Characterization of the Mechanical Properties Equivalents in Plates

Teresa Sofia Marques Mesquita
IDMEC/IST, Universidade Técnica de Lisboa (Lisbon)
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
Email: teresamesquita82@gmail.com

Abstract

Sandwich structures are widely used in automotive and aerospace industry. Existing structures on the market that brings the underlying manufacturing processes often require high costs, or have a core with very specific applications. This leads to an innovative model with high coefficient rigidity / weight called OPEN CELL® (real model), developed by PLY Engineering, as case study for this paper. The aim is to develop a theoretical model that interacts between computational software (ANSYS and MATLAB) that can even approach as nearly as possible the natural frequencies and deformed from the real structure, optimizing by an iterative algorithm (FMINSEARCH), material properties and thickness, using the classical theory of plates and the finite element method. Then we have a theoretical model previously known to be able to approximate the real model, responding quickly and therefore reduced costs to various situations. The main conclusions relate to the fact that the theoretical model obtained depend heavily on boundary conditions applied, it is so telling in advance the applications to implement the actual structure. It also depend on the behavior of the material as isotropic or orthotropic (and this brings better) and the geometry of the structure to a uniformly distributed load. The model meets the criteria of convergence of design variables.

Keywords: Sandwich Structures, Classical Theory of Plates, Computational model, Optimization of material properties, Dynamic analysis, Finite element method

1. Introduction

At present there are many structures and different models. One of genres that stand out are the Sandwich structures. It was the 2nd World War that began to produce large quantities of Sandwich laminates for the manufacture of Mosquito aircraft. [1] A conventional sandwich structure is usually composed of three basic elements, the two faces and core, these elements being joined by an adhesive or by welding. The faces are usually made of high strength material while the core materials are typically used with low specific weight.
The applications of this technology are very different: airplanes, water sports (on skis, surfboards or snowboards), means of transportation such as boats, ships, military, automotive, bicycles, trains, cargo containers, cabin wind turbines, construction civil, or heat exchangers. For example, a train in Sandwich structures becomes lighter and with better insulation, thus saving the energy used to move the train and its climate, which is very relevant in these days, as the Bombardier Talent in Germany, as illustrated in Figure 1.

The main advantages of sandwich structures are generally: high mechanical strength, good flexural behavior, relative low density, good impact resistance and favorable thermal and acoustic insulation. The main disadvantages are the low possibility of recycling, thus making a major intervention in terms of cost of raw materials. In addition to high costs associated with manufacturing processes, among which are the rolling, extrusion and cutting and bending through one puncture.

2. The OPENCELL® Concept

Recently was developed an innovative technique to produce structures Sandwich, in which a metal plate is cut and folded to form the core of the panel. This new solution promises to overcome the traditional metal structures Sandwich because it holds a higher stiffness and lower weight may be shared by several application fields. The inventor of this amazing innovation is António Valente, Mechanical Engineer of PLY Engineering (small company that develops other projects). [4] A patent has prompted in Finland, the United States and Portugal, and enhances the solution to the many constraints of traditional structural panels, with differential benefits such as:

- Isotropy when compared with traditional sandwich plate and similar solutions;
- No addition of material to join the boards of the panel which significantly increases the coefficient rigidity / weight;
- Use of 100% recyclable materials;
- Add extra functionality in addition to the structural function;
- Adaptation to specific needs of each sector and each customer.

This paper discusses a study of the structures developed by PLY Engineering, as illustrated in Figure 2.

Figure 2 – Real OPENCELL® analyzed

3. Objectives

The objective is to develop a theoretical computer model of plaque that can even closer as much as possible the natural frequencies and deformed to the real model optimizing the material properties and plate thickness, thus predicting its behavior. In future, we will have an equivalent model that will surely respond with speed and reduced costs for different situations.

