Controlo de um Forno Solar para Testes com Variações Rápidas de Temperatura

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Abstract

Solar furnaces are devices used to study, test and transform materials through exposure to highly concentrated solar energy. In this report, a study on automatic control strategies for solar furnaces is done, with the goal of being able to produce fast temperature variations, usually required to perform thermal stress tests. The process to be controlled brings about many interesting control challenges, such as nonlinearities in the sample’s thermal dynamics and disturbances caused by the natural variation of the available solar power.

A cascaded controller architecture is explored, separating the problem into two smaller ones. With this architecture, an inner controller is responsible for assuring a desired radiation flux on the furnace’s focus, compensating the variations on the available power. The outer controller handles the actual control of the temperature, having as output the desired radiation flux which is assured by the inner controller.

The variation of the sample’s dynamics with temperature suggests the use of a multi-model adaptive control approach, resulting in the design of a set of local controllers for different working temperatures. Root-locus based design of a PI controller, pole placement control based on polynomial techniques and linear quadratic optimal control approaches are considered for the design of these controllers. It is shown that the last two approaches prove to be able to assure desired performance for many temperature ranges without resorting to multiple controllers.

Sensitivity and robustness trials are done so as to perceive which approaches best handle model uncertainty and rejection of disturbances.

Keywords: Solar furnace, supervised multi-model adaptive control, exact linearization, polynomial control, optimal control.
Resumo

Os fornos solares são dispositivos utilizados para estudo, teste e transformação de materiais por exposição dos mesmos a elevadas concentrações de energia solar. Neste trabalho, é realizado um estudo de estratégias de controlo destes dispositivos com o intuito de produzir variações rápidas de temperatura, necessárias para realizar testes de stress térmico. O processo a controlar providencia desafios interessantes no âmbito do desenho de sistemas de controlo, dadas as propriedades não lineares da termodinâmica da amostra bem como as perturbações provocadas pela variação natural da potência solar disponível.

É explorada uma arquitectura de controlo em cascata, separando o problema em dois menores. Um controlador interno é desenhado por forma manter um fluxo de radiação constante no foco do forno, compensando as variações da potência solar. O controlador externo realiza o controlo da temperatura da amostra, providenciando o controlador interno com o fluxo de radiação desejado.

A variação da dinâmica da amostra com a temperatura sugere que seja explorada uma técnica de controlo adaptativo multi-modelo, resultando no projecto de um conjunto de controladores locais para diferentes regiões de funcionamento. Para o seu projecto, são exploradas as técnicas de PI baseado em root-locus, colocação de pólos baseada em técnicas polinomiais e controlo óptimo linear quadrático. É demonstrado que, com estas duas últimas abordagens, é possível manter a performance desejada para várias gamas de temperatura sem recorrer a múltiplos controladores.

São realizados ensaios de sensibilidade e robustez por forma a determinar quais as soluções menos susceptíveis a perturbações e incertezas no modelo utilizado.

**Palavras-Chave:** Forno solar, controlo adaptativo supervisionado multi-modelo, linearização exacta, controlo polinomial, controlo óptimo.
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List of Acronyms

**ARMAX**  Autoregressive Moving Average model with eXogenous inputs

**PI**  Proportional Integral controller

**PID**  Proportional Integral Derivative controller

**SMMAC**  Supervised Multi Model Adaptive Control

**LQ**  Linear Quadratic

**ZOH**  Zero Order Hold
Chapter 1

Introduction

Due to high demands on different, renewable and cleaner energies, solar energy has been in the spotlight of investigation in Mankind’s quest for the ultimate, unlimited source of power. Alongside wind, wave and tidal power, its potential is undeniably vast, not only available virtually anywhere on the planet but naturally free to harvest for anyone.

Since ancient times, Man has resourcefully found numerous ways to harness this source of power and applied it in order to solve various necessities. Beginning with applications such as sun watches, harvest of salt from seawater and illumination, to name a few, science brought about a broad and extensive collection of technologies to make use of solar energy. Namely, in the past century, research on the field increased tremendously, opening paths to new ways of house acclimatization, water heating, electrical energy production and material processing.

As the price of oil reaches levels that threaten international economy, as well as resulting in evermore significant climate and environmental deterioration, the use of solar energy for high power applications has played a major role in the past decade. As electricity production goes, photovoltaic and solar thermal generation have been the main players in the field, the latter resorting to high concentration of solar radiation through the use of parabolic mirrors upon high capacity heat accumulators, such as water and other fluids. These are used to provide a heat source for water boilers or steam powered engines, such as the Sterling Engine, which can either be used for electrical energy production or powering local machinery.

Concerning material testing, processing and synthesis, solar furnaces provide the means to concentrate solar radiation on samples placed at their foci. As a free and highly available source of power, it opens new possibilities of submitting materials to highly concentrated bursts of solar radiation of high energy density. This allows for both thermal stress testing and material transformation. For example, submitting tiles of specialized materials to this kind of radiation simulates what happens in a spacecraft’s reentry into atmosphere. Likewise, it is possible to test the resistance of nuclear reactor container materials to extreme temperature variations and study their aging when subject to long exposures\[3\]. Manufacturing of specialized silicon solar cells\[19\] and sintering of metals (thermal treatment of powders or compact mixtures under the melting point of the main constituent)\[17\] have also been possible through the use of solar furnaces. A wide variety of materials that can be manufactured is then
possible, in conditions that are otherwise difficult or expensive to recreate.

In Odeillo, France, a solar complex plant comprises different types of solar furnaces that are used for such experiments. Particularly, a 6KW furnace in the complex is capable of focusing a total of 5500KW/m² in a 0.5cm wide focus. In this document, the work developed to obtain an automatic control system to be applied to this system is presented. Obtaining a robust and reliable temperature control system for this device is of significant importance for material research, as it allows for repeatability of experiments as well as accurate ways to subdue samples to desired temperature profiles. The aim of this work is to design a control system that is capable of producing large gradients of temperature on the tested sample, submitting it to high levels of thermal stress.

1.1 The 6KW Solar Furnace

The solar furnace for which a control system is to be designed is comprised of four major components, each of them having their own influence on the dynamics of the plant, according to their relationship with the incoming solar radiation. These components are made up by the heliostat, the parabolic mirror, the shutter and the temperature control system.

The heliostat, shown in figure 1.1a, is a mirror located on the ground that reflects the direct solar radiation onto the parabolic mirror, which is on an upper floor on the complex. A control system allows the heliostat to track the position of the Sun along the day, aiming to guide the sun beams to the concentrator always in the same direction. This guarantees that the focus of the furnace remains static. It is assumed in this work that this device tracks the Sun correctly, not influencing in any way the amount of power that reaches the sample. The parabolic mirror in figure 1.1b or concentrator, focuses the incoming light from the heliostat on a small area, the focus of the furnace, where the sample to be tested is placed.

In between the focus and the concentrator, a shutter is used to manipulate the amount of incident radiation that reaches the focus. This device plays a critical role as it acts as an actuator for the temperature control system, and it is itself a closed loop system, as its motor is connected to a driver which receives desired positions for the opening of the shutter blades. A picture of the shutter subsystem is shown in 1.2.
In order to measure the temperature of the sample, a thermocouple is attached to its base. There is also a pyrometer, which measures the radiation emitted by the sample to estimate its temperature, but is only able to measure temperatures above 700K. In this work, only the readings from the thermocouple will be considered, despite its limitation of only measuring the temperature on the bottom side of the sample, which may of course differ from the temperature on the rest of the body of the sample.

The controller that will result from the present work will use the measured temperature, together with the desired temperature defined by the operator, to compute a command for the shutter. Given this signal, the shutter adjusts its aperture in order to control the amount of solar radiation that reaches the sample, therefore acting on its temperature. Thereafter, the controller will need to perform in order to readjust the shutter so that the operator’s commands are followed, robustly, under various types of disturbances.

1.2 State of the Art

As far as automatic control of solar furnaces goes, research on the matter has begun rather recently. Starting in 1999 with [1], efforts were made to model the process, particularly the thermal dynamics of the sample under test and the non linear relation between the angle of the shutter’s blades and the aperture it displays. By assuming that the changes in the parameters of the heated sample with temperature could be disregarded, it was appropriate to use a linearized model for the process. A feedforward controller was designed in order to compensate variations on the available solar power, which makes up the most influencing disturbance of the process.

In the same work, a PI controller was designed which had its gains chosen so that the system would have double real poles in closed loop while admitting a maximum change rate on the actuation, as the shutter does not displace its blades instantaneously. It was then shown that the dependency of the parameters on the working temperature makes a controller with fixed gains a poor choice to tackle the problem, as the performance it was designed for was not guaranteed for all temperature ranges. An adaptive PI controller was then used to counter these difficulties, having the predictable consequences for this type of approach: the initial adaptation transient of the controller can be extremely harsh, especially for bad initial estimates of the system’s parameters, sometimes even
leading to erroneous evolution of the estimation process. Supervisory mechanisms were
developed in order to improve the identification process's performance, which include
restrictions on abrupt changes on the estimated parameters and filtering of the signals
used for identification, having been shown that an incorrect choice for these filters may
lead to spikes on estimates' value. Nevertheless, interesting results were obtained and
the feasibility of a feedback control approach for solar furnaces was demonstrated.

In [3] a study on the shutter’s dynamics is developed, starting with a theoretical
model based on the components of the device: a DC motor, gears and the physical
properties of the blades. Also, identification methods are used on actual shutter data
for the same purpose, revealing that the device presents some non-linear behavior. The
same methods are used to obtain a model for the sample’s thermal dynamics which, in
closed loop with a PI controller and the shutter model, demonstrated that the shutter
dynamics are several times faster than the thermodynamics of the sample. This allows
for temperature controller design without taking into account the shutter’s presence,
which can be modeled by a static gain.

A cascade controller architecture is explored in [2], where an inner controller acts
directly upon the shutter subsystem and the outer controller aims at controlling the
sample temperature. The inner controller is obtained using the exact linearization
technique, assuming a first order model for the shutter dynamics. Its purpose was to
compensate variations on the solar power as well as linearizing the actuation of the
process, simplifying the design of the outer controller.

The temperature controller in this approach was again a PI approach with a constant
integral gain and a proportional gain that varied with the working temperature. The
stability of the closed system with both controllers is proved through the use of the
singular perturbation method based on Lyapunov theory. It is demonstrated that this
theory imposes a minimum shutter speed for which the closed loop is stable that is
somewhat conservative, has it is demonstrated through simulation that relatively slow
dynamics for the shutter still provide stable closed loop systems.

In [4], the same authors explore a control law for temperature obtained through
the exact linearization technique. The derived controller is able to compensate vari-
ations on the available solar power and the model’s parameters are adaptive through
parameterized laws of their own, which are chosen to guarantee estimate boundedness.

Fuzzy control has also been studied as a possible candidate for controlling solar
furnaces [17]. Depending on the process being developed, many different uncertainties
may have to be taken into account other than the variability of the available power
from the sun, such as material transformation, reflexivity changes in the mirrors and
unaccounted non-linearities, which make fuzzy logic control a plausible solution for
such a problem.

Finally, [18] explores controller design based on Disturbance Accommodating Con-
trol theory, providing a solution that handles both strong disturbances and process
nonlinearities efficiently.
1.3 Aims and Objectives

As previously mentioned, the goal of the work described in this document is to develop a system that not only accurately tracks the user desired temperature for the heated sample, but is also able to induce large variations of temperature for short periods of time, submitting the sample to high levels of thermal stress. Also, because the actual process to be controlled is subject to some level of uncertainty, given the properties on which the used models are built on are themselves obtained through experimentation, it is quite important to develop controllers that are robust to modelling errors. Another issue is the fact that the amount of solar power may change along time, even being subject to sudden drops as clouds block the direct sunlight on the heliostat. As such, different approaches for the design of such controllers are explored in this work, having in mind that disturbance rejection and robustness to uncertainty are key factors.

In Part I it will be shown that the system to be controlled behaves nonlinearly, both on the shutter subsystem and regarding the thermal properties of the sample. Because of the variations of the properties of the sample as its temperature rises, a controller with fixed gains should be unable to guarantee the desired performance for all ranges of working temperatures. As such, a possible approach to work around this problem is to divide the range of working temperatures into several regions, each being approximated by a linear model.

With good enough linear models, a controller can be designed for each region, using typical linear control techniques. A supervisor algorithm chooses the model that best resembles the behavior of the plant on a given instance and selects the controller most suited for that case. This is the main principle of the supervisory based adaptive control algorithm, which was considered for solving the problem.

1.4 Report Outline

After this introduction, this report is divided into three major parts. Part II focuses on the procedure of obtaining mathematical models of the process to be controlled. A chapter is dedicated to modelling the shutter subsystem, as both a linear approximation for its dynamics and a nonlinear switching model based on actual data from the shutter are designed. A second chapter describes the deduction of a model for the sample’s thermal dynamics based on an energy balance. Due to the nonlinear properties of this model, a linearized approximation around a working point is obtained. Finally, the sensor noise is modeled following a Gaussian distribution, followed by the design of a low-pass filter to reduce its effect.

Part III is centered on the design and simulation of controllers. A cascaded control architecture is explored, as an inner controller is designed to compensate sun power disturbances and shutter nonlinearities, while an outer controller handles temperature reference tracking. For the latter, different techniques are explored, providing a set of comparable solutions.

Finally, Part IV is comprised of a set of robustness and sensitivity trials, allowing to infer which of the followed control approaches best suits the problem at hand. Conclusions are then drawn on the matter, as well as an overall appreciation of the accomplishments of this work. Some possibilities of future developments to be done on
the subject are also presented.
Part I

Modelling
Chapter 2

Shutter Model

A quick description of the properties of this subsystem is firstly done, followed by the approach taken in order to obtain a model that can cope with said properties.

