CP Violation and Flavour-Changing Neutral Currents with an additional Vector-like Quark

Hugo Ricardo Colaço Ferreira

Under supervision of Joaquim Inácio da Silva Marcos

Dep. Physics, IST, Lisbon, Portugal

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Abstract

In this paper, we investigate the pattern of quark masses and mixings in an extension of the Standard Model which includes a down-type vector-like quark, in the framework of Universality of Strength for Yukawa Couplings (USY). Our main result is to show that the presence of new Physics of only one extra vector-like quark contributes significantly to diverse quantities which measure CP violation. After briefly discussing the theoretical background, we present the model and analyse three ansätze for the quark mass matrices, in the framework of USY. After obtaining an ansatz-scheme which leads to very good phenomenological results, we finally do a fine numerical scan of the input parameter space and find a region where all the computed physical quantities are in agreement with the experimental data.

Keywords: Universality of Strength for Yukawa Couplings, Yukawa couplings, quark mass matrices, CP violation, vector-like quark.

1 Introduction

At present, the Standard Model (SM) of Particle Physics is the theory which best describes all the experimental data concerning elementary particles and their strong and electroweak interactions [1]. However, the SM faces several theoretical difficulties, such as the fact that the flavour structure of the Yukawa interactions is not constrained by any symmetry, thus leading to arbitrary Yukawa couplings. There have been many attempts to constrain the Yukawa couplings. One of the most important examples is the hypothesis of the Universality of Strength for Yukawa couplings (USY), which was first introduced in 1990 by Branco, Silva-Marcos and Rebelo [2]. In this hypothesis, all the couplings have equal moduli but different flavour-dependent phases, leading to quark mass matrices of the form:

$$(M_u)_{ij} = c_u\, e^{i\phi^u_{ij}}\,, \quad (M_d)_{ij} = c_d\, e^{i\phi^d_{ij}}\,, \qquad (1)$$

where c_u and c_d are real numbers and ϕ^u_{ij} and ϕ^d_{ij} are the USY phases.

Although it was possible to fit all the experimental values of the quark masses and elements of the CKM matrix [2, 3, 4], it soon became apparent that ansätze in the framework of USY do not provide sufficient CP violation. Indeed, it was shown [5] that relaxing somewhat the USY proposal, so that some of its entries do not have moduli equal to 1, allows additional CP violation. This modification of the mass matrices and consequent improvement of CP violation can be described in the framework of an extension to the SM with vector-like quarks [6, 7], which are quarks whose left-handed and right-handed components transform in the same way under the SM gauge group.

Recently, Higuchi, Senami and Yamamoto [8] studied the case of extra down-type vector-like quarks in the framework of USY. In their specific scheme, it was found that the correct CP violation can be obtained if one considers at least two down-type vector-like quarks with masses in the TeV range.

In this paper, we present another view on this subject, by considering a model in the framework of a minimal extension of the Standard Model (SM) with only one additional down-type vector-like quark and a singlet complex scalar Higgs field. The content of the model is:

- 3 left-handed doublets $Q_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix}$;
- 1 charge $-1/3$ left-handed singlet D_L;
- 3 charge $2/3$ right-handed singlets $U_{R_i} \equiv u_{R_i}$;
- 4 charge $-1/3$ right-handed singlets D_{R_α}, with $D_{R_i} \equiv d_{R_i}$, and $D_{R_4} \equiv D_R$.
- 1 Higgs doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$;
- 1 complex scalar Higgs singlet S,

with $i = 1, 2, 3$ and $\alpha = 1, 2, 3, 4$. Additionally, we impose a $\mathbb{Z}_2$ symmetry under which all the fields of the SM transform trivially while the new fields D_L, D_R and S are odd.