4. Classical Theory of Plates

The plates are structural elements bounded by two planar surfaces spaced by a quantity called depth. In the case of the thickness dimension being much smaller than other dimensions, the plate is called thin plate on which it is possible to establish the Classical Theory of Thin Plates, developed by Lagrange in 1811 which the assumptions of Kirchhoff reported in [6]. The bending moments for orthotropic material are given by [7, 9]:

\[
M_x = \int_0^h \frac{h^3}{2} \sigma_{xx} z \, dz = -D_x \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) = -\frac{E_x h^3}{12(1-\nu_x \nu_y)} \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right)
\]  

(1a)
$M_y = \int_0^h \frac{h}{2} \sigma_{yy} z \, dz = -D_y \left( \frac{\partial^2 w}{\partial y^2} + v_x \frac{\partial^2 w}{\partial x \partial y} \right) = -\frac{E_y h^3}{12(1-v_x v_y)} \left( \frac{\partial^2 w}{\partial y^2} + v_x \frac{\partial^2 w}{\partial x \partial y} \right) \quad (1b)$

$M_{xy} = \int_0^h \frac{h}{2} \tau_{xy} z \, dz = -2D_t \frac{\partial^2 w}{\partial x \partial y} = -D_{xy}(1 - \sqrt{V_x V_y}) \frac{\partial^2 w}{\partial x \partial y} = -D_{xy}(1 - v_{xy}) \frac{\partial^2 w}{\partial x \partial y} \quad (1c)$

And $D_t = \frac{G_s h^3}{12}$ is the torsional rigidity and $v_{xy} = \sqrt{V_x V_y}$ e $M_{xy} = M_{yx}$.

And the shear forces are:

$Q_x = -D_x \frac{\partial^2 w}{\partial x^2} - B \frac{\partial^3 w}{\partial x^2 \partial y^2} \quad (2a)$

$Q_y = -D_y \frac{\partial^2 w}{\partial y^2} - B \frac{\partial^3 w}{\partial x^2 \partial y} \quad (2b)$

$B = \frac{1}{2} \left( v_x D_x + v_y D_y + 4D_t \right) \quad (3)$

For isotropic material we have the same equations with:

$D_x = D_y = D_{xy} = D; E_x = E_y = E; v_x = v_y = v_{xy} = v.$

5. Dynamic Analyses

There are many situations where the structures are subjected to vibrations which can result in deformities or instabilities, thus arises the need to conduct a study of the dynamic behavior of structures. Then we present the fundamental equations of dynamic equilibrium.

a. Fundamental Equations of dynamic equilibrium

The differential equation of equilibrium dynamics of orthotropic plates or Lagrange equation for free vibrations and harmonics [5] in its simplified form is given by:

$D_x \frac{\partial^4 w}{\partial x^4} + 2B \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} - \rho h \omega^2 w = 0 \quad (4)$

$W$ depends only on position coordinates.

For isotropic material is:

$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) - \rho h \omega^2 w = 0 \quad (5)$

The function $w$ ($\omega$ is expressed in rad/s) for free vibration is given by:

$w = W \cos \omega t \quad (6)$

b. Boundary Conditions

Since the equation of balance boards is set up in the 4th grade are required two boundary conditions at each end of the plate. There are many situations where the structures are subjected to vibrations which can result deformities or instabilities, thus arises the need to conduct a study of the dynamic behavior of structures. Then we present the fundamental equations of dynamic equilibrium.

