SAILBOT
AUTONOMOUS MARINE ROBOT OF EOLIC PROPULSION

CARLOS MANUEL MOREIRA ALVES

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Juri
President: Professor Carlos Jorge Ferreira Silvestre
Advisor: Professor António Pedro Rodrigues de Aguiar
Co-Advisor: Engenheiro Luís Amorin Coelho Sebastião
Examiner: Professor Carlos Baptista Cardeira

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Abstract

At the present time, the research in the area of clean energy is one of the most relevant issues, which together with the recent advances in the field of marine robotics, presents a promising subject and an excellent experimental field for development of new technologies to be used on another scale for freight transportation with low costs and pollution levels.

As the use of wind energy for propulsion is a very promising field of research, new concepts for sailboat’s systems are emerging, including new methods for the utilization of the wind as an energy source, in order to overcome the usual restrictions of course direction.

Motivated by this fact, this thesis addresses the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously. A detailed description of the sailboat, its layout system, and its mathematical model composed by the kinematics, dynamics and applied forces and moments are presented. After the derivation of the model of the applied forces and moments and its nonlinear dynamic equations of motion, simulation results are provided to illustrate the behavior of the sailboat. In particular, the velocity’s polar prediction (VPP) diagram that represents the maximum speed that the sailboat can reach for each wind speed and direction is computed. For develop a control system, the main sailing restrictions are presented, as the three sailing modes to outline them: upwind, downwind and direct sailing modes. Based on the VPP, we present a control strategy that exploiting the tack and jibe maneuvers and actuating only on the sail and rudder orientations allows the sailboat to make progress in any given direction (on a zoom out scale) independent of the wind direction. Simulation results that illustrate the proposed control strategy are presented and discussed.

With this thesis results is possible to conduct autonomous maneuvering tests in a sailboat.

Keywords: Autonomous sailboat, Autonomous surface marine vehicle, Wind-propelled vessel, Sailing guidance and control systems.
Resumo

A investigação na área de energias renováveis é um tópico muito presente na actualidade, o qual aliado à robótica marinha, permite o desenvolvimento de métodos e tecnologias que, mais tarde, podem vir a ser utilizados noutras escalas, como para transporte de mercadorias com baixos custos e níveis de poluição.

A utilização de energia eólica como sistema de propulsão é um tópico de pesquisa muito próspero, de forma que novos conceitos para o estudo de sistemas de barcos à vela estão constantemente a surgir, incluindo métodos de utilização desta fonte de energia que permitem superar as restrições usuais deste tipo de barcos no planeamento de rotas.

Com base neste factos, esta tese detalha o estudo e desenvolvimento de um barco à vela robótico, com a capacidade de executar o complexo processo de navegação à vela autonomamente. Desta forma, uma descrição do barco à vela, do layout do sistema, e do seu modelo matemático composto pelas equações de cinemática, dinâmica e forças e momentos aplicados são apresentadas. Após a derivação do modelo das forças e momentos aplicados e das equações não-lineares da dinâmica do movimento, resultados de simulações computacionais são apresentados para ilustrar o comportamento do barco, com particular destaque para o diagrama velocity’s polar prediction (VPP), o qual apresenta a velocidade máxima que o barco pode atingir para cada velocidade e direcção do vento. Para desenvolver um sistema de controlo, as restrições da navegação à vela são apresentadas, assim como as manobras que permitem ultrapassá-las (tack e jibe). Através da definição de diferentes modos de navegação, e dado um destino, o sistema de controlo opera as suas duas saídas, o ângulo da vela e do leme. Os resultados das simulações que ilustram a estratégia de controlo proposta, são apresentados e discutidos.

Com este sistema, o barco à vela consegue navegar de forma autónoma, e deste modo, com os resultados desta tese é possível a realização de testes de manobras autónomas num barco à vela.

Palavras Chave: Barco à vela autónomo, Veículo marinho de superfície autónomo, Veleiro autónomo, Sistema de controlo e navegação de barco à vela.
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Chapter 1

Introduction

Usually, in fuel propelled vessels, the fraction of energy necessary to provide motion is significant. Autonomous sailboats depend on wind to provide propulsion, but it also needs electrical energy to the sensors, electronics, actuators and to charge a small set of backup batteries. Although, a sailboat equipped with a micro wind turbine and/or solar cells to generate power for these elements can theoretically achieve infinite autonomy.

For fuel-propelled vessels in isotropic and stationary environments, where a straight line is the shortest path to the desired destination, both in terms of time and distance, the identification of the optimal path is trivial and is established by the computation of the optimal heading to reach the target. This is significantly different in a sailboat, where a straight line route may not be navigable if the desired destination is located upwind.

This thesis details the project of an autonomous GPS-guided wind-propelled sailboat. There are many possible applications for autonomous sailboats, such as

- Environmental data collection, surveying, mapping, and water ecological studies at low costs;
- Transportation of goods at low costs and low pollution levels;
- Specific missions at far reaches or dangerous regions.

1.1 Motivations

Many tasks that cannot currently be performed with an autonomous fuel propelled surface vehicle, mainly due to autonomy constrains, can be accomplished by an autonomous sailboat, which with the wind as propulsion and a micro wind turbine or/and solar cells becomes equivalent to a marine vehicle with infinite autonomy. In general, autonomous vehicles can be useful due to the ability to remove humans from dangerous environments, relieve them of tedious tasks, or simply go to locations otherwise inaccessible. Besides the autonomous sailboat can be used for transportation from one point to another, it can be used as an observation platform for environmental monitoring.

Motivated by the above considerations, this thesis addresses the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously.
1.2 Problem Statement

An autonomous sailboat is able to autonomously navigate to any given desired destination without human intervention. In a sailboat, the thrust force is produced by the wind that blows over the sail. This force is very sensitive to the angle between the sail and the apparent wind and can oscillate quite fast in a short period of time, making the overall process a nonlinear time-variant process, what difficult the development of the sailboat’s dynamic model.

In this thesis, the problem was simplified dividing the modeling of the dynamics into two parts. Firstly, an appropriate dynamic model for the sailboat is established in absence of wind, current and wave disturbances. Then, the resulting forces and moments from these disturbances are superimposed on previous sailboat’s dynamic model.

Hence an accurate model for the sailboat’s dynamics and environmental forces (wind, waves and current) is established, the model of the sailboat is simulated by setting the wind’s velocity and direction and simulating the computational model for each possible desired orientation of the sailboat between a range of [-180°,180°]. With a discretization of the sail’s angle, for each desired orientation, the computational system is simulated for each of the discretized values of the sail’s angle, storing the variables of the situation when the sailboat acquires his maximum velocity. This task computes a speed polar diagram.

The data from the velocity’s polar prediction (VPP) diagram is discretized in a look-up table, which is heavily used by the control system.

Since sailboats operate in highly dynamic environments, it must respond quickly to changing environmental conditions. To tackle the control problem, one of the main concerns is the generation of a feasible path compatible with the environmental conditions and sailing constraints. Based on the VPP, we present a control strategy that exploits the tack and jibe maneuvers and actuates only on the sail and rudder angles allowing the sailboat to make progress in any given direction (on a zoom out scale) independent of the wind direction. Roughly speaking, the outer level of the control system continuously selects the correct heading for a desired destination, deciding if and when to tack or jibe. The heading and the sail angle (also determined by the outer loop) are then viewed as reference input signals to the inner-loop controller.

Among other solutions, the control system can be tested using a model of the sailboat, which allows the prediction of its behavior by computing its position and motion, given the environmental conditions measured by sensors.

The control system and sailboat’s model tests were implemented in a time-domain simulation software (Simulink). The simulation in this tool allows a rapid test of the behavior of the sailboat model and control system.

To perform tests of the control system in a real sailboat, an onboard computer is required. It acquires the environmental conditions from sensors and controls the rudder and sail angles through a control system.

In a first stage of the experimental tests, the sailboat must be remotely controlled in order to test if the actuators and the radio-frequency communication system are working well. The communication between a user and the sailboat can be done through TCP/IP communication, using a remote communications
device that allows the establishment of a local network (radio-modem). Then, a GPS-based guidance and control systems can be installed, with the option to remotely control the sailboat when necessary.

1.3 Objectives

The main objectives of this study are

\( i \) develop skills in marine robotics;

\( ii \) develop concepts for study the wind energy as a propulsion system;

\( iii \) develop skills in modeling, guidance and control systems applied to marine robotics.

To accomplish these objectives, the knowledge of Newton's laws, hydrodynamics of a body placed in a fluid, wind propulsion systems, modeling process for ASVs (or autonomous surface vehicle), nonlinear control methods, system’s simulation tools, architecture and identification, among others, is fundamental.

The required work can be implemented in accordance with the following stages

\( i \) analysis and development of the sailboat’s model;

\( ii \) development and implementation of guidance and control systems;

\( iii \) application of the developed mathematical model and control strategy to test the system in a simulation environment.

On the scope of this thesis, a sailboat is instrumented, and a guidance and control systems are implemented in a controlled and autonomous way.

1.4 Previous Work and Contributions

Automatic steering for vessels has its origin at the beginning of the century and was prompted by the introduction of the gyrocompass. Until the earlier 70’s, almost all autopilots for vessels were based on proportional-derivative-integral (PID) controllers. With the development in technology, the hardware of autopilots changed from purely mechanical devices to electronic systems. The fast development of small and inexpensive microcomputers made the implementation of traditional autopilots practically realizable for a wider range of marine vessels.

Automatic control systems for fuel propelled vessels research had been active for many years, producing a wealth of results, both of theoretical and practical importance. In comparison, much less attention was given to control strategies for vessels whose means of propulsion are clean energies, like the wind. Fortunately some information of the research in the fuel propelled vessels can be applied to sailboats, mainly the study of the hydrodynamics. Most of these studies, such as [Yeh and Bin, 1992] and [Aartrijk et al., 1999], consider Artificial Intelligence based techniques for the control strategies, and do not make
use of the available dynamic models that would allow further analysis of stability and performance. Other references, [Xiao and Austin, 2000], or [Elkaim and Boyce, 2006], adopt a more traditional strategy, where the control design is mostly based on a linear model structure, not allowing the study of the sailboat’s dynamics and specific maneuvers.

In recent years, a growing interest in the use of underwater vehicles for ocean exploration has been witnessed. This trend can be reflected in the number of international projects dedicated to this area and the number of scientific publications and conference presentations about the development of such vehicles. The evolution of the research in this area is mainly due to the need to use these vehicles in inspection tasks in underwater environments. Among possible scenarios for the use of such vehicles are the integrated missions in marine biology, oceanography, geology, oil industry.

In the research of strategies to control autonomous marine vehicles, the study and development of control systems to multiple autonomous marine vehicles developed by [Sérgio Carvalhosa, A. Pedro Aguiar, and A. Pascoal, 2010] and [Francesco Vanni, A. Pedro Aguiar, and Antonio M. Pascoal, 2008] is highlighted. Despite this thesis focus in a marine surface vehicle, some AUV’s researches can help the development of the computational model and control system of a sailboat.

1.4.1 Autonomous Underwater Vehicles (AUVs)

Small AUVs are widely applied to hydrographic and oceanographic data gathering. Despite all evolution concerning this type of vehicle, the most recent researches neglect the effects of the wave effects in the dynamic model of the AUVs, mainly because the AUV usually steers in deep water, where the effect of the waves can be neglected. Unlike the AUVs and large ASVs cases, in small sized sailboats, the effect of the waves is much more significant and can influence its behavior.

Reporting to the AUV, several works had been developed, such as [Dunbabin et al., 2005] and [Warren et al., 2007], which are examples of more recent researches in AUVs, where monitoring and surveying functions had been exploited. In these studies, the models are described in equations with six degrees of freedom, which are divided into three sub-systems for speed control, steering and diving. In addition to the dynamic model of the AUV, the first and second order wave forces disturbances, i. e. the Froude-Kriloff and diffraction forces are introduced. As the AUV approaches the free surface, several complexities are introduced into its computational model.

Another contribution is the strategy to trajectory tracking and path-following of underactuated AUV’s by [A. Pedro Aguiar and João P. Hespanha, 2007] and [A. Pedro Aguiar and António M. Pascoal, 2007].
1.4.2 Autonomous Surface Vehicle (ASVs)

Autonomous surface vehicles (ASVs) provide opportunities in surveillance, monitoring and oceanographic research. Most of AVS’s previous studies focused, under calm water conditions, in the motion in a horizontal reference plane. Traditionally, the control systems for autonomous surface vehicles are functionally divided into three subsystems [Fossen, 1994]:

i) a guidance system, which continuously computes the desired position, velocity and acceleration of the vehicle to be used by the navigation and control system;

ii) a navigation system, which directs the vehicle by determining its position and course;

iii) a control system, which computes the forces and moments to be provided to the vehicle in order to satisfy a certain control objective.

Reporting to the ASV, several works had been developed such as [Abkowitz, 1964] who presented a significant development of the forces acting on a ship in surge, sway, and yaw motions, where the hydrodynamic forces are expressed as Taylor series expansions. The formulation resulted in an unlimited number of parameters, which can model forces and moments to an arbitrary degree of accuracy and which can be reduced to linear and extended to nonlinear equations of motion. Later, [Hwang, 1980] and [Kallstrom and Astrom, 1981] provided different approaches to estimate the coefficients of these models. [Son and Nomoto, 1982] extended the work of [Abkowitz, 1964] to include ship roll motion in the forces and moments acting on the ship, and used the Kirchhoff’s equations [Kirchhoff’s, 1869] to obtain the equations of motion from the derivatives of the system kinetic energy. These equations are special cases of the Euler-Lagrange equations, which also gives the Coriolis and Centripetal forces.

Results of missions executed including the three subsystems, guidance, navigation and control system, are described in [Healey and Marco, 1992], while in [Papoulias, 1993] a theoretical analysis of the nonlinear dynamic involved in guidance is performed, concentrating also in the stability.

Through some research in ASVs, it can be seen that the trajectory tracking problem for fully actuated systems is now well understood and satisfactory solutions can be found, but when it comes to underactuated vehicles (where the vehicle has less actuators than the state variables to be tracked) the problem is still a very active topic of research.

1.4.3 Sailing Boats

The history of sailing boats is relevant to this work as it demonstrates how the problem of wind propulsion was solved in the past. Sailing boats have evolved over many thousands of years through a huge range of shapes, sizes and technologies. All of these vessels, until the last few years have been sailed by humans with varying amounts of mechanical assistance, ranging from simple rope, through manually operated and steam-powered capstans, to modern electric and hydraulic winches on large modern yachts. A wide range of innovative designs have been experimented with over the years and some of these
designs have shown great promise. Modern junk rigs, wing sails and kites are good examples of these technologies.

A sail expert can explain the basic sailing skills and the rules to steer the rudder and the sail according to the direction of target and wind. The actual research is aiming to transform the sailor's knowledge into a control system, while achieving dynamic stability, robustness to uncertainties and a good overall performance of the sailboat. Robust behavior in reaching the target is a necessary condition for successful applications of autonomous sailboats.

The current generation of sailing robots requires a small number of essential components in order to function successfully. These include some kind of sail and a device for detecting the direction of the wind in order to ensure that the angle of attack of the sail is suitable for the course to be sailed. These two devices present some of the most difficult engineering and control system challenges in building sailing robots.

There are an extensive number of investigations about the sail characteristics in order to develop more efficient results. By these researches, flexible fabric sails have a number of useful properties. They can be reduced in area easily by either conventional reefing or by exchanging sails, they can be relatively easily repaired and modified, and their shape and camber can be altered by tensioning and releasing control lines. They also have a number of problems, such as they are prone to wearing and tearing when incorrectly set and they tend to twist which leads to different angles of attack at different points on the sail, reducing the sailing efficiency.


1.4 Thesis Outline

This thesis describes all the approaches and decisions done to conclude this project. For the sake of brevity and readability, some details were omitted. The general scope of the project should, however, remain clear. The outline of this thesis is structured as follows, where a brief description of each chapter is done.

Chapter 2 introduces a brief description of the vehicle and a general introduction to the system layout, the communication systems, the sensors and the actuators.

Chapter 3 describes the notation and coordinate systems, and introduces an explanation of the kinematical and dynamical model in a general reference framework.
Chapter 4 presents a study of the kinematics and dynamics in terms of the Newtonian and Lagrangian formalism, to derive the nonlinear dynamic equations of motion in 6 DOF.

Chapter 5 develops a strategy to define all the forces and moments applied to the sailboat. The final nonlinear dynamic equations of motion containing all the contributive forces and moments are developed.

Chapter 6 presents the computational simulation system and the respective simulation results of the sailboat’s model in open-loop.

Chapter 7 is devoted to the sailboat’s guidance and control systems. The main sailing restrictions and the maneuvers to outline them are presented. The implemented strategies to guidance and control systems are defined.

Chapter 8 describes the simulations performed to test the guidance and control systems applied to the computational model.

Chapter 9 summarizes the results obtained and proposes some future topics to develop.
Chapter 2

Vehicle’s description

The computational model of the sailboat was developed for a real sailing boat prototype (see Figure 2.1) property of ISR (Institute of Systems and Robotics) of the Instituto Superior Técnico, Lisbon.

Figure 2.1 - The GPS autonomous sailboat on a first radio test of the sail and rudder actuators, in Jamor at May 2010
In Figure 2.2 the main components of a conventional sailboat, hull, keel, rudder, mast and sails, are depicted.

The characteristics of the hull and sails directly influence the maximum speed that a sailboat can achieve. Basically there are two types of hulls, those that plane or skip across the fluid and those that displace and move through the fluid. The characteristics of this prototype’s hull resemble the second type. In this case, the maximum speed of any marine vessel moving through fluid is mainly determined by the length of the hull. Its shape is also important, as it influences the friction resistance opposed to the movement through the fluid. The influence of the sail in the maximum reachable velocity will be approached in chapter 5, along with the forces and moments produced by the sail in presence of wind.

To achieve a sufficient righting moment and stability during heavy weather situations, the sailboat has a keel, what is a crucial component for the performance, allowing the boat to sail in relative hard winds without capsizing.

