SAILBOT
Autonomous Marine Robot of Eolic Propulsion

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Abstract
At the present time, the research in the area of clean energy is one of the most relevant issues, which together with the recent advances in the field of marine robotics, presents a promising subject and an excellent experimental field for development of new technologies to be used on another scale for freight transportation with low costs and pollution levels. Motivated by this fact, this paper addresses the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously. A detailed description of the sailboat, its layout system, and its mathematical model composed by the kinematics, dynamics and applied forces and moments are presented. After the derivation of the model of the applied forces and moments and its nonlinear dynamic equations of motion, simulation results are provided to illustrate the behavior of the sailboat. In particular, the velocity's polar prediction (VPP) diagram that represents the maximum speed that the sailboat can reach for each wind speed and direction is computed. Based on the VPP, we present a control strategy that exploiting the tack and jibe maneuvers and actuating only on the sail and rudder orientations allows the sailboat to make progress in any given direction (on a zoom out scale) independent of the wind direction. Simulation results that illustrate the proposed control strategy are presented and discussed.

Keywords: Autonomous sailboat, Autonomous surface marine vehicle, Wind-propelled vessel, Sailing guidance and control systems.

1 Introduction

A sailboat equipped with a micro wind turbine and/or solar cells to generate power for sensors, electronics, actuators and to charge a small set of backup batteries, can theoretically achieve infinite autonomy. Some references in this research area includes solutions for the aerodynamics and hydrodynamics of sailing ([C. A. Marchaj, 1988]), approaches to the guidance and control systems through logic based control strategies ([Edge C. Yeh and Jenn-Cheng Bin, 1992], [T.W.Vaneck, 1997] and [Sang-Min Lee and Kyung-Yub Kwon, 2004]), approaches for optimization of the simulation and control system ([Fabian Jenne, 2010]) and algorithms for a navigation system ([Gion-Andri Busser, 2009]).

There are many possible applications for autonomous sailboats, such as
– Environmental data collection, surveying, mapping, and water ecological studies at low costs;
– Transportation of goods at low costs and low pollution levels;
– Specific missions at far reaches or dangerous regions.

Motivated by the above considerations, this paper addresses the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously. To this effect, a detailed description of the sailboat, its system layout, and its mathematical model composed by the kinematics, dynamics and applied forces and moments are first presented.

After the derivation of the model of the applied forces and moments and its nonlinear dynamic equations of motion, simulation results are provided to illustrate the behavior of the sailboat. In particular, the velocity's polar prediction (VPP) diagram that represents the maximum speed that the sailboat can reach for each wind speed and direction is computed.

To tackle the control problem, one of the main concerns is the generation of a feasible path compatible with the environmental conditions and sailing constraints. Based on the VPP, we present a control strategy that exploiting the tack and jibe maneuvers and actuating only on the sail and rudder orientations allows the sailboat to make progress in any given direction (on a zoom out scale) independent of the wind direction. Roughly speaking, the outer level of the control system continuously selects the correct heading for a desired destination, deciding if and when to tack or jibe. The heading and the sail angle (also determined by the outer loop) are then viewed as reference input signals to the inner-loop controller. Therefore, the required work can be implemented in accordance with the following stages
i) analysis and development of the sailboat’s model;
ii) development and implementation of guidance and control systems;
iii) application of the developed mathematical model and control strategy to test the system in a simulation environment.

The outline of this paper is structured as follows. Section 2 introduces a brief description of the vehicle and a general introduction to its system layout. Section 3 describes the notation and coordinate systems. Section 4 derives the nonlinear dynamic equations of motion. Section 5 proposes a strategy to determine all the applied forces and moments. Section 6 presents the computational simulation system and simulation results in open-loop. Section 7 is devoted to the sailboat’s guidance and control systems. In Section 8 simulation results are presented to illustrate the effectiveness of the control strategy. Conclusions are in Section 9.

2 Vehicle Description

In Figure 1 the main components of a conventional sailboat, hull, keel, rudder, mast and sails, are depicted.

![Figure 1 – Sketch of a sailboat main components](image1)

The hull is fitted with a moving rudder for steering and a fixed keel to achieve passive stability in roll and reduce lateral drift. In the particular case of the prototype under development in the scope of this project (see Figure 2), the main and head sail control cables are connected together to a single servo actuator to rotate the sails accordingly to the wind direction. A second servo is installed to rotate the rudder.