Plates Simply Supported (SS_SS_SS_SS)

The boundary conditions for the plate simply supported all around [5, 6, 7, 8] are zero displacement and zero bending moment (applying the above equations both cards):

$w = 0 \quad , \quad M_x = 0 \quad \text{to} \quad x = 0 \quad e \quad x = a$

$w = 0 \quad , \quad M_y = 0 \quad \text{to} \quad y = 0 \quad e \quad y = b$

Plates Simply Supported and Free (SS_F_SS_F)

The boundary conditions for $x = 0$ and $x = a$ are the displacement zero and bending moment too for $y = 0$ and $y = b$ zero bending moment and zero shear, as follows [5, 6, 7, 8]:

$w = 0 \quad , \quad M_x = 0 \quad \text{to} \quad x = 0 \quad e \quad x = a$

$M_y = 0 \quad e \quad V_y = Q_y + \frac{\partial M_{xy}}{\partial y} = 0 \quad \text{to} \quad y = 0 \quad e \quad y = b$

c. Analytical Method for Natural Frequencies

Navier’s method applies to rectangular plates and supported on four edges and becomes the expression of the vibration modes [5, 7, 8] in the form:

$w(x, y) = W_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \quad (7)$
The expression above represents the shape of the vibration modes of the plate (sinusoidal), where $W_{mn}$ vibration amplitudes are determined by the equilibrium equation of plate $(m, n)$ refer to the number of half sine waves in x and y direction and they are integers, $(a, b)$ are respectively the length and width of the plate. The natural frequencies for isotropic material are obtained by the following expression:

$$\omega_{mn} = \pi^2 \beta \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$$  \hspace{1cm} (8)

$$\beta = \sqrt{\frac{D}{\rho h}}$$  \hspace{1cm} (9)

$$D = \frac{E h^3}{12(1-\nu^2)}$$  \hspace{1cm} (10)

For orthotropic material and substituting equation 7 into equation 4, gives:

$$\omega_{mn} = \pi^2 \beta \left[ 2D_{xy}m^2 n^2 \left( \frac{a}{b} \right)^2 + D_{x}n^4 \left( \frac{a}{b} \right)^4 \right]$$  \hspace{1cm} (11)

The method of Levi applies only to rectangular plates with two opposite ends supported, the others being arbitrary. For this case the vibration modes represent themselves through the following expression [5, 8, 9] developed in Fourier series:

$$w(x, y) = \sum_{m=1}^{\infty} Y_m(y) sen \frac{m\pi x}{a}$$  \hspace{1cm} (12)

For this boundary condition (SS, F SS, F) there is no explicit solution for the natural frequencies.

7. Problem Formulation

In the optimization design of the structures there are certain steps that are guides to the emergence of an optimal result. The main steps are exposed below.

**a. Presentation of OPENCELL**

The structures analyzed in this paper (actual model) consisting of stainless steel are shown in figures 3 and 4:

![Rectangular Plate](image)

Figure 3 – OPENCELL® Rectangular Plate

![Square Plate](image)

Figure 4 – OPENCELL® Square Plate

**b. Definition of variables project**

The design variables are the parameters that will optimize. In this approach consider the following: $h$ - plate thickness uniform; For isotropic material: $E$ - Modulus of elasticity; For orthotropic:
$E_x$ – Modulus Elasticity of $x$;  
$E_z$ – Modulus Elasticity of $z$;  
$G_{xz}$ – Shear Modulus in the plane $xz$;  
$\nu_{xz}$ – Poisson Coefficient in the plane $xz$

c. Objective Function

The objective function depends on the design variables defined previously, and seeks to minimize it to obtain optimal parameters. Using the equations 8.9 we obtain the natural frequencies for the theoretical model. The number of frequencies to define is the resulting of the theoretical model behavior in terms of vibration modes compared with the actual model (OPENCELL'). For orthotropic and isotropic material we have then the objective functions that are presented below ($A_i$ are the weights of each factor of the objective function):