In the hull is also fitted a rudder and mast with a mainsail and a headsail. Both rudder and mainsail are operated by two servos that control their rotation angles. In a quite simple form, the rudder servo allows the control of the sailboat’s rotation, as the sail is responsible for thrust.
In Figures 2.3 and 2.4 are represented the sailboat’s elevations side and top, respectively. The sailboat’s measures, used through the development of this thesis, are presented in the Table 2.1.

<table>
<thead>
<tr>
<th>Table 2.1 – Sailboat technical data</th>
</tr>
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<tbody>
<tr>
<td>Length over all [m]</td>
</tr>
<tr>
<td>Width [m]</td>
</tr>
<tr>
<td>Height [m]</td>
</tr>
<tr>
<td>Weight [Kg]</td>
</tr>
<tr>
<td>Draft [m]</td>
</tr>
<tr>
<td>Sail area [m²]</td>
</tr>
</tbody>
</table>

The relatively sailboat’s small size allows an easy transportation and handling on shore, while also guarantees low construction costs.
2.1 Onboard Components

This sailboat’s control systems are composed by a network of sensors, microcontrollers and actuators. The control systems define the set of necessary actions to achieve a target. For this purpose, an onboard computer is necessary. With the data measured in real-time by the onboard sensors, as the environmental conditions, sailboat’s position and velocities, the control system implemented in the onboard computer can develop the set of actions that the sailboat must perform to accomplish the target. These set of actions are composed by the control of the rudder’s and sail’s angle.

The rudder and sail are driven electronically by a brushed DC-motor. The change of information between the onboard computer and the servo can be made through a dedicated \textit{PWM} servo control board.

As the sailboat operates in highly dynamic environments, the incoming data from sensors (e.g. GPS, compass, anemometer, etc.) must be analyzed in real-time by the intelligent control system. Optionally, weather forecasts and sea maps can be used for long course routing to detect stationary obstacles, while in short course routing a radar could be used to detect dynamic obstacles.

![Figure 2.5 – Sailboat’s onboard components](image)

Figure 2.5 describes the components to make possible the autonomous navigation of a sailboat. In order to power all these components, the sailboat must be equipped with a set of batteries, a ballast and a system to charge the batteries. One important requirement is that all the sensors must be low energy consumers.

As the sailboat is relatively small sized, the size of the volume occupied by the sensors, the onboard computer, the microcontrollers and the servos has to be reduced, in order to insert all these components into the sailboat. The component’s weight must also be analyzed, since the weight of these components cannot be very significant when compared with the sailboat’s weight.
2.1.1 Onboard Computer

The onboard computer must be capable of reading the measured data by sensors and control the state of the servos, through computations done by the control system. To make the measured data readable by the onboard computer and the computed action readable by the servos, a conversion from real world values, such as degrees or radian degrees, into values understood by the sensors or servos, such as voltage levels or Pulse Width Modulation timings must be performed.

The chosen system is a *Gumstix* (computer-on-module), which is an onboard computer with low cost, high performance and production ready. It contains a *TI OMAP* processor running at 600Mhz, an onboard RAM and Flash at 256MB, and it runs a slimmed down version of *Linux*, which still offers all the advantages of an operating system, such as processes and threads, file systems, device drivers and network stacks. The use of such operating system, *Linux*, allows remote login to the system over the network and the compilation of the code directly on the sailboat.

In order to store measured data and computational decisions from the control system, a micro SD card slot can be installed for additional storage space.

2.1.2 Wind Sensor

To compute the optimal course and the sail position, measured data about the wind direction is a key requirement. As for wind speed measured data, this is less important than the wind direction, but may be useful to know whether the attempt to sail is, or is not futile, due to too much or too less wind speed.

Essentially, three are three main classes of wind sensors:

1) pure mechanical sensors, which use a potentiometer to measure wind direction;
2) contactless mechanical sensors, which use magnets and hall effect sensors to compute the wind direction;
3) ultrasonic sensors, which measures the wind direction by the detection of the movement of air inside the sensor.
For this project, the solution consists in measure the magnitude of the wind velocity with an anemometer and its relative direction with a wind vane. The rotational speed measured by the anemometer is not proportional to the wind’s velocity, but rather proportional to the dynamic pressure of the flow in the anemometer surface (see Figure 2.6). As for the wind direction, the traditional approach is to attach a wind vane to a continuous rotation potentiometer. This wind vane is positioned in the top of the anemometer and operates simply, through a drag-rudder method, where the comparatively large surface generates a stabilizing force when the wind vane is not aligned into the apparent wind vector. Since the moment of inertia is very small, this force quickly returns the wind vane to point directly into the apparent wind. This approach offers a cheap and simple method for measure the wind’s direction. In Figure 2.7 is shown a commercial anemometer and weathervane.
2.1.3 Speed Sensor

A solution to measure the velocity and direction in respect to the water is through a hull speed paddle wheel transducer (see Figure 2.8). Functionally, the hull speed transducer resorts to the drag of the submerged portion of the paddle wheel to force it to turn.

An example of a hull speed transducer is a four armed paddle wheel made out of a magnetic ceramic material. The paddle wheel housing is such, that, the paddle remains semi-submerged in the fluid that passes the hull, and every time that one of the paddles passes near the magnetic detector, this device generates an electrical signal. The rate of pulses of this electrical signal is proportional to the inverse of the water velocity.

2.1.4 AHRS

One key component of the control architecture is the Attitude and Heading Reference System (AHRS). The set of output variable varies from manufacturer to manufacturer but in general all provide roll, pitch, yaw and angular velocities. It consists of two blocks:

i) an Inertial Measurement Unit (IMU) to measure the triaxial angular velocities, accelerations, and earth magnetic field components;
ii) a navigation filter which is fed with the IMU data and generates the AHRS output.

An AHRS is a complex system prone to cross coupled errors. For example, in the case of the heading computation, the measure element that gives the information about the sailboat’s heading is a tri-axial magnetometer which is subject to the boat’s rolling and pitching. Thus, errors in the roll and pitch
estimates, causes the North and East components of the measurement to be skewed by the downward component of the magnetic field, imparting significant errors in the yaw estimation.

2.1.5 GPS

Guidance systems need to know the vehicle’s position and velocity. Although the GPS only measures the vehicle position, it is possible to estimate the velocity vector. Ideally a measurement system should be quick to update the sailboat’s position, in order to allow the control system to quickly correct an eventual error of the sailboat’s heading.

The GPS receiver provides position measurement, which can be computed, providing information about the velocity of the vehicle. Its antenna should be small, lightweight and mounted on top of the mast. The GPS electronic unit is located in the hull. To represent GPS points on a map, a coordinate transformation is necessary.

\[
\begin{align*}
    x &= R_e \cos(lat) \cdot \frac{\pi}{180^\circ} \cdot lon \\
    y &= R_e \cdot \frac{\pi}{180^\circ} \cdot lat
\end{align*}
\]  

(2.1)

In local planning a transformation of the coordinates measured by the GPS, the latitude and longitude, into meter coordinates, can be made by the system of equations (2.1), where \(R_e\) is the local earth radius, and \(lat\) and \(lon\) are the GPS coordinates in degrees. This transformation assumes a flat water surface.

2.1.6 Communication System

An operator onshore can connect to the sailboat’s system to gather measured data by the sailboat’s sensors, monitor its position and velocity, monitor the computed strategies of the control system, and send new waypoints to the path planning system. Therefore the sailboat should possess a communication system, which combines WLAN and UMTS/GPRS to allow continuous real-time access from a user. At the same time, it has to be ensured that the system switches dynamically between the available communication channels, in order to use the most appropriate channel for each situation. For short distances a WLAN link connects the operator and the sailboat. If GPRS/UMTS infrastructure is available it can be used up to approximately 20 km offshore. For longer distances, the navigation system could use a satellite communication system. As the GPRS/UMTS have a base and a connection fee, the main communication link between the sailboat’s system and an operator is through WLAN.
Vessel’s Maneuvering Principles

The knowledge of the kinematical and dynamic characteristics of the physical system is essential to its control system design. Experimental works suggest that it is difficult to predict the maneuvering characteristics of a vessel, from a computational model test, due to the lack of precise knowledge of the interaction between steering and roll [Blanke and Jensen, 1997]. Thus, a great research effort has been made to analyze the dynamics involved in this interaction. The knowledge of the dynamics associated to roll, yaw and sway are, not only, useful to improve computational models of vessel’s maneuvering, but also, essential to the application of rudder roll damping, since the performance of this technique relies to a great extent on dynamic couplings between roll, yaw, and sway.

The design of guidance systems to modern marine surface vessels requires knowledge of vector kinematics and dynamics, hydrodynamics, aerodynamics, guidance and navigation systems, and control theory, among other fields of investigation. To be able to design a high performance control system, a precise mathematical model (developed through the physical model of the sailboat (see Figure 3.1) of the vehicle’s kinematics and dynamics is fundamental. In next section, the notation and coordinate systems used in this modeling process are described.
Figure 3.1 - Steps in the transformation of the real model of the sailboat into a computational model

3.1 Reference Frames

Following standard strategies, the kinematical and dynamic equations of the vehicle’s motion can be developed using proper cartesian reference frames, such as
- Earth-fixed inertial frame, \{I\}. The origin of \{I\} reference frame is located at a local tangent plane in the area of interest;
- Body-fixed reference frame, \{B\}. This is a moving reference frame, where the origin is, usually, chosen to coincide with the vehicle’s center mass (CM).

These two reference frames as described in the Figure 3.2, where the motion of a sailboat is defined in \{B\} relatively to \{I\}.
For marine vessels is assumed that the accelerations of a point on the surface of the Earth can be neglected. Indeed, this is a good approximation since the motion of the earth hardly affects low speed marine vehicles. \( \{I\} \) is an inertial reference frame, what suggests that the position and orientation of the vehicle is described relatively to this reference frame, while the linear and angular velocities of the vehicle are expressed in \( \{B\} \).

### 3.2 Vector Definitions

Generally, for the study of the motion of a vessel the use of 6 independent coordinates is indispensable, in order to define its position and orientation. Therefore, the motion of the sailboat is described in \( \{B\} \) by 6 Degrees Of Freedom (DOF). In Table 3.1 and Figure 3.3, the notation to define the vehicle’s motion and position is introduced.
Table 3.1 – *SNAME* notation used for marine vessels

<table>
<thead>
<tr>
<th>DOF</th>
<th>Motion/Rotation</th>
<th>Forces/Moments</th>
<th>Linear/Angular Velocities</th>
<th>Positions and Euler Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>motion in x-direction (surge)</td>
<td>$\dot{X}$</td>
<td>$u$</td>
<td>$x$</td>
</tr>
<tr>
<td>2</td>
<td>motion in y-direction (sway)</td>
<td>$Y$</td>
<td>$v$</td>
<td>$y$</td>
</tr>
<tr>
<td>3</td>
<td>motion in z-direction (heave)</td>
<td>$Z$</td>
<td>$w$</td>
<td>$z$</td>
</tr>
<tr>
<td>4</td>
<td>rotation about x-axis (roll)</td>
<td>$K$</td>
<td>$p$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>5</td>
<td>rotation about y-axis (pitch)</td>
<td>$M$</td>
<td>$q$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>6</td>
<td>rotation about z-axis (yaw)</td>
<td>$N$</td>
<td>$r$</td>
<td>$\psi$</td>
</tr>
</tbody>
</table>

In Table 3.1, the first three coordinates and their time derivatives describe the position and translational movement along the x, y and z-axes, while the three last coordinates and their time derivatives are used to describe the orientation and the rotational motion of the vessel.

In marine terms, the 6 DOF are conveniently named as surge, sway, heave, roll, pitch and yaw, where the surge, sway and heave are different types of displacement motions, and the yaw, pitch, and roll define angular motions, as shown in Figure 3.3.

![Figure 3.3 - Motion components used in *SNAME* notation](image-url)
Generally, to represent the position, velocities and forces, in SNAME notation, a set of vectors is defined as in (3.1).

\[ \eta = [\eta_1^T, \eta_2^T]^T; \quad \eta_1 = [x, y, z]^T; \quad \eta_2 = [\phi, \theta, \psi]^T \]
\[ \nu = [\nu_1^T, \nu_2^T]^T; \quad \nu_1 = [u, v, w]^T; \quad \nu_2 = [p, q, r]^T \]
\[ \tau = [\tau_1^T, \tau_2^T]^T; \quad \tau_1 = [X, Y, Z]^T; \quad \tau_2 = [K, M, N]^T \]

(3.1)

According to SNAME notation the use of the vectors in (3.1) is very useful to represent the position, velocities and forces in marine vessels. Here, \( \eta \in \mathbb{R}^3 \times S^3 \) denotes the position and orientation vector with coordinates in \{I\} (where \( \mathbb{R}^3 \) is the Euclidean space of dimension 3 and \( S^3 \) denotes a torus of dimension 3), \( \nu \in \mathbb{R}^6 \) denotes the linear and angular velocity vectors with coordinates in the body-fixed reference frame and \( \tau \in \mathbb{R}^6 \) is used to describe forces and moments acting on the vehicle in \{B\}. 
Figure 3.4 – The rotation sequence according to the xyz-convention

(a) Rotation over heading angle $\psi$ about z-axis

(b) Rotation over pitch angle $\theta$ about y-axis

(3) Rotation over roll angle $\phi$ about x-axis

$u_0$, $u_1$, $v_0$, $v_1$, $w_0$, $w_1$
Three Euler angles, $\phi, \theta, \psi$, define the orientation of \{B\} relatively to the reference of \{I\} through the definition of rotation, as shown in Figure 3.4. Considering the rotations in the Figure 3.5, when the reference of the \{I\} coincides with the reference of the \{B\}, the yaw angle can be set by rotating the \{I\} around the $z_B$-axis, the pitch angle can be set by rotating the \{I\} around a reference $y_B$-axis and the roll angle can be set by rotating the \{I\} around the $x_B$-axis.

In the analysis of the maneuverability of marine vessels is common to divide the modeling of the vessel’s kinematics and dynamics in two reference frames, the horizontal \{H\} and the vertical \{V\}. The motions in these two reference frames can be described neglecting the couplings between types of motion in different reference frames. This approximation can be reasonably accepted if the coupling effects are insignificantly small. In the Figure 3.5 is drawn a sketch of \{H\}.

The linear velocities of a marine vessel are measured in \{B\}, and when considering the \{H\}, only the surge and sway velocities are measured, because the heave linear velocity belong to the \{V\}.

If a marine vessel has a velocity given by $\vec{V}$, then the components of the velocity according to $x_B$ and $y_B$-axis are given by the equations (3.2).

\[
\begin{align*}
    u &= \vec{V} \cos(\beta) \\
    v &= \vec{V} \sin(\beta)
\end{align*}
\]  

(3.2)
Chapter 4

Kinematics and Dynamics

The development of a marine vessel mathematical model is made through the computation of the equations of motion, which are given by a set of nonlinear and complex differential equations. These equations describe the motion in 6 DOF: surge, sway and heave for translational motions, and roll, pitch and yaw rates for rotational motions. The computational models used to represent the physics of the real world will differ as the control objectives change. These control objectives can be roughly divided into slow speed positioning or high speed steering. Slow speed monitoring or dynamical positioning includes station keeping, position mooring and slow speed reference tracking (see [Strand, 1999] and [Lindegaard, 2003] for dynamical positioning references). In this case, the 6 DOF mathematical model is reduced to a simpler 3 DOF model, where the kinematics are linear. As for high speed steering, it includes automatic course control, high speed position tracking and path following (see, per instance, [Lefebre et al., 2003] and [Fossen et al., 2003]).

For surface vessels, comparable the case in study, it is common to decouple the surge dynamics from the steering. In some cases, the heave, pitch and roll modes are neglected under the assumption that these motion variables are small.

In this chapter the equations that express the kinematics and dynamics of a rigid body are deduced. Kinematic equations consist in equations that relate the position time derivative with the velocities, while the dynamic equations relate the velocities time derivative with the forces. For the derivation of these equations of motion, the elimination of forces acting between individual elements of mass is possible by the assumption that the vehicle is a rigid body.

4.1 Kinematics

The kinematic models characterize the transformation of motion variables (position, velocity and accelerations), forces and moments between different reference frames, as per instance, the transformation of a motion variable from \( \{B\} \) to an \( \{I\} \).
The kinematic equations relates the first time derivative of $\eta_1$ with $\nu_1$, and the first time derivative of $\eta_2$ with $\nu_2$. For that purpose, a relation between these variables must be established. The relation between these variables is possible through the deduction of rotation matrices, which allows the expression of vectors given in one reference frame into another. To deduce the kinematics equations, it is simpler to explore separately the translational and rotational movement.

4.1.1 Reference Frame Transformation

Since the linear and angular velocities are expressed in the $\{B\}$, the vessel’s route relative to the $\{I\}$ can be given by the velocity transformation

$$\dot{\eta}_1 = R(\eta_2)\nu_1$$

(4.1)

where $R(\eta)$ is a transformation matrix of Euler angles functions.

Considering the definition of rotation, the motion of a rigid body in a reference frame $\{R\}$ relatively to another reference frame $\{T\}$, can be called as a simple rotation of $\{R\}$ in $\{T\}$, when there is a line, $L$, called axis of rotation, whose orientation relative to both $\{R\}$ and $\{T\}$ remains unaltered through the motion. This can be fundament by the Euler theorem on rotation, which argues that: Every change in the relative orientation of two rigid bodies or reference frames $A$ and $B$ can be produced by means of a simple rotation of $B$ in $A$.

The rotation matrix $R$ between two reference frames, $\{R\}$ and $\{T\}$, is denoted as $R^R_T$, and it is an element in $SO(3)$. $SO(3)$ is the special orthogonal group of order 3 given by

$$SO(3) = \{ R | R \in \mathbb{R}^{3 \times 3}, \text{ $R$ is orthogonal and } \det R = 1 \}$$

(4.2)

The group $SO(3)$, is a subset of all orthogonal matrices of order 3, i.e., $SO(3) \subset O(3)$, where $O(3)$ is defined as

$$O(3) = \{ R | R \in \mathbb{R}^{3 \times 3}, RR^T = R^T R = I \}$$

(4.3)

Rotation matrices are useful when developing the kinematical equations of motion for a marine vessel. As a consequence of equations (4.2) and (4.3), it can be stated that a rotation matrix $R \in SO(3)$ satisfies
\(RR^T = R^T R = I\) and \(\det R = 1\), which implies that \(R\) is orthogonal. Hence, the inverse rotation matrix is given by \(R^{-1} = R^T\).

