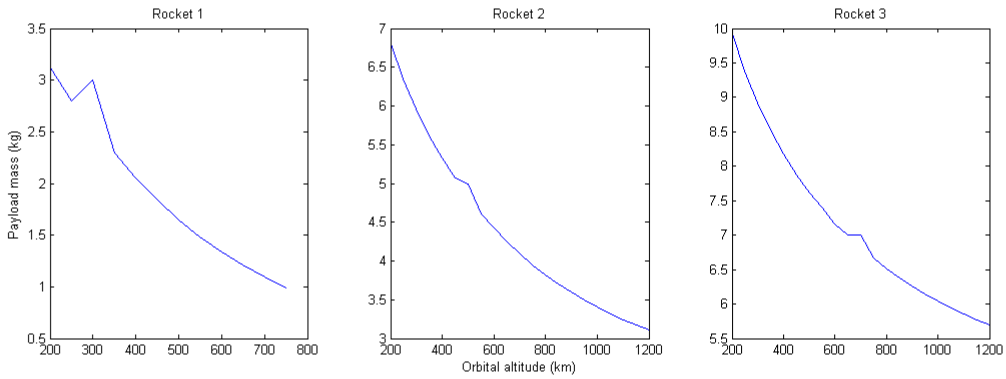

Figure 4-2: Rockets' capacities

Some considerations are in order before continuing. First, the calculation method used for the inverse calculations is slightly different from the optimizer and, being a secondary objective of the work, it does not provide as high accuracy as the main calculations. Second, each rocket is optimized for a given mission, which presents itself as a blip in the plot, the visible irregularity that rises up from the curve. The influence of the optimization is visible in the plots, as the payload capability for the optimum point is higher than the value that could be extrapolated from the rest of the curve. Finally, it was decided to consider results under 1 kg of payload mass irrelevant and they were trimmed.

Except for the blip representing the optimized point, the evolution of the payload mass with altitude is very similar for the three vehicles, logically decreasing with final altitude. Rocket 1, being the least capable, is the most sensitive to changes in the mission, as doubling the final altitude more than halves the payload mass (1.34091 kg to 600 km, under 45% of the initial mass). For the other rockets the reduction isn't as extreme, the second rocket being capable of taking 3.4025 kg (about 68% of the initial mass) to 1000 km, and rocket number 3 5.4396 kg to 1400 km (or 78% of the initial payload), this last result calculated merely for comparison as that orbit is out of the expected useful range. Reducing the orbital altitude to be half the design value, we obtain a much closer evolution, rocket 1 taking 3.699 kg to 150 km (about 1.23 times the original mass), rocket 2 is capable of lifting 6.35 kg to 250 km (or 1.27 times the initial mass) and rocket 3 can launch 8.522 kg to a 350 km altitude (an increase to 1.22 the initial mission).

4.2.2 Performance improvement

Should the carrier plane be capable of a higher performance than the selected as the baseline for the project, it would bring advantages in terms of the payload capacity. As a rough estimate of what may be expected, the final payload for rocket 3 (as it is the one the least sensitive to changes in mission) at the design point was calculated varying the initial conditions. In figure 4-3, the first plot shows the payload value and the second plot the payload value as a percentage of the design payload capacity.