Two important characteristics define the shutter, the first one being its static function and the other one its dynamic behavior. While the static function defines the relation between shutter aperture and the amount of radiation flux that is allowed through the shutter, the dynamic behavior of the shutter defines the positioning and speed of the shutter blades, as a response to a position input command. It is this positioning that will define the aperture of the device.

In this section, two models are developed. The first one consists of a simple linear model that does not precisely represent the dynamics of the shutter but will, however, allow for easier controller design.

As the shutter dynamics present some non-linearity, the second model developed in this section is composed of several linear models and a switching mechanism, attempting to model the different behaviors of the shutter dynamics for different operating regions. This model is not practical but provides a more accurate and realistic simulation of how the real shutter would behave, hence being useful for accurate robustness tests for designed temperature controllers.

2.1 Shutter Description

The shutter device consists of a closed-loop system - shutter blades, motor and PID controller - with fast dynamics when compared to the sample's thermal behavior.

It is composed by a set of 10 aluminum blades, 2.0mm thick and 0.5m long. The blades have a circular shape and their axles are connected to a single gear, which is moved by a brushless motor, connected to a controller. This controller receives reference values for the desired position for the blades, which is measured by an encoder on the motor. This position is measured as the angle formed between the blades and the horizontal plane.

As the blades overlap each other when in resting position, as their width is larger than the distance between their axles, there is a minimum angle of opening of the shutter below which there is no light passing through. This angle, $\theta_0$, has been determined
to be $25^\circ$.

Also, because of the way the blades are arranged, it admits that the relation between the percentage of solar radiation going through the open shutter and the blade’s angle can be approximated by equation (2.1), based on geometric considerations. This relation translates the static function mentioned previously.

$$S(\theta(t)) = \begin{cases} 
0 & \text{if } 0^\circ \leq \theta \leq 25^\circ \\
1 - \frac{\cos \theta}{\cos \theta_0} & \text{if } 25^\circ < \theta \leq 90^\circ 
\end{cases} \quad (2.1)$$

### 2.2 Linear Model for the Shutter

The data used to identify the shutter subsystem is composed by several step responses, each of them resulting from commanding the system to increase or decrease the angle of the blades by $10^\circ$. Two sets of data were used: one containing data for the opening of the shutter angle and the other describing its closure. These data sets are represented respectively in figures 2.1a and 2.1b.

Notice that, for the acquisition of the data, the resting position for the shutter was considered at $0^\circ$, meaning the data has a $25^\circ$ offset. This is helpful for identification purposes, and afterwards this offset will be added to the output of the model.

The sampling time of the used data is $10.0ms$. It can also be inferred from the data that the system’s delay is one sampling interval. Notice that there is some steady state error which implies that the integral part of the shutter’s controller is not properly tuned.

Also, if close attention is paid to the transient response of the shutter, the dampening factor of the various step responses appears to increase as the angle increases, which suggests that the system has nonlinear properties. This is most likely due to the fact that the motor torque varies in a non-proportional way as the blades become perpendicular to the horizontal plane. Because the controller applied has static parameters, the power applied on the motor will not be adjusted properly for the various positions.
As the goal in this section is simply to approximate these dynamics through a linear model, a section of the data was randomly taken from the one of the sets to design this model, which will be considered to be a good sample of the true dynamics on all possible working angles.

The model structure selected was a standard second order linear system with unitary gain, as the one presented in eq. (2.2).

\[
\theta(s) \theta_r(s) = \frac{\omega_n^2}{s + 2\xi \omega_n s + \omega_n^2}
\]  

(2.2)

The values for the dampening factor and natural frequency for this model were chosen as \(\xi = 0.7\) and \(\omega_n = 20\), respectively. Figure 2.2 validates this model for the region 20° – 30°, giving a mean square error of 5.61%.

![Figure 2.2: Comparison between the actual shutter data and the second order model.](image)

2.3 Non Linear Model for the Shutter

As was described in 2.2, dynamics of the shutter change as the blade’s angle increases. To tackle this issue, the workspace that makes up the various possible positions for the blades was divided into separate regions, each being covered by a linear model. A switching mechanism was then developed in order to decide when to commute between each model, according to the current angle of the blades.

To identify the models for each region, MATLAB’s Ident toolbox was used, assuming discrete time models (with sample time 10ms) with an ARMAX architecture. Each of the obtained models are incremental models, meaning they are linearizations of the real model around a specific working point. This working point was chosen as the mean value between the limits that defined each region of the workspace.

Initially, the models were obtained by considering each step response from each of the data sets as a region to be covered by a linear model, which means having six
models for the opening motion of the shutter and six other for the opposite motion. As the obtained models were of very high order, efforts were done in order to design models that would cover more than one of those regions, reducing the size of the set of models.

However, having models for the opening motion and closing motion would make the algorithm for the switching mechanism too complex to code and computationally heavy. In order to simplify the task, the two sets of data in figure 2.1 were concatenated, which was done by taking each step response from the one set of data and concatenating it with the step response on the same working region from the other set of data.

Using the resulting six sets of data, each representing a working region for the shutter, six models were obtained through the same method of identification. A criterion for accuracy, in order to know when a model was of high enough order to replicate the desired dynamics, was used when testing various model orders: assuming that a 1% error on the amount (percentage) of radiation passing through the shutter is acceptable, it is possible to compute the corresponding limit for the acceptable angle deviation from the desired flux going through.

For each of the stationary positions of the shutter in the data, the corresponding percentage of radiation flux going through was computed with (2.1). With the latter, the corresponding values for the shutter angle 1% above and below were computed by using the inverse equation. These values were set as upper and lower limits for the output interval, outside which it was no longer useful to precisely model the system’s response.

On table 2.1 the orders for the obtained ARMAX polynomial structures (its numerator, denominator and colored noise polynomials), referred to as $\text{na}$, $\text{nb}$ and $\text{nc}$, are presented. In figure 2.3 a performance comparison between the obtained model for the $30^\circ - 40^\circ$ region and the data that generated it is presented, where the above mentioned accuracy criterion is also represented.

Modelling of the shutter dynamics for the smaller aperture levels required higher order polynomials, as the output of the real device appears to oscillate a great deal more for the smaller blade angles. This is, again, most likely due to the amount of torque the motor controller was designed for, which might not be constant as the blades reach higher angles.

With this final set of six incremental models, a Simulink S-Function was developed in order to be able to simulate the shutter dynamics along its entire workspace. The main goal was to devise a way to properly commute between the models as deemed necessary, while still preserving continuity. The colored noise terms of the models were discarded in this section, for they later proved not contribute for a more accurate
Let $\Omega$ be the entire workspace of possible angles for the blades, built up by the various regions it was divided into, and $M$ the set of obtained model polynomials. For each computed output produced by the incremental model in use, a sub-routine is performed to check whether or not the model output will be beyond the current working region. If so, the appropriate incremental model is selected and a new output is produced. The idea is explained in detail in **Algorithm 1**. Given the output $x_k$ produced by a selected model $i$, with coefficient vectors $A_i$ and $B_i$, the program will determine whether or not it is necessary to commute to another model. Model $i - 1$ is selected if $x_k$ is on the range of angle values immediately before the current zone, $R_{i-1}$, or model $i + 1$ for the range immediately after, $R_{i+1}$. If it does belong to one of this ranges, the model output is recomputed using the correct model.

**Algorithm 1** Incremental model switching mechanics

**Require**: $k \geq 0$, $x_k$, $i$. $\{A_j, B_j\} \in M$, $R_j \subset \Omega$, $\forall j \in 1, \ldots, 6$

$x_{k+1} = A_i x_k + B_i u_k$

if $x_{k+1} \in R_{i-1}$ then

$x_{k+1} = A_{i-1} x_k + B_{i-1} u_k$, $i - 1 \geq 1$

else if $x_{k+1} \in R_{i+1}$ then

$x_{k+1} = A_{i+1} x_k + B_{i+1} u_k$, $i + 1 \leq 6$

end if

If the limits between each working region are used to determine whether the switching of models should occur, it is likely to that instability by constantly changing between models is induced. This may happen, for instance, if the output of one model would linger very close to such a limit and oscillate.

In order to prevent such undesirable behavior, the threshold which causes switching to the model for the region above was defined by taking the upper limit of a region and slightly raising it by $1.5^\circ$ (which is approximately the maximum value of the step
response overshoot on all models). On the other hand, the exiting value from one region to the region below was defined by decreasing its lower limit by the same amount. This gives the the system some hysteresis that will solve the model commutation instability issue.

In figure 2.4 the obtained switched model system is validated by exciting the system with the same input as the one used for obtaining identification data, comparing the results with the latter. The switching between the different models is also shown in this figure.

![Graph](image1)

(a) Shutter opening validation.

![Graph](image2)

(b) Shutter closing validation.

![Graph](image3)

(c) Model switching while opening.

![Graph](image4)

(d) Model switching while closing.

Figure 2.4: Performance of the obtained switching model compared with the actual shutter data and selected model sequence.

The results are quite satisfactory, as not only does the switched model preserve continuity when switching between regions, but on each region most of the real shutter dynamics are well covered. The most significant part of the shutter dynamics is its overshoot, which is shown to be well simulated by the model; the small, uncovered oscillations should not influence the actual radiation output of the device.

Special attention should of course be taken when analyzing the evident difference between the switching model’s performance and the real shutter dynamics, during the initial large step of $60^\circ$ presented in [2.4b]. Here, the obtained model seems to have a faster performance as well as a large overshoot when compared to the real device.
This may at first seem harmless since for other step responses, both upwards and downwards, mdl6 correctly simulates the shutter. Alas, because the shutter will most likely be required to perform such wide movements in order to produce large temperature gradients on the sample, it is important to model the shutter behavior well for this situation. It is reasonable, however, to consider these differences small enough when seen in a complete system perspective: the shutter is always several times faster than the thermal dynamics of the sample, thus a difference of a few milliseconds on the shutter’s response will be hardly noticed on the final closed loop system. As for the overshoot, the static function presented in (2.1) will help reduce its effect by smoothing its peak.
Chapter 3

Thermal Dynamics Model

In this chapter of the Modelling part of the report, a mathematical representation of the temperature dynamics of the sample is developed. The model obtained has nonlinear properties and hence linearization techniques will be used to obtain a linear model, allowing application of linear control theory. It will be assumed in this work that the sample being tested is a disk of alumina ($\text{Al}_2\text{O}_3$), allowing the specification of certain matter properties required to test the developed model. This material is commonly used in the types of experiments that will be considered.

3.1 Nonlinear Model

As in [1], a balance of energy will be used to study the thermal dynamics of the sample. All parameters in the equations in this section are defined in table 3.1.

Let us firstly consider the energy of the sample, given its temperature $T_s(t)$ in Kelvin degrees. It can be approximated by:

$$E = C_p m T_s(t) \quad (3.1)$$

The energy of the sample increases with the incoming power passing through the shutter subsystem, which is modeled by equation

$$P_i = \alpha_s A_s g f h_r G_s(t) S(\theta(t)) \quad (3.2)$$

where $S(\theta(t))$ describes the percentage of open area of the shutter given by equation [2.1], defining the amount of concentrated radiation that is allowed to reach the focus. The $h_r$ term represents the combined factor of reflexiveness of both the heliostat and the concentrator, as both of them can be affected by dust and moisture that can deteriorate the amount of reflected light. It will be admitted, for now, that this factor is 1, meaning optimal working conditions. The variable $G_s(t)$ is the solar flux that reaches the parabolic mirror.

Concerning the power losses due radiation and convection, these can be modeled using standard thermodynamical definitions by (3.3) and (3.4), respectively.

---

Data obtained from reports on the SOLCONTROL/SOLFACE/SFERA projects.
Table 3.1: Parameters for the thermal dynamics model. The sample is admitted to be of $\text{Al}_2\text{O}_3$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p[J.Kg^{-1}.K^{-1}]$</td>
<td>Specific heat</td>
<td>$eq. (3.7)$</td>
</tr>
<tr>
<td>$m[Kg]$</td>
<td>Mass of the sample</td>
<td>$2.651 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Emissivity of the material</td>
<td>$eq. (3.8)$</td>
</tr>
<tr>
<td>$\sigma[W.m^{-2}.K^{-4}]$</td>
<td>Stephan-Boltzmann constant</td>
<td>$5.67 \times 10^{-8}$</td>
</tr>
<tr>
<td>$A_s[m^2]$</td>
<td>Exposed area of the sample</td>
<td>$7.068 \times 10^{-4}$</td>
</tr>
<tr>
<td>$h_{conv}[W.m^{-2}.K^{-1}]$</td>
<td>Convection factor</td>
<td>$eq. (3.6)$</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Solar absorptivity of the material</td>
<td>0.14</td>
</tr>
<tr>
<td>$g_f$</td>
<td>Solar furnace gain</td>
<td>2525</td>
</tr>
<tr>
<td>$h_r$</td>
<td>Heliostat and concentrator mirror gain</td>
<td>1</td>
</tr>
<tr>
<td>$G_s(t)$</td>
<td>Solar radiation flux (max. value)</td>
<td>1KW/m²</td>
</tr>
</tbody>
</table>

\[ P_r = \epsilon \sigma A_s [T_s^4(t) - T_{\text{sur}}^4(t)] \]  
\[ P_c = h_{\text{conv}} A_s [T_s(t) - T_{\text{air}}(t)] \]  
\[ P_i = \alpha_s A_s g_f h_r G_s(t) S(\theta(t)) \]

Where $T_{\text{air}}(t)$ and $T_{\text{sur}}(t)$ are the temperature of the air and the temperature of the surroundings, respectively.