If \(v_B^0\) is a vector in \(\{B\}\), and \(v_I^0\) a vector in the \(\{I\}\), the vector \(v_I^0\) can be expressed in terms of the vector \(v_B^0\), when the unit vector \(\lambda = [\lambda_1, \lambda_2, \lambda_3]^T\) is parallel to the axis of rotation, \(|\lambda| = 1\) and \(\beta\) is the angle that \(\{I\}\) is rotated. Therefore this rotation is described by

\[
v_I^0 = R_I^b v_B^0, \quad R_I^b := R_{\lambda, \beta}
\] (4.4)

where \(R_{\lambda, \beta}\) is the rotation matrix corresponding to a rotation \(\beta\) about the \(\lambda\)-axis described by

\[
R_{\lambda, \beta} = I_{3\times3} + \sin(\beta) S(\lambda) + (1 - \cos(\beta)) S^2(\lambda)
\] (4.5)

where \(I_{3\times3}\) denotes the identity matrix and \(S(\lambda)\) is the skew-symmetric matrix defined as

\[
S(\lambda) = -S^T(\lambda) = 
\begin{bmatrix}
0 & -\lambda_2 & \lambda_3 \\
\lambda_2 & 0 & -\lambda_1 \\
-\lambda_3 & \lambda_1 & 0
\end{bmatrix}
\] (4.6)

Consequently, \(S^2(\lambda) = \lambda \lambda^T - I_{3\times3}\), since \(\lambda\) is a unit vector.

The Euler angles can be used to decompose the body-fixed velocity vector \(v_1\) in \(\{I\}\). If \(R_b^I(\eta_z)\) denote the Euler angles rotation matrix, then \(v_B^0\) is given by

\[
v_B^0 = R_b^I(\eta_z)v_1
\] (4.7)

The principal rotation matrices, or one axis rotation matrix, can be obtained setting \(\lambda = [1,0,0]^T\), \(\lambda = [0,1,0]^T\) and \(\lambda = [0,0,1]^T\) to correspond to the \(x\), \(y\), and \(z\) axes, with \(\beta = \theta\), \(\beta = \phi\) and \(\beta = \psi\) respectively, in the expression for \(R_{\lambda, \beta}\), (4.5). This yields to

\[
R_{x,\phi} = 
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix},
R_{y,\theta} = 
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix},
R_{z,\psi} = 
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (4.8)
Usually $R_B^I(\theta_2)$ is described by three principal rotations about $x, y,$ and $z$ axes. The order in which these three rotations are carried out is not arbitrary. In guidance and control applications it is common to use the zyx-convention to describe the motion in $\{I\}$ specified in terms of the Euler angles $\phi, \theta$ and $\psi$, for the rotations. This matrix is described by $R_B^I(\theta_2) = R_B^I(\theta_2)^T$. The transpose matrix implies that the same result is obtained by transforming a vector from the $\{B\}$ to the $\{I\}$, i.e., reversing the order of the transformation, this rotation sequence is mathematically equivalent to

$$R_B^I(\theta_2) := R_{Z,\psi} R_{Y,\theta} R_{X,\phi}, \quad R_B^I(\theta_2)^{-1} = R_I^B(\theta_2) = R_{X,\phi} R_{Y,\theta} R_{Z,\psi}$$

(4.9)

Expanding (see [Fossen, 1994]) yields to

$$R_B^I(\theta_2) = \begin{bmatrix}
\cos \psi \cos \theta & -\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \cos \phi \sin \theta \\
\sin \psi \cos \theta & \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi & -\cos \psi \sin \phi + \sin \theta \sin \psi \cos \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}$$

(4.10)

The body-fixed velocity vector $v_1$ is now decomposed in the $\{I\}$ as

$$\vec{n}_1 = R_B^I(\theta_2) v_1$$

(4.11)

where $\vec{n}_1$ is the velocity vector expressed in the $\{I\}$.

The body-fixed angular velocity vector $v_2 = [p, q, r]^T$ and the Euler rate vector $\eta_2 = [\phi, \theta, \psi]^T$ are related through a transformation matrix $T_{\eta_2}(\theta_2)$, according to

$$\eta_2 = T_{\eta_2}(\theta_2)v_2$$

(4.12)

It should be noted that the angular velocity vector $v_2 = [p, q, r]^T$, in the $\{B\}$ cannot be integrated directly to obtain actual angular coordinates. This is due to the fact that $\int_0^t v_2(\tau) d\tau$ does not have any immediate physical interpretation. However, the vector $\eta_2 = [\phi, \theta, \psi]^T$ represents proper generalized coordinates. The angular velocity vector in the body-fixed reference frame can be represented as
The transformation matrix $T_{\eta_2}(\eta_2)$ can be derived, leading to several definitions, as for instance

$$
T_{\eta_2}^{-1}(\eta_2) = \begin{bmatrix}
1 & 0 & -\sin \phi \\
0 & \cos \phi & \cos \theta \sin \phi \\
0 & -\sin \phi & \cos \theta \cos \phi
\end{bmatrix} 
= \begin{bmatrix}
\sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
$$

(4.14)

4.1.1.1 Linear Velocity Transformation

The translational kinematic equations allow a linear velocity transformation between two different reference frames. The relation between $\dot{\eta}_1$ and $v_1$ can be given by

$$
\dot{\eta}_1 = {R}'(\eta_2) v_1
$$

(4.15)

It is custom to describe $R'(\eta_2)$ by three rotations of $\{I\}$, obtaining the $\{B\}$ coordinate-system $x_B y_B z_B$.

This rotation sequence is written as

$$
{R}'(\eta_2) = R_{x,\psi}^{T} R_{y,\theta}^{T} R_{x,\phi}^{T}
$$

(4.16)

Expanding (4.16) expression yields to

$$
{R}'(\eta_2) =
\begin{bmatrix}
\cos \psi \cos \theta & -\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \cos \phi \sin \theta \\
\sin \psi \cos \theta & \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi & -\cos \psi \sin \phi + \sin \theta \sin \psi \cos \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
$$

(4.17)
4.1.1.2 Angular Velocity Transformation

The body-fixed angular velocity vector \( \mathbf{\dot{v}}_2 \), and the Euler rate vector \( \mathbf{\dot{\eta}}_2 \), are related through a transformation matrix \( T_{\eta_2}(\mathbf{\eta}_2) \) according to

\[
\mathbf{\dot{\eta}}_2 = T_{\eta_2}(\mathbf{\eta}_2) \mathbf{\dot{v}}_2
\]

(4.18)

According to [Fossen, 1994], the transformation matrix, \( T_{\eta_2}(\mathbf{\eta}_2) \) is given by

\[
T_{\eta_2}(\mathbf{\eta}_2) = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}
\]

(4.19)

4.1.2 Generalized Kinematic Equations

Summarizing, the 6 DOF kinematical equations can be expressed in vector form as

\[
\mathbf{\dot{\eta}} = f(\mathbf{\eta})\mathbf{v} \iff \begin{bmatrix}
\mathbf{\dot{\eta}}_1 \\
\mathbf{\dot{\eta}}_2
\end{bmatrix} = \begin{bmatrix}
R_{\phi}(\mathbf{\eta}_2) & 0_{3\times3} \\
0_{3\times3} & T_{\eta_2}(\mathbf{\eta}_2)
\end{bmatrix} \begin{bmatrix}
\mathbf{v}_1 \\
\mathbf{v}_2
\end{bmatrix}
\]

(4.20)

Although \( T_{\eta_2}(\mathbf{\eta}_2) \) is undefined for a pitch angle \( \theta = \pm 90^\circ \), the sailboat is not foreseen to, in normal sailing conditions, operate close to this singularity.

As said before, for surface vessels simplification to represent the kinematical model in a 3 DOF system can be made. This simplification is made on the assumption that \( \phi \) and \( \theta \) are small, what is true for most conventional surface vessels. Hence, neglecting the heave, roll and pitch elements, \( R_{\phi}(\mathbf{\eta}_2) = R_{x,\psi}R_{y,\theta}R_{x,\phi} \approx R_{x,\psi} \) and \( T_{\eta_2}(\mathbf{\eta}_2) \approx I_{3\times3} \), yielding to

\[
\mathbf{\dot{\eta}} = R(\psi)\mathbf{v}
\]

(4.21)

where \( R(\psi) = R_{x,\psi} \), while \( \mathbf{v} = [u, v, r]^T \) and \( \mathbf{\eta} = [x, y, z]^T \).
4.2 Dynamics

The dynamic model of a rigid body can be obtained through the application of Newtonian and Lagrangian formalism. The Newton's second law relates the acceleration of a rigid body with the forces acting on it. According to this law, a body at rest will acquire motion only if there is a force applied to it, otherwise, a body in motion will reach the rest state only if a force in the opposite direction to its motion is applied. The equations of dynamics relates the temporal derivative $\dot{\mathbf{r}}$ with $\mathbf{r}$. As in the kinematics, the deduction of these equations is made separating the dynamics of translational from the rotational dynamics.

Figure 4.1 – The definition of the Center of Mass reference frame relative to $\{B\}$ and $\{I\}$

For the study of the dynamics is important to define the center of mass, CM, relative to the two reference frames, $\{B\}$ and $\{I\}$, (see Figure 4.1).
4.2.1 Dynamic Equation of translation motion

According to Newton’s Second Law

\[ m \left. \frac{d\mathbf{v}_{CM}}{dt} \right|_{(I)} = F \quad (4.22) \]

where \( m \) is the sailboat’s mass and \( \mathbf{v}_{CM} \) is the linear velocity of the vehicle’s CM measured in \( \{I\} \). To express the equation (4.22) in \( \{B\} \), the derivatives in the \( \{I\} \) must be transformed to \( \{B\} \), as defined by

\[ \left. \frac{d\mathbf{v}}{dt} \right|_{(I)} = \omega_B \mathbf{v} + \left. \frac{d\mathbf{v}}{dt} \right|_{(B)} \quad (4.23) \]

where \( \omega_B \) is the angular velocity of \( \{B\} \) measured in \( \{I\} \). Applying (4.23) to (4.22), yields to

\[ m \left( \omega_{CM} \mathbf{v}_{CM} + \left. \frac{d\mathbf{v}_{CM}}{dt} \right|_{(B)} \right) = F \quad (4.24) \]

The linear velocity \( \mathbf{v} \), from any point with position \( \mathbf{r}_0 \), is expressed by

\[ \mathbf{v} = \mathbf{v}_0 + \omega (\mathbf{r}_0 - \mathbf{r}_g) \quad (4.25) \]

where \( \mathbf{v}_0 = \dot{\mathbf{r}}_0 \), \( \omega \) is the angular velocity in \( \{B\} \) measured in \( \{I\} \) and \( \mathbf{r}_g \) is the position of the CM relative to \( \{B\} \), as in the Figure 4.1. Finally (4.24) can be written as

\[ m \left( \omega \mathbf{v}_0 + \omega \mathbf{r}_g + \left. \frac{d\mathbf{v}_0}{dt} \right|_{(B)} + \left. \frac{d\omega}{dt} \right|_{(B)} \mathbf{r}_g \right) = F \quad (4.26) \]

Representing (4.25) in \( \{B\} \), and using the \( SNAME \) notation, the equations of motion, considering the conservation of linear and angular moments can be obtained by
\[ \begin{align*}
m[\ddot{u} - vr + wq - x_{CM}(q^2 + 2) + y_{CM}(pq - \dot{r}) + z_{CM}(pr + \dot{q})] &= X \\
m[\ddot{v} - wp + ur - y_{CM}(r^2 + p^2) + z_{CM}(qr - \dot{p}) + x_{CM}(qp + \dot{r})] &= Y \\
m[\ddot{w} - vq + vp - z_{CM}(p^2 + q^2) + x_{CM}(rp - \dot{q}) + y_{CM}(rq + \dot{p})] &= Z \end{align*} \] (4.27)

where \( X, Y \) and \( Z \) are the components of the generalized forces along \( x_B, y_B \) and \( z_B \)-axis, and \( m \) is the sailboat’s mass. By setting the CM of the sailboat to the coordinate origin (4.27) is simplified. The detailed deduction of these expressions can be found in [Fossen, 1994].

4.2.2 Dynamic Equation of Rotational Motion

By definition, the angular moment of a body, relative to a point \( 0_c \) is given by

\[ L_{0c} = \int_{\text{body}} (r - r_{0c})(v - v_{0c})dm \] (4.28)

where \( dm \) represents an elemental part of the body’s mass. As the body is rigid, (4.28) can be given by

\[ L_{0c} = \int_{\text{body}} (r - r_{0c})[\omega(r - r_{0c})]dm \] (4.29)

Expressing \( r - r_{0c} \) in the form \([x, y, z]^T\) and making use of the distributive property \( a \cdot b \cdot c = (a \cdot c)b - (a \cdot b)c \), yields to

\[ L_0 = I_0 \omega \] (4.30)

where \( I_0 \) is the inertia tensor matrix, which takes the form

\[ I_0 = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \] (4.31)

33
where \( I_x, I_y \) and \( I_z \) are the moments of inertia about the \( x, y \) and \( z \)-axis, and \( I_{xy} = I_{yx}, I_{xz} = I_{zx} \) and \( I_{yz} = I_{zy} \) are the products of inertia defined as

\[
\begin{align*}
I_x &= \int_V (y^2 + z^2) \, \rho_m \, dV; \quad I_{xy} = \int_V xy \, \rho_m \, dV = \int_V yx \, \rho_m \, dV = I_{yx} \\
I_y &= \int_V (x^2 + z^2) \, \rho_m \, dV; \quad I_{xz} = \int_V xz \, \rho_m \, dV = \int_V zx \, \rho_m \, dV = I_{zx} \\
I_z &= \int_V (x^2 + y^2) \, \rho_m \, dV; \quad I_{yz} = \int_V yz \, \rho_m \, dV = \int_V yz \, \rho_m \, dV = I_{zy}
\end{align*}
\]

where \( \rho_m \) is the mass density of the body.

Applying an identical approach as in the dynamic equation of translation, and deriving (4.28) in time

\[
\frac{\partial I_\omega}{\partial t} = M_0 - m \, \gamma \, \frac{dv_\omega}{dt} \tag{4.33}
\]

Through this derivation, the Euler’s law is obtained. According to (4.33), (4.29) can be given in \{B\} as

\[
\omega \, I_0 \, \omega + I_0 \, \frac{\partial \omega}{\partial t} = m \, \gamma \, \left( \omega \, v_0 + \frac{dv_\omega}{dt} \right) = M_0 \tag{4.34}
\]

Applying (4.34) in a generalized way to (4.32), yields to

\[
\begin{align*}
I_x \frac{p_r(l_z - l_y)q_r - (\dot{\gamma} + pq)l_{xz} + (r^2 - q^2)l_{yz} + (pr - \dot{q})l_{xy}}{\omega \, I_0 \omega} + m[y_{CM} \omega - uq + vr] - z_{CM} (\dot{\gamma} - wp + ur) & = K \\
I_y \frac{q_r(l_z - l_x)p_r - (\dot{\gamma} + qr)l_{zx} + (p^2 - r^2)l_{xz} + (qv - \dot{r})l_{yz}}{\omega \, I_0 \omega} + m[x_{CM} (\dot{\gamma} - wp + vr) - y_{CM} (\dot{\gamma} - wp + vr)] & = M \\
I_z \frac{q_r(l_y - l_z)p_r - (\dot{\gamma} + rp)l_{yz} + (q^2 - p^2)l_{xy} + (rq - \dot{p})l_{xz}}{\omega \, I_0 \omega} + m[z_{CM} (\dot{\gamma} - wp + vr) - y_{CM} (\dot{\gamma} - wp + vr)] & = N
\end{align*}
\]

Equations given by (4.35) are the result of the dynamic equations of rotational motion for a rigid body.
4.3 Generalized Equation of Dynamics

By Newton’s second law, and according to [Fossen, 1994], the 6 DOF rigid body dynamics of a marine vehicle can be expressed in a vector setting and in a compact form as

\[ M_{RB} \dot{v} + C_{RB}(v) \cdot v = \tau \quad (4.36) \]

where \( M_{RB} \) is the inertia matrix of the rigid body, \( C_{RB} \) include a Coriolis vector term \( v_2 \times r_b^0 \) and a centripetal vector term \( v_2 (v_2 \times r_b^0) \), with \( r_b^0 \) being the distance between 0 and the CM of the rigid body. Finally \( \tau \) is a generalized vector of external forces and moments given by (3.1).

The representation of the rigid body system inertia matrix \( M_{RB} \) is unique and satisfies

\[ M_{RB} = M_{RB}^T > 0, \quad \dot{M}_{RB} = 0_{6 \times 6} \quad (4.37) \]

with

\[
M_{RB} = \begin{bmatrix}
m I_{3 \times 3} & -m S(r_b^0) \\
m S(r_b^0) & I_0
\end{bmatrix}
= \begin{bmatrix}
m & 0 & 0 & mz_g & -my_g \\
0 & m & 0 & -mx_g & 0 \\
0 & 0 & m & my_g & -mx_g \\
0 & -mx_g & my_g & I_x & -l_{xy} & -l_{xz} \\
mx_g & 0 & mx_g & I_y & -l_{yx} & -l_{yz} \\
my_g & mx_g & 0 & -l_{zx} & I_z & -l_{xy}
\end{bmatrix} \quad (4.38)
\]

where \( I_{3 \times 3} \) is the identity matrix, \( S(r_b^0) \) is a skew-symmetric matrix, and \( I_0 = I_0^T > 0 \) is the inertia matrix about 0. The body’s inertia tensor, \( I_0 \), referred to an arbitrary \( \{B\} \) with origin 0 in \( \{B\} \) is defined as (4.31).