The Yukawa part of the Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} = &-\overline{Q_{L_i}}\left[(Y_d)_{ij}\,\Phi\, d_{R_j} - \overline{Q_{L_i}}\,(Y_u)_{ij}\,\tilde{\Phi}\, u_{R_j}\right] \\ &- \overline{D_L}\left[f_j\, S + f'_j\, S^*\right] d_{R_j} - \mu\, \overline{D_L}\, D_R \\ &+ \mathrm{h.c.}\,, \end{aligned} \qquad (2)$$

where Y^u and Y^d are arbitrary 3×3 complex matrices, f_j, f'_j, $j = 1, 2, 3$, and μ are arbitrary complex numbers.

The Higgs fields can be written in a basis such that:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}\,, \quad \langle S \rangle = \frac{v_S}{\sqrt{2}}\, e^{i\theta_S}\,, \qquad (3)$$

where, in principle, the scale of v_S may be larger than v, which is of the electroweak scale.

In this model we require that Y^u and Y^d are in the framework of USY. Concerning the remaining couplings, we allow them to be general complex numbers. Therefore, after spontaneous symmetry breaking, the mass matrices are such that:

$$(M_u)_{ij} = c_u\, e^{i\phi^u_{ij}}\,, \quad i,j = 1,2,3\,, \tag{4}$$

$$(M_d)_{ij} = c_d\, e^{i\phi^d_{ij}}\,, \quad i,j = 1,2,3\,, \tag{5}$$

$$(M_d)_{i4} = 0\,, \quad (M_d)_{4j} = k_j\,, \quad i,j = 1,2,3\,, \tag{6}$$

$$(M_d)_{44} = \mu\,, \tag{7}$$

where k_j are complex numbers of order v_S and μ is also of order v_S.

In this way, it will be possible to obtain sufficient CP violation with just one more quark, but with a mass under the TeV range.

2 Analysis of three ansatz-schemes

In this paper, three ansatz-schemes for the mass matrices are analysed, for both the cases in which there are either 3 or 4 down-type quarks. In short, the invariants for each mass matrix are calculated and related with the quark masses, from which the USY phases and other parameters of the mass matrix are written in terms of quark mass ratios. After that, it is possible to compute the CKM matrix and other physical quantities as functions of the quark masses and check the validity of the ansatz. The objective is to find an ansatz which leads to results in agreement with the phenomenological predictions, given appropriate input parameters.

The physical quantities which are computed are: the quark masses:

$$(m_u, m_c, m_t) = m_t^{\rm exp}\left(\sqrt{\frac{\lambda_u}{\lambda_t}}, \sqrt{\frac{\lambda_c}{\lambda_t}}, 1\right)\,, \tag{8}$$

$$(m_d, m_s, m_b, m_D) = m_b^{\rm exp}\left(\sqrt{\frac{\lambda_d}{\lambda_b}}, \sqrt{\frac{\lambda_s}{\lambda_b}}, 1, \sqrt{\frac{\lambda_D}{\lambda_b}}\right)\,, \tag{9}$$

where λ_u, λ_c and λ_t are the eigenvalues of H_u and λ_d, λ_s, λ_b and λ_D are the eigenvalues of H_d, with H_u and H_d being the Hermitian matrices:

$$H_u \equiv \frac{M_u \cdot M_u^\dagger}{\mathrm{Tr}\big(M_u \cdot M_u^\dagger\big)}\,, \quad H_d \equiv \frac{M_d \cdot M_d^\dagger}{\mathrm{Tr}\big(M_d \cdot M_d^\dagger\big)}\,; \tag{10}$$

the CKM matrix:

$$V_{ij} \equiv \big(W_u^\dagger\big)_{ik}\big(W_d\big)_{kj}\,, \tag{11}$$

where W_u and W_d are unitary matrices formed by the eigenvectors of H_u and H_d, respectively; the quantity $\rho \equiv |V_{13}|/|V_{23}|$; and the quantity $|J| \equiv |\mathrm{Im}(V_{12}V_{23}V_{13}^*V_{22}^*)|$, which is a measure of the strength of CP violation.