![Figure 2 – Prototype of the autonomous Sailboat](image2)

2.1 Onboard Components

This sailboat control system is composed by a network of sensors, microcontrollers and actuators. Figure 3 shows its components. The volume occupied by these components is constrained, its weight cannot be very significant and they must be low energy consumers. In order to power all these components, the sailboat must be equipped with batteries and a system to charge the batteries.

![Figure 3 – Sailboat’s onboard components](image3)

Onboard Computer

An onboard computer is essential to run the control algorithm that provides the set of actions that the sailboat must perform to accomplish a given mission. A possible option is to use a Gumstix, a computer-on-module, with low cost, high performance and production ready. It runs a slimmed down version of Linux.

Wind Sensor

To compute optimal sail angle and the vessel course, measured data about wind conditions is a key requirement. The magnitude of the wind velocity is determined by an anemometer and its relative direction by a wind vane.

Speed Sensor

A solution to measure the speed with respect to the water is through a hull speed paddle wheel transducer.

AHRS

The Attitude and Heading Reference System (AHRS) provides roll, pitch, yaw and angular velocities. It consists of two blocks:

i) an Inertial Measurement Unit (IMU) to measure the triaxial angular velocities, accelerations, and earth magnetic field components;

ii) a navigation filter which is fed with the IMU data and generates the AHRS output.

GPS

Guidance systems need to know the vehicle’s position and velocity. Although GPS intrinsically measures the vehicle position only, it is also possible to estimate the velocity vector and this is valuable information for guidance as well.

Communication Systems

An operator onshore can connect to the sailboat’s system to gather measured data by the sensors, monitor the
position, velocity, state and computed strategies, and to send new waypoints. To accomplish this, the envisioned communication system combines WLAN, UMTS/GPRS.

3 Notation and Reference Frames

Reference frames

The kinematical and dynamical equations of motion can be developed using proper cartesian reference frames, which in this case are (Figure 4)

- Earth-fixed inertial frame, \( \{I\} \). The origin of \( \{I\} \) reference frame is located at a local tangent plane in the area of interest.
- Body-fixed reference frame, \( \{B\} \). This is a moving reference frame, where the origin is, usually, chosen to coincide with the vehicle’s center mass (CM).

![Figure 4 – \( \{B\} \) and \( \{I\} \) reference frames](image4.png)

Vector Definitions

The motion of the sailboat is described in \( \{B\} \) by 6 Degrees Of Freedom (DOF). In Table 1 and Figure 5, the notation to define the vehicle’s motion and position is introduced. To represent the position, velocities and forces, a set of vectors are defined in (1).

![Figure 5 – SNAMES’s notation for motion components](image5.png)

\[
\begin{align*}
\eta &= [\eta_x, \eta_y]^T; \quad \eta_1 = [x, y, z]^T; \quad \eta_2 = [\phi, \theta, \psi]^T \\
v &= [v_x, v_y]^T; \quad \nu_1 = [u, v, w]^T; \quad \nu_2 = [p, q, r]^T \\
\tau &= [r_x, r_y]^T; \quad \tau_1 = [X, Y, Z]^T; \quad \tau_2 = [K, M, N]^T
\end{align*}
\]

4 Kinematics and Dynamics

The development of a dynamic model is made through the computation of the equations of motion.

4.1 Kinematics

The kinematic models characterize the transformation of the velocities between different reference frames. To deduce the kinematic equations, the translational and rotational motions are studied separately.

Linear Velocity transformation

Since, linear and angular velocities are expressed in the \( \{B\} \), to express in \( \{I\} \) one needs the velocity transformation

\[
\eta_1 = R_B^I(\eta_2)v_1
\]

where \( R(\eta) \) is a rotation matrix, given by

\[
R_B^I(\eta_2) = \begin{bmatrix}
\cos \phi \cos \theta & -\sin \phi \cos \theta + \cos \phi \sin \theta \sin \phi & \sin \phi \cos \theta + \sin \phi \sin \theta \cos \phi \\
\sin \phi \cos \theta & \cos \phi \cos \theta + \sin \phi \sin \theta \sin \phi & -\cos \phi \sin \theta + \sin \phi \sin \phi \cos \theta \\
-\sin \theta & \cos \theta & \sin \theta
\end{bmatrix}
\]

Angular Velocity Transformation

The angular velocity vector \( \nu_2 \) and the Euler rate vector \( \eta_2 \) are related through a transformation matrix \( T_{\eta_2}(\eta_2) \)

\[
\eta_2 = T_{\eta_2}(\eta_2)v_2
\]

where, \( T_{\eta_2}(\eta_2) \) is given by

\[
T_{\eta_2}(\eta_2) = \begin{bmatrix}
1 & \sin \phi \tan \theta & -\cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\]