As for the Coriolis and centripetal matrix, it can always be parameterized such that \( C_{RB} = -C_{RB}^T \). \( C_{RB}(v) \) can be given by

\[
C_{RB}(v) = \begin{bmatrix}
0_{3 \times 3} & -S(M_{11}v_1 + M_{12}v_2) \\
-S(M_{11}v_1 + M_{12}v_2) & -S(M_{21}v_1 + M_{22}v_2)
\end{bmatrix} \quad (4.39)
\]
where $S$ is a cross product operator in accordance to (4.4) and $M_{11}, M_{12}, M_{21}$ and $M_{22}$ are the elements of a 6x6 system inertia matrix $M$ defined as

$$M = M^T = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} > 0$$

(4.40)

where $M_{21} = M_{12}$. With $M = M_{RB}$ it is obtained the following expression for the $C_{RB}(v)$

$$C_{RB}(v) = \begin{bmatrix} 0_{3x3} & -mS(v_1) - mS(S(v_2)\eta_{q}) \\ -mS(v_1) - mS(S(v_2)\eta_{q}) & mS(S(v_1)\eta_{q}) - S(l_0v_2) \end{bmatrix}$$

(4.41)

for which it is noticed that $S(v_1)v_1 = 0$. Developing (4.41)

$$C_{RB}(v) = \begin{bmatrix} 0 & 0 & 0 & m(y_{CM}q + x_{CM}r) & -m(x_{CM}q - w) & -m(x_{CM}r + v) \\ 0 & 0 & 0 & -m(x_{CM}q - w) & m(x_{CM}q + u) & -m(x_{CM}r + u) \\ -m(y_{CM}q + x_{CM}r) & m(y_{CM}q + u) & m(x_{CM}q - w) & m(x_{CM}q + u) & -m(x_{CM}r + u) & -m(x_{CM}r + u) \\ m(x_{CM}q - w) & m(x_{CM}q + u) & m(x_{CM}q - w) & m(x_{CM}q + u) & -m(x_{CM}r + u) & -m(x_{CM}r + u) \\ m(y_{CM}q + x_{CM}r) & -m(x_{CM}q - w) & m(x_{CM}q - w) & m(x_{CM}q + u) & -m(x_{CM}r + u) & -m(x_{CM}r + u) \\ m(x_{CM}q - w) & m(x_{CM}q + u) & m(x_{CM}q - w) & m(x_{CM}q + u) & -m(x_{CM}r + u) & -m(x_{CM}r + u) \end{bmatrix}$$

(4.42)

The $M_{RB}$ and $C_{RB}(v)$ matrices are dependent of the sailboat’s mass, CM and inertia matrix. The mass of the rigid body can be defined as

$$M = \int_{V} \rho_m \, dV$$

(4.43)

With a conveniently chose of the location of \{B\} relative to the sailboat, it is possible to simplify the structure of $M_{RB}$ and $C_{RB}(v)$ matrices. Choosing the origin such as the origin of \{B\} coincides with the CM of the sailboat, and the respective $xy$ reference frame is the symmetry reference frame of the sailboat, leads into $x_{CM} = y_{CM} = z_{CM} = 0$ and $I_{xy} = I_{yz} = 0$, reducing $M_{RB}$ and $C_{RB}(v)$ to (4.44) and (4.45).
4.4 6 DOF Rigid Body Equations of Motion

In the kinematic and dynamic equations of motion, if the origin of \{B\} is chosen according to its origin coincide with the CM, \( r_{CM} = [0,0,0]^T \), and consequently \( l_c = diag \{ l_{xc}, l_{yc}, l_{zc} \} \). This simplification yields to the system of equations of motion given by (4.46).

\[
egin{align*}
    m(\dot{u} - vr + wq) &= X; & & l_{xc} \dot{p} + (l_{zc} - l_{yc}) qr = K \\
    m(\dot{v} - wp + ur) &= Y; & & l_{yc} \dot{q} + (l_{xc} - l_{zc}) rp = M \\
    m(\dot{w} - uq + vp) &= Z; & & l_{zc} \dot{r} + (l_{yc} - l_{xc}) pq = N
\end{align*}
\]
Chapter 5

Forces and Moments

In this chapter the set of all the forces and moments applied to the sailboat are subjected to analysis. The sailboat is subject to the action of the water, atmosphere and gravitational field of the earth. The analyzed forces are generally non-linear and time-variant.

The thrust force in a sailboat depends of the wind amount flowing through the sails and, therefore it depends from the environmental conditions to acquire thrust. This is principal difference between the aerodynamics and hydrodynamics of a sailboat and a conventional vessel.

The analysis of the forces and moments in the body (hull and keel), sail and rudder must count with all the influences on heading, including disturbances caused by wind, current, waves. With some simplifications, this analysis yields to the non-linear differential equations, which describe the surge, sway, heave, yaw, roll and pitch motions.

To compute the total forces and moments applied, each one of the forces and moments is analyzed separately for each one of the constitutive elements of the sailboat, as if they were isolated from each other. Then the total forces and moments applied to the sailboat are obtained by summing all the forces and moments, what also depends from their relative positions. Therefore it is considered that the generalized total force, $\tau_T$, applied to the sailboat is given by equation (5.1).

$$\tau_T = \tau_B + \tau_{cs} + \tau_D + \tau_G$$  (5.1)

where $\tau_B$ denotes the hydrodynamic and aerodynamic forces and moments acting on the Body (hull and keel) due to the current and wind, $\tau_{cs}$ the resultant forces and moments due to the control surfaces (sails and rudder), $\tau_D$ the forces and moments relative to disturbances (waves, current and wind) and $\tau_G$ the forces and moments due to the gravity. $\tau_B$ and $\tau_{cs}$ are given by

$$\tau_B = \tau_h + \tau_a$$  
$$\tau_{cs} = \tau_s + \tau_r$$  (5.2)
where \( \tau_h \) and \( \tau_a \) denotes the hydrodynamic and aerodynamics forces and moments acting on the Body, \( \tau_s \) denotes the aerodynamic forces and moments generated by sails, and \( \tau_r \) the hydrodynamic forces and moments that arise from the movement of the rudder in the water.

The model of the sailboat’s dynamics in 6 DOF is described by the block diagram from Figure 5.1, where the Rigid Body block represents the dynamics of the rigid body, the hydrodynamic and aerodynamics block computes \( \tau_B \) based on a static function of the velocity and acceleration and the Control Surfaces block computes \( \tau_{CS} \).

\( \tau_h \) can be developed for fluids with different characteristics, which originate different effects and results, such as

i) Motion in an ideal fluid without circulation. In this analysis, only the displacement is considered, revealing the added mass and inertia forces and moments. The added mass and inertia reflects the build-up of kinetic energy of the fluid as the hull moves through it. The motion of the fluid associated with the accelerations produces the sailboat to move with an equivalent added mass and inertia, although the fluid does not move along with it. In the model, this effect is described by terms proportional to the accelerations.

ii) Motion in an ideal fluid with circulation. In this analysis, the shape of the hull and keel is relevant. For a body with a profile comparable to an air wing, there is a force acting on it when it moves in the fluid. This reveals the existence of lift forces acting on the center of pressure (CP) of the hull.
Since the CP in sway motion is forward to the CM, there exist a moment that tends to increase the angle of attack.

iii) Motion in a viscous fluid. In this analysis, it is revealed the presence of hydrodynamic resistance. This resistance is constituted by a number of different components caused by the interaction of a variety of phenomena’s. For instance, the total calm water resistance can be assumed to be made up of three components [Lewis, 1988]: frictional resistance (due to the motion of the hull in viscous fluid), wave-making resistance (due to the energy carried away by the waves generated by the hull on the sea surface) and Eddy resistance (due to energy carried away by eddies shed from the hull and keel).

The forces and moments caused by the action of the fluids, $\tau_h$ (water) and $\tau_a$ (wind) in the hull and keel are extremely complex. For the mathematical formulation of this problem, one strategy is to use the equation of continuity and Navier-Stokes equations with their boundary conditions. The set of forces and moments would be obtained by integrating the tangential stress of the cut along the surface of the sailboat, and thus depend on the history of the movement of the sailboat, as well as the initial conditions of the fluid. According to this, it appears that $\tau_h$ and $\tau_a$ depends on a dynamic and nonlinear form from $\Re$, $\Re$ and $\Re\Re$. Given the complexity of these forces and moments, to compute $\tau_h$ some simplifications became necessary

i) amplitude of the existing waves in the sea is small relative to its wavelength;  
ii) wavelengths existing in the sea are large relative to the length of the sailboat, so that the velocity of the fluid can be considered locally uniform;  
iii) frequency of movement of the sailboat is low;  
iv) boundary layer of the sailboat is small and the separation is negligible.

According to the hypothesis i), the sea state can be described based on the linear theory of gravity waves, where the action of waves and currents affects the hydrodynamic forces and moments through the sailboat-fluid relative velocity and through the inclusion of the Froude-Kriloff forces, which depend only on the acceleration of fluid. The Froude-Kriloff force is introduced by the pressure field generated by waves. The Froude–Kriloff force together with the diffraction force make up the total non-viscous forces acting on a floating body in regular waves. The diffraction force is due to the floating body that disturbs the waves.

Assumptions ii) and iii) make the surface of the fluid substantially flat, enabling the rejection of the memory effects associated with the fluid surface gravity waves. Taking in account iv), the pressure forces can be computed from the model of an ideal and irrotational fluid, assuming the quasi-stationary regime for the forces of viscosity. Thus the memory effects are small and the fluid forces and moments applied to the vessel can be expressed in terms of instantaneous values of position, velocities and accelerations. Neglecting the roll and pitch motions by assuming that those are small, and considering that $z \approx 0$, a 3 DOF dynamic model is developed for control purposes. By neglecting the pitch motion, it is assumed that it does not affect the behavior of the rudder, i.e. the rudder is always in the water. This 3 DOF dynamic model is the equivalent model of the sailboat in $\{H\}$. Through a division of the modeling of the dynamics
in two reference frames, \( \{H\} \) and \( \{V\} \), in \( \{H\} \) the sailboat is predominantly excited by the forces of propulsion, while in \( \{V\} \) are mostly excited by the waves. As the gravity and hydrostatic pressure do not influence the horizontal modes, only the forces and moments of hydrodynamic and aerodynamic origin counts for the dynamic model. As the frequency of the wave amplitude is not very high, this implies low velocities and accelerations in \( \{V\} \). Therefore, this mode will be neglected for the development of the dynamics equations of the sailboat.

Along with a model of the environmental disturbances, this dynamic mode is useful for computational simulations, predictions of maneuvering qualities and for the design of the control system.

This way, in 3 DOF, or in \( \{H\} \), a generic vector \( \tau \) is given by

\[
\tau_{\text{generic}} = \begin{bmatrix} X_{\text{generic}} \\ Y_{\text{generic}} \\ N_{\text{generic}} \end{bmatrix}
\] (5.3)

## 5.1 Forces and Moments in the Control Surfaces

The computation of the forces and moments acting in the control surfaces can be obtained resorting to the Finite Wing Theory, where the rudder and sail are modeled as finite length wings.

![Figure 5.2 – Sketch of a wing and an airfoil](image)

![Figure 5.3 – Sketch of the most relevant characteristics of an airfoil](image)
The cross-sectional shape obtained by the intersection of a wing with its perpendicular plane is called airfoil (see Figure 5.2). The major design feature of an airfoil is the mean camber line, which is the locus of points halfway between the upper and lower surfaces of the airfoil measured perpendicular to the mean camber line itself (see Figure 5.3). The straight line connecting the leading and the trailing edge is the chord line of the airfoil, and the precise distance from the leading to the trailing edge measured along the chord line is simply designated by the chord of the airfoil, given by the symbol $c$.

The camber, the shape of the mean camber line and the thickness distribution of the airfoil control the lift force characteristics of the airfoil. The pressure and shear stress distributions over the wing surface by the fluids create an aerodynamic force that can be divided into two parts, a drag and a lift force. The drag force is always defined as the component of the aerodynamic force parallel to the apparent fluid, and the lift force is perpendicular to the drag force. Their magnitudes are related with the drag and lift coefficients, $C_L$ and $C_D$.

![Figure 5.4 – Sketch of the Drag and Lift forces in an airfoil](image)

In the sailboat, there are two control surfaces, where the acting forces and moments will be considered as if these control surfaces are an approximation of a finite wing. The rudder can be compared with a rigid symmetric wing, as the sail can be compared to a flexible cambered wing. Each section of the rudder and sail experiences lift and drag forces defined as the forces acting normal and parallel to the local apparent current or wind, respectively, at each radial location on the rudder and sail. As for an airfoil, the elemental forces can be defined from the fundamental drag and lift equations. In order to analyze these forces is essential to study the Reynold’s number effects and the wing’s camber.

### 5.1.1 Reynold’s Number Effect

In fluid mechanics, the Reynold’s number, $Re$, is a dimensionless number that gives a measure of the ratio of kinematic or inertial forces to the viscous forces in the fluid, or by other means the ratio of force required to push the fluid out of the way. This dimensionless number characterizes the flow’s ability to
negotiate the curves of a section without separation, quantifying the relative importance of these two types of forces relative to the flow conditions.

In order to maximize the efficiency of a wing, the Reynold’s number effect of the wing section design must be taken into consideration, or otherwise, ignoring it, can result in sections with poor performance. 

\[ R_e = \frac{\rho f V_f c}{\mu_f} \]  

(5.4)

where \( \rho_f \) denotes the density of the fluid, \( V_f \) the velocity of the fluid, \( c \) the characteristic length or the chord length and \( \mu_f \) the viscosity of the environment.

Typically, low Reynolds number range corresponds to small model airplanes, usually gliders. Both models, the glider and the sailboat, require a high lift/drag (L/D) ratio. In a glider, this corresponds to glide distance or a minimum sink condition, as in a sailboat, this corresponds to the ability to point upwind. Both models also require a high maximum lift \( c_L \). In the case of a sailboat, the maximum speed is configured while sailing across or downwind. Low Reynold’s numbers produce several effects that make design of high lift sections difficult. The flow about an airfoil, at low Reynold’s numbers, is almost entirely laminar. In the case of airfoil sections, the flow separates, but then reattaches causing a laminar separation bubble whose Eddy’s flow results in a very large increase in the drag of the section. Furthermore, as soon as the section is put at an increased angle of attack, the laminar separation bubble bursts, causes large scale flow separation and effectively limits the maximum lift coefficient attainable.

Some background research suggests that the lift coefficient for a trimmed jib and mainsail, due to their sharp leading edge, tend to be insensitive to Reynolds number variation, unlike the lift coefficient for the rudder.

### 5.1.2 Wing’s Camber

Airfoils can be symmetrical or asymmetrical. Symmetric characteristics, in sailing vessels are important, because the sailboat needs to sail equally, on both port and starboard.

An asymmetrical airfoil section can achieve higher maximum lift coefficients and lift/drag ratio, than a symmetric airfoil section. Symmetric airfoil sections have the benefit of identical lift characteristics for positive and negative angles of attack. The camber is related with the asymmetry between the top and the bottom curves of an airfoil.
5.1.3 Drag and Lift Forces in a Wing

The analysis of forces and moments acting in a wing is developed in a reference frame supportive in the wing, \{W\}, where the origin lies in the middle of the wing’s chord, with the x-axis guided through the wing. To compute the drag and lift forces, the wing is considered finite.

![Figure 5.5 – Sketch of the drag and lift forces produced by the existence of an apparent fluid through the wing](image)

Figure 5.5 shows a wing in an inertial reference frame supportive in the wing, \{W\}, the apparent velocity of the fluid, \(V_{af}\), its components in surge and sway, \(u_{af}\) and \(v_{af}\), respectively, and the direction of the drag and lift forces according to the direction of the apparent velocity of the fluid. In this situation, the forces applied by the apparent fluid to the wing can be expressed as a combination of a drag force (in the direction of the apparent fluid velocity) and a lift force (normal to the apparent fluid velocity), both applied in a center of effort, CE. According to this analysis, the drag and lift can be expressed by

\[
L = \frac{1}{2} \rho_0 S_w c_L(\alpha_w)V_{af}^2
\]

\[
D = \frac{1}{2} \rho_0 S_w c_D(\alpha_w)V_{af}^2
\]

(5.5)

where \(\rho_0\) denotes the density of the of the fluid (\(\approx 1,026 kg.m^{-3}\) for the water and \(\approx 1,2928 kg.m^{-3}\) for the air, at normal conditions of temperature and atmospheric pressure), \(S_w\) the apparent area of the wing \((S_w = c.p, \text{ with } c \text{ as the wing’s chord and } b \text{ as the wing’s span})\), \(c_D\) and \(c_L\) are the drag and lift coefficients, which are influenced by the \(Re\) and angle of attack, \(\alpha_w\).

\(V_{af}\) is the apparent velocity of the fluid, which is defined as in the Figure 5.6 and given by
\[ V_{af} = V_f - \vec{V} \] (5.6)

where \( V \) denotes the total velocity of the sailboat and \( V_f \) the fluid’s velocity, both measured in the same reference frame, in this case \( \{W\} \).

Figure 5.6 – Definition of the apparent velocity of the fluid

One of the main objectives of aerodynamics is the prediction of appropriated values for the coefficients \( c_L \) and \( c_D \), through the basic concepts of physical science.

A large bulk of experimental airfoil data was compiled over the years, first by the National Advisory Committee of Aeronautics (NACA), and then by the National Aeronautics and space Administration (NASA). In these experiments, drag and lift coefficients were systematically measured for many airfoil shapes in low-speed sub-sonic wind tunnels. With the help of these compiled data, the computation of the drag and lift coefficients becomes more modest.