Now, in order to combine these equations, the energy balance for the heated sample is defined by $dE/dt = -P_r - P_c + P_i$. Substituting the previous equations on the latter yields:

\[ C_pm \frac{dT_s(t)}{dt} = -\epsilon \sigma A_s [T_s^4(t) - T_{\text{sur}}^4(t)] \]
\[ - h_{\text{conv}} A_s [T_s(t) - T_{\text{air}}(t)] \]
\[ + \alpha_s A_s g_f h_r G_s(t) S(\theta(t)) \]  
\[ (3.5) \]

Both the convection factor, specific heat capacity and emissivity of the sample are temperature dependent. They are described in equations (3.6), (3.7) and (3.8).

\[ h_{\text{conv}}(T_s) = 1.32 \left( \frac{\|T_s - T_{\text{sur}}\|}{L_c} \right)^{0.25} \]  
\[ C_p(T_s) = (1.0446 + 1.742 \times 10^{-4} T_s - 2.796 \times 10^{4} T_s^{-2}) \times 10^3 \]  
\[ \epsilon(T_s) = -3.3333 \times 10^{-4} (T_s - 800) + 0.65 \]  
\[ (3.6), (3.7), (3.8) \]

The gain of the solar furnace was computed using data presented taken from the Odeillo Solar Complex website. As the furnace is able to concentrate 6KW onto a $55mm$ wide focus, the amount of power per square meter in that focus is:

\[ P_f = \frac{6 \times 10^3}{(55/2 \times 10^3)} \approx 2525 \times 10^3[W/m^2] \]
Assuming the average available solar power of 1000\,\text{W/m}^2, the gain of the furnace is then \( g_f = 2525 \).

For the sake of simplified notation, equation (3.5) is rewritten as the variation of the temperature and parameters \( \alpha_1, \alpha_2 \) and \( \alpha_3 \), which depend on the temperature, are defined.

\[
\frac{dT_s(t)}{dt} = -\alpha_1[T_s^4(t) - T_{\text{sur}}^4(t)] - \alpha_2[T_s(t) - T_{\text{air}}(t)] + \alpha_3 G_s(t) S(\theta(t)) \tag{3.9}
\]

\[
\alpha_1 = \frac{\epsilon \sigma A_s}{C_p m} \tag{3.10}
\]

\[
\alpha_2 = \frac{h_{\text{conv}} A_s}{C_p m} \tag{3.11}
\]

\[
\alpha_3 = \frac{\alpha_s A_s g_f h_r}{C_p m} \tag{3.12}
\]

It should be noted that the equations presented above assume that the sample is thin enough to admit that the energy within its mass is uniformly distributed (temperature homogeneity). This allows the assumption that the temperature reached at the upper surface of the sample, which is exposed to radiation, is the temperature measured by the thermocouple attached to its bottom surface. It also allows the assumption that there is a single surface, represented by \( A_s \), that is responsible for the power losses and radiation absorption.

In figure 3.1, the open loop response of the process model is represented, along with the shutter percentage of opening that was used as input.

The effect of the fourth power non-linearity in (3.9) is quite evident, as for the same increase in the shutter aperture (10\% each), the corresponding increase in temperature of the sample is ever smaller. The figure also shows that the system responds very much like a first order one, only that it appears to change its static gain and time constant along different temperature ranges.

The model developed has also been tested before actual data from the Odeillo 6\,\text{KW} Solar Furnace in [1], validating the developed mathematical approximations.

### 3.2 Linear Model

In order to apply linear control techniques to the obtained model in 3.1, equation (3.9) must be linearized.

Let \( U(t) = G_s(t) S(\theta(t)) \), the amount of radiation that is allowed to reach the sample (without the furnace gain), be the input for the system. If the system is in the equilibrium point \((U_0, T_0)\), where the energy variation on the sample is null, and if a small perturbation on the input \((U = U_0 + u, T_s = T_0 + \delta)\) is induced, a linearized model of the thermal balance is obtained. Using Taylor’s Theorem for smooth nonlinear functions [14], the following equation defines a linear approximation for \( T_s(t) \):
Figure 3.1: Open loop response of the temperature of the sample with the percentage of open shutter area. The solar radiation flux for this simulation was 1 $kW/m^2$

\[
\dot{T}_s(t) \approx \dot{T}_s(t) \bigg|_{(U=U_0,T_s=T_0)} + \frac{\partial \dot{T}_s(t)}{\partial T_s} \bigg|_{(U=U_0,T_s=T_0)} (T_s - T_0) + \frac{\partial \dot{T}_s(t)}{\partial U} \bigg|_{(U=U_0,T_s=T_0)} (U - U_0)
\]

(3.13)

Together with (3.9), the linear differential equation for the temperature dynamics is obtained.

\[
\dot{\delta}(t) = -4\alpha_1 T_0^3 \delta - \alpha_2 \delta + \alpha_3 u
\]

(3.14)

Now, considering initial null conditions, the Laplace Transform may be applied, with \((u(t) \rightarrow u(s), \delta \rightarrow T_s(s))\), the resulting transfer function for the linearized model presented in (3.15).

\[
T_s(s) = \frac{\alpha_3}{s + 4\alpha_1 T_0^3 + \alpha_2}
\]

(3.15)

Given that the input of this model is a perturbation of an equilibrium of the system, and that its output represents the corresponding perturbation on the output of the non-linear system, this model is an incremental model that closely represents the actual system behavior around said equilibrium point. The parameters $\alpha_1$, $\alpha_2$ and $\alpha_3$ are considered constant for this linear model, evaluated at $T_0$, and it should be noted that the linearized model no longer depends on the surroundings and air temperatures.

In order to further simplify the notation, (3.16) will be used to represent the linearized model for the sample’s thermal dynamics, where $a = 4\alpha_1 T_0^3 + \alpha_2$ and $b = \alpha_3$.

\[
\frac{T_s(s)}{u(s)} = \frac{b}{s + a}
\]

(3.16)
Figure 3.2: Linear model parameter values for different working temperatures.

Of course parameters $a$ and $b$ of the above model are temperature dependent, though considered constant for the working temperature range the model intends to represent. Figure 3.2 shows the variation of these parameters for different working temperatures $T_0$.

Much like in figure 3.1, it is possible to observe that plant’s gain decreases with the temperature of the sample, while its time constant, given by $1/a$, decreases as well.

3.2.1 Linear Model Validation

In order to validate the linear model, simulations are run to compare the obtained local model with the nonlinear one, for different working temperatures. In figures 3.3, 3.4 and 3.5, three distinct working temperatures are considered to compare both models, as well a perturbations of 1 and 5% on shutter aperture. The available solar power for these simulations was considered to be $1000W/m^2$ and figure 3.6 shows the Simulink schematic used to perform simulations.

In these simulations, it was necessary to drive the nonlinear model to the equilibrium point around which the corresponding local model is meant to work. To do this, equation (3.9) was used considering thermal equilibrium, $\dot{T}_s(t) = 0$, reached on a desired reference temperature $T_s(t) = T_r$. These conditions lead to equation (3.17), allowing computation of the stationary input (shutter aperture) for the system to reach and remain at temperature $T_r$.

$$S_{stationary} = \frac{\alpha_1(T_r^4 - T_{sur}^4) + \alpha_2(T_r - T_{air})}{\alpha_3 G_s(t)}$$  \hspace{1cm} (3.17)

From these simulations, a few important conclusions are reached:

- For medium to high working temperatures around which the linear models are derived, the linear models become more accurate and closely resemble the nonlinear model response. For very high temperatures, these models start to deviate from the nonlinear response, though not significantly.

- As expected, responses for small perturbations on the input are are better covered by the local model than larger ones. This is specially true for lower temperatures,
Figure 3.3: Comparison between the non-linear model and the local model for perturbations around $T_r = 600K$.

Figure 3.4: Comparison between the non-linear model and the local model for perturbations around $T_r = 900K$. 
Figure 3.5: Comparison between the non-linear model and the local model for perturbations around $T_r = 1200K$.

Figure 3.6: Simulink schematic used for validating the linear local model.
where differences in the amount of radiation hitting the sample have greater effect than in higher working temperatures, as was observed in figure 3.4.

- From the information on parameter dependency on temperature, one may conclude that it is very likely that more local models will be needed to approximate the temperature dynamics for the higher temperature ranges, as the system’s pole changes faster in those ranges.

3.3 Noise and Filtering

In order to model the noise on the measurement signals, a random process sampled from a Gaussian distribution is added to the output of the models. From measurements obtained from the actual plant, the observed noise from the thermocouple had most of its values bouncing within an interval of $[-25, 25]$ Kelvin degrees around a mean value. This implies a variance of $69.4K^2$, following the three sigma rule for a Gaussian distribution.

Necessarily, to allow the design of a wide range of controllers, a filter must be connected to the output of the sensors. This is mainly because this already high level of noise would be amplified by the controller and cause high frequency variations on the input current of the shutter motor, drastically reducing the life expectancy of the device. As such, a simple low-pass filter was designed, following the heuristic rule that the filter should have its pole ten times as faster than the dynamics of the plant. Thus, the following transfer function define the designed low-pass filter for the system:

$$\frac{T_f(s)}{T_o(s)} = \frac{\omega_f}{s + \omega_f}$$

With $\omega_f = 10 \times (4\alpha_1T_0^3 + \alpha_2)[rad.s^{-1}]$, $T_f$ being the filtered temperature. For the design of the filter, the temperature chosen for linearization, $T_0$, was considered 1000K. This proved to greatly reduce the noise at the output of the process model without affecting significantly its dynamics, even for greater values of $T_0$ where the dynamics are faster. Figure 3.7 shows a simulation comparing both the unfiltered output of the process and filtered one.

![Figure 3.7: Comparison between the filtered and unfiltered outputs of the process.](image-url)
Though the order of the system to be controlled is increased by adding this filter to the process output, it is now possible to use controllers with a wider range of proportional components, which allows for faster closed loop responses.
Part II

Controller Architecture and Design
As previously mentioned, the main goals for the developed controller are to achieve high gradients of temperature on the sample for short periods of time, in order to test stress resistance capabilities of materials. Given this, it will be a constant goal to achieve as fast as possible responses to step variations of the desired reference temperature. However, as many of the performed tests in these kinds of facilities are done working very close to the samples’ melting temperatures, it is quite important to avoid overshoot on the closed loop responses, as these may very likely lead to the destruction of the sample under test.

In Part I, it was shown that some important characteristics of the plant to be controlled come into play in its dynamics. Mainly, the shutter subsystem’s nonlinear static function makes the actuator on the control loop behave nonlinearly, which is limiting when using standard linear control techniques. Furthermore, the actuation signal (i.e., the incident radiation on the sample) also depends on the available solar power $G_s$, which varies along time.

To cope with these challenges, a cascaded architecture for the closed loop control of the temperature is developed, consisting of an inner and outer control loops. The outer loop controller receives as inputs the desired value for temperature and the current temperature of the sample, designating the required radiation flux that must hit the sample’s surface. The inner loop controller receives this radiation flux reference and, together with the available sun power, current radiation outputted by the shutter and its current blade positioning, adjusts the shutter aperture so that the desired radiation flux is achieved. The latter, as do most closed loop control systems, creates a linear response between input and output, hence linearizing the actuation of the plant. The schematics of this architecture are represented in figure 3.8. In [2], the stability of this control architecture is demonstrated.

![Figure 3.8: Cascade control architecture.](image)

The inner loop controller development is firstly presented, as it allows for simple controller design for the outer loop. The design of the outer controller, as previously mentioned, makes use of the linear approximations of the thermal dynamics, intended to represent the behavior of the sample’s temperature around the working point the linearization is considered. Linear control techniques are applied to these models, making up a bank of local controllers, which are to be used according to the sample’s current temperature. To design the local controllers, *root-locus*, pole placement and optimal control techniques were considered.
Chapter 4

Inner Controller

As mentioned previously, the inner control loop goals are twofold: transform the actuation of the system into a linear signal from the point of view of the outer control loop and compensate variations on the incoming solar radiation. By doing this, the control signal for the temperature controller can take values between 0 and the maximum available power from the Sun, as it is now the actual desired incident power that reaches the sample. Also, the radiation on the output of the shutter of this signal should no longer vary with $G_s(t)$, given that the inner controller counters these variations. The schematics for this controller are presented in figure 4.1.

\[
\begin{align*}
\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \omega_n^2 z(t) &= \omega_n^2 u_{shut}(t) \\
S(z(t)) &= \cos \left( \theta_0 + \frac{z(t)(90 - \theta_0)}{100} \right) / \cos \theta_0
\end{align*}
\]

(4.1)

(4.2)

The notation $\dot{z}(t)$ denotes the first derivative of signal $z$ in order to time.

Figure 4.1: Inner loop controller schematics.

The exact linearization technique was used to achieve this, where the desired closed loop dynamics are firstly defined and, through algebraic manipulation and differentiation, it is possible to obtain the control law that forces the output of the shutter to have said dynamics. This approach is based on the work done in [2], where the same technique was used for a first order model that simulated the shutter dynamics.

The dynamics of the shutter used for the design of this controller were the ones shown in (2.2). Instead of considering the dynamics of the actual angle of the blades $\theta(t)$, in this section, the same dynamics will be used to represent the response of a normalized angle $z(t)$, ranging from 0 to 100. As such, the equations for the shutter dynamics, as well as the static function that translates the opening of the blades into aperture, are shown below:

\[
\begin{align*}
\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \omega_n^2 z(t) &= \omega_n^2 u_{shut}(t) \\
S(z(t)) &= \cos \left( \theta_0 + \frac{z(t)(90 - \theta_0)}{100} \right) / \cos \theta_0
\end{align*}
\]

The notation $\dot{z}(t)$ denotes the first derivative of signal $z$ in order to time.
The signal $u_{\text{shut}}(t)$ in (4.1) is the command for the normalized angle received by the shutter subsystem. To apply exact linearization, one must first define the desired dynamics for the closed loop system. It will be admitted in this case that the closed loop will perform with the same dynamics as the shutter subsystem, that is to say:

$$\ddot{r}(t) + 2\xi\omega_n\dot{r}(t) + \omega_n^2 R(t) = \omega_n^2 R(t)$$

(4.3)

Where $r(t)$ is the amount of radiation flux at the output of the shutter subsystem and $R(t)$ is the desired value for this quantity, defined by the outer controller.