Generalized Kinematic Equations

The 6 DOF kinematical equations are expressed by

Table 1 – SNAMES notation used for marine vessels

<table>
<thead>
<tr>
<th>Motion/Rotation</th>
<th>Forces/Moments</th>
<th>Linear/Angular Velocities</th>
<th>Positions and Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>motion in x-direction (surge)</td>
<td>( X )</td>
<td>( u )</td>
<td>( x )</td>
</tr>
<tr>
<td>motion in y-direction (sway)</td>
<td>( Y )</td>
<td>( v )</td>
<td>( y )</td>
</tr>
<tr>
<td>motion in z-direction (heave)</td>
<td>( Z )</td>
<td>( w )</td>
<td>( z )</td>
</tr>
<tr>
<td>rotation about x-axis (roll)</td>
<td>( K )</td>
<td>( p )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>rotation about y-axis (pitch)</td>
<td>( M )</td>
<td>( q )</td>
<td>( \theta )</td>
</tr>
<tr>
<td>rotation about z-axis (yaw)</td>
<td>( N )</td>
<td>( r )</td>
<td>( \psi )</td>
</tr>
</tbody>
</table>
\[ \dot{\eta} = f(\eta) \Rightarrow [\eta_2] = \begin{bmatrix} R^2(\eta_2) \\ 0_{3 \times 3} \\ T_{\eta_2}(\eta_2) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \]

### 4.2 Dynamics

Considering the sailboat to be a rigid body, the dynamic model is obtained through the application of Newtonian and Lagrangian formalisms.

**Dynamic Equation of Translational Motion**

Resorting to the Newton’s second law and applying the conservation of linear and angular moments, the components of the generalized forces along \( x, y \) and \( z \)-axis, are obtained by

\[ m[\ddot{u} - vr + wq - x_{CM}(q^2 + r^2) + y_{CM}(pr - q) + z_{CM}(qp - r)] = X \\
 m[\ddot{v} - wp + ur - y_{CM}(r^2 + p^2) + x_{CM}(qr - p) + z_{CM}(pq + r)] = Y \\
 m[\ddot{w} - vq + vp - z_{CM}(p^2 + q^2) + x_{CM}(vp - q) + y_{CM}(pq + r)] = Z \]

where \( m \) is the sailboat’s mass and \( x_{CM}, y_{CM}, z_{CM} \) are the coordinate origin of the center of mass, CM.

**Dynamic Equation of Rotational Motion**

The angular moment of a body is given by

\[ L_0 = I_0 \omega \] (8)

where \( I_0 \) is the inertia tensor matrix, which takes the form

\[ I_0 = \begin{bmatrix} I_x & -I_yz & -I_zx \\ -I_yx & I_y & -I_yz \\ -I_yz & -I_zy & I_z \end{bmatrix} \] (9)

where \( I_x, I_y \) and \( I_z \) are the moments of inertia about the \( x, y \) and \( z \)-axis, \( I_{xy} = I_{yx}, I_{xz} = I_{zx}, \) and \( I_{yz} = I_{zy}. \) This result conducts to

\[ I_x(\ddot{r} - \dot{\phi} \dot{q}) - (\dot{r} + p \dot{q}) + (q^2 - r^2) \dot{\phi} + (pr - q) \dot{\phi} + m[y_{CM}(w - q + v) + x_{CM}(v - w + p)] = K \\
 I_y(\ddot{r} - \dot{\phi} \dot{q}) - (\dot{r} + p \dot{q}) + (p^2 - r^2) \dot{\phi} + (qr - p) \dot{\phi} + m[x_{CM}(u - v + w) - y_{CM}(u - v + w)] = M \\
 I_z(\ddot{r} - \dot{\phi} \dot{q}) - (\dot{r} + p \dot{q}) + (q^2 - p^2) \dot{\phi} + (pq - r) \dot{\phi} + m[z_{CM}(u - v + w) - x_{CM}(u - v + w)] = N \]

### 4.3 6 DOF Rigid Body Equations of Motion

Assuming the previous simplification, yields to

\[ m(\ddot{u} + vr + wq) = X; \quad I_{x\phi} + (l_{x\phi} - l_{y\phi}) \ddot{\phi} = K \]

\[ m(\ddot{v} - wp + ur) = Y; \quad I_{y\phi} + (l_{y\phi} - l_{z\phi}) \ddot{\phi} = M \]

\[ m(\ddot{w} - vq + vp) = Z; \quad I_{z\phi} + (l_{z\phi} - l_{x\phi}) \ddot{\phi} = N \]