Selecting a NACA airfoil profile for each one of the “finite wings”, rudder and sail, the lift coefficient is given in the profile, for angles of attack, \( \alpha_w \), between zero and the stall angle. Then, the drag coefficient is given by the airfoil profile for this value of the lift coefficient and \( R_c \).
Figure 5.7 – Sketch of the lift coefficient as a function of the angle of attack for a symmetric and a cambered wing

Figure 5.7 exhibits a sketch of a typical lift coefficient curve for a symmetric and a cambered airfoil, from where can be determined that when the angle of attack is increased, the adverse pressure gradient on the top surface of the airfoil will become stronger, and at the stalling angle of attack, the flow becomes separated from the top surface. When this separation occurs, the lift coefficient decreases drastically and the drag coefficient increases suddenly.

Until the stall angle, the drag and lift coefficients, for a generic wing, can be computed through the data from the NACA airfoils profiles by the expressions

\[
\begin{align*}
    c_L &= \frac{k \alpha_w}{1 + (2/A\text{Re})} \\
    c_D &= c_{d_0} + \frac{c_l^2}{2/A\text{Re}}
\end{align*}
\]

where \( k \) denotes the slope of the lift of the section with angle of attack \( \alpha_w \), \( \alpha_w \) denotes the angle of attack, \( A\text{Re} \) is the effective aspect ratio of the airfoil, \( c_L \) the lift coefficient, \( c_D \) the drag coefficient, \( c_{d_0} \) the profile drag coefficient and \( \frac{c_l}{\pi e A\text{Re}} \) the induced drag coefficient.
In Figure 5.8 is defined the angles of attack of the rudder and the sail, $\alpha_r$ and $\alpha_s$, respectively. In this figure, $V_{af}$ is the apparent velocity of the fluid defined by (5.6), $\beta_{af}$ is the side slip angle of the apparent velocity’s vector measured from the $x_B$-axis to the relative apparent velocity’s vector, and $\delta_r$ and $\delta_s$ are the angles of rudder and sail measured in the $x_B$-axis.

Through some tests of the sailboat in the sea, where the apparent velocities of the water and air, and their respective angles can be measured, as the rudder and sail angles, the values of $c_L$, $c_{d_0}$ and $c_D$ can be computed obtaining approximate values of these coefficients for each angle of the rudder and sail, which together with the apparent current and wind angles, allows the computation of the angles of attack of the rudder a sail, respectively. With the data given by this test, a NACA airfoil profile can be choose according to the characteristics of each “finite wing”.

As the relative flows from both fluids, air and water, can come from any direction, this makes the angles of attack vary between $-180^\circ$ and $180^\circ$. The curves for the lift coefficient from NACA airfoil profiles can only be used between the range $[-\alpha_{stall}, \alpha_{stall}]$. For values of angles of attack above these range, an estimation for the lift coefficients of the remaining angles of attack must be developed.
Figure 5.9 – Sketch of the lift coefficient as a function of the angle of attack for a symmetric wing, where \[ |C_{l\text{max}}| > |C_{l\text{min}}| \]

Figure 5.10 – Sketch of the drag coefficient as a function of the angle of attack for a symmetric wing

In the Figure 5.9, for negative angles of attack the lift coefficient curve is symmetrical to itself for positive angles of attack. The same happens with the coefficient of drag curve (Figure 5.10).

5.1.1 Rudder Analysis

Rudders are devices which develop large lift forces. Since the rudders are at the stern of the boat, the force induces a moment around the vertical axis and thus affects the boat’s heading. So, the rudder provides the necessary yaw moment to, either cause the sailboat to diverge from a straight course, or to return the sailboat to a proper course. In a sailboat, the rudder may also serve as a lifting foil to counteract leeway.
Using the standard approximation for airfoils in a moving fluid, the components of the forces and moment components of the rudder can be represented as a combination of a drag and lift forces (see Figure 5.11). The lifting action of the rudder comes from the difference in the average pressure of the water over the upper and lower surface of the rudder.

As for the drag, in order to minimize its value, the rudder should have the smallest possible area. However, if the rudder area is too small, it would difficult the control of the sailboat at lower speeds, where the drag is more important than the lift. Theoretically, an equivalent rigid rudder with elliptical chord distribution, give an acceptable level of control, mainly on the torque.
Considering the rudder from Figure 5.12, where \( a_r \) is half the major axis of the rudder’s foil and \( b_r \) is the minor axis of the rudder’s foil, its principal characteristics needed to the computation of the lift and drag coefficients are given by

\[
S_r = \frac{\pi a_r b_r}{4},
\]

\[
ARg_r = \frac{a_r^2}{S_r},
\]

\[
ARe_r = 2 \ ARg_r
\]

where \( ARg_r \) denotes the geometrical and \( ARe_r \) the effective aspect ratios of the rudder, and \( S_r \) is the surface area of the rudder foil. In order to compute the drag and lift coefficients given by (5.7), the area of the foil and the geometric aspect ratio, which can be assumed to be twice the geometric aspect ratio, due to the free surface boundary, must be determined, what can be achieved by (5.8). Through some field tests, the approximate curves for \( c_{ir}(\alpha_r) \) and \( c_{dr}(\alpha_r) \) could be optimize, leading to a more accurate model of the rudder.

In Figure 5.13, the apparent current velocity, \( V_{ac} \), and direction, \( \psi_{ac} \), and other useful angles for the computation of the drag and lift forces, as \( \beta_{ac}' \) and \( \beta_{ac}'' \), are defined.
Finally, with the values of the coefficients $c_r$ and $c_{dr}$ for each angle of attack, $\alpha_r$, in a range between $[-180^\circ, 180^\circ]$ it is possible to compute the forces, $X_r$, $Y_r$, and moment, $N_r$, through

\[
X_r = -|D_r| \cdot \cos(\beta_{ac}) + |L_r| \cdot \cos(\beta_{ac}^\prime) \\
Y_r = |D_r| \cdot \sin(\beta_{ac}) + |L_r| \cdot \sin(\beta_{ac}^\prime) \\
N_r = \frac{c_r}{4} Y_r
\]  

(5.9)

As the lift force is normal to the drag, its angle relative to $x_\beta$-axis, $\beta_{ac}^\prime$ (defined in the Figure 5.13), can be $\beta_{ac} + \frac{\pi}{2}$ or $\beta_{ac} - \frac{\pi}{2}$, depending in the position of the rudder, $\delta_r$, and the direction of the apparent current velocity, $\beta_{cr}$, in according to the data in Table 5.1. The rudder’s angle of attack, depends on the direction of the oncoming apparent fluid (Figure 5.8), and can be defined as, $\alpha_r = (\beta_{ac} - (\delta_r + \pi))$, between the range $-\pi \leq \alpha_r \leq \pi$. The angle of the rudder is limited within the range $-55^\circ < \delta_r < 55^\circ$, due to the physical model limitations. For this purpose, all the angles are measured in the clockwise direction, as shown in Figure 5.13.

<table>
<thead>
<tr>
<th>$\beta_{ac}^\prime$</th>
<th>$\delta_r &lt; \beta_{ac}^\prime &lt; \delta_r + \pi$</th>
<th>$\delta_r + \pi &lt; \beta_{ac}^\prime &lt; \delta_r + 2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{ac}$</td>
<td>$\beta_{ac} + \frac{\pi}{2}$</td>
<td>$\beta_{ac} - \frac{\pi}{2}$</td>
</tr>
</tbody>
</table>

Table 5.1 – Direction of the lift force in the rudder

5.1.2 Sail Analysis

The complex nature of the aerodynamics on a sail can make any sort of precise control very difficult to accomplish. In order to achieve precision, the disturbances generated by the propulsion system must be minimized.

To simplify the analysis of the forces acting in the sail, it can the can be considered as an infinite series of discrete airfoil sections, by approaching the clothed sail by a cambered wing. If the sailboat had a rigid sail, a symmetric wing could be chosen. However, with a clothed sail, when the sail’s angle of attack is null, there exists some lift force, due to the fact that all airfoils with camber have to be pitched to some negative or positive angle of attack to the flow, depending if the camber is positive or negative, respectively. Also, in this situation, a great amount of drag force is developed. Despite when the sail’s angle of attack is null, there exists some lift force, due to the camber of the clothed sail, this effect is neglected, for achieve symmetry on port and starboard motion.
Using this standard approach for airfoils in a moving fluid, the components of the forces acting in the sail can be modeled as a combination of a drag and a lift force, as in the Figure 5.14.

![Figure 5.14 – Drag and lift forces in the sail](image)

For simplicity of the problem, the sail can be considered equivalent to a triangle rectangle, Figure 5.15, where $a_s$ is half the major axis of the sail’s foil and $b_s$ is the minor axis of the sail’s foil. Exploiting this approximation, the area of the sail’s foil, the geometric aspect ratio and the effective aspect ratio, needed to compute the drag and lift of a sail’s foil is given by (5.10), where $S_s$ is the surface area of the sail foil, $ARg_s$ is the geometric aspect ratio and $ARE_e$ is the effective aspect ratio, both from the sail foil.

![Figure 5.15 – Sketch of a sail with triangular chord distribution](image)
In order to establish these forces, the measurement of the apparent wind is required. The apparent wind is the actual flow of air acting on the sail and can be computed in a similar way, from what was done to the apparent current, and it is defined in Figure 5.16, as its direction, $\psi_{aw}$, and other useful measures for the computation of the drag and lift forces in the sail.

![Figure 5.16 – Useful velocities and respective angles to computation of lift and drag forces on the sail](image)

Identically to the rudder, some tests must be made to compute an approximate curve for the lift and drag coefficients, $c_L(\alpha_s)$ and $c_D(\alpha_s)$, for the sail. Finally, with the values of the coefficients $c_L$ and $c_D$ for each angle of attack, $\alpha_s$, it is possible to compute the forces, $X_s$, $Y_s$, and moment, $N_s$, acting in the sail through:

$$
\begin{align*}
X_s &= |D_s| \cdot \cos(\beta'_{aw}) + |L_s| \cdot \cos(\beta''_{aw}) \\
Y_s &= |D_s| \cdot \sin(\beta'_{aw}) + |L_s| \cdot \sin(\beta''_{aw}) \\
N_s &= \frac{l_s}{4} Y_s
\end{align*}
$$

(5.11)
where $D_z$ and $L_z$ are computed applying (5.5), and $\beta_{aw}', \beta_{aw''}$ and $l_z$ are defined in the Figure 5.16.

By (5.11), the torque generated by the sail, $N_z$, can be computed through the value of $l_z$, which is a vector that links the position of the center of effort, $CE$, with $CM$.

As the lift force is normal to the drag, its angle relative to $x_B$-axis, $\beta_{aw}'$ (defined in the Figure 5.16), can be $\beta_{aw}' + \frac{\pi}{2}$ or $\beta_{aw}' - \frac{\pi}{2}$ depending in the position of the sail, $\delta_z$, and the direction of the apparent velocity of the wind, $\beta_{aw}$, in accordance with the data in Table 5.2. The sail’s angle of attack, depends on the direction of the oncoming apparent fluid (Figure 5.8), and can be defined as, $\alpha_s = (\beta_{aw} - (\delta_s + \pi))$, between the range $-\pi \leq \alpha_s \leq \pi$.

<table>
<thead>
<tr>
<th>$\beta_{aw}'$</th>
<th>$\delta_s &lt; \beta_{aw}' &lt; \delta_s + \pi$</th>
<th>$\delta_s + \pi &lt; \beta_{aw}' &lt; \delta_s + 2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{aw}'$</td>
<td>$\beta_{aw}' + \frac{\pi}{2}$</td>
<td>$\beta_{aw}' - \frac{\pi}{2}$</td>
</tr>
</tbody>
</table>

5.1.3 Total Control Surface’s Forces and Moments

Applying (5.9) and (5.11) to (5.2) yields to

$$\tau_{cs} = \begin{bmatrix} X_{cs} \\ Y_{cs} \\ N_{cs} \end{bmatrix} = \begin{bmatrix} X_r + X_s \\ Y_r + Y_s \\ \left[\frac{c_r}{4}, Y_r\right] + \left[\frac{c_s}{4}, Y_s\right] \end{bmatrix}$$

(5.12)

5.2 Hydrodynamic and Aerodynamic Forces and Moments in the Body

In this section the forces and moments generated by the movement of a Body (hull and keel) in a fluid, $\tau_B$, which is the result of several hydrodynamic and aerodynamic components, are developed. For an ideal fluid, the main forces and moment components are

i) added mass,

ii) damping effects,

iii) Froude-Kriloff and Diffraction,

iv) restoring forces due to Archimedes (weight and buoyancy).
Restoring forces are only important in the case of mooring, and for the considered 3 DOF model these forces are neglected. The damping effects includes radiation-induced potential damping, skin friction, wave drift damping and damping due to vortex shedding [Faltinsen, 1990]. These damping effects are functions with many factors, including water and air density, viscosity, surface tension, pressure, vapor pressure and the motions of the body.

The Froude-Kriloff and Diffraction forces are the forces in the body when it is restrained from oscillating and there are incident waves.

The forces and moments acting in the body due to the fluids can be split into two types, a first type where the fluid is the current, for the submerged part of the hull and keel, and a second type where the fluid is the wind, for the part of the hull that is in contact with the atmosphere. Due to this fact, $\tau_h$ is given by (5.2). As the resistive force, $\tau_a$ is much smaller than $\tau_h$, $\tau_a$ will be neglected. Despite $\tau_a$ is not computed for the dynamic model of the sailboat, the develop of this force and moment follow a parallel logic from the computation of $\tau_h$, with the exception that the air offers a much lower resistance to the movement of the sailboat, than the sea.

5.2.1 Hydrodynamic Forces and Moments in the Body

This section presents the use of hydrodynamic coefficients as a form for predicting the hydrodynamic response to the motion of a Body in the water. According to [Fossen, 1994], in basic hydrodynamics it is common to assume that the hydrodynamic forces and moments in a body can be linearly superposed by considering the added mass and damping effects, and the Froude-Kriloff and Diffraction forces. Descriptions of non-linear hydrodynamic forces and moments for a body, where the coefficients are estimated theoretically and experimentally can be found, per instance, in [Lewis, 1989] and [Ogawa et al., 1978].

The analysis of the hydrodynamic forces and moments in the body are made basis in a reference frame supportive with the Body, which origin is situated in the CM of the Body, and with the $x_B$-axis directed to the bow.

$$\tau_h = \begin{bmatrix} X_h \\ Y_h \\ N_h \end{bmatrix} = \begin{bmatrix} X_{RB} + X_A + X_D \\ Y_{RB} + Y_A + Y_D \\ N_{RB} + N_A + N_D \end{bmatrix}$$

(5.13)

Following the description made in [Inuoe et al., 1981], the total forces and moment of the body can be given by the sum of the hydrodynamics of the rigid body with the added mass and damping forces (5.13). In this equation, $X_{RB}$, $X_A$ and $X_D$ are the force along $x_B$-axis relative to the hydrodynamic force of the rigid body, to the added mass terms and to the damping force, respectively.
It could, also, be possible, in a way similar to what was done with the wing, with the exception that the body cannot generate high lift forces, to estimate the terms due to accelerations and rotations, and also the forces and moments that arise in the body, assuming that the body is moving in an ideal and irrotational fluid, but for reasons related to the accuracy of the sailboat’s model, the description made in [Inoue et al., 1981] was chosen. Therefore, \( \tau_{RB} \) is modeled as a nonlinear function of the accelerations \( \dot{\nu} \), velocities \( \nu \), and Euler angles included in \( \eta \), and can be expressed in a series expansion that is affine in the parameters or coefficients, yielding to

\[
X_{RB} = \frac{1}{2} \rho_w A_{UT} V_{rc}^2 (X' u_{rc}^2 + X' v_{rc}^2 + X' u_{rc} r_{rc} + X' v_{rc} r_{rc} + X' r_{rc}^2)
\]

\[
Y_{RB} = \frac{1}{2} \rho_w A_{UL} V_{rc}^2 (Y' u_{rc}^2 + Y' v_{rc}^2 + Y' u_{rc}^2 + Y' v_{rc}^2 + Y' r_{rc}^2 + Y' v_{rc}^2 + Y' r_{rc}^2)
\]

\[
N_{RB} = \frac{1}{2} \rho_w A_{UL} L^2 V_{rc}^2 (N' u_{rc}^2 + N' v_{rc}^2 + N' u_{rc}^2 + N' v_{rc}^2 + N' r_{rc}^2 + N' v_{rc}^2 + N' r_{rc}^2)
\]

where \( u_{rc}^i, v_{rc}^i, r_{rc}^i \) are dimensionless velocities, \( X'_{uu}, X'_{uu}, \ldots \) \( N'_{rr} \) are dimensionless coefficients, \( \rho_w \) is the water density, \( L \) is the length of the sailboat, \( A_{UT} \) is the underwater transverse cross-sectional area, \( A_{UL} \) is the underwater longitudinal centerline area and \( r_{rc} \) is the relative yaw rate from the sailboat to the current, that when in approximation of the fluid by an irrotational fluid \( r_{rc} = r \), where \( r \) is the yaw rate of the sailboat, and \( V_{rc} \) is the relative velocity of the sailboat to the fluid. In the case of the hydrodynamics computation, the \( V_{rc}, u_{rc} \) and \( v_{rc} \) are defined in the Figure 5.17, and given by

\[
\bar{V}_{rc} = \bar{V} - V_c
\]

\[
\bar{u}_{rc} = \bar{V}_{rc} \cos (\beta_{rc}) = u - u_c
\]

\[
\bar{v}_{rc} = \bar{V}_{rc} \sin (\beta_{rc}) = v - v_c
\]
Figure 5.17 – Definition of the relative velocity from the sailboat to the current, $V_{rc}$ and its direction, $\beta_{rc}$

The dimensionless coefficients of the model, $X'_{uu}$, $X'_{vv}$, ..., $N'_{rr}$, can be computed from the Taylor Expansion Series by (5.16).

$$f(x) = f(x_0) + \frac{\partial f(x_0)}{\partial x}(x - x_0) + \frac{1}{2!} \frac{\partial^2 f(x_0)}{\partial x^2} (x - x_0)^2 + \cdots$$  \hspace{1cm} (5.16)

$$f_x = \frac{\partial f(x_0)}{\partial x}$$

$$f_{xx} = \frac{1}{2!} \frac{\partial^2 f(x_0)}{\partial x^2}$$  \hspace{1cm} (5.17)

Using the notation from (5.17), in some cases where the formal Taylor Series is meaningless, the notation is still clear. (5.16) and (5.17) leads to the writing of, per instance, a drag force as

$$F = \frac{1}{2} \rho C_d A u |u| = F_{ulu}|u|$$  \hspace{1cm} (5.18)

Furthermore, the dimensionless coefficients of the body in equation (5.14), are the indicated in the Table 5.3, developed based in the geometrical parameters of the hull and keel of the sailboat, and the dimensionless velocities in Table 5.4.
Table 5.3 – Dimensionless coefficients of the sailboat’s body

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X'_{uu}$</td>
<td>-0.12</td>
<td>-0.244</td>
<td>-0.2</td>
</tr>
<tr>
<td>$X'_{uv}$</td>
<td>-0.089</td>
<td>-0.9</td>
<td>-0.044</td>
</tr>
<tr>
<td>$X'_{ur}$</td>
<td>0.07</td>
<td>0.079</td>
<td>0.07</td>
</tr>
<tr>
<td>$X'_{rr}$</td>
<td>0.05</td>
<td>-0.25</td>
<td>-0.6</td>
</tr>
<tr>
<td>$X'_{rr}$</td>
<td>0.011</td>
<td>-0.087</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Table 5.4 – Relation between the current relative dimensional and dimensionless velocities

<table>
<thead>
<tr>
<th>Velocities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$u'_{rc}$</td>
<td>$\frac{u_{rc}}{v_{rc}}$</td>
</tr>
<tr>
<td>$v'_{rc}$</td>
<td>$\frac{v_{rc}}{v_{rc}}$</td>
</tr>
<tr>
<td>$r'_{rc}$</td>
<td>$\frac{r_{rcL_{rc}}}{v_{rc}}$</td>
</tr>
</tbody>
</table>

5.2.2 Added Mass

The concept of added mass is well known and its effects have been included in all accurate ship simulation models.