Rectangular Plate $SS_{SSSS}$:

$$f = A_1 * \left( \frac{75312.20 - 11.79725 \cdot E \cdot h^3}{75312.20} \right)^2 + A_2 * \left( \frac{86763.08 - 13.78787 \cdot E \cdot h^3}{86763.08} \right)^2 + A_3 * \left( \frac{104219.2 - 17.45028 \cdot E \cdot h^3}{104219.2} \right)^2 + A_4 * \left( \frac{124912.2 - 23.30151 \cdot E \cdot h^3}{124912.2} \right)^2 + A_5 * \left( \frac{143975.9 - 32.06542 \cdot E \cdot h^3}{143975.9} \right)^2 + A_6 * \left( \frac{152490.2 - 44.67271 \cdot E \cdot h^3}{152490.2} \right)^2 + A_7 * \left( \frac{6.46 \cdot 4 - 3.63776 \cdot E \cdot h^3}{6.46 \cdot 4} \right)^2$$

Rectangular Plate $SS_FSS_F$:

$$f = A_1 * \left( \frac{3141.60 - w_{Theoretical}}{3141.60} \right)^2 + A_2 * \left( \frac{11945.62 - w_{Theoretical}}{11945.62} \right)^2 + A_3 * \left( \frac{0.011337 - \delta_{Theoretical}}{0.011337} \right)^2$$

Square Plate $SS_{SSSS}$:

$$f = A_1 * \left( \frac{14246.81 - 0.551515 \cdot E \cdot h^3}{14246.81} \right)^2 + A_2 * \left( \frac{34514.21 - 3.446968 \cdot E \cdot h^3}{34514.21} \right)^2 + A_3 * \left( \frac{34551.37 - 3.446968 \cdot E \cdot h^3}{34551.37} \right)^2 + A_4 * \left( \frac{0.003223 - 91.2384 \cdot E \cdot h^3}{0.003223} \right)^2$$

Square Plate $SS_FSS_F$:

$$f = A_1 * \left( \frac{6491.41 - w_{Theoretical}}{6491.41} \right)^2 + A_2 * \left( \frac{11737.76 - w_{Theoretical}}{11737.76} \right)^2 + A_3 * \left( \frac{0.005788 - \delta_{Theoretical}}{0.005788} \right)^2$$

8. Optimization Criterion

a. FMINSEARCH and the algorithm of Nelder-Mead

In this paper we used the FMINSEARCH, a method of searching for iterations to optimize the parameters of the theoretical model. The optimization method uses a FMINSEARCH simplex search method. This is a direct search method and not used numerical or analytic gradients. If $n$ is the length of $x$, the simplex in $n$-dimensional space is characterized by the $n + 1$ distinct vectors that are its vertices. In two-dimensional space, the simplex is a triangle in space is a pyramid. At each stage of the research, a new point on / near the current simplex is generated. The value of the function at new point is compared with the function values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified
tolerance. This method is known as the simplex algorithm of Nelder-Mead [10].

b. Flow Chart of Computer Simulation

The computer simulation is carried out through the following diagram:

```
START
Inversion for variables of project (Eigenvalue & H) in the MATLAB

Generates a program at PKS in ANSYS with the properties of the linear model by means of E & h provided by MATLAB

ANSYS calculates the natural frequency and O (deformed) with the values of E & h

The MATLAB reads O and calculates T function for optimization

Is material? Compare with the values of OPENCCEL

Yes

No

Optimization Method PRIMERA
```

Figure 5 – Sequence diagram of operations performed computationally

9. Results

a. Case studies

To effectively characterize the properties of the theoretical model some case studies were discussed, table 1.

Each of these case studies presents numerical values of natural frequencies and deformed. The aim is to bring the best possible uniform model to OPENCCEL*, table 2.

