Calculation of the dynamic behavior of a falling plate or disk in a fluid

by

Vitor Fernando Rosa Caetano

December 2010

Abstract

In this paper Computational Fluid Dynamics (CFD) was applied together with the Computational Rigid Body Dynamics (RBD) to the study of moving of 2D and 3D bodies immersed in a fluid environment. A Fortran subroutine using non-inertial reference frames was developed to model the 6 degrees-of-freedom (6dof) rigid body dynamics. This routine was coupled to the commercial CFD software package Star-CD, using the available Star-CD subroutine interfaces. All the interactions between the RBD and CFD models including the moving mesh techniques were fully tested and validated for parallel processing.

A set of simulations was conducted to investigate the 2D motion of falling plates under several different conditions: initial orientation and density ratio with ambient fluid. The tumbling and fluttering movement patterns were identified to appear in different flow regimes, in accordance to reported results in the scientific literature.

The results indicate that the initial plate orientation does not influence the terminal velocity and the density ratio may display tumbling, fluttering or chaotic regime behavior.

1. Introduction

Foliage tree’s seeds or oscillating cards (fluttering and tumbling movement patterns) are wonderful examples of solids moving immersed in a fluid. Foliage’s falling paths also generate inspired view for both physicists and poets. The study of falling plates started before the classical aerodynamic development, J. C. Maxwell already described the so-called tumbling phenomena [1].
The dynamic behavior of free falling bodies inside a fluid is dominated by fluid-structure interactions where the full coupling between the solid and fluid dynamics leads to an unsteady complex behavior, see e.g. [2-6]. The fluttering and tumbling motions of falling cards or disks [7-10] is essential in understanding locomotion techniques used by insects and fishes to move in fluids [11-13] and important also to study high Reynolds number Re fluid–solid interactions in launching of artillery projectiles, deep sea research, and seed dispersion, bubble dynamics etc, see e.g. [13-16].

To deal with this difficulty in numerical simulations, Different numerical techniques have been proposed, to deal with the coupling between fluid and solid equations on moving boundaries whose position is itself the result of the coupled dynamics. Fluid–solid solvers using fitted moving grids [10, 11] or immersed boundary methods [20, 21].

The Star-CD was used in the present study and complementary software was developed with rigid body dynamics, RBD, with 6 degree’s of freedom in a non-inertial reference frame to model the body movement. The developed subroutines were coupled with the Star-CD allowing to capture the relevant physics of the interaction.

The main objective of the present work is to verify the predictions of free falling plates against reference calculations and to conduct a parametric study of the density ratio between the solid and the fluid and also on the inclination plate angle at t=0. In addition the 2D, methodologies was ported to 3D and the prediction of a the free falling disk problem showed very good qualitative agreement with simple experiments of an aluminium disk in water.

Next section describes the numerical treatment of the two-way coupling fluid-structure problem. Section 3 is devoted to the verification of the present predictions with reported reference data. This is followed in section 4 with the results of the parametric study related with density ratio and initial plate angle as well as the disk. The resume ends with summary conclusions and future work.

2. Problem Formulation

In the present study we will consider dynamical systems consisting of fluid flow interacting with moving immersed bodies. Figure 1, shows a body that interacts with the flow. In this setting, the dynamic of the fluid and structure are described by different sets of equations, which need to be solved as a coupled system. On the side of the fluid Navier-Stokes equations for incompressible flow are solved:

\[
\frac{\delta U_i}{\delta t} = 0
\]

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + v \frac{\partial^2 U_i}{\partial x_j \partial x_j}
\]

where \(x_i (i = 1, 2, 3)\) are the Cartesian coordinates \(U_i\) are the velocity components in the corresponding direction, \(P\) is the pressure.
The motion of a set of rigid bodies within the fluid domain is governed by the Euler equations of the form (3):

\[
\begin{align*}
M_x &= l_{xx}\dot{\omega}_x + (l_{xy} - l_{yx})\omega_y\omega_x \\
M_y &= l_{yy}\dot{\omega}_y + (l_{yx} - l_{xy})\omega_x\omega_y \\
M_z &= l_{zz}\dot{\omega}_z + (l_{yz} - l_{zy})\omega_x\omega_y
\end{align*}
\] (3)

where the \(M_i\) are the moment component in the Cartesian coordinates, \(l_i\) are the moment of inertia and \(\omega_i\) are the angular velocity. Equations that model the rotational motion of a rigid body about a fixed point. In the present modeling the aeronautical version of the Euler's angles has been used, Figure 1. And the relations will provide us an estimation of the rotation matrix (4), which will determine the body's orientation.

\[
[R] = \begin{bmatrix}
\cos \psi \cos \theta & \cos \psi \sin \theta \sin \varphi - \sin \psi \cos \varphi & \sin \psi \cos \theta \sin \varphi + \sin \psi \sin \varphi \\
\sin \psi \cos \theta & \sin \psi \sin \theta \sin \varphi + \cos \psi \cos \varphi & \cos \psi \sin \theta \cos \varphi - \cos \psi \sin \varphi \\
-\sin \theta & \cos \theta \sin \varphi & \cos \theta \cos \varphi
\end{bmatrix}
\] (4)

Figure 1 - (a) Systems coordinate, (b) Euler's angles

The computational domain is shown in Figure 2. The plate has thickness to length ratio equal to 8 and is allowed to rotate with a sliding mesh interface. Consequently there are mainly 3 sub-domains. The first is fixed to the plate and comprises a very refined region in the plate vicinity expanding to a circular shape that slides with as a rotor with the second computational domain as shown in Figure 3 a), b) and c).
For the 3D case of the disk with thickness to diameter ratio equal to 0.1 the computational domain box is 100 diameters wide and 200 diameters height. A sliding spherical surface mesh allows the disk to have rotational movement. The mesh comprises 969320 control volumes.