A body in a movement set in a stationary fluid, generates motion to the particles of the fluid that surround it. The acceleration of these particles induces forces and moment opposed to the movement of the body [Techet, 2005]. Consequently, the fluid will possess kinetic energy produced by the work of the sailboat on the fluid. The kinetic energy expression of the fluid is

$$T_A = \frac{1}{2}v^T M_A v$$  \hspace{1cm} (5.19)

where $M_A$ is the 6x6 system inertia matrix of added mass terms and $v$ is the 3 DOF vector of motion given by $v = [u, v, r]$.

Furthermore, the equations of motion must take into account the kinetic energy given to the fluid. This is performed through the sum of the added mass terms, which functions as an apparent increase of the mass
of the sailboat, to the equations of motion of the sailboat. So, added mass should be understood as pressure-induced forces and moments proportional to the acceleration of the body, due to its motion. Therefore the forces and moments that actuate in the rigid body through the added mass effect can be computed as

\[ \tau_A = -M_A \dot{v}_{rc} - C_A(v) v_{rc} \]  

(5.20)

where \( v_{rc} = [u_{rc}, v_{rc}, r_{rc}] \)  \( M_A \) is the 6x6 system inertia matrix of added mass terms and \( C_A(v) \) is the added mass Coriolis and centripetal matrix. \( M_A \) terms are given by

\[ M_A = -\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (A_{ij} \in R^{3x3}) \]

(5.21)

In (5.21), the NAME notation is also. For instance, the hydrodynamic added mass force \( Y \) along the \( y_B \)-axis due to acceleration \( \ddot{u} \) in the \( x_B \)-axis direction is written as

\[ Y = -Y_u \ddot{u} \]  

(5.22)

where

\[ Y_u = \frac{\partial Y}{\partial u} \]  

(5.23)

As for the added mass Coriolis and centripetal terms, considering an ideal fluid, \( C_A(v) \) can be parameterized such that
\[ C_A(v) = -C_A^T(v), \quad \forall v \in \mathbb{R}^6 \] (5.24)

by defining

\[
C_A(v) = - \begin{bmatrix}
0_{3 \times 3} & -S(A_{11}v_1 + A_{12}v_2) \\
-S(A_{21}v_1 + A_{22}v_2)
\end{bmatrix} = - \begin{bmatrix}
0 & 0 & 0 & Z_w v Y_o v \\
0 & 0 & 0 & Y_o v X_u u \\
0 & 0 & 0 & N_r r M_q q \\
Z_w w_0 X_u u & 0 & 0 & K_q p \\
Y_o v & X_u u & 0 & M_q q K_p p \\
Y_o v & X_u u & 0 & M_q q K_p p
\end{bmatrix}
\] (5.25)

Considering (5.21), (5.25) and a 3 DOF model, \( \tau_A \) can be computed as

\[
\tau_A = \begin{bmatrix}
X_u \\
Y_u \\
N_u
\end{bmatrix} = \begin{bmatrix}
X_u & X_v & X_r \\
Y_u & Y_v & Y_r \\
N_u & N_v & N_r
\end{bmatrix} \begin{bmatrix}
\dot{u}_{\tau c} \\
\dot{v}_{\tau c} \\
\dot{r}
\end{bmatrix} + \begin{bmatrix}
0 & 0 & -(Y_o v + Y_r r) \\
0 & 0 & X_u u \\
Y_o v & Y_r r & -X_u u
\end{bmatrix} \begin{bmatrix}
\dot{u}_{\tau c} \\
\dot{v}_{\tau c} \\
\dot{r}
\end{bmatrix}
\] (5.26)

For surface vessels the added mass coefficients are dependent of the wave circular frequency, and as the vehicle will move at a low speed, the contribution from of the off-diagonal elements in the added mass matrix can be neglected. This approximation has been made due to the fact that the off-diagonal elements of a positive matrix of inertia will be much smaller than their diagonal counterparts.

The added mass terms can be derived applying strip theory [Fossen, 1994]. Based on the assumptions of the inviscid fluid, no circulation and double-body theory, which allows an analysis based on the separation of the submerged and emerged parts of the body and the division of the submerged part of the body into a number of strips, the hydrodynamic coefficients for added mass in two-dimensional can be computed for each strip and summarized over the length of the body to yield to the three-dimensional coefficients. This yields to

\[
X_u = \frac{\alpha}{2 - \alpha} \frac{4}{3} \pi \epsilon_x \epsilon_y \epsilon_z \rho
\]

\[
Y_v = \frac{\beta}{2 - \beta} \frac{4}{3} \pi \epsilon_x \epsilon_y \epsilon_z \rho
\]

\[
N_r = \frac{1}{5} \frac{(\epsilon_z^2 - \epsilon_x^2)^2 (\beta - \alpha)}{2(\epsilon_z^2 - \epsilon_x^2) + (\epsilon_z^2 + \epsilon_x^2)(\alpha - \beta)} \frac{4}{3} \pi \epsilon_x \epsilon_y \epsilon_z \rho
\] (5.27)
where

\[ \alpha = \epsilon_x \epsilon_y \epsilon_z \int_0^\infty \frac{d\lambda}{(\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2) \Delta} \]

\[ \beta = \epsilon_x \epsilon_y \epsilon_z \int_0^\infty \frac{d\lambda}{(\epsilon_y^2 + \lambda) \Delta} \]

\[ \Delta = \sqrt{(\epsilon_x^2 + \lambda) + (\epsilon_y^2 + \lambda) + (\epsilon_z^2 + \lambda)} \]

(5.28)

To the computation of the added mass forces and moment from (5.27), the body can be approximated by a semi-ellipsoid, with an increase of 25% in the dimensions, being that its semi-axes \( \epsilon_x, \epsilon_y \) and \( \epsilon_z \) are given by

\[ \epsilon_x = 1.25 \frac{L}{2} \]

\[ \epsilon_y = 1.25 \frac{B}{2} \]

\[ \epsilon_z = 1.25 T \]

(5.29)

where \( L \) is the length and \( B \) is the width of the sailboat. The reason of this choice is to obtain values of the added mass terms closer with the values given by empirical formulas, which can be found in [Lewis, 1989]. Finally, the computed values for the added mass terms are given in the Table 5.5.

<table>
<thead>
<tr>
<th>Added mass terms</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_a ) [kg]</td>
<td>0.231</td>
</tr>
<tr>
<td>( Y_a ) [kg]</td>
<td>9.95</td>
</tr>
<tr>
<td>( N_r ) [kg.m^2]</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Table 5.5 – Added mass term values
5.2.3 Damping Forces and Moment

When a Body has motion, the sea offers resistance to the movement of submerged part of this body. One type of resistance is called by hydrodynamic damping. As mentioned previously, the hydrodynamic damping in mainly caused by

i) radiation-induced potential damping due to force body oscillations,

ii) linear skin friction due to laminar boundary layers and quadratic skin friction due to turbulent boundary layers,

iii) wave drift,

iv) damping due to vortex shedding.

Consequently the total hydrodynamic damping matrix can be written as a sum of these components, such that

\[ \tau_D = -D_{rt}(v_{rc})v_{rc} - D_{sf}(v_{rc})v_{rc} - D_{wd}(v_{rc})v_{rc} - D_{vs}(v_{rc})v_{rc} \]  (5.30)

where \( D_{rt}(v_{rc}) \) denotes the radiation-induced potential, \( D_{sf}(v_{rc}) \) to skin friction, \( D_{wd}(v_{rc}) \) wave drift, \( D_{vs}(v_{rc}) \) vortex shedding and \( D_T(v_{rc}) \) total damping matrices. \( v_{rc} \) in the studied 3 DOF system is given by

\[ v_{rc} = v - v_c = [u_{rc}, v_{rc}, r_{rc}] \]  (5.31)

These different damping terms, from (5.30), contribute to both linear and quadratic damping. As it is, in general, difficult to separate these effects, for a body moving through an ideal fluid, an approximate matrix of all these damping effects, \( D_T \), will be computed. This matrix is real, non-symmetric and strictly positive, and can be written by

\[ D_T = \begin{bmatrix} X_u & X_p & X_w & X_p & X_q & X_r \\ Y_u & Y_p & Y_w & Y_p & Y_q & Y_r \\ Z_u & Z_p & Z_w & Z_p & Z_q & Z_r \\ K_u & K_p & K_w & K_p & K_q & K_r \\ M_u & M_p & M_w & M_p & M_q & M_r \\ N_u & N_p & N_w & N_p & N_q & N_r \end{bmatrix} \]  (5.32)
Considering the 3 DOF system, for low speed vessels, the surge and the steering modes (sway and yaw) can be decoupled. As the off-diagonal terms in the general expression for the total damping coefficients are complex to compute, usually it is convenient to write the total hydrodynamic damping, $D_T$, as

$$
D_T(v_{rc}) = -\begin{bmatrix}
X_u & 0 & 0 \\
0 & Y_v & Y_r \\
0 & 0 & N_v & N_r
\end{bmatrix}
$$  \hspace{1cm} (5.33)

There are various methods to estimate the total damping coefficients, from experimental technics with models of reduced scale diagrams to numerical methods. Approaching the damping force by a quadratic drag force \cite{Faltinsen, 2005}, per instance, along the $x_B$-axis

$$
X_{D_T} = -\frac{1}{2} \rho w C_{D_d} A_{yz} u_{rc} |u_{rc}|
$$  \hspace{1cm} (5.34)

where $\rho w$ is the density of the water, $C_{D_d}$ is the drag coefficient of the total damping and $A_{yz}$ is the projected area of the body along the reference frame $y_Bz_B$. The drag coefficient can be obtained using empirical expressions. As the drag coefficient depends on the Reynold’s number, for this case, with $l \approx 1.5m$, $v = 1.005e-6$ m$^2$.s$^{-1}$ for a temperature $T = 20^\circ C$, and a typical velocity value of $V = 1$ m.s$^{-1}$, $Re = Vl/v = 1.5e6$. Computing $C_{D_d}$ through the laminar flux theory \cite{Hoerner, 1965} to an ellipsoidal body with nonzero speed, and applying (5.34) to (5.30), (5.36) and the values of Table 5.6 are obtained.

$$
X_u = -\frac{1}{2} \rho w C_{D_d} A_{yz}
$$

$$
Y_v = -\frac{1}{2} \rho w C_{D_d} A_{xx}
$$

$$
Y_r = -\frac{1}{2} \rho w C_{D_d} A_{xx}
$$

$$
N_v = -\frac{1}{2} \rho w C_{D_d} A_{xy}
$$

$$
N_r = -\frac{1}{2} \rho w C_{D_d} A_{xy}
$$  \hspace{1cm} (5.36)
Table 5.6 – Damping coefficients

<table>
<thead>
<tr>
<th>Damping coefficients</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ [kg.m$^{-1}$]</td>
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</tr>
<tr>
<td>$Y_1$[kg.m$^{-1}$]</td>
<td>50.8</td>
</tr>
<tr>
<td>$Y_2$[kg.m]</td>
<td>1.83</td>
</tr>
<tr>
<td>$N_1$[kg]</td>
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</tr>
<tr>
<td>$N_2$[kg.m$^{-2}$]</td>
<td>2.31</td>
</tr>
</tbody>
</table>

5.3 Disturbances

A marine vessel is influenced by wind, waves and currents. They appear as disturbances in the equation of motion. It is assumed that the disturbances can be modeled by forces and moments. The following disturbances will be discussed

− Current,
− Wind,
− Waves.

5.3.1 Current

Currents in the upper layer of the ocean are mainly generated by the atmospheric wind system over the sea surface. Besides the wind-generated currents, the exchanges of heat at the sea surface develop an additional sea current. As the study case is an ASV, it is considered a two-dimensional current model, described by two parameters, the current speed, $V_c$, and the direction of the current, $\psi_c$. 
The current is assumed to be constant and homogeneous, and it only influences the inertial velocity of the sailboat, and not the hydrodynamic forces and moments. Relating to Figure 5.18 the velocity components of the current are obtained as following

\[
\begin{align*}
    u_c &= \bar{V}_c \cos(\psi_c - \psi - \pi) \\
    v_c &= \bar{V}_c \sin(\psi_c - \psi - \pi)
\end{align*}
\]  

(5.37)

Thus, the influence of the current on the sailboat’s motion is given by the forces and moments in (5.38).

\[
\begin{align*}
    X_{\text{current}} &= m v_c \dot{\psi} \\
    Y_{\text{current}} &= m u_c \dot{\psi} \\
    N_{\text{current}} &= m x_{CG} u_c \dot{\psi} = 0
\end{align*}
\]

(5.38)

These forces and moment are added to the model of the sailboat.
5.3.2 Wind

The wind effects, which determine the dynamical response of the sailboat, can be modeled by a combination of a mean wind speed and a turbulent wind component, describing the effect of gusting. The mean component of wind exposes the sailboat to a quasi-steady force although non-stationary in a long period of time, while the turbulent component is random in magnitude and direction, and it can be characterized by an appropriate spectrum. The turbulent component of the wind in magnitude and in direction can be determined each time instant from the realization of a stochastic process of known spectral density. [Davies, 1996] proposes a first order approximation of the wind spectral density, which is used during the simulation study, in order to account for the random nature of the environmental wind.

\[
V_{rw} = \sqrt{u_{rw}^2 + v_{rw}^2} \\
\psi_{rw} = \arctan \left( \frac{v_{rw}}{u_{rw}} \right) \\
u_{rw} = V_w \cos(\beta_w) - u \\
v_{rw} = V_w \sin(\beta_w) - v
\]

(5.39)

Based on [Isherwood, 1972] the influence of the wind on the sailboat's motion is given by the following forces and moments
\[
X_{\text{wind}} = \frac{1}{2} \rho_{\text{air}} V_R^2 c_X (\beta_R) A_T
\]
\[
Y_{\text{wind}} = \frac{1}{2} \rho_{\text{air}} V_R^2 c_Y (\beta_R) A_L
\]
\[
N_{\text{wind}} = \frac{1}{2} \rho_{\text{air}} V_R^2 c_N (\beta_R) A_L L
\]

where \( \rho_{\text{air}} \) is the density of air \((\rho_{\text{air}} = 1.23 \text{kg/m}^3)\), \(L\) the vessel length, \(A_T\) and \(A_L\) are the transverse and lateral projected areas. For the computation of the forces and moment, \(c_X\), \(c_Y\) and \(c_N\) can be determined using recent studies about the identification of vessels parameters, like \([\text{Astrom and Kallstrom, 1975}]\) and \([\text{Isherwood, 1972}]\). These data can also be found in \([\text{Fossen, 1994}]\).

Based on the arguments shown before, this disturbance was neglected in the sailboat’s model, due to its small influence relatively to the other disturbances.

### 5.3.3 Waves

The surface waves introduce inertia and drag hydrodynamic forces. The inertia force is the sum of two components. The first is a buoyancy force acting on the body due to a pressure gradient generated from the flow acceleration. The buoyancy force is equal to the mass of the fluid displaced by the body, multiplied by the acceleration of the flow. The second, the inertia component is due to the added mass, which is proportional to the relative acceleration between the body and the fluid.

The drag force is the sum of the viscous and pressure drags produced by the relative velocity between the body and the flow. This type of hydrodynamic drag is proportional to the square of the relative velocity.

The influence of incident waves of arbitrary direction along the body is the change of the average submerged shape defined by the instantaneous position of the wave.

As waves are random in both time and space, to find a reasonable characterization of the waves, a simplified model describing regular waves is used. It considers a simple two-dimensional wave train progressing over an infinite water surface with an infinite depth. Assuming that the wave amplitude is small compared with the wavelength and the water is incompressible, necessary condition in order to guarantee that the velocity of the fluid can be considered uniform all over the sailboat, a wave profile is obtained

\[
\xi(x_0, t) = \frac{h}{2} \cos(kx_0 - \omega t)
\]

where \(\xi(x_0, t)\) is the wave amplitude, \(z_0\) is coordinate of the sea level at \(x_0\) and time \(t\), and \(h\) is the wave height of Figure 5.20.
The wave number, \( k \), is defined as \( k = \frac{2\pi}{\lambda} \), with \( \lambda \) as the wavelength. The theory of gravity waves gives the relation

\[
k = \frac{4\pi^2w_e^2}{g}
\]  

(5.42)

with \( g \) as the acceleration of gravity and where \( w_e = w_w - kV\cos(\chi) \) is the encounter frequency of the wave by the sailboat when it advances with speed \( V \), with \( \chi \) as the wave incidence. From (5.41), the wave slope \( s(x_0, t) \) can be obtained by

\[
s(x_0, t) = \frac{\partial \xi}{\partial x_0} = -(kh/2) \sin(kx_0 - \omega t)
\]  

(5.43)

To derive an expression for the forces and moment induced by the waves, two assumptions must be made

i) the forces and moment only result from pressure,

ii) the wave field is not disturbed by the presence of the vessel.