Table 2 - Standard Table of natural frequencies (rad/s) and deformed for all case studies.

b. Results of Isotropic Material

Using the computational model shown in Figure 5 we obtained the results for several case studies. Analyzing the equations described previously concluded that the objective function for isotropic materials depends exclusively on the factor $E \cdot h^3$. Initial conditions for optimizing (must be well defined because they define the way to go by the algorithm) shows that the points belonging to the valley minimum of the function are hyperbole whose great mathematical equation is: $E \cdot h^3 = K$, and $k$ is the value to optimize. Then we present all results of hyperbole for a linear isotropic material.

```
Table 1 - Definition of case studies for optimization of parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>Isotropic Material</th>
<th>Orthotropic Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>Rectangular Plate simply supported in all edges, SS, SS, SS - Isotropic Material</td>
<td>Rectangular Plate simply supported in all edges, SS, SS, SS - Orthotropic Material</td>
</tr>
<tr>
<td>Case B</td>
<td>Rectangular Plate simply supported and free SS, SS, SS - Isotropic Material</td>
<td>Rectangular Plate simply supported and free SS, SS, SS - Orthotropic Material</td>
</tr>
<tr>
<td>Case C</td>
<td>Square Plate simply supported in all edges, SS, SS, SS - Isotropic Material</td>
<td>Square Plate simply supported in all edges, SS, SS, SS - Orthotropic Material</td>
</tr>
<tr>
<td>Case D</td>
<td>Square Plate simply supported and free SS, SS, SS - Isotropic Material</td>
<td>Square Plate simply supported and free SS, SS, SS - Orthotropic Material</td>
</tr>
</tbody>
</table>

```

Table 6 - Hyperbolic optimal (case A) - Different and equal weights

Figure 6 - Hyperbolic optimal (case A) - Different and equal weights
For case A we can see from Figure 10 that the optimization algorithm used (FMINSEARCH) converges well for the optimal values where the objective function is minimal. The same happened to other cases.

We can see by the tables that follow the detailed results for each of the cases presented in Table 1 and 2, and the percentage error when compared with OPENCELL.

Table 3 - Values for Optimal Rectangular Plate SS_SS_SS_SS - Case A

Table 4 - Values for Optimal Rectangular Plate SS_F_SS_F - Case B

Table 5 - Values for Optimal Square Plate SS_SS_SS_SS - Case C
Table 6 - Values for Optimal Square Plate SS_F SS_F - Case D

c. **Results of Orthotropic Material**

For all cases orthotropic (cases E, F, G, H) we have the following constraints:

\[ 10 \text{ GPa} < E_x < 1000 \text{ GPa} \]

\[ 10 \text{ GPa} < E_z < 1000 \text{ GPa} \]

\[ 0.1 < v_{xz} < 0.4 \]

\[ 3 \text{ GPa} < G_{xz} < 160 \text{ GPa} \]

For orthotropic material the objective function is the same as in isotropic. The fixed parameters are summarized in Table 7. These are the ones that do not influence the dynamic behavior and static structure, so are not considered design variables.

<table>
<thead>
<tr>
<th>Fixed Parameter</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_x )</td>
<td>200 GPa</td>
</tr>
<tr>
<td>( v_{xy} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( v_{yx} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( G_{xy} )</td>
<td>80 GPa</td>
</tr>
<tr>
<td>( G_{yz} )</td>
<td>80 GPa</td>
</tr>
</tbody>
</table>

Table 7 - Fixed parameters not influential

The results are detailed in the tables that follow for three different starting points of optimization which are defined in Table 8. Where \( G = \frac{E}{2(1+v)} \) for isotropic material.

<table>
<thead>
<tr>
<th>Starting Points of Optimization</th>
<th>( h ) (mm)</th>
<th>( E_x ) (GPa)</th>
<th>( E_z ) (GPa)</th>
<th>( v_{xz} )</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case E</td>
<td>Ponto 1</td>
<td>2</td>
<td>150</td>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 2</td>
<td>5</td>
<td>150</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 3</td>
<td>10</td>
<td>150</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Case F</td>
<td>Ponto 1</td>
<td>2</td>
<td>150</td>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 2</td>
<td>5</td>
<td>150</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 3</td>
<td>10</td>
<td>150</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Case G</td>
<td>Ponto 1</td>
<td>2</td>
<td>150</td>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 2</td>
<td>5</td>
<td>150</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 3</td>
<td>10</td>
<td>150</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Case H</td>
<td>Ponto 1</td>
<td>2</td>
<td>150</td>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 2</td>
<td>5</td>
<td>150</td>
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<td>60</td>
</tr>
<tr>
<td></td>
<td>Ponto 3</td>
<td>10</td>
<td>150</td>
<td>10</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 8 - Starting points for the optimization of the cases E, F, G, H