In table 1, we present the main results of this analysis. The lowest-order results for the physical quantities are presented. We note that only the two last ansätze provide acceptable results for the entries $|V_{12}|$ and $|V_{23}|$ of the CKM matrix, although both ρ and $|J|$ are not in agreement with the experimental values (see [9] and PDG).

M_u/c_u	M_d/c_d	$\lvert V_{12}\rvert$	$\lvert V_{23}\rvert$	ρ	$\lvert J\rvert \times 10^5$
$\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{i\xi} & 1 \\ 1 & 1 & e^{i\eta} \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{i\beta} & 1 \\ 1 & 1 & e^{i\alpha} \end{pmatrix}$	$\frac{1}{\sqrt{3}}\frac{m_d}{m_s}$ 0.027960	$\frac{1}{\sqrt{2}}\frac{m_s}{m_b}$ 0.018619	0.0700	0.000219
	$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & e^{i\beta} & 1 & 0 \\ 1 & 1 & e^{i\alpha} & 0 \\ k & k & k & M \end{pmatrix}$	$\frac{1}{\sqrt{3}}\frac{m_d}{m_s}$ 0.027961	$\frac{1}{\sqrt{2}}\frac{m_s}{m_b}$ 0.018618	0.0700	0.00116
$\begin{pmatrix} e^{-i\xi} & 1 & 1 \\ 1 & e^{i\xi} & 1 \\ 1 & 1 & e^{i\eta} \end{pmatrix}$	$\begin{pmatrix} e^{-i\beta} & 1 & 1 \\ 1 & e^{i\beta} & 1 \\ 1 & 1 & e^{i\alpha} \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.15166	$\frac{1}{\sqrt{2}}\frac{m_s}{m_b}$ 0.018763	0.569	0.0179
	$\begin{pmatrix} e^{-i\beta} & 1 & 1 & 0 \\ 1 & e^{i\beta} & 1 & 0 \\ 1 & 1 & e^{i\alpha} & 0 \\ k & k & k & M \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.15165	$\frac{1}{\sqrt{2}}\frac{m_s}{m_b}$ 0.018764	0.569	0.0739
$\begin{pmatrix} 1 & 1 & e^{i(\eta-\xi)} \\ 1 & 1 & e^{i\eta} \\ e^{i(\eta-\xi)} & e^{i\eta} & e^{i\eta} \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & e^{i(\alpha-\beta)} \\ 1 & 1 & e^{i\alpha} \\ e^{i(\alpha-\beta)} & e^{i\alpha} & e^{i\alpha} \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.20792	$\sqrt{2}\,\frac{m_s}{m_b}$ 0.035606	0.197	0.0986
	$\begin{pmatrix} 1 & 1 & e^{i(\alpha-\beta)} & 0 \\ 1 & 1 & e^{i\alpha} & 0 \\ e^{i(\alpha-\beta)} & e^{i\alpha} & e^{i\alpha} & 0 \\ k & k & k & M \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.20790	$\sqrt{2}\,\frac{m_s}{m_b}$ 0.035603	0.197	0.0273

Table 1: Summary table with the results for the three ansätze, where the lowest order results for positive USY phases (η, ξ, α and β) and $1/M$ were used.

These two last quantities can be improved in two separate ways. First, the results presented in table 1 were obtained using positive signs for the USY phases, but in reality there are 16 solutions, one for each combination of positive or negative solutions. If we calculate the physical quantities for each combination, we conclude that we obtain a value of ρ in agreement with its experimental value only in the cases where the phase β is negative and in the opposite case.

Second, in a first attempt, we have set the first three entries of the last row of the down-type quark mass matrix equal to a positive constant k, although there is no such requirement. In fact, the entries can be different complex numbers. It can be shown that introducing complex phases in those entries may add a significant contribution to $|J|$.