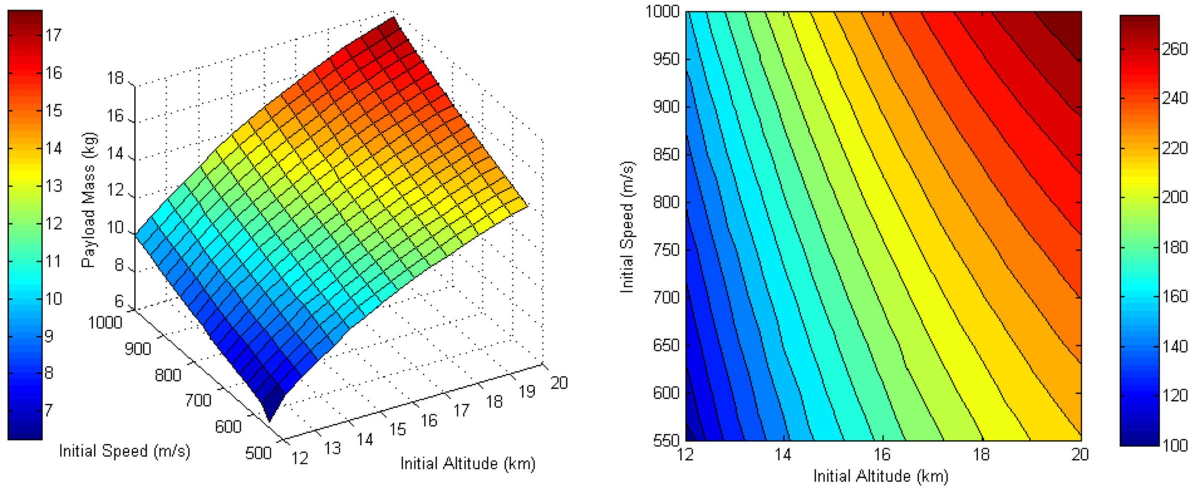

Figure 4-3: Performance improvement with different initial conditions

An improvement of the initial conditions always leads to a higher performing rocket. It is much more beneficial to increase the initial altitude than the initial velocity; an increase in 25% of either clearly shows that, as increasing the velocity to 700 m/s leads to a carrying capacity of 7.7649 kg (11% increase) while incrementing the initial altitude by the same proportion, to 15 km, allows for a payload mass of 10.423 (49% extra).

This demands further thought. First of all, the factors are not really directly comparable, even increasing them by the same proportion does not represent an equal increase in terms of effect. Second, the difference in efficiency may arise from the direct effect of each parameter; while increasing the initial altitude is always beneficial, by reducing the potential energy difference required and reducing drag by launching at a thinner atmosphere, the increase of the initial velocity has contradicting effects: a higher value signifies a higher initial kinetic energy, but it also leads to increased drag.

4.3 Family/Developments

As is usual in space launchers design, it is possible to fulfil a wider range of missions by developing a family of launchers, using a common core and modifying some components. The most frequent changes are the use of a varying number of parallel boosters such as Atlas V family which can use up to 5 boosters (20), different fairings for different payload volumes (again the Atlas V or the Ariane V (8)) or even different engines (Ariane V).

Any developments in the propulsion system would bring benefits for the final performance of the rocket. As proven in the Ariane V rocket, which now has a cryogenic upper stage, this is a method that ensures an increase in payload capacity. Given the propellant choices made, there is also the possibility of increasing the quality of the hydrocarbon (maybe by using a more refined mixture) which will also bring the desired advantages. There is also the option of using strap on boosters which provide extra thrust in the initial phases of flight. While useful in terms of increasing the payload capacity, this adds complications especially in terms of the interaction with the carrier plane (as mentioned in the configuration studies).

A commonly used option is that of different nosecones, usually increasing the interior volume. However, since the designed nosecone is already extremely large for the payload mass and as it is optimized for the launch it seems that there would be little to be gained.

4.4 Use considerations

Even though the current work is a preliminary project, it is still useful to take some time to consider some aspects of the use of a vehicle like the one designed.

4.4.1 Handling and risks

When studying a project whose results are desired to have a global use, special care has to be taken regarding the risks in both use and handling, as a requirement for very tight or specialized procedures will woefully reduce the number of possible users. That was already considered in the fuel selection, where it was selected a non toxic option, which also has the added advantage of facilitating and reducing the risk of the transport operations of the vehicle; it may be carried empty and filled with propellant on location. There is always a certain degree of risk when working with rockets, it is inherent to space exploration and will most certainly never be fully eliminated. However the choices made, from a non toxic fuel and to non hypergolic propellants suggest a low and admissible risk rate in terms of ground handling and storage.