### 5 Forces and Moments

The sailboat is subject to the action of the water, atmosphere and the gravitational field of the earth. The analysis of the forces and moments in the sailboat’s main elements, sail, body (hull and keel) and rudder is done by a separated analysis of each one. Considering that a total force, \( \tau_T \), is applied to the sailboat

\[ \tau_T = \tau_B + \tau_{cs} + \tau_D + \tau_G \]

where \( \tau_B \) denote respectively the forces and moments due to disturbances (waves, current and wind), \( \tau_G \) due to gravity, and \( \tau_B \) and \( \tau_{cs} \) are the

\[ \tau_B = \tau_h + \tau_a \].

\[ \tau_{cs} = \tau_t + \tau_r \]

\( \tau_B \) can be decomposed as \( \tau_h \) and \( \tau_a \) denotes the hydrodynamic and aerodynamic forces and moments, acting on the Body. As for Control Surfaces, \( \tau_s \) denotes the aerodynamic forces and moments generated by sails and \( \tau_r \) the hydrodynamic forces and moments that arise from the movement of the rudder in the water.

The 6 DOF model of the sailboat is depict by the block diagram in Figure 6.

**Generalized Equation of Dynamics**

The 6 DOF rigid body dynamic’s is expressed by

\[ M_{RB} \ddot{\phi} + C_{RB}(\phi) \dot{\phi} = \tau \] (11)

where \( M_{RB} \) is the inertia matrix of the rigid body and \( C_{RB}(\phi) \) includes Coriolis and centripetal terms. Choosing the origin of \( \{B\} \) to coincide with the CM of the sailboat, \( M_{RB} \) and \( C_{RB}(\phi) \), are given by

\[ M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & -I_{xz} \\ 0 & 0 & 0 & I_y & 0 & I_z \\ 0 & 0 & -I_{xz} & 0 & I_z \end{bmatrix} \] (12)

and

\[ C_{RB}(\phi) = \begin{bmatrix} 0 & -mr mq & 0 & 0 & 0 & 0 \\ -mr mq & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -l_{xq} + l_{r} - l_{q} & 0 & 0 \\ 0 & 0 & 0 & l_{dp} - l_{r} & -l_{r} + l_{dp} & I_{r} \\ 0 & 0 & 0 & l_{dp} - l_{r} & -l_{r} + l_{dp} & I_{r} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] (13)

**Figure 6 – Diagram blocks of the Sailboat’s model in 6 DOF**

Neglecting the roll and pitch motions by assuming that those are small and considering that \( z \approx 0 \), we obtain for control purposes a 3 DOF dynamic model.
5.1 Control Surfaces

The computation of the forces and moments acting in the control surfaces can be obtained resorting to the Finite Wing Theory, where the rudder and sail modeled as finite length wings.

Wing & Airfoil

The cross-sectional shape obtained by the intersection of a wing with its perpendicular plane is an airfoil. Pressure and shear stress distributions over the wing surface by the fluids create an aerodynamic force that can be divided into two parts, a drag and a lift force (Figure 7).

![Figure 7 – Drag and lift forces in an airfoil](image)

The drag force is the component parallel to the apparent fluid velocity and the lift is perpendicular to the drag force. To analyze drag and lift forces is necessary to study the Reynold’s number effects and the wing’s camber. The Reynold number, \( R_e \), is a dimensionless number that provides a measure of the ratio of inertial forces/viscous forces. As for the camber, it is related with the asymmetry between the top and the bottom curves of an airfoil. An asymmetrical airfoil section can achieve higher maximum lift coefficients and lift/drag ratio (Figure 8).

![Figure 8 – Lift coefficient as function of the angle of attack for a symmetric and a cambered airfoil.](image)

Let \( V_{af} \) be the apparent velocity of a generic wing with respect to the fluid where it is immersed

\[
\overline{V_{af}} = \overline{V_f} - \overline{V}
\]

where \( V_f \) and \( V \) are the velocity of the fluid and the sailboat, respectively. The drag and lift forces for a generic wing are expressed by

\[
L = \frac{1}{2} \rho D_{uw} c_L (\alpha_w) V_{af}^2
\]

\[
D = \frac{1}{2} \rho D_{uw} c_D (\alpha_w) V_{af}^2
\]

where \( \rho \) denotes the density of the fluid, \( S_w \) the apparent area of the wing, \( c_D \) and \( c_L \) the drag and lift coefficients and \( \alpha_w \) the angle of attack.