More precisely, in the case where all entries are equal positive numbers k, the 3×3 upper-left submatrix of the CKM matrix will be given approximately by:

$$V \simeq W_d \cdot O_{23}\,, \tag{12}$$

where W_d is the unitary matrix which diagonalizes the 3×3 down-type mass matrix for the case in which there are only 3 down-type quarks and O_{23} is an orthogonal matrix whose Euler angle is of order m_s^2/m_d^2.

If, instead of (k, k, k), we choose (ik, ik, $-k$) for those entries, the same 3×3 submatrix of the CKM matrix will be approximately:

$$V \simeq W_d \cdot K_3 \cdot O'_{23}\,, \tag{13}$$

M_u/c_u	M_d/c_d	$\|V_{12}\|$	$\|V_{23}\|$	ρ	$\|J\| \times 10^5$
$\begin{pmatrix} 1 & 1 & e^{i(\eta-\xi)} \\ 1 & 1 & e^{i\eta} \\ e^{i(\eta-\xi)} & e^{i\eta} & e^{i\eta} \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & e^{i(\alpha-\beta)} & 0 \\ 1 & 1 & e^{i\alpha} & 0 \\ e^{i(\alpha-\beta)} & e^{i\alpha} & e^{i\alpha} & 0 \\ ik & if & -g & M \end{pmatrix}$	0.22431	0.039443	0.0795	1.51

Table 2: Summary table with the results for the best ansatz, using positive η, ξ and α and negative β.

where $K_3 = \mathrm{diag}(0, 0, e^{i\varphi})$ is a unitary matrix, with φ being an important phase contribution, and O'_{23} is an orthogonal matrix whose Euler angle is of order $m_s/m_d \gg m_s^2/m_d^2$. We conclude that, in this second case, the contribution to CP violation is indeed much more significant.

In table 2, we present again the results for the last ansatz-scheme, but incorporating the changes discussed. We see that all the computed physical quantities are in good agreement with the experimental values (see [9] and PDG).

3 Numerical scan of the input parameter space

In this section, we do a fine numerical scan of the input parameter space, which consists of the parameters (η, ξ, α, β, k, f, g, M), around the values computed in the last section, and calculate the important physical quantities for each value of the input parameters.

The physical quantities which are computed are the same as for the last section. In addition, we calculate: the neutral mixing matrices for the up and down-type quarks:

$$V_0^u \equiv V \cdot V^\dagger \,, \quad V_0^d \equiv V^\dagger \cdot V \,, \tag{14}$$

which give us a measure of the flavour-changing neutral currents (FCNC); the quantity $\sin(2\beta)$, where:

$$\beta \equiv \arg\left(-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right) \;; \tag{15}$$

and the quantity:

$$\gamma \equiv \arg\left(-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right) \;. \tag{16}$$

We present one example of the several scans done. The input is as follows:

Begin	End	Step
$\eta_i = 0.0170$,	$\eta_f = 0.0176$,	$\Delta\eta = 0.0002$,
$\xi_i = 0.00095$,	$\xi_f = 0.00106$,	$\Delta\xi = 0.00002$,
$\alpha_i = 0.120$,	$\alpha_f = 0.127$,	$\Delta\alpha = 0.0002$,
$\beta_i = -0.0335$,	$\beta_f = -0.0324$,	$\Delta\beta = 0.0002$,
$k_i = 160$,	$k_f = 260$,	$\Delta k = 10$,
$f_i = 200$,	$f_f = 300$,	$\Delta f = 10$,
$g_i = 350$,	$g_f = 440$,	$\Delta g = 20$,
$\mu_i = 350$,	$\mu_f = 440$,	$\Delta\mu = 20$.

With this input, 2 352 000 points in the parameter space were analysed and 15 338 points were in agreement with experimental values. Using these points, 2 example plots were drawn, in figures 1 and 2.

We choose one of the points of the parameter space in agreement with the experimental values as an example.