4.4.2 Storage and reliability

The vehicle is designed to be kept in storage ready to launch when required. The use of earth storable propellants was fundamental in this matter, as the rocket may be stored completely fuelled for a significant period of time (estimated to be around 10 years for the oxidizer (16)). Since the design choices were made with simplicity in mind, avoiding complex mechanisms whenever possible, there is no reason to expect relevant reliability issues, especially if the vehicle is adequately stored and periodically checked.

4.4.3 Quick response estimate

Two of the most interesting and useful aspects of an air launch are the reduced dependence on launch windows, and the absence of weather interference. In practical terms this represents the ability to operate on a short notice, a rising demand in the field of space launches. By having the rocket in storage fuelled and even with the payload already inside (possible if it is a standard satellite), the time required for the launch is reduced to the time required to prepare the carrier for launch and the ferry to launch point – hours, as opposed to the weeks, months or even years missions usually take. This allows for a quick response to events, such as terrorist attacks or natural disasters, and a much easier real time monitoring of situations.

5 Conclusions and final considerations

The development of small satellite launching systems was considered an interesting and relevant issue to study since it hasn't accompanied the development of nanosatellites. Results in the literature suggest that the use of an air launch may bring advantages for launching these type of small systems.

Most of the developed work related to this subject tend to focus on one particular issue of the problem, with the air launch system appearing only as an example or case study of the particular aspect studied instead of as being the ultimate motivation. Conversely, in the present work a detailed analysis of the system as a whole was developed, reviewing its most important aspects. Several trade-off analyses were developed towards obtaining a near optimal solution that fulfils the objective of designing a fairly simple and affordable system capable of launching nanosatellites to low Earth orbit in a responsive way.

To establish a framework for the analysis, an initial assessment of the problem was conducted where the main intervening parameters were identified and categorized: Carrier (initial conditions), Trajectory and Rocket. A dependency chart was also created to assess how all these factors relate. While we do not claim to have considered every relevant factor, as that would not be feasible during the preliminary design phase of the system, which is the scope of this work, we took into account the most influential, and all of the main factors have been discussed even when not deeply analyzed.

To be able to understand the impact variations in the parameter have on the design of the system, simulations were developed that model the problem and allow for the analysis of the parameters both individually and in correlation with each other. Several hundred simulations were performed to find optimal points for the various parameters, often involving trade-off studies between several simultaneous factors. This promoted a deeper understanding of the problem as a whole, and in particular of the factors that most influence the performance of the vehicle.

Regarding the carrier, we were limited by the capabilities of existing aircraft. After taking several possibilities into consideration, we selected a fighter-type plane for its superior performance and greater availability. Having considered a supersonic launch to be feasible, a baseline carrier (as an average of the most widespread fighters) was defined as having an initial speed of 550 m/s and initial altitude of 12 km. The best initial angle for the problem was determined to be 62.5º after a trade-off analysis with the propulsion system. Tests using the same rocket with different initial conditions (to represent the use of a better than average carrier) have shown that while superior plane performance always leads to superior rocket performance, increasing the initial altitude is more advantageous than increasing the initial speed.

Trajectory optimization is a very complex problem and was never one of the goals of the project. The type of trajectory used is merely a realistic approximation of a real flight path, consisting of two distinct burns (each associated with a stage), the first providing the necessary energy to reach orbital altitude and the second accelerating the payload to orbital speed. The first burn contains an atmospheric segment which leads to severe structural constraints, as a rocket is designed to withstand loads mainly in the axial direction, the solution to which was to perform a gravity turn trajectory (12) thus ensuring the thrust and velocity vectors are aligned. The second burn is simply a straight ahead acceleration.

We focused our study in the definition of the rocket: aerodynamics, structure and propulsion were analyzed separately taking into consideration possible correlations between them.

The aerodynamics were simulated using a simplified model for rockets (17). It is a simple analytical model, proven to be accurate by comparison with sturdier and industry accepted software, and usable for a Mach number between 1.5 and 6. As the rocket has no aerodynamic control surfaces, the only optimization available is the finesse ratios of both the nosecone and the cylindrical body. The first was optimized with a value of 4.6, which was shown to minimize required initial weight, while the second was optimized in the structural section.