Selecting a NACA airfoil profile for the wing, the lift and drag coefficients can be computed for angles of attack between zero and the stall angle (Figure 8), by

\[
c_L = \frac{k \alpha}{1 + \left( \frac{2}{AR_e} \right)}
\]

\[
c_D = c_{d0} + \frac{c_L^2}{2AR_e}
\]

where \( k \) denotes the slope of the lift of the section with angle of attack \( \alpha \), \( AR_e \) the effective aspect ratio of the airfoil (\( AR_e = 2 a^2 / S_w \), where \( a \) denotes the half the wing span) and \( c_{d0} \) the profile drag coefficient.

In a sailboat, \( \alpha \in [-180°, 180°] \). Therefore, the used curves for the dimensionless lift and drag coefficients are given by Figure 10.

![Figure 10 - Lift (a) and drag (b) coefficients as a function of the angle of attack for a symmetric wing](image)
The rudder is at the stern of the sailboat, which implies that the forces acting on it induces a moment around the vertical axis, affecting the sailboat’s heading.
Comparing the rudder with a rigid symmetric wing with elliptical chord distribution ($S = \pi a_r b_r/4$, where $a_r$ denotes the half the major axis and $b_r$ the minor axis of the rudder’s foil), the forces and moment components, are a combination of a drag and lift forces. The forces and moment components, are given by

\[ X_r = -|D_r| \cdot \cos(\beta_{ac}^\prime) + |L_r| \cdot \sin(\beta_{ac}^\prime) \]
\[ Y_r = |D_r| \cdot \sin(\beta_{ac}^\prime) + |L_r| \cdot \cos(\beta_{ac}^\prime) \]
\[ N_r = \frac{c_r}{4} Y_r \]

where $D_r$ and $L_r$ are computed through (18), replacing $V_{af}$ with $V_{ac}$. The angle $\beta_{ac}^\prime$ is defined according to Table 2.

Table 2 – Direction of the lift force in the rudder

<table>
<thead>
<tr>
<th>$\beta_{ac}^\prime$</th>
<th>$\beta_{ac}^\prime + \frac{\pi}{2}$</th>
<th>$\beta_{ac}^\prime - \frac{\pi}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_r &lt; \beta_{ac}^\prime &lt; \delta_r + \pi$</td>
<td>$\delta_r + \pi &lt; \beta_{ac}^\prime &lt; \delta_r + 2\pi$</td>
<td>$\delta_r + 2\pi &lt; \beta_{ac}^\prime &lt; \delta_r + 3\pi$</td>
</tr>
</tbody>
</table>

In spite of the fact that a clothed sail has some camber there exists no lift force for null angle of attack because the sail will be luffing (flapping), therefore the camber effect is neglected. The final forces in the sail are given by

\[ X_s = |D_s| \cdot \cos(\beta_{aw}^\prime) + |L_s| \cdot \cos(\beta_{aw}^\prime) \]
\[ Y_s = |D_s| \cdot \sin(\beta_{aw}^\prime) + |L_s| \cdot \sin(\beta_{aw}^\prime) \]
\[ N_s = \frac{L_s}{4} Y_s \]

where $D_s$ and $L_s$ are computed through (18) replacing $V_{af}$ with $V_{aw}$. The torque, $N_s$, is computed with $L_s$, which is a vector that links the position of the center of effort, $CE$, with $CM$, $\beta_{aw}^\prime$, is defined by the data in Table 3.

Table 3 – Direction of the lift force in the sail

<table>
<thead>
<tr>
<th>$\delta_s &lt; \beta_{aw}^\prime &lt; \delta_s + \pi$</th>
<th>$\delta_s + \pi &lt; \beta_{aw}^\prime &lt; \delta_s + 2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{aw}^\prime$</td>
<td>$\beta_{aw}^\prime + \frac{\pi}{2}$</td>
</tr>
</tbody>
</table>

Control Surfaces Total Forces

\[ \tau_s = \begin{bmatrix} X_{cs} \\ Y_{cs} \\ N_{cs} \end{bmatrix} = \begin{bmatrix} X_r + X_s \\ Y_s \\ \frac{c_s}{4} Y_{aw} \end{bmatrix} \]

5.2 Hydrodynamics and Aerodynamics

As the resistive body aerodynamic force, $\tau_a$ is much smaller than hydrodynamic part $\tau_h$, $\tau_a$ will be neglected.