Since the radiation that goes through the shutter is given by $r(t) = G_s(t)S(z(t))$, the following equation for the second derivative of this signal holds true:

$$\ddot{r}(t) = \ddot{G}_s(t)S(z(t)) + 2\dot{G}_s(t) \frac{dS(z)}{dz} \bigg|_{z(t)} \dot{z}^2(t) + G_s(t) \left( \frac{d^2S(z)}{dz^2} \bigg|_{z(t)} \ddot{z}(t) + \frac{dS(z)}{dz} \bigg|_{z(t)} \dot{z}(t) \right)$$

(4.4)

Now, replacing $\dot{z}(t)$ by (4.1) and $\ddot{r}(t)$ by (4.3), it is possible to rewrite the resulting equation in order to the control variable, defining the control law\(^2\):

$$u_{\text{shut}} = \frac{\omega_n^2 (R - r) - 2\xi\omega_n\dot{r} - \ddot{G}_s S - \dot{z}S' [2\dot{G}_s + G_s (S'' \ddot{z} - S' \omega_n)] + zG_s S' \omega_n^2}{G_s S' \omega_n^2}$$

(4.5)

Where $S = S(z) \bigg|_{z(t)}$ $S' = \frac{dS(z)}{dz} \bigg|_{z(t)}$ and $S'' = \frac{d^2S(z)}{dz^2} \bigg|_{z(t)}$. The signals $\dot{G}_s(t)$ and $\ddot{G}_s(t)$ can be obtained by filtering the solar radiation signal.

It is, of course, necessary to take into account that $\dot{G}_s(t)$ and $\ddot{G}_s(t)$ will be very noisy if $G_s(t)$ is not filtered before estimating the derivative. This can be done with a simple low-pass filter that may have a very low cutoff frequency, since the solar power does not vary rapidly along the day. However, its cutoff frequency must not be too small, as it may dampen excessively the effect of perturbations such as clouds blocking the sunlight.

Simulations were run in order to evaluate performance for this controller, with results shown in figure 1.2. One can see that during the interval [5, 15]s, because there is not enough available power to satisfy the desired value, the shutter remains fully open and the curve of the available solar power can be seen at the output of the device. The controlled system displays good performance for the rest of the simulation, as it is able to keep a steady flux on the output despite constant variation of the available power, while still keeping track of the reference with significant speed.

Another interesting case that should be taken into account is the case where there is an abrupt drop of available solar power. Such a case could be exemplified by the passing of a cloud over the path of the radiation flux hitting the heliostat. In figure 1.3, an example of a power drop of such magnitude is simulated, assuming a steady desired radiation flux of 500W/m\(^2\) and a rapid descent of available power. The shutter

\(^{2}\)The time dependency of the signals was removed in order not to overburden the notation.

\(^{3}\)In all simulations on this section, the plots for $G_s(t)$ represent a filtered solar power flux, which is given as input to the inner controller. The unfiltered signal has added white noise with variance 20W\(^2\)/m\(^4\).
increases its opening in order to maintain the outgoing power, which is kept at the desired level.

As a second order system, the shutter will obviously filter the outer controller’s incoming signal, slowing and smoothing rapid variations. However, it will also filter out high frequency noise which may result from the proportional components of the outer controller, assuring a steady radiation flux hitting the sample. Figure 4.4 demonstrates this fact, where the high variance noise on the reference for the power flux is greatly reduced on the shutter’s output, which can also be seen on the smooth signal for the percentage of shutter aperture. The variance for this simulations was chosen as worst and improbable scenario where the reference would quickly oscillate within a 200W/m² interval.

Nevertheless, this noise reduction effect comes at the price of putting high frequency noise on the shutter’s commanding current, which is undesirable.

Now, using this inner loop within the temperature control loop, working as the actuator, it is possible to design a controller which does not take into account the nonlinearities of the shutter nor the solar power that is available at any given instance. This outer controller will generate desired radiation power at the focus, which will now saturate between 0 and the maximum available solar power.
Figure 4.3: Inner controller performance for an abrupt power loss. Reference signal, radiation flux output, shutter aperture and available solar radiation flux.

Figure 4.4: Inner controller performance for a noisy reference signal. Reference signal, radiation flux output, shutter aperture and available solar radiation flux. The noise power used is $1111K^2$. 

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Chapter 5

Outer Controller

The design of the controller system for reference temperature tracking is covered in this chapter. As was explained in 3.2 because of the nonlinear properties of the sample’s dynamics, the parameters of the linearized model for these dynamics depend on the working temperature of the sample. As such, the techniques presented in this section will allow designing of local controllers for the temperature, i.e. for each value taken by the parameters in (3.16), a different controller is attained.

For now, the controllers developed will focus only on achieving some desired performance, at least on the temperature range the model used do design them is meant to cover. This section is partitioned into three subsections, each covering a different approach for designing the local controllers. The first one tackles the problem with a classic root-locus approach for a PI controller, whilst the second subsection uses pole-placement based on polynomial techniques and the third one resorts to an optimal quadratic regulator solution.

Before designing a controller, it is important to define the specifications of the desired closed-loop response for the controlled system. As was mentioned in the beginning of this chapter, material tests in solar furnaces often involve working close to the fusion temperatures of the samples, as well as submitting them to rapid temperature increases for thermal stress testing. These applications help defining the following set of goals for the closed-loop system’s response:

- **Stationary Error**: Zero error tracking of the reference temperature.
- **Overshoot**: Minimal overshoot in order not to destroy the sample.
- **Rise Time**: Fast rise time for step inputs to assure large gradients of temperature.

5.1 Root-Locus Approach

Let us first consider the design of a PI controller, described by equation (5.1). The signal $u(t)$ is the controller’s output, taken as the radiation flux reference by the inner controller developed in Chapter 4. The signal $e(t)$ represents the tracking error, computed by subtracting the current (filtered) sample temperature from the desired reference value, $T_r(t) - T_f(t)$. 

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\[ u(t) = K_p[e(t) + K_i \int e(\tau)d\tau] \] (5.1)

with corresponding transfer function

\[ \frac{u(s)}{e(s)} = K_p[1 + K_i \frac{1}{s}] \] (5.2)

Though a simple and commonly used approach, the PI is a robust and well performing controller, having been shown to be able to suffice in the issue of controlling a solar furnace \[1, 3\]. While the integrator part of the controller guarantees zero error tracking of the desired temperature and robustness to perturbations on the actuation signal, the proportional part allows fast responses for rapid variations temperature. Recall that the signal to be controlled is the filtered temperature of the sample, which implies that the transfer function of the plant may be described by

\[ \frac{T_f(s)}{u(s)} = \frac{b\omega_f}{s^2 + (a + \omega_f)s + a\omega_f} \] (5.3)

with poles at \(-a\) and \(-\omega_f\), considering the linearized model for the sample’s dynamics \[3.16\] around a working temperature \(T_0\). When considering the open-loop system \(\text{PI-Plant}\), the controller imposes an extra pole in the origin and a zero placed at \(-K_i\), which may be arbitrarily chosen. The closed loop system will then be of third order with one zero.

Using the classic root-locus technique, it is possible to study the variations of the closed loop poles of the controlled system for various values of the proportional gain \(K_p\). Considering the performance goals mentioned in the beginning of this section, one possibility of assuring fast responses and no overshoot is to drag the slowest poles (the thermal dynamics and the integrator poles) further down the real axis on the left half-plane of the pole map, allowing the system to respond faster to high frequency inputs such as step changes in the reference, while still guaranteeing that no overshoot occurs. This can be achieved by placing the added zero in between the plant’s two poles, closer to the system’s natural pole \(-a\). This means selecting a value for \(K_i\) within the interval \([a, \omega_f]\).

Increasing the controller’s proportional gain, the two slowest poles are shifted beyond the selected zero position, as seen in figure 5.1 \((K_i = 1.1a)\). Using MATLAB’s rlocus function, it is possible to find the appropriate value of \(K_p\) that places the poles in the fastest possible condition with no overshoot, which corresponds to double real poles.

For a double real pole system, and considering a working temperature of \(T_0 = 900K\), the required value for \(K_p\) is 0.582, resulting in the step response for \(T_r = 50K\) shown in figure 5.2 along with the corresponding controller output. This simulation was done using the linearized model of the plant, with resulting closed loop poles located at \(\{-0.0415, -0.0415, -0.3157\}\).

Though the system is driven towards the desired value, the double real pole response proves to be too slow for the type of application being considered, as the desired value is reached more than 1 minute after the step input is induced. This gives the system a maximum temperature gradient of \(2.71Ks^{-1}\).
In order to improve the performance, the controller’s gain may be further increased, such that the slowest pole’s effect is nullified by the controller’s zero and the two fastest poles are merged into a pair of conjugate complex poles. The resulting response is shown in figure 5.3, where $K_p = 1.4$ and $K_i$ remains the same. The closed system poles are now $\{-0.0322, -0.1832 \pm 0.0837i\}$.

In this case, a small 1.53% overshoot occurs, making the temperature surpass the reference value by less than 1 Kelvin degree. This is quite acceptable for a local controller, where shifts in temperature reference are not expected to be larger than 100K. The temperature gradient is now $3.417Ks^{-1}$, corresponding to a 26% increase when compared with the previous test, which is an acceptable compromise.

It is, of course, necessary to test the performance of this local controller with the nonlinear model of the plant, validating the results obtained in the previous simulations. A simulation was done with the PI controlled nonlinear model of the plant, stabilized
at the working temperature $T_f(t) = 900K$ with the previously used controller gains. The same step increase of 50K on the reference input is considered, with the respective performance shown in figure 5.4. The inner control loop developed in section 4 was connected between the PI controller and the plant, assuming a constant available solar power of $G_s = 1000W/m^2$. Also, the sensor noise modeled in 3.3 was taken into account.

It can be seen that the same performance is achieved with the more detailed and complex model of the plant, as the desired reference temperature is achieved within 25 seconds after the reference step change. This not only shows that results on controller performance for the linear model are transposable to the non-linear case, but it also demonstrates that indeed the inner loop dynamics are fast enough not to degrade the desired outer loop performance. Also, a preventive anti-windup scheme was added to the controller, avoiding possible problems generated by the lower and upper bounds of usable solar power. The selected gain for this anti-windup scheme was $10 \times K_p K_i$.

Finally, it is interesting to observe the behavior of the designed controller along the various temperature ranges. In figure 5.5 the developed controller was used on the [600, 900]K and [900, 1300]K temperature ranges. It can be seen that the desired

---

\[ T_R(t), T_f(t) [K] \]

\[ u(t) [W/m^2] \]

Figure 5.3: Closed loop system response for the PI controlled system for $T_0 = 900K$, $K_p = 1.4$ with complex conjugate poles. Reference temperature, filtered temperature and incident radiation input.

Figure 5.4: PI controller performance on nonlinear plant model. Reference temperature, filtered temperature and incident power.

\[ T_f(t) = 900K \]

\[ G_s = 1000W/m^2 \]

\[ 10 \times K_p K_i \]

\[ \text{This was done during the first 200 seconds of simulation which are not presented in the figure.} \]
Figure 5.5: PI controller performance on nonlinear plant model for two different temperature ranges, with $K_p$ and $K_i$ fixed. Reference temperature, filtered temperature and incident power.

Performance is only achieved on the [900, 1000] K range of the test. For temperatures below the range for which the controller was designed, the system’s response is faster and has more overshoot, while for higher temperatures the system becomes slower and has smaller temperature gradients.

This performance change can be explained analytically by studying how the closed loop transfer function for the linearized plant model, presented in (5.4), varies with temperature.

\[
\frac{T_r(s)}{T_f(s)} = \frac{K_p b \omega_f (s + K_i)}{s^3 + (a + \omega_f)s^2 + (a \omega_f + K_p b \omega_f)s + K_p K_i b \omega_f}
\]  

(5.4)

Figure 5.6 shows the displacement of the closed loop poles for the above transfer function for different working temperatures, while keeping the controller gains $K_p$ and $K_i$ constant. It can be observed that, although the pair of conjugate complex poles increases its imaginary component, the real pole gets smaller with each temperature increase, dominating the dynamics of the system as its effect is no longer nullified by the system’s zero. For temperatures below the range the controller was designed for, the system’s zero is located closer to the origin than all three poles, which means the system will have an uncompensated derivative of the input reference. This causes an abrupt variation on the system’s output, which explains the fast rise time and overshoot [14].

As was explained during the controller’s design, this zero is defined solely by the controllers integral gain $K_i$, which does not change with temperature.

It is then clear that one PI controller is unable to assure the desired performance for different working temperatures. The possibility that this problem may be overcome by using several, appropriately tuned, PI controllers for each working temperature range motivates the study done in Chapter 6.
Figure 5.6: Pole locations of the PI controlled system for different working temperatures. Fixed controller gains $K_p = 1.4$, $K_i = 1.1a$, designed for $T_0 = 900K$.

5.2 Pole Placement

In this section of the design of the outer controller, a pole placement approach based on polynomial techniques\[13\] is applied, hoping to provide a different and more robust solution for the design of this controller. The main advantages of this technique, when compared with the root-locus technique used previously, are the following:

- **Algorithmic technique:** The design of controllers through polynomial techniques is done in an algorithmic fashion. This allows fast designing of controllers for several different plants without having to search for parameter values that assure the desired performance for the closed loop system (as was done in 5.1). This is possible due to the fact that, assuming a fixed structure/order for the plant to be controlled and a fixed desired closed loop structure, the algorithm is reduced to solving a set of linear equations.