Regarding the structure the rocket was considered to be composed by a parabolic nosecone followed by the cylindrical section with the nozzle in the end, which is the basic form of a generic rocket. As we considered a rocket with two stages, proven to be much more efficient, the cylindrical section is comprised of the first stage body, second stage body and a linkage ring covering the second stage nozzle. Inside the body, space has been calculated to house the propellant tanks and the pressurizing gas tank, other components fitting around these as everything has been overdesigned in terms of volume. Structurally, the propellant tanks are simulated by 3 circular sections. Having the data for every moment throughout the flight, the maximum stresses were calculated both in the longitudinal and transverse directions. With this the minimum sizing required for each material option is obtained and the lightest solution selected, shown to be the titanium option. An optimization of the structural finesse ratio was also performed, resulting in the final definition of the structure.

Due to the large number of options available for the propulsion system, an effort was developed to limit the number of analysis by opting for reasonable solutions when possible. After a heuristic analysis and literature review (10), we selected a liquid fuel option as it provided the best performance, choosing a bipropellant fuel as it balances good characteristics with availability and moderate complexity. The use of a cryogenic solution has constraints that are not compatible with the desired objectives of simplicity and promptness, which has led us to an earth storable solution using nitrogen tetroxide as a propellant and Rp-1 as fuel. As a pressurizing system and given the small size of the engine, a tank with pressurized hydrogen, which is a very simple solution, was selected. Finally, the mass flow rates were set at 17 kg/s and 0.5 kg/s for each stage respectively, since analyses showed that these values minimize the required fuel mass.

With all the parameters defined we simulated the whole system to obtain good, near optimal, rocket solutions (within our performance constraints) for a range of mission objectives, the payload mass ranging from 1 to 10 kg (nanosatellite class) and the orbital altitude (for a low Earth orbit) from 250 to 750 km. Results show that the increase in payload mass has a greater influence in the rocket than the orbital altitude – while not directly comparable, doubling the payload leads to a heavier rocket than doubling the final altitude.

We chose 3 design points as examples for an extended analysis. Rockets 1 through 3 with, respectively, 3,5, and 7 kg of maximum payload, and a service orbit of 300, 500, and 700 km of altitude. As expected, the rockets grow from 1 to 3 in terms of mass and size with the first stage following this trend, but the second stage becomes smaller from 1 to 3. The overall growth is explained by the need of a bigger launcher to provide the energy required to take a heavier load higher, while the reduction in size of the second stage is due to the higher horizontal velocity at the end of the first stage burn (as it is longer) as well as from a lower orbital velocity at higher altitude. This is also defined by the trajectory, and a different plan may change the mass evolution of the stages.

An inverse calculation was performed to estimate what missions the rockets would be able to fulfil beyond their optimal point. Besides the obvious conclusions that it is possible to carry a heavier load to a lower altitude and vice-versa, we found that rocket 1 is the most sensible to variations in mission. This is visible in the results as they clearly show the optimal point as a discontinuity in the curve, with the variation being much smoother for the larger rockets. This may be explained by the fact that rocket 3 is much more powerful, thus being able to more easily accommodate small changes in the mission. We also tested the effects of using a higher performing carrier. It always leads to a better carrying capacity, with an increase in the launch altitude bringing more advantages than an increase in the launch speed. As increasing the altitude brings two positive effects (higher potential energy and lower drag) and increasing the speed brings contradicting effects (higher kinetic energy and higher drag) this is reasonable.

This work may be used as a framework upon which to build a more detailed analysis or even a complete engineering project. Having studied the problem from a global perspective preliminary solutions were obtained, which was the main goal of the present work. The results can be used as starting point for a more detailed design and full optimization processes.

There are some limitations arising from the methods used. First of all the use of the heuristic analysis, while valid, introduces a bias in the results. While our decisions have always been justified and adequately researched, there is always the possibility of having ignored some options that would have led to better solutions.