Hydrodynamics

For an ideal fluid, the main force and moment components are: added mass, damping effects, Froude-Kriloff and Diffraction, and restoring forces.

In the 3 DOF model the restoring forces are neglected. Therefore, for this sailboat, the total forces and moment are given by

\[ \tau_h = \tau_{RB} + \tau_A + \tau_D \]
where \( \tau_{RB}, \tau_A \) and \( \tau_D \) are the vector of forces and moment relative to the hydrodynamic force of the rigid body, added mass terms and damping force, respectively.

### Rigid Body Forces and Moments

\( \tau_{RB} \) is modeled as a nonlinear function of the accelerations \( \dot{v} \), velocities \( v \), and Euler angles included in \( \eta \). This yields to

\[
X_{RB} = \frac{1}{2} \rho_w A_{UL} V_{rc}^2 \left( X'_{u\dot{u}} + X'_{v\dot{v}} + X'_{r\dot{r}} + X''_{u\dot{u}} + X''_{v\dot{v}} + X''_{r\dot{r}} + X''_{r\dot{r}} \right)
\]

\[
Y_{RB} = \frac{1}{2} \rho_w A_{UL} V_{rc}^2 \left( Y'_{u\dot{u}} + Y'_{v\dot{v}} + Y'_{r\dot{r}} + Y''_{u\dot{u}} + Y''_{v\dot{v}} + Y''_{r\dot{r}} + Y''_{r\dot{r}} \right)
\]

\[
N_{RB} = \frac{1}{2} \rho_w A_{UL} V_{rc}^2 \left( N'_{u\dot{u}} + N'_{v\dot{v}} + N'_{r\dot{r}} + N''_{u\dot{u}} + N''_{v\dot{v}} + N''_{r\dot{r}} + N''_{r\dot{r}} \right)
\]

where \( u', v', r' \) denotes dimensionless velocities, \( X', Y', N' \) dimensionless coefficients, \( \rho_w \) the water density, \( L \) the total length of the sailboat, \( A_{UL} \) the underwater transverse cross-sectional area, \( A_{UL} \) the underwater longitudinal centerline area. \( V_{rc} \) is the relative sailboat’s velocity to the current as

\[
\begin{align*}
V_{rc} &= V - V_c \\
u_{rc} &= V_c \cos(\beta_{rc}) = u - u_c \\
v_{rc} &= V_c \sin(\beta_{rc}) = v - v_c
\end{align*}
\]

where \( V \) is the vehicle velocity and \( V_c \) is the current velocity.

### Added Mass

A body in motion in a stationary fluid generates motion to the particles of the fluid that surround it, which induces forces and moment opposed to the movement of the body. The forces and moments in the rigid body due to added mass are computed as

\[
\tau_A = -M_A \dot{v}_{rc} - C_A(v) v_{rc}
\]

where \( v_{rc} = [u_{rc}, v_{rc}, r_{rc}] \), \( M_A \) is the system inertia matrix of added mass terms and \( C_A(v) \) the added mass Coriolis and centripetal matrix. To compute the added mass forces and moments, the body is approached by a semi-ellipsoid. Given that the vehicle moves at a low speeds, \( \tau_A \) is given by

\[
\tau_A = \begin{bmatrix}
X_A \\
Y_A \\
N_A
\end{bmatrix} = \begin{bmatrix}
X_A & 0 & 0 \\
0 & Y_A & 0 \\
0 & 0 & N_A
\end{bmatrix} \begin{bmatrix}
u_{rc} \\
v_{rc} \\
r_{rc}
\end{bmatrix} + \begin{bmatrix}
0 & 0 & -(Y_{u\dot{u}} + Y_{v\dot{v}}) \\
Y_{u\dot{u}} + Y_{v\dot{v}} & -X_{u\dot{u}} & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
u_{rc} \\
v_{rc} \\
r_{rc}
\end{bmatrix}
\]

where

\[
\frac{Y_u}{u} = \frac{\partial v}{\partial u}
\]