- **Two degree of freedom control:** The polynomial technique revolves around designing both a reference-processing structure and a feedback structure, allowing for more ways to achieve the desired performance than through simply processing the tracking error.

This section is divided into two subsection. The first one provides a simple introduction to the algorithm to be used and its requirements for causality and uniqueness of solution, whilst the second one demonstrates its application for the case in study.

5.2.1 Introduction

Let us first start by assuming a generic plant to be controlled, defined by the transfer function:

$$\frac{y(s)}{u(s)} = \frac{B(s)}{A(s)}$$  \hspace{1cm} (5.5)
where $A(s)$ and $B(s)$ are two polynomials with no common roots. It is intended that this plant be controlled by the following control law:

$$R(s)u(s) = S(s)y(s) + T(s)r(s)$$  \hfill (5.6)

This defines the closed-loop scheme shown in figure 5.7. $R(s)$, $S(s)$ and $T(s)$, are three polynomials to be defined. Evidently, in order to assure causality within the loop, the following inequalities must hold true:

$$\partial R(s) \geq \partial S(s)$$  \hfill (5.7)
$$\partial R(s) \geq \partial T(s)$$  \hfill (5.8)

Where $\partial R(s)$ denotes the order of polynomial $R(s)$. It is desired that the closed loop system behaves according to the transfer function defined by (5.9).

$$\frac{y(s)}{r(s)} = \frac{B_m(s)}{A_m(s)}$$  \hfill (5.9)

Now, given the control law (5.6), the closed loop system transfer function is given by:

$$\frac{y(s)}{r(s)} = \frac{B(s)T(s)}{A(s)R(s) + B(s)S(s)}$$  \hfill (5.10)

The problem of designing the controller is reduced to finding the set of polynomials $R(s)$, $S(s)$ and $T(s)$ such that:

$$\frac{BT}{AR + BS} = \frac{B_m}{A_m}$$  \hfill (5.11)

In order to guarantee correct tracking of the reference, as well as proper rejection of perturbations on the control action, it is necessary to assure the existence of integral action in the feedback chain. To do this, it is enough to guarantee that polynomial $R$ is able to be described by:

\footnote{For the sake of notation simplicity, the polynomials’ dependency on the complex frequency-domain variable $s$ will be discarded for the rest of this introduction.}
\[ R = s^\lambda R' \]  

(5.12)

Where \( \lambda \) is the order of the integral action, or the number of integrators in the loop. \( R' \) represents the remaining components of \( R \), to be specified.

It is assumed that there is a polynomial \( A_0 \), named observer polynomial. It will be used in order to be able to increase the controller’s order as deemed necessary, having as an imposed condition that it must be a stable polynomial, i.e. have its roots on the left half complex plane. In order not to disturb the closed loop system’s dynamics, it is imposed that \( A_0 \) be a common factor between polynomials \( R S \), and \( T \), guaranteeing the following equality:

\[
\frac{BT}{AR + BS} = \frac{B_m A_0}{A_m A_0} \quad (5.13)
\]

If numerator \( B \) is only partially (or not at all) a factor of numerator \( B_m \), than some of its components should be canceled by denominator \( RA + BS \). Let us then consider that polynomial \( B \) is able to be factored as:

\[ B = B^+ B^- \]  

(5.14)

Considering polynomial \( B^+ \) as the components of the numerator to be canceled out and \( B^- \) the ones that are meant to be left uncompensated. In order to achieve zero cancellation, \( B^+ \) must be a common factor in \( RA + BS \). Naturally, it is not recommendable that \( B^+ \) is allowed to have roots in the right half plane, as that would imply unstable poles on the closed loop denominator, which may cause unstable internal modes in the loop.

Now, because \( B^- \) is not meant to be canceled, it must be a component in the desired numerator \( B_m \). As such:

\[ B_m = B^- B'_m \]  

(5.15)

Where \( B'_m \) is a polynomial chosen at will. On the other hand, as \( B^+ \) is already in \( BS \) because of (5.14), it falls to \( R \) to have it as a factor as well. This implies redefining the latter as:

\[ R = s^\lambda B^+ R'_1 \]  

(5.16)

With \( R'_1 \) being the remaining components of \( R \) to be specified. Finally, by combining the definitions in (5.14), (5.15) and (5.16), equality (5.13) may be rewritten as:

\[
\frac{B^+ B^- T}{B^+(A s^\lambda R'_1 + B^- S)} = \frac{B'_m A_0}{A_m A_0} \quad (5.17)
\]

Which can be simplified into (5.18):

\[
\frac{T}{s^\lambda R'_1 A + B^- S} B^- = \frac{B'_m A_0}{A_m A_0} B^- \quad (5.18)
\]
From this result, equations (5.19) and (5.20) are defined:

\[ T = B'_m A_0 \]  

\[ s^{\lambda} R'_1 A + B^- S = A_m A_0 \]  

Where (5.20) is the so called Diophantine Equation, which allows computing of the unknown remaining terms for control polynomials \( R \) and \( S \). Now, for this equation to have a unique solution, the following rules for the orders of \( R'_1 \) and \( S \) are imposed:

\[ \partial S < \lambda + \partial A \]  

\[ \partial R'_1 = \partial A_0 + \partial A_m - \partial A - \lambda \]  

These conditions assure that there will be no unknown coefficients on the left hand side of (5.20) with no corresponding terms on the right hand side of the equation, forcing the existence of a unique solution for the problem. Lastly, so as to guarantee causality, the following inequality defines the minimum order of the observer polynomial:

\[ \partial A_0 \geq 2\partial A - \partial A_m \partial B^+ + \lambda - 1 \]  

The controller is then implementable using the schematic shown in figure 5.8, where causality is guaranteed in each of the blocks that make up the controller.

**5.2.2 Application on the Solar Furnace Plant**

Having introduced the polynomial control approach, it is now simple to use the several equations and order conditions for the polynomials to obtain a control law for the linearized solar furnace plant. The dynamics to be controlled, on the notation previously described, are:

\[ \frac{y(s)}{u(s)} = \frac{B(s)}{A(s)} = \frac{b \omega_f}{s + (a + \omega_f) + a \omega_f} \]  

Assuming working temperature \( T_0 = 1000 K \).
First, it is necessary to specify the desired closed loop dynamics. Unlike with the \textbf{PI} controller, the closed loop dynamics will not necessarily be of third order, so a second order behavior is selected. Using the following set of equations, it is possible to define the parameters for the desired closed loop system in (5.27) by firstly defining the desired overshoot and peak time:

\begin{align}
\text{Overshoot}[\%] &= \exp\left(-\frac{\pi \xi_{cl}}{1 - \xi_{cl}^2}\right) \quad (5.25) \\
\text{Peak Time}[s] &= \frac{\pi}{\omega_{cl} \sqrt{1 - \xi_{cl}^2}} \quad (5.26) \\
\frac{T_f(s)}{T_r(s)} &= \frac{B_m(s)}{A_m(s)} = \frac{\omega_{cl}^2}{s^2 + 2 \xi_{cl} \omega_{cl} s + \omega_{cl}^2} \quad (5.27)
\end{align}

The values for the peak time and overshoot were chosen as 20 seconds and 1.5\%, respectively. $B_m$ was chosen so that the closed loop system will have static gain. Also, it is desirable to have integral action on the control loop, for which $\lambda = 1$.

Now, one can compute the necessary order for the \textit{observer} polynomial using (5.23):

\[ \partial A_0 \geq 2 \times 2 - 2 - 0 + 1 - 1 = 2 \quad (5.28) \]

If, for the sake of simplicity, the polynomial’s roots are considered all at the same value, the \textit{observer} polynomial is defined by (5.29).

Choosing $a_0 = 5 \times a$ assures the control system is able to quickly track variations on the plant’s output. However, choosing a fast \textit{observer} polynomial also implies some lack of noise rejection by the controller. In this case though, because $a$ is such a slow pole, it will be demonstrated further on that this choice does not result in closed loop performance degradation.

\[ A_0 = (s + a_0)^2 \quad (5.29) \]

Since the open loop system has no zeros, using the factorization in (5.14) does not require any decision to be made. The $B^+$ and $B^-$ terms are there defined by:

\begin{align}
B^+ &= 1 \\
B^- &= b \omega_f \quad (5.30) \\
A_0 &= (s + a_0)^2 \quad (5.31)
\end{align}

The results so far allow for full definition of the numerator parts in (5.13), $B_m$ and $T$.

\begin{align}
B_m &= B'_m B^- \Rightarrow B'_m = \frac{\omega_{cl}^2}{b \omega_f} \quad (5.32) \\
T &= B'_m A_0 = \frac{\omega_{cl}^2}{b \omega_f} (s + a_0)^2 \quad (5.33)
\end{align}
Now, the orders for the controller polynomials \( R \) and \( S \) must be defined. This is done through the uniqueness conditions in (5.21) and (5.22).

\[
\begin{align*}
\partial S < 1 + 2 & \Rightarrow \partial S = 2 \\
\partial R' = 2 + 2 - 2 - 1 = 1 
\end{align*}
\]

(5.34) (5.35)

Polynomials \( S \) and \( R' \) may be defined by the following pair of unknown coefficient polynomials:

\[
\begin{align*}
S &= \gamma_2 s^2 + \gamma_1 s + \gamma_0 \\
R' &= r_1 s + r_0
\end{align*}
\]

(5.36) (5.37)

Polynomial \( R \) is then defined through (5.16).

\[
R = s(r_1 s + r_0)
\]

(5.38)

With all above definitions, the Diophantine Equation in (5.20) is written as:

\[
s(r_1 s + r_0)[s + (a + \omega_f) + a \omega_f] + b \omega_f (\gamma_2 s^2 + \gamma_1 s + \gamma_0) = (s + a_0)^2(s^2 + 2 \xi \omega_c s + \omega_c^2)
\]

(5.39)

By expanding the products in both sides of the equation, the problem is reduced to a simple coefficient matching between sides, as was guaranteed by the imposed conditions for polynomial orders. The following set of equalities provide solutions for each of the unknown controller coefficients.

\[
\begin{align*}
r_1 &= 1 \\
r_0 &= 2 \xi \omega_c + 2a_0 - (a + \omega_f) \\
\gamma_2 &= \frac{\omega_c^2 + 4a_0 \xi \omega_c + a_0^2 - (a + \omega_f) r_0 - a \omega_f}{b \omega_f} \\
\gamma_1 &= \frac{2a_0 \omega_c^2 + 2a_0^2 \xi \omega_c - a \omega_f r_0}{b \omega_f} \\
\gamma_0 &= \frac{a_0^2 \omega_c^2}{b \omega_f}
\end{align*}
\]

In figure 5.9, the results of a simulation run with a Simulink schematic based on 5.8 is presented. Notice that the desired peak time and overshoot are reached with precision, which is expected as this simulation was done with the appropriate linear model for the plant. The maximum temperature gradient achieved in with this performance was 6.235K s\(^{-1}\).

It can be seen, however, that the amount of power used by the controller could be far greater, possibly reaching larger temperature gradients. For instance, if the actual plant system were stationary at 1000K, the amount of steady incident power would
be around 140W/m², which means that if higher temperatures were to be reached, the controller would have as much as 860W/m² to use before saturating the actuation signal (assuming an available average sun power of 1000W/m²). This suggests that faster dynamics could be obtained, without endangering performance.

In figure 5.10 a new simulation is presented with the performance of the same plant, now controlled with polynomials computed using the same solutions for the polynomial coefficients, assuming a more demanding desired closed-loop dynamics. The peak time selected at 10 seconds and the maximum overshoot 3%.

The incident radiation imposed by the controller stayed well within the assumed upper bound of 1000W/m² and a much faster response was obtained, reaching a maximum temperature gradient achieved was of 7.262Ks⁻¹. Naturally, as the actuator is likely to saturate, an anti-windup scheme should be implemented for the polynomial controller. This can be done by rearranging the positioning of the block containing the controller’s integral component. Figure 5.11 shows one such rearrangement.

Notice that, when working on the linear zone, the transfer function between the controller’s output and the actual radiation that hits the sample is given by:

$$\frac{u_s}{u} = \frac{1}{1 - \frac{A_0 - R}{A_0}} = \frac{A_0}{R}$$

(5.40)

Finally, as was done in 5.1 the local controller is tested with the nonlinear plant, taking the process noise as well as inner closed loop dynamics into consideration. The results are shown in figure 5.12 where again it is possible to observe how the closed loop performance obtained when testing with the linearized system is replicated on the non-linear system, around the linearization working point. The peak time is 10 seconds and the overshoot is below 4%.
Figure 5.10: Polynomial control performance for $T_0 = 1000K$. Desired dynamics with peak time 10 seconds and overshoot 3%. Reference temperature, filtered temperature and incident radiation input.

Figure 5.11: Polynomial control schematic with anti-windup mechanism. $u_s$ is the saturated controller output.
Also, simulations were run with this controller for different temperature ranges. However, the performance of the controlled system does not change the same way it did with the PI controller. In fact, for the temperature ranges in $[800, 1500]K$, it can be observed that the peak time is kept between 10 and 13 seconds, whilst the overshoot is dampened from 6% to nearly 0%. Given that these ranges are the ones most significant for temperature stress tests, it is possible to consider the global use of this controller, as opposed to just locally, which can be seen in figures 5.13a and 5.13b.