The simplifications that were used also contribute towards this. The structural layout of the vehicle, as well as the stress analysis, was streamlined towards faster calculations with a consequent reduction in precision. Another item that was simplified was the trajectory. The one chosen is realistic and would be possible to use in a live launch, but a more optimized solution is extremely likely to exist. The calculation of that solution was not an objective of this work, and for the study of the interaction of the various parameters the one chosen was more than adequate.

These limitations do not invalidate the results obtained. The interactions between the parameters remain the same regardless of the optimization level, as do most if not all of the engineering decisions.

This work has shown that an air launch is a viable solution to the nanosatellite launch problem, being possible to obtain a responsive and relatively simple system by making use of broadly available technology, putting such system within reach of many countries and organisations. From this point of view, this type of systems together with the development of the nanosatellites themselves, can promote a broader access to space and its advantages.

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7 Appendix

Here we present a description of the working of the numerical software both overall and each particular module. It is presented here only as a complement to the information already displayed, as the numerical aspects of the work are not too relevant.

A.1 Overall Layout

The code has been developed from scratch using Matlab and is split amongst several modules, each with its own M-file(s). The modules are described below.

- Main Module – Controls the other modules' work, calling them as required, as well as setting the global variables for the runs. Dependencies:
  - Variables file
  - Gravity turn module
  - Orbital module
  - Structural module
  - Trajectory propagator
  - Writefile
- Variables file – Not exactly a module, it contains most of the variables required for the simulations, both in numerical/computational terms as well as the vehicle and mission properties. Dependencies:
  - Nozzle module
- Gravity turn module – Performs the calculations from launch until first stage burn out, calculating the required propellant mass. Dependencies:
  - Propagator A
- Orbital module – Calculates the propellant mass required for the second stage burn.
- Structural Module – Given the required propellant mass, calculates the dimensions for each stage and the respective mass of all components. Dependencies:
  - Nozzle module
- Trajectory propagator – Propagates the full trajectory.
- Writefile – Exports all the results in txt format.
- Atmdata – Calculates the atmospheric data for a given altitude.
- DragCalc – Calculates the drag values.

The inverse module was created to calculate the capabilities of a rocket for different missions.

- Inverse Module – Given a rocket's data, calculates payload capability. Dependencies:
  - Propagator D
  - Propagator E

- Orbital module 2

There are also 2 post processing tools: Readfile, which opens the results file and organizes the relevant information and Analyzedata, which organizes data for plotting.

A.2 Main Module

The main module is stored in the file Maincall.m, and calls all the other functions starting with the variables file, thus loading all the program variables. The code has been prepared to run in batch mode, with a series of for cycles varying the parameters as required. The iterative process is then run inside a while cycle controlled by the following conditions:

- Number of iterations under the limit (30)
- Either the horizontal error, the vertical error or the mass variation remains over 1.5%.

The value for the maximum error was considered adequate seeing as the nature of the present work is more geared towards studying the interaction of the various parameters than obtaining a completely accurate solution. Not only that, but given the nature of the project - a low cost solution for satellite launches – it is reasonable to expect a lower overall accuracy when compared to standard solutions.

The function calls the Orbital module, which calculates the propellant mass for the second stage. Given that, the Gravity turn module is run, calculating the propellant mass for the first stage as well as the flight details (accelerations, velocities, positions...) which are then used to calculate the forces applied at each instant on the vehicle, both in the longitudinal and transverse axis. The maximum value is chosen as the limit load: and then used as input for the structural module, the main module receiving the results and arranging them for use. The mass variation from the previous iteration is then calculated, and the trajectory propagator run to calculate the vertical (orbital altitude) and horizontal (speed) errors. The cycle is repeated or finished according to these results. Should the results converge, the writefile function is called to save the results and a new simulation is started.

A.2b Variables File

This file, vars.m, contains the variables required for the simulations. The variables are organized in classes:

- Global variable definitions – For those variables that are used in several modules.
- Program variables – Variables that control the computational execution of the program.
- Universal constants – Earth’s radius, gravitational constants...
- Initial conditions – The launch conditions: speed, angle and position.
- Objective – Orbital altitude and payload.
- Vehicle variables – Number of stages, propellant mass flow rates, fuel selection and Isp. The Isp is calculated by a call to the Nozzle module.