### Damping Forces And Moments

The damping effects include radiation-induced potential damping, \( D_{rf}(v_{rc}) \), skin friction, \( D_{sf}(v_{rc}) \), wave drift damping, \( D_{wd}(v_{rc}) \), and damping due to vortex shedding, \( D_{es}(v_{rc}) \). Consequently the total hydrodynamic damping matrix can be written as a sum of these components, such that

\[
\tau_D = -D_{rf}(v_{rc}) v_{rc} = -D_{sf}(v_{rc}) v_{rc} = -D_{wd}(v_{rc}) v_{rc} - D_{es}(v_{rc}) v_{rc}
\]

(29)

For a body moving with low speed through an ideal fluid, usually the total hydrodynamic damping, \( D_T \), can be simplified to

\[
D_T(v_{rc}) = -\begin{bmatrix}
X_u \\
Y_v \\
N_r
\end{bmatrix} \begin{bmatrix}
V_c \\
V_c \cos(\beta_{rc}) \\
V_c \sin(\beta_{rc})
\end{bmatrix}
\]

(30)

### 6. Computational Model

![Figure 13 – Sailboat’s simplified model](image)

The computation of the velocities in the 3 DOF, \( \upsilon_{(H)} = [u, v, r]^T \) is made through

\[
\upsilon_{(H)} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
\frac{V_{rc}}{r} \cos(\psi + \beta_{rc}) + \frac{V_c}{r} \cos(\psi_c) \\
\frac{V_{rc}}{r} \cos(\psi + \beta_{rc}) + \frac{V_c}{r} \cos(\psi_c) \\
\end{bmatrix}
\]

(31)

where \( \frac{V_{rc}}{r} \) is defined by (25) and \( \beta_{rc} \) is the angle of \( V_{rc} \) in \( \{B\} \), \( \frac{V_c}{r} \) and \( \psi_c \) are the velocity and direction of the current in \( \{I\} \) and \( \psi \) is the heading of the sailboat in \( \{I\} \). For the acceleration equations, \( \upsilon_{(H)} = [u, v, r]^T \) is given by

\[
\dot{\upsilon}_{(H)} = M^{-1} \tau_{(H)} - M^{-1} Cv_{(H)}
\]

(32)

where \( M \) is the total mass and inertia matrix (including added mass terms), \( C \) is the Coriolis and centripetal matrix (including added mass terms), both in 3 DOF, and \( \tau_{(H)} \) is given by

\[
\tau_{(H)} = \begin{bmatrix}
X_{r} \\
Y_{r} \\
N_{r}
\end{bmatrix} = \begin{bmatrix}
X_B + X_A + X_{Cs} \\
Y_B + Y_A + Y_{Cs} \\
N_B + N_A + N_{Cs}
\end{bmatrix}
\]

(33)

Through the integration of the state model equations (31) and (32), a prediction the behavior of the sailboat is possible.

### 6.1 Simulation in Open-loop

#### Velocity Prediction Program, VPP

The velocity polar diagram represents the maximum speed the sailboat can reach for each wind speed and direction. To compute it the domain of possible wind directions is discretized and for each angle a simulation is run, optimizing the sail angle to achieve the maximum speed in that direction. This process is repeated for each
wind speed. To maintain the desired orientation during the simulation, a heading controller (PID) was used. A VPP is a table with the discretized results of the velocity polar diagram for a range of wind speed values, which allows the prediction of the maximum speed that the sailboat can reach in a given direction, depending on the wind’s velocity and direction.

Figure 14 - Velocity’s polar diagram for $V_w = 1$ m. s$^{-1}$ and $V_w = 2$ m. s$^{-1}$

Examining Figure 14 it is possible to verify that, while no direct course is possible straight into the wind, the maximum speed is usually obtained downwind at about $\psi = \pm 120^\circ$.

7. Sailboat’s Control

This section describes the control strategy to maneuver autonomously the sailboat. The control system should be able to generate and follow a desired trajectory, considering the restrictions of sailing boats maneuverability. The control algorithms will responsible for taking sailing decisions and issuing appropriate commands to the rudder’s and sail’s servos.

Sailing Modes

Figure 15 - Sailing modes in different headings

To understand the proposed control algorithms for a sailing robot we first describe the general rules of sailing. As the kinematics and dynamic constraint limits do not allow to sail at all wind angles, several sailing modes and perform maneuvers need to be considered to overcome bearings not directly navigable. Basically there are three main modes (Figure 16) to sail a boat

i) Upwind sailing (tack maneuver),
ii) Downwind sailing (jibe maneuver),
iii) Direct sailing (wind angle in white zone, Figure 15).