By analyzing the pole displacement along the various temperature ranges, one can observe some relevant facts. Through the polynomial $T$, the controller imposes two fixed zeros on the closed loop transfer function, also adding two other poles through polynomials $R$ and $S$. In the working temperature the controller is designed for, $T_0$, the two zeros are placed such that two of the poles are compensated, while the other two remaining poles are left to define the desired closed loop dynamics. In figure 5.14, the pole positioning for three different working temperatures is displayed.

One can see that the poles that are to be nullified by the zeros start closer to the origin and, when the temperature reaches $T_0$ (1000K for this example), they merge on the real axis together with the zeros, leaving the two conjugate poles on the left defining the dynamics. For higher temperatures, the nullified poles drift apart on the real axis and on opposite directions. However, the presence of the two zeros is enough to compensate the effect of the slowest pole, denying its dominance over the dynamics. This is possible because the slowest pole does not move far enough from the zeros, unlike what happened with the PI example. The same two zero effect is felt for the $[800, 1000]K$ range.

Given that the most significant test in terms of thermal stress would be to induce a large temperature increase on the sample, a simulation was done subjecting the sample to several abrupt temperature variations on the $[800, 1500]K$ range, with results shown in figure 5.15. It can be seen the controller is able to accurately track the desired reference only if it is constant. Also, one can see that it imposes fast variations of
Figure 5.13: Polynomial control on various temperature ranges. $T_0 = 1000K$.

Figure 5.14: Closed loop pole and zero positioning for different temperatures, using a polynomial controller designed for $T_0 = 1000K$. 
temperature with no overshoot, reaching a temperature gradient of $59.4\text{K}\text{s}^{-1}$ without causing overshoot.

To summarize, it is clear that the polynomial method provides more versatility when it comes to choosing the desired performance, when compared with the PI approach. Its algorithmic procedure allows for fast design of new controllers for different desired performances or different plant characteristics. Most importantly, as has been shown, it is able to provide a good performance for different temperature ranges, despite of the linear model’s parameter variation with temperature.

The method lacks, however, of the PI method’s straightforwardness in implementation on the actual plant.

### 5.3 Optimal Control

An optimal control design approach is considered in this section, with an obtained control law based on linear quadratic regulator theory. Again, the controller design will be based on the linearized model of the plant shown in (3.16), assuming a working temperature $T_0$ and that experiments are done with an alumina sample. Firstly, because the plant to be controlled is of second order and access to the system states will be required, the plant comprised by the sample’s dynamics and noise filter will be described by the following state space representation:

\[
\begin{bmatrix}
  \dot{x}_2 \\
  \dot{x}_1
\end{bmatrix} =
\begin{bmatrix}
  -\omega_f & \omega_f \\
  0 & -a
\end{bmatrix}
\begin{bmatrix}
  x_2 \\
  x_1
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  b
\end{bmatrix} u
\]

\[y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} \] (5.41)
Or, in compact form:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

(5.42)

Where state \(x_1\) is the sample’s temperature and state \(x_2\) is the filtered sample’s temperature, making up the state space vector \(x = [x_2 \ x_1]^T\). This is a valid and appropriate consideration to make, as in practical terms these two states can be physically accessed. Throughout this section, the state’s and control signal \(u\) dependency on the independent time variable \(t\) will be omitted.

Let us now consider the Linear Quadratic Regulator problem. In essence, the problem can be stated as finding the control law \(u\) that minimizes the infinite horizon quadratic cost functional \(J\), defined by:

\[
J = \int_{t_0}^{\infty} (x^T Q x + u^T R u) dt
\]  

(5.43)

Where matrices \(Q\) and \(R\) are semi-definite and positive definite, respectively. Vector \(x\) is subject to the dynamics defined in (5.42).

By establishing that the control law \(u\) is defined by:

\[
u = -K x
\]  

(5.44)

The problem is reduced to finding the gain matrix \(K\) that minimizes the functional in (5.43). With the above mentioned conditions for matrices \(Q\) and \(R\), and given that the time horizon is considered infinite, the solution is well known to be given by solving the Algebraic Riccati Equation [15]. However, because this problem formulation is meant to optimally drive the system’s state to zero, a slight modification must be considered.

Given that the controller must track a given temperature reference, an integrator will be used to process the tracking error. This leads to adding an extra state \(x_i\), which is integrated signal of this error. With this extra state, the state space vector is expanded into \(z = [x_2 \ x_1 \ x_i]^T\).

The control law is now defined by \(u = -K z\) with \(K = [K_2 \ K_1 \ K_i]\), which is implemented through the scheme shown in figure 5.16. The signal \(T_r\) represents the desired temperature reference and \(T_f\) the plant’s output to be controlled.

Given that \(\dot{x}_i = T_r - x_2\), the system’s dynamics are now described by the following pair of equations:

\[
\begin{bmatrix}
\dot{x}_2 \\
\dot{x}_1 \\
\dot{x}_i
\end{bmatrix} =
\begin{bmatrix}
-\omega_f & \omega_f & 0 \\
0 & -a & 0 \\
-1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_2 \\
x_1 \\
x_i
\end{bmatrix} +
\begin{bmatrix}
0 \\
b \\
0
\end{bmatrix} u +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} T_r
\]

(5.45)

\[
y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\begin{bmatrix}
x_2 \\
x_1 \\
x_i
\end{bmatrix}
\]
 Represented in compact form as:

\[
\dot{z} = \bar{A}z + \bar{B}u + \bar{B}_i T_r
\]

\[
y = \bar{C}z
\]

With this new state space definition, the quadratic cost to be minimized is defined by:

\[
J = \int_{t_0}^{\infty} (z^T Q z + u^T R u) dt
\]

considering \( T_r = 0 \) and \( Q \) with appropriate dimensions. By using the function \textit{lqi} from MATLAB’s \textit{Control Systems} toolbox, the values of gain matrix \( K \) are quickly obtained for this configuration. The selected weighting matrices \( Q \) and \( R \) were the following:

\[
Q = \begin{bmatrix}
10 & 0 & 0 \\
0 & 10 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad R = 0.1
\]

Which were chosen so that the closed loop system shown in Figure 5.16 would have a quick step response with no overshoot, while keeping the manipulated input \( u \) (the amount of incident radiation) from saturating. This was done using the linearized model for sample’s thermal dynamics around \( T_0 = 1000 K \) while assuming a constant available solar power of \( G_s = 1000 W/m^2 \). The simulation results with the linearized model are presented in figure 5.17. The gains returned by the aforementioned MATLAB function, given the specified linear system and weighting matrices, were the following:

\[
K_2 = 9.4095, \quad K_1 = 13.0851, \quad K_i = -3.1623
\]

5.3.1 Noiseless System Model

Considering the nonlinear model for the plant and the inner loop from section 4 without the presence of noise on the thermocouple’s signal \( (T_s(t)) \), a simulation is done for a
Figure 5.17: Optimal controller performance for the linear system, $T_0 = 1000K$. Reference temperature, filtered temperature ($x_2$), sample temperature ($x_1$) and incident power.

Figure 5.18: Optimal controller performance for the nonlinear system for the temperature range $[800, 1500]K$. Reference temperature, filtered temperature, and incident power. $G_s = 1000W/m^2$

reference input that spans the temperature working region $[800, 1500]K$. The results, shown in figure 5.18, show a very promising result as the closed loop performance is maintained for the whole range of working temperatures.

Again, to understand this behavior, an analysis on the closed loop pole variation with the sample’s working temperature is undertaken. First, let us consider the system shown in (5.46), which in closed loop with control law $u = -Kz$ yields:

$$
\dot{z} = (\bar{A} - \bar{B}K)z + \bar{B}_i T_r \\
y = \bar{C}z 
$$

(5.49)

By computing the eigenvalues of matrix $\Phi = (\bar{A} - \bar{B}K)$, the closed loop poles are obtained. Table 5.1 shows a few values of the poles for different working temperature and hence, different values of the linearized model, $a$ and $b$. It is important to mention that this control scheme imposes a numerator in the closed-loop transfer function with insignificant coefficients, other than the zero order one. This means that the system
can be considered not to have any zeros.

Before $T_0 = 1000K$, poles 1 and 2 are complex conjugate, but placed below the value of pole 3, which mainly defines the dynamics of the system. For $T_0 = 1000K$, the temperature for which the controller is tuned, the three poles take real values, yet pole 3 is still the slowest, imposing dynamics only slightly faster than for lower temperatures. For the temperatures above $T_0 = 1000K$, poles 2 and 3 form a complex conjugate pair that now dominates the dynamics, but the overshoot caused by this pair does not go beyond 1% for $T_0 = 1500K$ and the system’s rise time does not change much from its previous values.

In figure 5.19 a sequence of fast reference temperature changes is given to the same closed loop system and assuming again $G_s = 1000W/m^2$, simulating a thermal stress test. The system responds quickly with satisfying results, except when drops in the reference temperature are imposed, which make the controller quickly saturate the actuator and winding up the integrator. In order to improve this flaw in the performance, a simple anti-windup mechanism is added to the controller, with an anti-windup gain equal to the controller’s $K_i$. This improvement does indeed provide for better reference tracking when saturation of the actuation signal occurs, as shown in figure 5.20. It can be seen however that the mechanism does add a very small overshoot to the response.

Finally, if the thermocouple’s noise is taken into account, i.e. state $x_1$ is added the noise modeled in 3.3, simulations will still show a good performance despite some small

Table 5.1: Closed loops for the optimal control closed-loop system for temperatures in the range $[800,1500]K$.

<table>
<thead>
<tr>
<th>$T_0[K]$</th>
<th>Pole 1</th>
<th>Pole 2</th>
<th>Pole 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>-0.635 + 0.222i</td>
<td>-0.635 - 0.222i</td>
<td>-0.228</td>
</tr>
<tr>
<td>900</td>
<td>-0.606 + 0.144i</td>
<td>-0.606 - 0.144i</td>
<td>-0.244</td>
</tr>
<tr>
<td>1000</td>
<td>-0.669</td>
<td>-0.476</td>
<td>-0.274</td>
</tr>
<tr>
<td>1100</td>
<td>-0.745</td>
<td>-0.320 + 0.073i</td>
<td>-0.320 - 0.073i</td>
</tr>
<tr>
<td>1200</td>
<td>-0.786</td>
<td>-0.284 + 0.115i</td>
<td>-0.284 - 0.115i</td>
</tr>
<tr>
<td>1300</td>
<td>-0.814</td>
<td>-0.256 + 0.134i</td>
<td>-0.256 - 0.134i</td>
</tr>
<tr>
<td>1400</td>
<td>-0.836</td>
<td>-0.232 + 0.145i</td>
<td>-0.232 - 0.145i</td>
</tr>
<tr>
<td>1500</td>
<td>-0.853</td>
<td>-0.210 + 0.152i</td>
<td>-0.210 - 0.152i</td>
</tr>
</tbody>
</table>
steady state error. However, the control signal becomes extremely noisy, which is of course due to the fact that the controller gain $K_1$ greatly amplifies the already high powered noise from the sensor, as shown in figure 5.21. In practical terms, this would render the shutter subsystem useless after prolonged use.

Given this limitation, and in order to implement this controller, it is necessary to take into account optimal filtering techniques, such as the use of a Kalman Filter to provide good and less noisy estimates of states $x_1$ and $x_2$.

### 5.3.2 Optimal Filtering

Let us begin by defining the equations of a standard linear Kalman Filter, which shall be used to improve the estimates for the states used by the LQ controller. The filter works in a two step routine, composed by a prediction phase and a correction phase, which shall be very briefly described next and is thoroughly explained in [16]. The filter admits that the process whose states are to be estimated is modeled by the discrete state space model:
\[ x_k = A_d x_{k-1} + B_d u_k + D w_k \]
\[ y_k = C_d x_k + G v_k \]  

(5.50)

Where \( w_k \) and \( v_k \) are zero mean, white Gaussian noise signals, with covariance matrices \( Q_w \) and \( R_v \) respectively. Independent variable \( k \) represents the discrete time instance.

**Prediction** In the prediction step, the filter makes use the process model to *predict* the next value for the state vector estimate \( \hat{x}_{k+1} \), given the previous estimate \( \hat{x}_k \). The obtained result is known as the *a priori* estimate and is represented by \( \hat{x}_-^k \). Also in this step, a the covariance matrix \( P \) that describes the level of uncertainty of the estimates is propagated based on the process model. The equations used in this step are presented below.

\[
\begin{align*}
\hat{x}_-^k &= A_d \hat{x}_{k-1} + B_d u_k \\
P_-^k &= A_d P_{k-1} A_d^T + G Q_w G^T
\end{align*}
\]  

(5.51)

**Update** In the update step, the *a priori* estimate \( \hat{x}_{k+1} \) is corrected by making use of the current measurements. This correction is done by using the error between the measurements and predicted estimates, called *innovation*, which is multiplied by a gain \( K_k \), computed to be optimal given the levels of noise in the process, measurements and the predicted covariance \( P_- \). The equations for the optimal gain, obtained estimate and respective covariance are the following:

\[
\begin{align*}
K_k &= P_-^k C_d^T (C_d P_-^k C_d^T + D R_v D^T)^{-1} \\
\hat{x}_k &= \hat{x}_-^k + K_k (y_k - C_d \hat{x}_-^k) \\
P_k &= P_-^k - K_k P_- C_d
\end{align*}
\]  

(5.52)

Now, using the above equations, a Kalman Filter was designed for the estimation of the states on the solar furnace process. The state space model (5.42) was discretized through ZOH assuming a 0.5 second sampling time, generating the filter’s prediction model in (5.51) (\( G \) and \( D \) were considered identity matrices).

Given that the process model is considered a good approximation, and the variance of the noise on the thermocouple’s output modeled in 3.3, the following define the covariance matrices for noises \( w \) and \( v \):

\[
Q_w = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad R_v = \begin{bmatrix} 4.70 & -1.23 \\ -1.23 & 71.7 \end{bmatrix}
\]

Matrix \( R \) was obtained experimentally, by computing the covariance between states \( x_1 \) and \( x_2 \).