3. Verification of the Calculations

The reference data reported by Xu (2008) [4] and Wang et al experiments [7] was considered for the verification of the predictions obtained with Star-CD using the moving mesh methodology coupled with the Rigid Body Dynamics. The free fall of a plate of rectangular section with a length to thickness ratio equal to 8 was simulated with the physical parameters given in Table I, in which the Reynolds number Re* was defined with a velocity scale given by (5).
\[ Re^* = \frac{\rho_f L U_t}{\mu} \]  
\[ U_t = \sqrt{2hg \left( \frac{\rho_b}{\rho_f} - 1 \right)} \]

<table>
<thead>
<tr>
<th>Grandeza</th>
<th>Símbolo</th>
<th>Valor</th>
<th>Unidades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate length</td>
<td>( L )</td>
<td>0.00648</td>
<td>m</td>
</tr>
<tr>
<td>Plate thickness</td>
<td>( h )</td>
<td>0.00081</td>
<td>m</td>
</tr>
<tr>
<td>Time step</td>
<td>( \Delta t )</td>
<td>1/4000</td>
<td>s</td>
</tr>
<tr>
<td>Density of the fluid</td>
<td>( \rho_f )</td>
<td>1000</td>
<td>Kg/m(^3)</td>
</tr>
<tr>
<td>Density of the body</td>
<td>( \rho_b )</td>
<td>2700</td>
<td>Kg/m(^3)</td>
</tr>
<tr>
<td>Viscosity</td>
<td>( \mu )</td>
<td>0.000887</td>
<td>Ns/m(^2)</td>
</tr>
<tr>
<td>Acceleration of gravity</td>
<td>( g )</td>
<td>9.8</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>Initial velocity</td>
<td>( V_0 )</td>
<td>0.12615</td>
<td>m/s</td>
</tr>
<tr>
<td>Initial angle</td>
<td>( \varphi_0 )</td>
<td>45</td>
<td>[degree]</td>
</tr>
</tbody>
</table>

Table 1 - Physical parameters

Figure 4 shows the plate trajectories and their comparison against reference calculation and experiments. The results are very good up to \( X^* = 20 \). After that the prediction deviates a bit from the experiments. The prediction keep the tumbling motion in a more regular periodical motion than that observed in the experiments. The same occurred for the reference Xu (2008) predictions but for \( X^* > 25 \).
Figures 5 a) and b) show the prediction of the non-dimensional force, $F_i^* = \frac{F_i}{mg}$, as a function of the non-dimensional time $T^* = \frac{t}{L}$ and their comparison with reference computations by Xu (2008). The peak force amplitudes are in very good agreement as well as the forces near the zero horizontal velocity.

4. Results of the parametric study

The density ratio $B = \frac{\rho_0}{\rho_f}$ was varied from $2.7 \leq B \leq 27$ and all the other values were kept constants. Figure 6 shows the trajectories of the plate for the different density ratios considered and all of them were obtained from $t = 0$ to $t = 1$ s. Consequently the trajectories with higher density ratios are longer than for lower B's, meaning that they have higher terminal velocities. The trajectories denote that although the tumbling motion arise in the cases considered it happens that from $B=4$ to $B=5$ there was a dramatic decrease of the oscillation amplitude in the x-y trajectory diagram. For $B>5$ the plate enters in tumbling mode very quickly after $t = 0$ and for lower values the plate descends quite a lot before entering in rotation.
Figure 6 - Trajectories versus Density Ratio

Figure 7 – Trajectories versus initial angle. a) complete computed trajectories b) zoom of the initial instants.

Figure 7 a) shows the prediction for plate trajectories as a function of the initial plate inclination, $\varphi_0$. All the conditions were kept unchanged to the verification case, see Table I. Figure 7 b) shows the zoom of the initial instants of the plate acceleration. In the trajectories for $0.05\pi \leq \varphi_0 \leq 0.2\pi$ the plate rotates and stabilize for the left side and the opposite occurs for $0.25\pi \leq \varphi_0 \leq 0.45\pi$, where the plate after one rotation stabilizes towards the right side. During the first instants, where the first plate rotation occurs, the plate flight direction is ‘randomly’ stated.
Figure 8 a) shows vorticity contours at four time instants, corresponding to one cycle, of the tumbling trajectory and Figure 7 b) for the disk with the physical parameters listed in Table I. The disk diameter to thickness ratio is equal to 10 while the Reynolds number is similar to that of the plate case. The tumbling trajectory is present and details can be found in the Dissertation.
5. Conclusions

Two and three dimensional calculations of a plate or disk free fall were conducted with the Star-CD software for the fluid-structure interaction described by coupling of the Navier-Stokes and Rigid Body Dynamics.

A moving mesh methodology was employed based on sliding mesh for a rotational of the body. Translation of the mesh account for the body descent motion.

The calculations were verified against a reported case including calculations and also measurements of the plate trajectory and forces applied in. The agreement was considered satisfactory during several plate rotations in the tumbling trajectory.

The following conclusions can be withdrawn from the conducted parametric study:

- The initial plate angle does not influence the plate terminal velocity but the plate flight descend may change towards left or right in a random fashion.
- The density ratio has a tremendous influence in the first plate rotation and also in the amplitude of the plate center of mass motion, as well as terminal velocity.
- Some well known movement patterns have been found in different flow regimes, such as tumbling and fluttering. All this are in close agreement with the data reported in the scientific literature.
- The adopted numerical methodology provides physical realistic results from that we believe the coupling between CFD and RBD has been successfully employed.

Future studies should investigate the presence of shear winds or heat transfer and their effects in the body regimes and trajectories.

References