In the tack maneuver (Figure 16 (a)), a straight line route is not navigable, and the sailboat has to take a zig-zag path, turning its bow through the wind, so that the direction from which the wind blows changes from one side to the other. As for the jibe (Figure 16 (b)), although a sailboat can sail straight downwind, for some sailboats it is faster to follow a zig-zag path also.

7.1 Control System

In Figure 17, $\psi_{Tack}$ is the trajectory orientation, $\psi_d$ the desired heading, $\delta_s$ is the range that the angle of sail can vary, and $\delta_r$ is the rudder’s angle. The sailboat control system can be decomposed in three sub-systems, as in Figure 17.

i) Guidance system. Its role is to compute the desired heading. This computation takes into account the
sailing modes, as function of the position of the sailboat and the trajectory orientation.

ii) Sail Controller. This system is responsible to control the value of the sail’s range angle, through a comparison between the wind conditions and the sailboat’s orientation with the data from the VPP.

iii) Heading Controller. It continuously computes the rudder’s angle to maintain a desired heading.

Guidance System

The Guidance System computes the desired orientation, based on the wind velocity and direction, and the given trajectory orientation. It, also computes the appropriated sailing mode. To accomplish this type of control strategy, the limits of wind angles for each sailing modes must be defined. According to the data from the VPP, the following ranges of true wind angles for each sailing mode were defined (see Table 4).

Table 4 –True wind angles for each sailing mode, in {B}

<table>
<thead>
<tr>
<th>Sailing modes</th>
<th>True wind angle’s range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upwind mode</td>
<td>[-60°,60°]</td>
</tr>
<tr>
<td>Direct mode</td>
<td>[-150°,60°] and [60°,150°]</td>
</tr>
<tr>
<td>Downwind mode</td>
<td>[-180°,150°] and [150°,180°]</td>
</tr>
</tbody>
</table>

In the upwind or downwind modes, the desired heading cannot be directly sailed. In this case, the strategy is to define a beating band around the desired trajectory (see Figure 18) and compute the sailboat’s orientation to the next beating.

Sail Controller

This controller computes the optimal sail angle in order to reach the maximum thrust, by a comparison of the wind conditions and actual orientation, with the data from the VPP.

Heading Controller

For most of the sailboats, the rudder is efficient enough to make the sailboat follow the target direction as long as the sailing mode is not upwind. As the guidance system does not allow such a direction, this controller, only has to adjust the rudder’s angle, so that the sailboat follow a given trajectory orientation.

The input data for the heading controller is the sailboat’s orientation and the desired orientation. The difference between these two headings, gives the necessary path correction, which enters directly into a PID controller.

8. Simulations Results

Upwind Mode Simulation

For the simulation of the Upwind mode (Figure 19), the applied wind is defined by \( V_w = 1 \text{ m.s}^{-1} \) and \( \psi_w = 0^\circ \), with \( \psi_{traj} = 0^\circ \). In (a) it is shown the progress of the sailboat in space, as in (b) the surge and sway velocities and the yaw velocity and angle.

In the tacking maneuver simulation, as the initial velocities of the sailboat are null, the initial orientation of the sailboat was set to \( \psi = -70^\circ \), in order to facilitate the sailboat to gain velocity.
Downwind Mode Simulation

In Downwind Mode simulation, the applied wind is defined by \( V_w = 1 \text{ m.s}^{-1} \) and \( \psi_w = 180^\circ \), with \( \psi_{\text{traj}} = 0^\circ \). In figure 20, (a) shows the progress of the sailboat in space and (b) the surge and sway velocities and the yaw velocity and angle.

![Figure 20](image)

Figure 20 – Simulation results from in Downwind Mode

9. Conclusions

This paper focused on the study and development of a robotic sailboat, which should be capable to execute the complex process of sailing autonomously. For that effect, a detailed mathematical model of the vehicle is derived and a control strategy that exploits the tack and jibe maneuvers is proposed. Actuating only on the sail and rudder orientations the control system is able to steer the vehicle to any given direction (on a zoom out scale) independent of the wind direction. The proposed control system has been successfully implemented and simulated in the developed computational model of the sailboat. The simulator developed can be used as an efficient platform helping to optimize the control system. Extensive outdoor tests with real systems, as well as time and cost consuming operations, can be avoided by the use of this simulator. Another benefit of the simulator is that environmental conditions can be set arbitrarily, such that performance and functions can be systematically verified. Overall this project has managed to fulfill its aims in studying a software control system which is capable of controlling a sailing robot by steering it along a predetermined course direction. In the real world the software still requires additional development in order to run it in real time.

References