The measurements from sample temperature \( T_s(t) \) and filtered temperature \( T_f(t) \), along with the incident radiation \( u(t) \) are given as inputs to the Kalman Filter, which then provides estimates for the LQ based controller to use as state values \( x_1 \) and \( x_2 \).
A simulation was run with the complete non-linear system with noise, together with the newly designed optimal filter and in closed loop with the optimal controller designed for $T_0 = 1000K$, previously presented. The available solar power was once more considered constant at $G_s = 1000W/m^2$, with results shown in figure 5.22.

There is a clear significant improvement with the use of the Kalman Filter, as the control variable’s variance is now bounded to much smaller values. One can also see that the controller’s tracking error is now null, while performance wise the new addition did not cause deterioration.

This not only makes the LQ based control approach much more robust, it also makes it feasible when it comes to practical implementation.
Chapter 6

Switching Control

This chapter focuses on the study of an adaptive switching control method. The goal is to be able to improve the performance of the temperature controller designed in 5.1 such that, for different working temperatures, some desired performance is guaranteed.

This chapter begins with a quick introduction on supervised multiple model adaptive control, based on the rich literature that is available on the subject. The following sections cover the design and implementation of the approach in study to the case of solar furnace control with a set of fixed PI controllers, along with a series of tests to evaluate its effectiveness.

6.1 Introduction

Due to the fact that many processes that are to be controlled are subject to uncertainties or time varying properties, such as unknown parameter values in process models or nonlinearities, control engineers have long studied tackling the issue with different solutions on how to adapt the existing theory to these cases.

Adaptive control has provided a solution to this problem, but the high transients corresponding to the adaptation period have proven to make this approach unreliable, especially under conditions of fast changing process dynamics or poor initial parameters estimates. As an alternative approach, switching between an array of controllers, known as multiple model adaptive control, has been proven to be a good performing solution.

In its simplest form, supervised multiple model adaptive control (SMMAC) may be composed by a set of differently tuned controllers, each one being designed for stability and performance of closed loop response of the plant under certain conditions, called the bank of controllers. A supervisory mechanism processes an indicator variable obtained from the plant and, according to some defined criteria, selects which controller the plant should be connected to. This concept is shown in figure 6.1. For a situation where the variation of the plant is strictly defined by the indicator variable, the supervisor may simply define the appropriate controller parameters from a previously defined set, which hopefully accounts all possible plant behaviors. This is known as adaptive control through gain scheduling.

In [8], a formal introduction to the idea of various, pre-designed controllers for a
process with a switching logic implementation is done. The issues on how to draw the switching logic, or the number of necessary controllers to assure good performance for all possible plant conditions, have been the subject of many rigorous studies. In most cases, the number of controllers has been chosen to be the same as the amount of different process models that are able to describe the plant’s behavior for its different conditions. In some cases, studies were done in order to reduce the number of controllers by implementing model agglomeration techniques, which assure that the use of one controller for various models can produce good results [5, 6]. This technique makes use of Vinnicombe metrics to decide which models resemble each other enough.

Regarding the design of the supervisor process, two major approaches are usually considered. The supervisor may evaluate the closed loop performance of each available controller with the plant, by testing them out and measuring the outcome. The selected controller will then be the one which satisfies a set of maximal desirable outputs [11, 12].

The supervisor may also, given the plant’s output and its current input, make use of a bank of models to generate several performance estimates. These estimates are then compared with each other, being that the model that generates the best performance estimate should be the best model for the plant’s dynamics, hence making the supervisor opt for the controller designed for this model. This estimate may be, for instance, the squared prediction error of the models, which should be minimum for the best one. In [9], it is shown that a design of an observer for obtaining a filtered estimate of the process output may play a critical role on the performance of the supervised adaptive controller.

There is, of course, the issue of how many models there should be in this bank. Having a fixed number of models implies that, if the plant’s behavior is to change to an unexpected (unmodeled) one, no controller will be suited for the task. To handle this, a dynamical bank of models is many times considered [6, 7, 10], where learning techniques are implemented. As the process changes, adaptive models identify these modifications and may generate new models, for which adaptive controllers provide the
Table 6.1: Local PI controllers with corresponding working temperature, subregion and gains. Overshoot and peak time for simulations done with linear and non-linear plant models.

<table>
<thead>
<tr>
<th>$p \in P$</th>
<th>$[R_j[K], R_{j+1}[K]]$</th>
<th>$T_0[K]$</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$S[%]$</th>
<th>$P_t[s]$</th>
<th>$S[%]$</th>
<th>$P_t[s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[800,900]</td>
<td>850</td>
<td>1.40</td>
<td>0.0279</td>
<td>1.376</td>
<td>33.0</td>
<td>1.32</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>[900,1000]</td>
<td>950</td>
<td>1.35</td>
<td>0.0347</td>
<td>1.576</td>
<td>33.9</td>
<td>2.02</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>[1000,1100]</td>
<td>1050</td>
<td>1.25</td>
<td>0.0420</td>
<td>1.612</td>
<td>36.9</td>
<td>2.73</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>[1100,1200]</td>
<td>1150</td>
<td>1.15</td>
<td>0.0499</td>
<td>1.560</td>
<td>40.3</td>
<td>3.50</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>[1200,1300]</td>
<td>1250</td>
<td>1.15</td>
<td>0.0565</td>
<td>1.227</td>
<td>40.8</td>
<td>4.29</td>
<td>38</td>
</tr>
<tr>
<td>6</td>
<td>[1300,1400]</td>
<td>1350</td>
<td>1.15</td>
<td>0.0633</td>
<td>1.068</td>
<td>41.4</td>
<td>5.53</td>
<td>37</td>
</tr>
<tr>
<td>7</td>
<td>[1400,1500]</td>
<td>1450</td>
<td>1.10</td>
<td>0.0690</td>
<td>0.528</td>
<td>44.8</td>
<td>6.50</td>
<td>37</td>
</tr>
</tbody>
</table>

desired performance. In order to do appropriate management of the size of this bank, some ideas have been proven worth considering, such as defining a maximum time for which a model can be left without being used.

Finally, in order to prevent excessive switching between controllers, which in many cases leads to overall instability, some safety mechanisms are usually implemented. An example of such mechanisms is the imposing of a minimum time interval during which a selected controller must remain in closed-loop with the plant, known as dwell time.

### 6.2 Switching PI Controllers

Let us now consider the application of switching control to the solar furnace problem. Through the methods shown in 5.1 a bank $C = \{C_j, j = 1, \ldots, N\}$ of PI controllers is designed, each one appropriately tuned as to assure an acceptable performance in the temperature range it is designed for. Each selected controller $C_p \in C$, $p$ being the selection index outputted by the supervisor, should comply with a set of performance criteria which are meant to be kept throughout the different temperature ranges. Being that most of the relevant tests in thermal stress are performed in the [800, 1500]$K$ temperature range, this interval will be considered the entire temperature workspace $\mathcal{R} = \{R_j, j = 1, \ldots, N\}$.

In table 6.1 the bank of designed controllers is presented, having the performance criteria of keeping the closed-loop responses with a similar peak time amongst each other, while maintaining the overshoot within acceptable values. The temperature workspace was divided into subregions $R_j \subset \mathcal{R}$ of $\Delta R[K]$ each, with each corresponding controller being drawn assuming a working temperature valued at its mean. For each controller $C_p$, a simulation is done with the linearized model for its working temperature $T_0$, which provides values for the predicted overshoot ($S$) and peak time ($P_t$). A second simulation is then run using the non-linear model, so as to perceive what is the actual performance of the controller around its working point. With these results, it was observed that each PI held the previewed performance for approximately 50$K$ above and below its operating temperature, defining $\Delta R = 100K$.

It is easily noticeable how, as the working temperature increases, the results obtained for the linear model differ more significantly from the ones obtained with the non-linear one. This is due to the fact that the linearized model of the plant, for these temperature ranges, admits a smaller gain than the actual non-linear model, as was observed in figure.
Regarding the supervisor system, and given that the indicator variable in this case defines the changes in the process dynamics (the temperature), a design through gain scheduling is considered following Algorithm 2. By observing the value of the measured filtered temperature, and taking into account the current controller in use, the supervisor infers if the current temperature has either gone above, below or stayed within the current controller’s temperature range. The supervisor’s output, index signal $\sigma = p \in P$, is then the one corresponding to the correct controller for each of these three situations.

Because there is the possibility of overshooting and due to the presence of noise in the signal used by the supervisor to infer the current temperature range, it is important to add some hysteresis $h$ on the values that define the limits for each region. Assuming that a temperature range $R_p$ is defined by its upper and lower limits, $R^u_p$ and $R^l_p$, the supervisor’s decision logic is done in such a way that if the controller $C_j$ is selected, and if the temperature signal $T_f$ surpasses $R^l_j + 1 + h$, controller $C_{j+1}$ is selected. On other hand, if the temperature signal $T_f$ undergoes $R^u_j - 1 - h$, controller $C_{j-1}$ is selected. This prevents the effect of chattering (or excessive commuting) between controllers in bank $C$, ensuring closed loop stability.

Given the values of noise and maximum overshoot, the selected value for the hysteresis level was $10K$, which will be shown to guarantee the above mentioned conditions.

Algorithm 2  Supervisor’s switching logic

Require:  \( j \in P, \, R, T_f \)
\[
\sigma = j \\
\text{if} \quad T_f \leq R^u_{j-1} - h \quad \text{then} \\
\quad \sigma = j - 1, \, \sigma \geq 1 \\
\text{else if} \quad T_f > R^l_{j+1} + h \quad \text{then} \\
\quad \sigma = j + 1, \, \sigma \leq N \\
\text{end if}
\]

Now, considering the implementation aspect of this bank of controllers, a scheme where one single integrator is shared by all PI controllers is implemented. This scheme helps ensuring smoothness between controller transitions [5, 8], as if each of the controllers were to have an integrator of their own, the accumulated error on each of the integrators would always differ significantly, resulting in abrupt variations on the plant’s input once a controller transition occurred. Each PI controller inside the array $C$ must then generate two signals, one for the proportional component ($u_p$) and one for the integral component ($u_i$), which still needs to be integrated.

Figure 6.2 shows how the complete switching control scheme is assembled, where an anti-windup mechanism is included. The representative block labeled Plant comprises the sample’s dynamics, the low-pass filter for the temperature and the inner control loop for the shutter.

6.3 Simulation Results

Simulating the closed loop system with the developed bank and supervisor subsystems, the results shown in figure 6.3 were obtained, assuming a constant available solar power
of $G_s = 1000W/m^2$. For comparison purposes, a simulation of the control performance of local model $C_4$ for the same reference input is presented. Notice that the switching between controllers does not affect the smoothness of the output signal of the closed loop with switching control.

The closed loop output now shows how, with switching control, the overshoot and peak times are kept fairly constant for the whole working temperature range.

In figure 6.4, the closed loop performance of the switching controller is tested for a random sequence of step reference temperatures, comparing them with fixed gain controllers $C_1$ and $C_4$. It is shown that the switching controller scheme is able to ensure faster performance without relevant overshoot when compared with controller $C_1$. However, when compared with $C_4$’s performance, the switching control system does not provide a much better performance, except for the lack of overshoot during large temperature variations. Notice, however, that the variations considered in this test are done close to $C_4$’s region, which of course explains the clear supremacy of the switching control approach when compared with $C_1$ but not with $C_4$.

Finally, figure 6.5 shows a comparison between $C_4$ and the switching controller for a sequence of reference inputs including ramps. This simulation shows that, for non steady references, the performance of one control technique does not surpass the other. Notice, however, that the switching controller assures that no overshoot occurs when the reference is again steady, proving its worth when testing sample’s under steady temperature increase.

As a concluding remark, the results obtained in this chapter demonstrate that the use of switching control with local PI controllers does indeed provide a significant improvement when compared with single fixed PI solutions. This approach may, however, have a too big of a trade-off in complexity to be worth considering practical implementation. The complexity of having several controllers running at the same time, plus the existence of an independent supervisor process that handles commutation between the controllers in the bank would outmatch by far the requirements of installing a single industrial PID controller. However, it must be stated that indeed performance improvements are achieved through this approach, and the possibility of having tested not so well tuned local PI must be considered.

Furthermore, it is important to mention that switching in Simulink simulations is done instantaneously which, when considering the actual hardware that would be used to implement the switching control system, is an inaccurate assumption. The physical switching between the appropriate controllers impose a small delay that may have some
Figure 6.3: Switching controller performance for all temperature ranges compared with the performance of a single fixed gain PI controller. Reference temperature, filtered temperature and selected controller. The fixed controller’s performance is shown in green.

impact on the overall performance. However, were this approach to be implemented through computer control, these issues would not be considered.
Figure 6.4: Switching controller performance for random reference input compared with that of controllers $C_1$ and $C_4$. Reference temperature, filtered temperature, selected controller and incident power. The fixed controllers’ performance is shown in green.

Figure 6.5: Switching controller performance for ramp inputs compared with $C_4$. Reference temperature, filtered temperature, selected controller and incident power. The fixed controller’s performance is shown in green.
Part III

Comparison and Conclusions
Chapter 7

Comparative Results

The controllers developed in this work were designed and tested assuming certain conditions that may be far from realistic. In fact, not only was it admitted that the available solar power was constant, but it was always noiseless and considered at its ideal average of $1000\,W/m^2$. Of course, given the inner control loop developed in [4], disturbances in solar power are greatly dampened, but it would be most interesting to observe the behavior of the controllers in a situation where the inner controller could not fully compensate a loss of power.