Table 9 – Orthotropic Rectangular Plate SS_SS_SS_SS with different weights (A1=A2=A7=0.2; A3=A4=A5=A6=0.1) - Case E. (Note: E.M. = Equivalent Model)

Figure 11 - Convergence of the values of Modulus of elasticity – Case E (Ponto 1, table 9).

Figure 12 - Convergence of the values of \( h \), F and Poisson Coefficient – Case E (Ponto 1, table 9).

And for the case E where illustrated the graphics of convergence and for the remaining cases the same is true although not included in this paper. This proves the effectiveness of the optimization method used in MATLAB.
Table 10 - Orthotropic Rectangular Plate SS_F_SS_F with different weights (A1=A3=0.4; A2=0.2) - Case F.

Table 11 - Orthotropic Square plate SS_SS_SS_SS with different weights (A1=A4=0.3; A2=A3=0.2) - Case G.

Table 12 - Orthotropic Square plate SS_F_SS_F with different weights (A1=A3=0.4; A2=0.2) - Case H.

Note that for example, comparing Table 3 with Table 9 that differ only in the type of behavior (isotropic and orthotropic) would expect that the theoretical model orthotropic best approached, however this is not due to the fact that it does not split the same initial conditions of the project. Note the table 13 where this already happens, for these same cases A and E starting from the same point of optimization.

Table 13 - Comparison of the objective function for orthotropic and isotropic material (case A and E) considering the same initial conditions optimization for equal weights

Next will be presents the main conclusions for the whole analysis of the resulting tables, graphs and classical theory of plates.

10. Conclusions

This work is an asset to the extent that we have a theoretical model previously known to be able to approximate the real model, responding quickly and therefore reduced costs to various situations. The main conclusions derived from the results obtained are:

- The initial conditions define the optimal way to go by the optimization algorithm, which implies that we must decide the best place to begin to optimize;

- The optimal solutions depend strongly on the boundary conditions, or in other words, for an application in which we simply supported and free plates at opposite ends (SS_F_SS_F) we have a thickness greater / more rigid than the theoretical model over the boundary condition of simply supported in four edges (SS_SS_SS_SS);

- Depending on the application that is intended for the real model, we can work the objective function for example giving more importance to the first frequency and the deformed by increasing the weights assigned to their parameters;
For orthotropic material can obtain a better approximation to the real model (Table 13) compared to the isotropic material, although it is more difficult to obtain certainty as to the optimal parameters as given that there are more variables, the objective function will likely present more minimum valleys (some unknown), and yet easier to scale a certain thickness;

- For the case where the material is isotropic, it is possible to obtain optimal hyperbole that present a mathematical equation where will be easy to remove (by observation of the graphs or by calculation - \( E = \frac{k}{h^3} \)) the modulus of elasticity for a fixed thickness \( h \) or vice versa. However, summarizing what was said earlier, let us not forget that the vibration modes of the optimal theoretical model obtained must be equal to the real model. The same applies to the orthotropic material. It was found that the ends of the hyperbole (very low thickness and very high modulus of elasticity or vice versa) the behavior of the theoretical model in terms of vibration mode was totally different from the real model. The fact that this issue may be due to the finite element program ANSYS to present a theory of Mindlin (also applicable to thick plates) more sophisticated, which is different from that used in the objective function implemented in the computational model of this paper is based on the classical theory plate (thin plate).

- The computational model used is robust, since it effectively brings together the design variables, as shown in the previous charts. Also find local solutions more easily when compared by another method with derivative.

**References**