Thus, the forces and moment can be computed by

\[
X_{\text{wave}} = \rho gBLT \cos(\beta_c)s(t) \\
Y_{\text{wave}} = -\rho gBLT \sin(\beta_c)s(t) \\
N_{\text{wave}} = \frac{1}{24}\rho gBL(L^2 - B^2)Tk^2 \sin(2\beta_c)\xi(t)
\]  

(5.44)

where \( L \) is the length, \( B \) is the breadth, \( T \) is the draft and \( \rho \) is the density of the water.
Open-loop Simulation

A testing on a real sailboat is limited by weather conditions, however by simulation it is possible to predict the behavior of the sailboat under determined circumstances. In order to run these tests, it is necessary to develop and identify a suitable simulator system which needs access to basic information (mainly the wind’s direction and the sailboat’s heading), to compute the forces and moments and estimate the behavior of the sailboat.

The developed computational model is described in the Figure 6.1, where

- $\eta = [x \ y \ \psi]^T$ – sailboat’s position,
- $\nu = [u \ v \ r]^T$ – sailboat’s velocity,
- $\delta_r$ – rudder’s angle,
- $\delta_s$ – sail’s angle,
- $V_w, \psi_w$ – wind’s velocity and direction,
- $V_c, \psi_c$ – current’s velocity and direction.

Figure 6.1 – Sailboat’s simplified model
6.1 State Model

The computation of the velocities in the 3 DOF, $\mathbf{v}_{(H)} = [u, v, r]^T$ is made through

$$
\mathbf{v}_{(H)} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
V_{rc} \cos(\psi + \beta_{rc}) + \bar{V}_c \cos(\psi_c) \\
V_{rc} \cos(\psi + \beta_{rc}) + \bar{V}_c \cos(\psi_c) \\
\end{bmatrix}
$$

(6.1)

where $\bar{V}_{rc}^*$ is defined in the Figure 5.17 and by (5.15), $\beta_{rc}$ is the angle of $\bar{V}_{rc}^*$ in $\{B\}$, $\bar{V}_c$ and $\psi_c$ are the velocity and direction of the current in $\{I\}$ defined in the Figure 5.18 and $\psi$ is the heading of the sailboat in $\{I\}$.

For the acceleration equations, $\dot{\mathbf{v}}_{(H)} = [\ddot{u}, \ddot{v}, \ddot{r}]^T$ is given by

$$
\dot{\mathbf{v}}_{(H)} = M^{-1}\mathbf{r}_{(H)} - M^{-1}\mathbf{C}\mathbf{v}_{(H)}
$$

(6.2)

where $M$ is the total mass and inertia matrix (including added mass terms), $C$ is the Coriolis and centripetal matrix (including added mass terms), both in 3 DOF, and $\mathbf{r}_{(H)}$ is given by

$$
\mathbf{r}_{(H)} = \begin{bmatrix}
X_T \\
Y_T \\
N_T
\end{bmatrix} = \begin{bmatrix}
X_B + X_A + X_{CS} \\
Y_B + Y_A + Y_{CS} \\
N_B + N_A + N_{CS}
\end{bmatrix}
$$

(6.3)

The simulation of this state model can be simulated through the Euler approximation or integrating the output of the model, what gives $\eta$ and $\mathbf{v}$. Through the integration of the state model equations (6.2) and (6.3), a prediction the behavior of the sailboat is possible.

This model simulated in *Simulink* is constituted by four principal blocks (which are depicted in Figure 6.2)

i) Rigid Body, a dynamical block whose input is the vector of forces and moments applied to the sailboat, $\mathbf{r} = [X \ Y \ N]^T$, and whose outputs are the position, $\eta = [x \ y \ \psi]^T$, and velocity, $\mathbf{v} = [u \ v \ r]^T$;

ii) Waves and current generator, which is a static block which calculates the current velocity $\mathbf{v}_c$, its temporal derivative $\dot{\mathbf{v}}_c$, the direction of the current $\psi_c$ and the acceleration of the fluid $\chi_c$ in the sailboat's position $\eta$;
iii) Wind generator, which is a static block which calculates the wind velocity $v_w$, its temporal derivative $\dot{v}_w$, the direction of the wind $\psi_w$ and the acceleration of the fluid $\chi_w$ in the position $\eta$ where the sailboat is;

iv) Forces and Moments block, which is a static block that computes the total forces acting in the sailboat.

The blocks in the Figure 6.2 are described in the previous chapters 4 and 5. In order to reach a desired destination, knowing the sea and wind conditions, the system simulator should compute the input variables, $\delta_r$ and $\delta_s$, of the sailboat.

Before the presentation of the simulation results of the sailboat’s model, as in a real sailboat, the angle of the sail is not fixed to a determined value, but limited to a range of values. In open-loop, the sail angle is an input of the sailboat’s model, and if, e.g. the value forwarded to the model is 30°, then the sail will be limited between the range: $\delta_s\in[-30^\circ,30^\circ]$, defined as $\bar{\delta}_s = 30^\circ$. Through the data from the Table 6.1 and 6.2, the position of the sail for a given $\bar{\delta}_s$ and $\beta_{aw}$, is defined.
6.2 Open-loop Simulation Results

To develop a control system to make the sailboat fully autonomous, it is necessary to have a prediction of the sailboat’s behavior in the presence of wind. Open-loop simulation results become crucial in the design of the control system. To perform open-loop simulations, it is imperative to know the system hydrodynamics, added mass, damping, and disturbances parameters, as well as the rudder’s and sail’s drag and lift coefficients.
Considering a stall angle $\alpha_{\text{stall}} \approx 35^\circ$, the Figures 6.3 and 6.4 defines the sail’s and rudder’s, drag and lift coefficients, respectively. Applying these definitions for the drag and lift coefficients in the computational model of the sailboat, the essentials parameters can be adjust by approaching the behavior of the sailboat’s computational model to a desired behavior.

By applying an initial nonzero value to the surge, sway and yaw velocities, and test the computational model in absence of wind and current, the hydrodynamics, added mass and damping coefficients can be tuned for the sailboat in order to after a determined time, it reaches an equilibrium position were all the velocities are null. Through this, the time that the boat takes up to achieve the determined position with null velocities can be adjusted to an acceptable value. The result of this test can be verified through Figure 6.5, (a) and (b). In the Figure 6.5 (a), a $XY$ positioning graphic, a sailboat with a rudder and a sail are
drawn. This drawing is not scaled according to the sailboat’s real size (and the graphic scale). The intention is to help the visualization of the problem in question.

Through Figure 6.5 (b), it can be seen that the hydrodynamic, added mass and damping resistive forces and moments cancel the movement of the sailboat when there are no wind or current. The velocity in sway mode goes, almost instantly to zero, while in surge mode the velocity delays a little more time to be canceled as expected.

Some other results from the computational model’s simulation are shown in following Figures.

Figure 6.5 – Stability simulation results from a test with \( \varphi(0) = 0^\circ \), \( V_w = 0 \text{ m.s}^{-1} \), \( u(0) = v(0) = 2 \text{ m.s}^{-1} \), \( \delta_r = 0 \) and \( \delta_x = 0 \)

(a) shows the progress of the sailboat in space

(b) shows the surge and sway velocities and the yaw angular velocity and angle

Through Figure 6.5 (b), it can be seen that the hydrodynamic, added mass and damping resistive forces and moments cancel the movement of the sailboat when there are no wind or current. The velocity in sway mode goes, almost instantly to zero, while in surge mode the velocity delays a little more time to be canceled as expected.

Some other results from the computational model’s simulation are shown in following Figures.
Figure 6.6 – Simulation results from a test with $\psi(0)=0^\circ$, $V_w=1\text{ m.s}^{-1}$, $\psi_w=180^\circ$, $V_c=0\text{ m.s}^{-1}$, $\delta_r=0$, $\delta_s=40^\circ$ ($\delta_s(0)=-40^\circ$) and $t=60\text{s}$

(a) shows the progress of the sailboat in space,

(b) shows the surge and sway velocities and the yaw angular velocity and angle
Analyzing Figure 6.7, when the wind blows from behind the sailboat, \( \psi(0)=0^\circ \), \( V_w=1 \text{ m.s}^{-1} \), \( \psi_w=180^\circ \), \( \delta_s = 40^\circ \) (\( \delta_s(0) = -40^\circ \)), and \( \delta_r = 0 \) for \( t<30\text{s} \), and \( \delta_r = -10^\circ \) for \( 30\text{s} \leq t < 60\text{s} \). After 30 seconds, is applied in the rudder a fixed angle, \( \delta_r = -10^\circ \), what provokes a positive yaw angular velocity, \( \psi = 1.56^\circ\text{.s}^{-1} \). In the end of the, \( t = 60\text{s} \), the orientation is, \( \psi = 47^\circ \). When \( t \approx 46 \), the velocity in surge mode reaches the maximum in this simulation. This occurs when the angle between the sail and the apparent wind is approximately 50°.
Various attempts were made in order to accomplish a 360° turn, with a fixed wind angle of $\psi_w = 180^\circ$. From the results it was determined that the model of the sailboat only succeeds for $V_w > 0.94 \text{ m.s}^{-1}$. For wind velocities below this value, before the sailboat reaches $\psi = 180^\circ$, this maneuver in not succeed and it begins to go backward, due to the fact that the sailboat does not achieve enough velocity to complete the turn and when reaches the opposite direction of the wind begins to go backward. In order to accomplish this result, a simulation of a situation when the wind velocity is $V_w = 1 \text{ m.s}^{-1}$ is depicted in the Figure 6.8.

Persisting with the simulation for this wind velocity, after the fourth consecutive 360° turn, the sailboat fails to accomplish a fifth 360° turn, due to the fact that in this simulation, when it begins the first 360° turn, it has 20 seconds to gain velocity, which prevails until the fourth consecutive turn. After that it begins to go backward and enters in an instable zone.

![Figure 6.8](image)

**Figure 6.8 – Simulation results from a test with $\psi(0)=0^\circ$, $V_w=1 \text{ m.s}^{-1}$, $\psi_w=180^\circ$, $\delta_y = 40^\circ$ ($\delta_y(0) = -40^\circ$) and $\delta_r = 0$ for $t<40s$, and $\delta_r = -15^\circ$ for $40s \leq t < 120s$

(a) shows the progress of the sailboat in space,
(b) shows the surge and sway velocities, and the yaw angular velocity and angle
6.2.1 Velocity Polar Prediction (VPP) Diagram

![Velocity Polar Prediction Diagram]

The velocity polar prediction (VPP) diagram represents the maximum speed the sailboat can reach for each wind speed and direction. To compute it, the domain of possible wind directions is discretized and for each angle a simulation is run, optimizing the sail angle to achieve the maximum speed in that direction. This process is repeated for each wind speed. To maintain the desired orientation during the simulation, a heading controller (PID) was used. The points on a polar diagram are a succession of converged steady states. The polar diagram is obtained by giving the pilot a succession of constant instructions. Each step ends when the sailboat has a stationary movement.

Figure 6.9 – Velocity’s polar prediction diagram for $V_w = 1 \text{ m.s}^{-1}, V_w = 3 \text{ m.s}^{-1}, V_w = 5 \text{ m.s}^{-1}$ and $V_w = 7 \text{ m.s}^{-1}$
The VPP of the actual computational model for a case where the velocities of the wind are \( V_w = 1 \text{ m.s}^{-1} \), \( V_w = 3 \text{ m.s}^{-1} \), \( V_w = 5 \text{ m.s}^{-1} \), and \( V_w = 7 \text{ m.s}^{-1} \) are given in the Figure 6.9. Examining this figure it is possible to verify that, while no direct course is possible straight into the wind, the maximum speed is usually obtained downwind at about \( \psi = \pm 120^\circ \). As the wind velocity increases, for the same heading the sailboat, the maximum velocity will decrease, as proved by the VPP. This is due to forces opposed to the motion, which increases with the relative velocity of the boat to the fluid.

With the results from the VPP, a look-up table is developed with the discretized data from the VPP, which allows the prediction of the maximum speed that the sailboat can reach in a given direction, depending on the wind’s velocity and direction.

For the computation of the look-up table, a velocity’s polar diagram must be simulated a few times for a discretized range of wind velocity values. In this work, the wind velocities were discretized in steps of 0.2 m.s\(^{-1}\) between the range \([0,5] \text{ m.s}^{-1}\). In the Table 6.3 are presented some discretized results from the simulated VPP to \( V_w = 1 \text{ms}^{-1} \) and \( V_w = 5 \text{ms}^{-1} \), with the heading discretized in ranges of 15°. The simulated range of headings was between \([0^\circ, 180^\circ]\), due to the fact that from \([180^\circ, 360^\circ]\) the results are identical, the sail’s angle is equal in module but negative in signal.

<table>
<thead>
<tr>
<th>( \psi ) [°]</th>
<th>( \delta ) [°]</th>
<th>( u_{max} ) [ms(^{-1})]</th>
<th>( V_w = 1 \text{ms}^{-1} )</th>
<th>( V_w = 5 \text{ms}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>90</td>
<td>0.79</td>
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</tr>
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</table>
6.3 System Identification

With logged data from an outdoor real-time path could be used to evaluate the modeled system and adjust the parameters. Data such as the observed wind, the resulting velocities and the trajectory of the real system would be regarded. By simulating an equivalent path with equivalent wind data, the system parameters of the modeled sailboat are adjusted. Thus, the system could be identified and a similar behavior of the simulation compared to the real hardware established.
Chapter 7

Control

This section describes the control strategy to maneuver autonomously the sailboat. The control system should be able to generate and follow a desired trajectory, considering the restrictions of sailing boats maneuverability. The control algorithms will responsible for taking sailing decisions and issuing appropriate commands to the rudder’s and sail’s servos.

Basically the control system operates two output variables, the rudder’s and sail’s angle. With data from the sensors, the sailboat should be able reach a desired destination. To design the control system, both PID and logic based controllers were implemented.

The simulator system consists of four main sub-systems as represented in the Figure 7.1.

The Store System manages a shared memory between the individual processes, allowing the writing and reading global variables to all systems. The systems are connected to each other’s through the store. For instance, sensor drivers read out actual sensor data from the hardware and write it to the store. Any other
system that requires the sensor data, can now read the data from the store. As this data is stored, later it would be available for further analyzes and eventual corrections.

The sailboat is a highly non-linear system, and the straight line route to the desired destination may not be navigable, if it is located upwind. Per instance, if the sailboat is to go upstream (against the wind), it must follow a zig-zag path and perform a series of tacks. So it is necessary to study the sailboat constraints and the maneuvers that allow the overcome of some of them.

7.1 Sailing Maneuvers

![Sailing modes in different headings](image)

To understand the proposed control algorithms for a sailing robot we first describe the general rules of sailing. As the kinematics and dynamic constraint limits do not allow to sail at all wind angles, several sailing modes and perform maneuvers need to be considered to overcome bearings not directly navigable. The system has to differentiate between several states of sailing and perform maneuvers such as tack and jibe in order to overcome ranges not directly navigable. Depending on the required heading and wind’s angle, the control switches to the corresponding mode.

To perform maneuvers such as tack and jibe, a guidance system switch the sailing mode depending on the desired heading and apparent wind’s angle, is designed. In all the state cases, sails are trimmed to achieve maximum thrust.
Basically there are three main modes (Figure 7.3) to sail a boat

i) Close-hauled or upwind sailing (tack maneuver),

ii) Downwind sailing (jibe maneuver),

iii) Normal sailing (for the apparent wind angles in the white are in Figure 7.2).

If the sailboat is pointed too close to the wind for the sails generate any thrust (unless they are backed), the sail will be luffing (flapping) in the breeze and making noise like a flag. To sail towards this direction, the sailboat must tack across the wind with no more than 45°, between the desired direction and the apparent wind direction. So when sailing upwind, it is mandatory to maintain an apparent wind angle,
that allows the boat to sail at a good speed. In this case, heading information is used to estimate the velocity through the VPP.

To sail with the wind directly from behind the boat, the sail is allowed to run out as far as it can, so that the sail's angle is perpendicular to the wind direction (known as running). For all points of sail other than run, the sail needs to be out on the opposite side of the boat to the direction the apparent wind is coming from.

The truth is that through a jibe maneuver, the boat would sail substantially faster than when running downwind. So when sailing downwind the jibe’s strategy will be adopted, instead the running’s maneuver.

Through the VPP data, when the wind direction is perpendicular to the sailboat's course, the sail should be approximately half way out or at an angle of 45° between it and the center of the sailboat. This is known as a beam reach. This sailing strategy was implemented in the control system, due to the fact that this is generally the most efficient point of sail to use.

7.1.1 Tack Maneuver

If the target is against the wind, the straight line route is not navigable. In this case the sailboat has to take a zig-zag path against the wind, known as a tacking maneuver.

The tacking maneuver is a sailing maneuver by which a sailboat turns its bow through the wind, so that the direction from which the wind blows changes from one side to the other (see Figure 7.4). When a sailboat is tacking, it is moving both upwind and across the wind. Cross-wind movement is not desired, therefore the sailboat changes tack periodically, reversing the direction of cross-wind movement while continuing the upwind movement.