Furthermore, the model used for designing the controllers assumed correctness of all sample’s properties. It is well known however, that knowing accurately the properties of a given substance can not be given as granted, which stresses out the importance of testing these controllers under some uncertainty.

With these challenges in mind, this chapter aims at testing and comparing controller performance, robustness and sensitivity to parameter change. As an ultimate test, the controllers will also be given the task of controlling the temperature of a different type of sample, with properties that greatly differ from the sample their design was based on. The purpose of this last test is to evaluate the versatility of the obtained control systems.

In the sections that follow, simulations are done using the multiple-model based PI switching controller, polynomial based controller and LQ based optimal controller, all under exact same conditions. For easy identifications on the simulation graphs, they are respectively identified as SMMAC, PP and LQ. Figure 7.1 shows a comparison simulation aimed at evaluating the three controllers’ performance under the conditions they were previously tested.

As was already known, the polynomial controller provides the fastest response, as it was the only one that was easily configurable to have low overshoot and low peak time. These results will serve as a reference for the trials presented next.

7.1 Robustness

Let us first consider that the available solar power is not only noisy but also suffers sudden decreases, simulating the passing of clouds. Let us also assume that the power increases from $700\,W/m^2$ to $1000\,W/m^2$ along the simulation time, modeling the sunrise
during a morning period. The solar power hitting the heliostat was added a Gaussian white noise signal of variance $20W^2/m^4$, which is goes through a low pass filter as was done in chapter 4. For now, the parameters of the process are the same as the ones used to generate the controllers, as this is a test for robustness against actuation disturbances.

In figure 7.2, the simulation results for all three controllers under these conditions are presented. In 7.2b, the available solar power is shown along with the the radiation power used by the controllers.

The sudden shortage of power, to the eyes of the temperature controller, is but a change in the upper saturation boundary. Consequentially, the closed loop controllers will saturate the actuation signal much faster than they would before, particularly when rapid increases in temperature are required by the user. The actual effect this has on the response is that it becomes much slower as the capacity of the furnace to
heat the sample is greatly reduced.

Regarding a comparison of performance between the controllers, it can be seen that while the polynomial and optimal controllers provide essentially the same result, the SMMAC approach shows a slower response. It can be seen that during the interval where the shortage of power is simulated, a switch between controllers occurs as the amount of incident power is no longer on the saturation level, leading to a slower temperature gradient. The other two controllers keep the actuation signal at its maximum for a longer time period, hence resulting in faster temperature increases.

### 7.2 Sensitivity to Parameter Uncertainty

The data on sample characteristics and the laws that describe their dependency on temperature, such as the ones presented in chapter 3, are usually obtained through series of experiments. Therefore, the numerical values considered when modelling the process are subject to inherited experimental errors.

With this in mind, it is relevant to explore the capabilities of the control systems developed when applied to samples that differ on some level from the modeled ones. As a representative situation, an increase on the estimated emissivity value $\epsilon$ of the alumina is considered, which is done by increasing the values obtained through equation (3.8) by 10% and 30%.

This increase in emissivity results in a shift of the system’s original pole, approximately defined by the linearized model parameter $a$, which becomes larger. In figure 7.3 the resulting effects of this parameter change on the closed loop response for the three controllers is shown, assuming a constant $G_s = 1000\, W/m^2$.

![Figure 7.3: Simulation of the three controllers for an increase of 10 and 30% on the $Al_2O_3$ emissivity value. Reference temperature, filtered temperature. $G_s = 1000\, W/m^2$.](image)

It can be seen that, for both simulations, the closed loop response is slower for all controllers, though only significantly on the SMMAC one. This is expected to occur if the results obtained during the design phase of the three control approaches are taken into consideration.
As this parameter change results in a pole displacement to a larger value, much like what happened with temperature increase due to the plant’s non-linear properties, the plant’s static gain and time constant are decreased. It was shown in Part II that fixed PI controllers suffered performance degradation for system’s pole changes, while the polynomial approach and LQ based controllers had their performances hardly altered. For this reason, the various PI controllers with fixed gains that make up the SMMAC controller will be less appropriate for keeping the desired performance on their respective regions.

Having covered the effect of a pole changing parameter, it is then logical to study the effects of a change in the plant’s input gain, modeled by $\alpha_3$ in 3.1. The sample’s absorptivity value $\alpha_s$ is an example of a parameter which value could be subject to some experimental error.

In figure 7.4, variations of $\pm 20\%$ on $\alpha_s$ are considered for the process in closed loop with the controllers. Again, the available solar power is assumed constant at $G_s = 1000\, W/m^2$.

![Simulation of the three controllers for a $\pm 20\%$ variation on the $Al_2O_3$ absorptivity value. Reference temperature, filtered temperature. $G_s = 1000\, W/m^2$.](image)

Naturally, increasing $\alpha_s$ means that the controllers will be applying more gain than needed, leading to some more noticeable overshoot on the SMMAC controller’s response. For the polynomial and LQ based controllers however, the parameter $b$ of the linearized model of the plant (which changes with $\alpha_s$), has a more significant effect on the closed loop characteristic polynomial. Recall that, for the polynomial controller, the closed loop poles are the roots of the denominator polynomial of (5.10). In figure 7.5, it can be observed that for values of $b$ lower than the ones the controller is designed for, complex conjugate poles are obtained, explaining the very small overshoot in 7.4b. For larger values of $b$, like with temperature increases, the slowest pole’s effect is dampened by the presence of the two zeros.

The roots of the LQ optimal controller are the eigenvalues of matrix $\Phi = (\bar{A} - \bar{B}K)$ as was shown in 5.3.1 corresponding to the roots of the following polynomial:

$$s^3 + (a + K_1b + \omega_f)s^2 + \omega_f[a + b(K_1 + K_2)]s - \omega_f bK_i$$ (7.1)
Figure 7.5: Closed loop pole positioning for the polynomial controller for three values of parameter $b$. Controller designed for $T_0 = 1000K$

Figure 7.6 shows how, for $T_0 = 1000K$ (for which the controller was designed), the variations of parameter $b$ affect the closed loop poles. One can easily see how quickly these poles change with this parameter. For lower values of $b$ the two poles are complex conjugate poles that are not too far to the left of the slowest pole, which explains the overshoot in 7.4b. For higher values of $b$, the dominating pole becomes just slightly slower.

Figure 7.6: Closed loop pole positioning for the LQ optimal controller for three values of parameter $b$.

However, from both experiments, one can infer that the SMMAC controller is more sensitive to variations on sample parameters, as it seems to change its performance more drastically when compared to the other control approaches.
Table 7.1: Physical properties of the Silicon carbide sample used in simulation. The average working temperature for which these parameters apply is 1200K.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p [J.Kg^{-1}.K^{-1}]$</td>
<td>Specific heat</td>
<td>1260</td>
</tr>
<tr>
<td>$m [Kg]$</td>
<td>Mass of the sample</td>
<td>$2.191 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Emissivity of the material</td>
<td>0.85</td>
</tr>
<tr>
<td>$A_s [m^2]$</td>
<td>Exposed area of the sample</td>
<td>$7.068 \times 10^{-1}$</td>
</tr>
<tr>
<td>$h_{conv} [W.m^{-2}.K^{-1}]$</td>
<td>Convection factor</td>
<td>eq. (3.10)</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Solar absorptivity of the material</td>
<td>0.8</td>
</tr>
</tbody>
</table>

7.3 Different Types of Samples

Many experiments in solar furnaces are done with samples of silicon carbide (SiC), which differs significantly from alumina in its physical properties. Mainly, its absorptivity factor is quite larger when compared with the material under test so far. In table 7.1, the average values for some of this material’s properties are presented for a working temperature of 1200K.

Given the sensitivity test results of the previous section, and especially taking into account the significant difference in the absorptivity, it is of course expected that none of the controllers are to properly maintain the desires temperature levels. Simulations run proved this to be correct.

As such, it is clear that the temperature control systems are quite dependent on the material under test, as they prove not be universally practical. For a change in sample type, the optimal and polynomial controllers would be most recommended for their easiness to reconfigure, especially because though the characteristic properties of the sample may change, the equation for its dynamics is still described by eq. (3.5). Of course, it would be necessary to analyze if their ability to maintain a good performance for a wide range of working temperatures still holds for other materials, mainly because the way the closed loop system poles change with temperature variation might be more significant than what has been seen for the alumina.
Chapter 8

Conclusions and Future Work

This report presents the methodologies used to plan and design control strategies to be applied in a solar furnace. Starting with the modelling of the process in study, it was established that the temperature dynamics of the sample under test were subject to nonlinearities. A linear model for these dynamics was obtained by linearizing around an operating temperature, so as to be able to design linear controllers. Also it was seen that the shutter, the actuator of the system, had its dynamics described by a linear system and a fixed nonlinear relation between the angle of the blades and the amount of open area it displayed.

In order to linearize the actuation of the system and compensate the natural variations of available solar power due to day time and weather conditions, a controller that takes as reference the desired radiation flux that reaches the focus of the furnace was developed. This controller proved to provide good results and its dynamics were shown to be fast enough not to influence significantly the variations of the desired reference signal. The inner loop, comprised of the shutter subsystem and and this controller, was considered the actuation of the process from the perspective of a temperature controller.

Because of the fact that the linear model for the temperature dynamics of the sample was only valid for the temperature region it attempted to model, it was expected that a good approach to tackle the issue would be to design a set of local models for each region. Amongst the three considered approaches for designing these controllers, only the PI design based on root-locus proved to only be able to control the plant for a specific region. The controllers obtained through pole placement, based on a polynomial approach, and through linear quadratic regulator theory, proved to be able to maintain the desired dynamics for the entire temperature range, though they were both designed for a working temperature of $1000K$.

It was explained, through an analysis of closed loop pole displacement with temperature, that these two approaches had resulting changes in dynamics that were not significantly noticeable. However, an important remark must be made as these conclusions were taken for the temperature range in question, $[800, 1500]K$. For instance, concerning the polynomial controller, the behavior of the closed loop dynamics changes significantly for the lower temperature ranges, as the dynamics as mostly defined by a pair of closed loop poles located close to the origin.

The controller designed based on linear quadratic regulator theory demanded the use
of an optimal filtering technique, since this approach assumed full state feedback and the process’s noise was greatly amplified, possibly damaging the shutter’s components. With a Kalman Filter put to use, the controller still proved to perform very well, though it should be taken into account that this addition demands computational resources that other controllers would not.

Instead of discarding the PI controller approach, a switching multi-model adaptive control scheme was developed using a bank of local PI controllers, each expected to maintain desired dynamics in their corresponding temperature ranges. A supervisor process was designed which, according to the working temperature considered, selected the appropriate controller. A significant improvement when compared to single fixed PI controllers was observed, as the dynamics were maintained for the whole temperature range under consideration. A test with a ramp input proved this approach’s worth as it assured the same rising speed as a single PI controller but assured no overshoot was detected when the reference was gain constant. For material tests with steady temperature increases but ending up very close to the sample’s melting point, this behavior is most valuable.

With three plausible solutions for the obtaining temperature control with good performance, some trials were conducted so as to define which approaches were more robust to input disturbances, such as sudden solar power drops, and process model uncertainties. For the robustness to disturbance test, it was seen that a reduction of available solar power was comparable to a lowering of the upper bound of the actuation signal saturation. As such, the controllers were expected to quickly reach this upper control signal bound, limiting the speed of their step responses. However, a case was shown where the switching control scheme, when changing controllers, reduced the amount of power being allowed through the shutter, which caused a slower response when compared with the other two solutions.

Regarding the sensitivity to model parameter uncertainty, two cases were considered. The first one consisted of admitting that the emissivity of the material was subject to some experimental error. Because this term defines the position of the thermal dynamics’ pole, its effect on the LQ and polynomial controllers was similar to the effects of working in different temperatures, resulting in practically no degradation of performance. The SMMAC controller however, because of its local models designed for specific pole positions, suffered a more evident effect.

The second test was conducted by tampering with the absorptivity of the material. This parameter defines the input gain of the model and as such, reducing its value should make the closed loop responses slower, which was easily seen with the SMMAC controller. For the other two controllers, because of the larger gains used, changing this value resulted in a noticeable change of dynamics. Still, they both performed reasonably well, with responses staying within acceptable limits. It can then be concluded that, when it comes to robustness and sensitivity, the polynomial and optimal controllers were better options. Taking these results into consideration, it is then intuitive to classify the polynomial and LQ controllers as more suitable solutions for the problem at hand.

Regarding versatility with respect to different types of material under test, it was seen that the developed solutions were too dependent on the physical properties of the process. Because of the time demanding task of adjusting the gains of each local PI controller of the switching control approach, this technique is definitely less promising
when it comes to different types of samples. The polynomial and LQ controllers however, can be easily reconfigured in order to cope with the new dynamics. Particularly the polynomial controller, given its algorithmic technique, is the easiest to readjust. It must be said, however, that the capability of covering various regions of these two control approaches needs to be demonstrated through simulation for new types of samples, as this property is very dependent on the way the process’s parameters change with temperature.

Recall that, during the modelling of the sample’s thermal dynamics, the assumption of an homogeneous distribution of temperature on the sample’s body was made. This is a crude approximation, as the temperature of the upper surface of the sample, which is directly hit by the concentrated radiation, will reach higher temperatures than its lower surface, to which the thermocouple is attached. Having this into consideration, a possible further development of this work could consist of better modelling the sample’s dynamics to cover this property. Another possible improvement to the present work would be to enhance the switching control technique considered, taking into account for instance some of the ideas mentioned during the short literature review on the matter. The possibility of a having a dynamical model bank would be quite beneficial for the solar furnace case, as the materials under test undergo transformations that drastically change their dynamics, which suggests that this would be a very promising approach.
Bibliography