- Extra variables – Mostly concerned with program functioning.

A.3 Gravity turn module

This module is housed in two separate files, mver.m and mver2.m, and calculates the required propellant mass for the first stage. Mver.m is run first, calling the Propagator A successively with increasing initial propellant masses until the payload's final mechanical energy associated with the vertical movement is above the orbital potential energy variation. When the value is obtained, further analyses are performed until a convergence is reached, considered as a value within 0.5% of the required energy variation. Mver2.m is then called, repeating the calculations with corrections for Drag.

A.3b Propagator A

Given initial conditions, the Propagator uses the motion equations to calculate the full flight path of the vehicle. It runs a cycle until convergence is achieved, considered to be when the residue in both the x and y directions is lower than 10^{-6}. The trajectory is divided into a linear mesh with a time resolution defined by the user, and then the equations are solved for each moment. The other values cannot be calculated directly from an equation, and are calculated using a simple first degree backwards finite differences scheme. The Drag forces are calculated as shown in section 3.3.3.

A.4 Orbital Module

This module calculates the fuel required for the second burn, which accelerates the payload up to orbital velocity. Given that the burn is in space, and thus without drag, we chose to use Tsiolkovsky's rocket equation which provides a near perfect solution.

A.5 Structural module

The structural module performs the calculations regarding the physical dimensions of the main components as well as the respective masses. It is stored in files StrMass(2), StrCalc(2) and varsstruct, the subscript corresponding to the number of stages, the final file storing the necessary variables. The first file works as a main function, calling the others and performing the final calculations, starting with a call to Nozzle module which returns the propulsion system's data. The second file is then called and the calculations performed.

A.6 Nozzle module

This module uses isentropic relations to calculate the nozzle thrust as well as physical characteristics. It is stored in the file nozzlecalc.m and varsnozzle.m, the second one containing the required variables.

A.7 AtmData

This function uses the 1976 version of the U.S. Standard Atmosphere model to calculate atmospheric data with altitude.

A.8 DragCalc

The Drag module uses the equations described in section 3.3.3 to obtain the Drag forces.

A.9 Trajectory propagator

This module propagates the full trajectory, including both burns. The equations are the same as in the Propagator A and will not be repeated here. The differences arise from the need to perform a full profile run, instead of just the first burn, and are mentioned below.

- First flight part: first stage burn, the same as Propagator A.
- Second flight part: small coast section, the same with Impulse=0.
- Third flight part: second stage burn, shedding of mass and all the impulse is in the x direction, only change is shown. Also, drag is considered 0, for spaceflight.

A.10 Inverse Module

The inverse module is used to calculate the payload capacities of a given rocket for differing conditions, usually orbital altitude. It is stored in InvCalc.m, and works mostly as a main function, managing and calling the other parts of the code.

A.10b IteratorD

This module, contained in IteratorD.m, handles the calculations for the gravity turn part of the trajectory. It works in two different ways, depending on the desired final altitude value. Should it be lower than the design altitude, it calculates the first stage burn required to obtain orbital energy, using the remaining fuel (together with the second stage) to increase velocity until orbital speed. In case the desired altitude is higher than the design value, a burn from the second stage is used to reach orbit, the remaining fuel used to accelerate. Seeing as the equations used are the same as the prior iterators, there is no need to repeat them.

A.10c IteratorE

This is stored in IteratorE.m and propagates the full trajectory for plotting and error calculation. As before, the equations will not be repeated.

A.10d Orbital Module 2

This module, mhor2.m, is in all similar to the Orbital Module, except that it calculates the maximum payload for the given burn, having the structural mass value fixed. The functioning and equations used are the same as the previously mentioned module.

A.11 Other Files

Writefile, Readfile and analyzedata are purely computational modules and thus, while an important part of the work, have little interest for analysis and thus are not presented in depth.