Beating is another sailing term that refers to the procedure by which a ship which moves on a zig-zag course to make progress directly in to the wind (upwind). A sailboat that is beating will sail as close to the wind as possible (this position is known as close-hauled). In general, the closest angle to the wind that a ship can sail is around 35° to 45°.
When tacking, as in the Figure 7.4, the following procedure happens

i) The sailing boat is sailing close-hauled,

ii) The sailboat initiates the tacking maneuver by moving the rudder, where it begins to turn in order to achieve a desired orientation,

iii) When the sailboat is pointed directly into the wind, the rudder moves to the central position, heeling decreases below the desired range of values due to lack of lateral wind force, thus the sail control system tightens the sheets to reach the desired heeling. Although the rudder is in the central position, due to mass inertia the sailboat keeps turning towards the desired orientation,

iv) When new desired direction is reached, lateral wind forces increases again, hence heeling increases above the desired heeling. So sheets are eased off in order to reach desired heeling.

Tacking means that the boat is no longer travelling towards its desired heading, but it is alternating between the wind direction ±45°. This can be achieved by comparing the desired heading with the apparent wind direction. If tacking is necessary, then the desired heading must be adjusted to either ±45°, and this value must alternate on a regular basis to ensure that the sailboat still travels in the desired direction. In practice, the sails are set at angle of 45° to the wind for conventional sail ships and the tacking course is kept as short as possible, before a new tack is set in.

### 7.1.2 Jibe Maneuver

A jibe is a sailing maneuver where a sailboat (which is sailing before the wind) turns its stern through the wind, such that the wind direction changes from one side of the boat to the other (see Figure 7.5).

![Figure 7.5 - Jibe maneuver](image)

A jibe is a sailing maneuver where a sailboat (which is sailing before the wind) turns its stern through the wind, such that the wind direction changes from one side of the boat to the other (see Figure 7.5).

Jibe is a less common technique than tacking, since a sailboat can sail straight downwind, unlike when tacking, where it cannot sail directly into the wind. Although, the jibe maneuver is a bit more difficult
than the tack, due to the fact that in jibe the optimal timing is more important, many sailboats are significantly faster sailing on a broad reach than sailing straight downwind.

It is common to, only, practice jibe when the wind was already coming from directly behind the boat. When performing a jibe maneuver, as shown in Figure 7.5, the following procedure happens

i) The boat initiates the maneuver sailing broad reach,

ii) The jibe maneuver is initiated by moving the rudder, similar to what happens in the initialization of the tack maneuver. Moving the rudder, performs a turn of the sailboat to reach a desired orientation,

iii) When the stern is turned through the wind significantly, the sail gets tightened temporarily in order to move to leeward side,

iv) When the sailboat broad reach on starboard side, the sails are eased off completely again, due to the desired heeling is too low. So the rudder angle decreases because the desired destination is almost reached,

v) As due to mass inertia the sailboat keeps turning, the rudder is then putted in the center position,

vi) When desired orientation is reached, sheets are eased off to reach the desired heeling.

7.2 Sailboat Control System

There are several control techniques, which have been applied to compute solutions to similar problems, such as PID controllers, optimal control, non-linear control by sliding means, robust control, etc. Some descriptions of these types of control can be found in [Fossen, 1994].

Figure 7.6 - Architecture of the Sailboat’s Control System
In Figure 7.6, $\psi_{traj}$ is the trajectory orientation given to the guidance system, $V_w, \psi_w$ are the velocity and direction of the true wind, $\psi_d$ the desired orientation, $\delta_r$ is the range that de angle of sail can vary, $\delta_r$ the rudder’s angle, $(x, y)$ and $\psi$ are the position and the heading of the sailboat.

In common sailing practice, different persons are able to control rudder and sail independently without the need for communication. Therefore, in the present work, the sail and rudder controllers are two independent systems, that only depend from the data computed by the guidance system.

The sailboat’s control system can be decomposed in three sub-systems, as in Figure 7.6

1) Guidance system. Its role is to compute the desired heading. This computation takes into account the sailing modes, as function of the position of the sailboat and the trajectory orientation.

2) Sail Controller. This system is responsible to control the value of the sail’s range angle, through a comparison between the wind conditions and the sailboat’s orientation with the data from the VPP.

3) Heading Controller. It continuously computes the rudder’s angle to maintain a desired heading.

In this control system, the main control input for thrust force, and therefore, velocity, is the sail angle, whereas the rudder angle controls the orientation.

7.2.1 Guidance System

The Guidance System computes the desired orientation, through the given wind velocity and direction, and the given trajectory orientation. This system, also, computes the appropriated sailing mode, developing a route according to it and the data from the VPP, in order to assure that the sail is adjusted to reaches the maximum possible.

For these computations, the system assumes that the true wind is the same all over the area between sailboat and the target, due to the fact that the wind velocity and direction sensor, only measures the wind in the sailboat area. Therefore, the guidance system is continuously generating the desired orientation, so when there is a change of the wind direction, it can update the sailing mode and the desired heading for the new wind conditions.

7.2.1.1 Sailing Modes

To accomplish this type of control strategy, the limits of the angles of the wind must be defined for each one of the sailing modes. The closest proposed direction of sailboat movement against the wind is predefined by the shape of the velocity polar diagram.
To accomplish this type of control strategy, the limits of wind angles for each sailing modes must be defined. According to the data from the VPP, the following ranges of true wind angles for each sailing mode were defined (see Table 7.1).

<table>
<thead>
<tr>
<th>Sailing modes</th>
<th>True wind angle’s range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upwind mode</td>
<td>[-60°, 60°]</td>
</tr>
<tr>
<td>Direct mode</td>
<td>[-150°, -60°] [and] [60°, 150°]</td>
</tr>
<tr>
<td>Downwind mode</td>
<td>[-180°, -150°] [and] [150°, 180°]</td>
</tr>
</tbody>
</table>

When the guidance system assumes an upwind or downwind sailing mode, the desired heading cannot be directly sailed, according to the followed strategies. Therefore, with the objective to generate as much velocity as possible, it converts the next beating into a local waypoint. Through this local waypoint, this system computes the desired heading, which is delivered to the heading controller.

In order to do not miss the desired trajectory between two beatings, it generates a rectangular beating band, where a constant beating parameter is proportional to the width of the defined beating band, and when the sailboat approaches the beating band wall, the system orders an instantaneous beating (Figure 7.7).

When in a direct normal sailing mode, the guidance system does not have to do any computation. It simply forwarded the trajectory orientation to the heading controller.
7.2.2 Sail Controller

For this controller, the orientation is assumed to be successfully controlled and will not be considered in this task. This controller, computes the optimal sail angle in order to reach the maximum thrust. To accomplish this goal, the chosen control strategy consists in finding the optimal sail’s angle through the VPP.

For safety reasons, as the wanted angle of attack can changes dynamically depending on the wind speed, when the wind speed increases, the wanted angle of attack can approaches zero, what would reduce the force generated by the wind in the sails. With this behavior, the boat remains steerable even in strong winds.

7.2.3 Heading Controller

For most of the sailboats, the rudder is efficient enough to make the sailboat follow the target direction as long as the sailing mode is not upwind. As the guidance system does not allow such a direction, the
Heading controller will be simple. This controller, only has adjust the rudder’s angle, so the sailboat follow the given trajectory orientation. Therefore, if the actual boat orientation deviates from the trajectory direction, the system adjusts the rudder position in order to bring the boat to the desired course, according to the rudder control strategy in the Figure 7.8.

![Figure 7.8 - Rudder control strategy](image)

The input data for the heading controller are the sailboat’s position and the desired orientation. The difference between these two headings, gives the necessary path correction, which enters directly into a PID controller.

A PID controller is a commonly used method in control systems and makes use of the errors observed in a system in order to perform corrections. In this case the error is determined by the difference between the current heading and the desired heading. A PID controller is made up of three parts known as bands, which are known as the Proportional, Derivative and Integral bands. The Proportional band, as its name suggests makes corrections in proportion to the size of the error, this alone is enough to control many systems however it may encounter problems such as when the sailboat is nears its destination, because the applied force can be so low that the desired course is never quite achieved. Integral control takes the sum of all errors and multiplies them by the amount of time elapsed. In doing this, the steady state problem can be overcome as eventually the integral control will cause enough correcting force to hit the desired course. Finally derivative control acts as a form of break to prevent the rate of change occurring too quickly and thus overshooting the desired course. This is determined by measuring the rate of change in the error. The final value of a PID controller is calculated by multiplying the output of each band by a pre-determined constant known as the gain and then taking the sum of all of the three bands. The gain constants are dependent upon the system and a PID controller must be tuned in order to find optimal settings for the gain constants.
Chapter 8

Closed-loop Simulation Results

This chapter presents the simulation results of the computational model controlled by the systems described in the previous chapter. Several computational tests were made to verify and optimize the control systems.

To optimize the tack and jibe maneuvers, some simulation tests were made to correct Beating Band. As the controller is directed straight to short-path-planning, the optimization of the timing between two tacks was optimized to this situation. For a long-course, the optimization could run to different values.

The heading controller adjusts the rudder position in order to bring the sailboat to the desired course through a PID controller, as referred in the previous chapter. To adjust the gains of the PID to values that gives an acceptable control of the heading of the sailboat, the model of the sailboat, with the heading controller is simulated. The goal is that the heading control approaches and follows a desired orientation in the shortest time. The computed gains for the PID are shown in the Table 8.1.

<table>
<thead>
<tr>
<th>Gain type</th>
<th>Dimensionless gain value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$ - proportional band gain</td>
<td>0.3</td>
</tr>
<tr>
<td>$K_i$ – integral band gain</td>
<td>0.005</td>
</tr>
<tr>
<td>$K_d$ – derivative band gain</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 8.1 – PID gain parameters
8.1 Upwind Mode Simulation (Tacking Maneuver)

Figure 8.1 shows the simulation of the sailboat system with the following conditions:

- \( \psi_w = 0^\circ \), start in the origin of the \( \{I\} \) and \( \psi_{\text{traj}} = 0^\circ \). In this case, the guidance system has correctly chosen an upwind sailing mode, performing a tacking maneuver to follow the heading trajectory.

In the tacking maneuver simulation, as the initial velocities of the sailboat are null, the initial orientation of the sailboat was \( \psi = -70^\circ \), in order to facilitate the sailboat to gain velocity faster. For this test, the width of the beating band was 20 meters, as is easy to identify by the Figure 8.1b. When the sailboat...
passes the imaginary vertical line with $y = -10$ or $y = 10$ the sailboat immediately runs a beating, in order to not diverge from the desired trajectory.

As in the chapter 6, in the $XY$ graphic of the simulations of the sailboat system that is drawn in this Figures are not scaled according to its real size (and the graphic scale), because it is drawn with the intention to help the visualization of the problem in question.

### 8.2 Downwind Mode Simulation (Jibe Maneuver)

Figure 8.2 – Simulation results from a jibe maneuver test where is applied wind with $V_w = 1 \text{ m.s}^{-1}$ and $\psi_w = 180^\circ$, during 60s, and where $\psi_{traj} = 0^\circ$

(a) shows the progress of the sailboat in space,

(b) shows the surge and sway velocities and the yaw velocity and angle
Figure 8.2 shows the simulation of the sailboat system with the following conditions: $V_w = 1 \text{ m.s}^{-1}$, $\psi_w = 180^\circ$, start in the origin of $\{I\}$ and desired trajectory $\psi_{\text{traj}}=0^\circ$. In this case, the state control system has correctly chosen downwind sailing mode, performing a jibe maneuver to reach the destination.

Performing a comparison between the tacking and the jibe simulations, can be seen that in the jibe maneuver the sailboat achieves a an average speed higher than in the tacking maneuver, what would be the expected result. With wind from behind, the sailboat, according with the polar diagram from Figure 6.8, can reach velocities above $1 \text{ m.s}^{-1}$ for a wind with velocity $V_w=1 \text{ m.s}^{-1}$, unlike in the tacking maneuver, what allows the sailboat to reach the destination faster. In this simulation the width of the beating band was 10 meters. As can be seen in (b) the guidance system obeyed to the rule.

8.3 Direct Mode Simulation

![Diagram of sailboat and wind direction](image)

(a) shows the progress of the sailboat in space, (b) shows the surge and sway velocities and the yaw velocity and angle

Figure 8.3 – Simulation results from when in normal sailing mode is applied wind with $V_w=1 \text{ m.s}^{-1}$ and $\psi_w=90^\circ$, during 60s, with $\psi_{\text{traj}}=0^\circ$
In the situations where the wind blows laterally within the navigable range, a straight line route is supported by the guidance system, as it happened in the Figure 8.3. In this simulation can be seen that the route followed by the sailboat isn’t a straight line, but also isn’t far way of it. This deviance is due to the fact that, with lateral wind, the force along \( y_B \) and the moment applied to the sail isn’t null, and tries to deviance the sailboat from going straight up. In this situation then heading controller automatically adjust the rudder to correct the heading.

8.4 Upwind Mode-Direct Mode Simulation

![Diagram](image)

Figure 8.4 – Simulation results from when in Upwind mode with \( V_w = 1 \text{ m.s}^{-1} \) and \( \psi_w = 0^\circ \), until \( t=70\text{s} \). For \( t>70\text{s} \), \( V_w = 1 \text{ m.s}^{-1} \) and \( \psi_w = 90^\circ \), \( \psi_{\text{traj}} = 0^\circ \).

(a) shows the progress of the sailboat in space,

(b) shows the surge and sway velocities and the yaw velocity and angle.
In Figure 8.4, it can be seen that until \( t=70s \), the sailboat is operating in Upwind mode with a beating band with width 10 meters. After that, due to the alteration of the wind direction to \( \psi_w=90^\circ \), it enters in Direct mode, as expected.
Chapter 9

Conclusions and Further Research

9.1 Summary

This thesis focused on the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously. For that purpose an architecture for an autonomous sailboat, through the setting up of the equations of the dynamics of the sailboat and of the principal physical phenomena that influence their behavior, has been presented. The non-linear models describe the dynamic response of the sailboat very accurately, and therefore can be exploited for testing control strategies. A technique is presented to determine suitable boat headings in order to reach any target. The method works without knowledge of future weather conditions and does not consider yet obstacles like land masses or extreme weather phenomena. This is advantageous especially for short term path-planning, where no accurate weather forecasts are available.

This project has shown that it is possible to achieve some level of control over a sailing robot using only a guidance and a control system that identify the sailing mode and then compute the rudder’s and sail’s angles in order to approach the desired destination. The proposed control system has been successfully implemented and simulated in the developed computational model of the sailboat. The method is simple and easy to implement even on an embedded system.

The simulator developed can be used as an efficient platform helping to optimize the control system. Extensive outdoor tests with real systems, as well as time and cost consuming operations, can be avoided by the use of this simulator. Another benefit of the simulator is that environmental conditions can be set arbitrarily, such that performance and functions can be systematically verified.

By the results of simulation in close-loop, the strategy to impose a beating band when operating in Upwind or Downwind modes function well.

Further work is still required to improve the accuracy of the simulator.

Overall this project has managed to fulfill its aims in studying a software control system which is capable of controlling a sailing robot by steering it along a pre-determined course direction. In the real world the software still requires additional development in order to run it in real time.
9.2  Future Directions

9.2.1  Experimental Tests

First experimental tests of the real prototype in a real environment must be done in order to identify the Body (hull and keel), rudder and the parameters, and then an experimental testing of the control system to ensure the durability of the robot.

9.2.2  Power Sources and Consumption Optimization

In order to approach a fully autonomous model, electric power consumption, not approached in the present work, is one of the great concerns in an autonomous sailboat. For a small boat, the reasonable sources of electric energy for long term navigation are micro wind turbines and solar cells. In both cases, the availability of energy always depends on the weather conditions, which still have a high degree of uncertainty. For this reason, the electronic system must consume the lowest possible energy and whenever possible adapt its behavior to the power budget available at each stage. In theory, the solar cells provide enough power for the sailboat’s systems, but in bad weather the possibility of a fuel cell should be analyzed. Another possibility is to use some kind of energy recovery strategy similar to regenerative breaking in hybrid cars or kinetic watches.

In the conservation of battery’s power, when power is low the sailboat can react rationing the power supplies. As power levels drop, non-vital systems are shut down and the power to other systems is reduced (possibly by making steering movements less frequently or making smaller moves until power levels increase).

9.2.3  Sail Optimization

Further work needs to be undertaken in the sail, in order to optimize its performance. The sail wing hypothesis could be analyzed and compared with the actual sail, in order to optimize the performance of the sailboat. Wing’s shapes and sizes studies can be performed to test the full potential of each type of sail, as the use of multiple sails to improve performance and stability.

The need to have a reasonable size sail is important, in order to obtain efficiency in the sailing, however at the same time the software would need to understand the need to deliberately reduce sailing efficiency, during high winds in order to reduce strain on the sail and prevent the boat from capsizing.
9.2.4 Rudder Optimization

The developed rudder controller is based on a kinematic model in a two-dimensional plane. Considering that the boat rotates around its $x_B$-axis, it may happen that the rudder is above the water surface. The idea is to find a dependency of the rudder area in the water and the angular position of the sailboat around its $x_B$-axis, and to implement this relation into the rudder controller, obtaining a three-dimensional rudder control. The possibility of a two rudder system could be analyzed, integrating this possibility in the computational model of the sailboat, and testing it in the simulation system.

9.2.5 Sailboat Dimensions

Given the sailboat’s small size, it is unlikely to be able to sail effectively in storm conditions at seas. In order to solve this problem, the entire boat could be scaled to a suitable size. However this should not require any major changes to the software.

9.2.6 Dynamic Obstacle Avoidance

The current implementation makes no attempt at all to avoid collisions. In long-term autonomous marine systems, an important problem to be solved is obstacle detection and avoidance. Static obstacles such as landmasses can be predefined on the sea map as a basis for the routing system. A combination of multiple techniques, such as thermal imaging, radar, camera, and automatic identification system, can be used to detect dynamic obstacles. Research in this field has been carried out for autonomous underwater vehicles and motorized autonomous surface vehicles. The obstacle avoidance task is different for sailing vessels, as they cannot navigate in any direction directly, depending on wind conditions. This condition could be studied and implemented in the sailboat’s software.


Jerome Jouffroy. On Steering a Sailing Ship in a Wearing Maneuver. University of Southern Denmark, 8th IFAC International Conference on Manoeuvring and Control of Marine Craft, September 2009