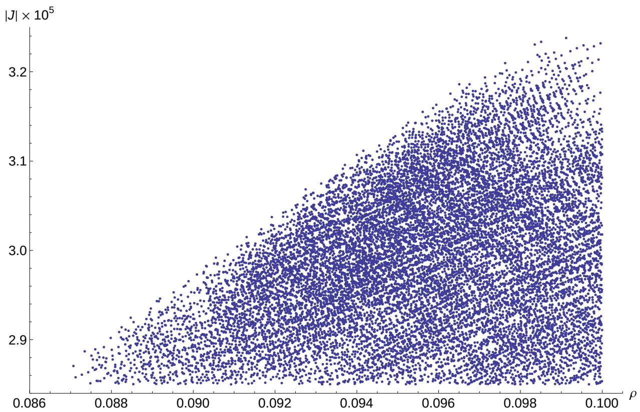

Figure 1: Plot of $|J|$ as a function of ρ.

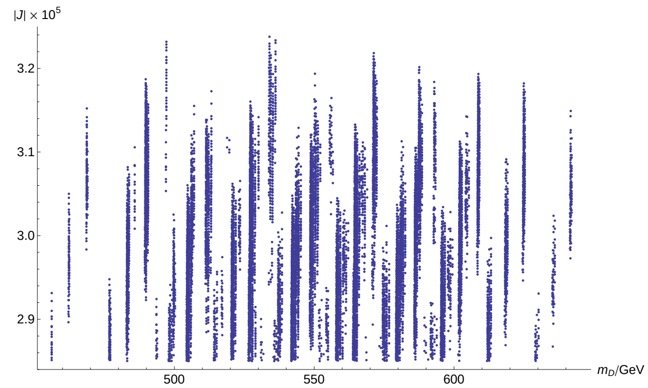

Figure 2: Plot of $|J|$ as a function of m_D.

Input:

$$\eta = 0.019 \qquad k = 155$$
$$\xi = 0.00102 \qquad f = 145$$
$$\alpha = 0.11 \qquad g = 295$$
$$\beta = -0.0305 \qquad \mu = 265.$$

Output at m_Z scale ($\epsilon = 10$):

$$m_u = 1.74 \text{ MeV} \qquad m_d = 3.82 \text{ MeV}$$
$$m_c = 724 \text{ MeV} \qquad m_s = 84.3 \text{ MeV}$$
$$m_t = 181 \text{ GeV} \qquad m_b = 3.00 \text{ GeV}$$
$$m_D = 543 \text{ GeV}.$$

$$|V| = \begin{pmatrix} 0.97506 & 0.22190 & 0.003675 & 0.000034 \\ 0.22173 & 0.97435 & 0.038527 & 0.00014 \\ 0.009557 & 0.037503 & 0.99924 & 0.0038 \end{pmatrix},$$

$$|V_0^u| = \begin{pmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{pmatrix},$$

$$|V_0^d| = \begin{pmatrix} 1.00 & 3.63\epsilon^{-10} & 7.20\epsilon^{-9} & 1.93\epsilon^{-6} \\ 3.63\epsilon^{-10} & 1.00 & 6.73\epsilon^{-7} & 1.80\epsilon^{-4} \\ 7.20\epsilon^{-9} & 6.73\epsilon^{-7} & 0.99999 & 3.74\epsilon^{-3} \\ 1.93\epsilon^{-6} & 1.80\epsilon^{-4} & 3.74\epsilon^{-3} & 1.40\epsilon^{-5} \end{pmatrix},$$

$$\rho \equiv |V_{13}|/|V_{23}| = 0.0954\,, \qquad |J| = 3.05 \times 10^{-5}\,,$$

$$\sin(2\beta) = 0.693\,, \qquad \gamma = 95.0^{\text{o}}\,.$$

Some conclusions can be drawn from these results. If the results are compared with the experimental data (see [9] and PDG), we see that most of the calculated quantities are in very good agreement with their experimental bounds. Furthermore, it is again clear the advantage of having an extra vector-like quark: $|J|$, in addition to $\sin(2\beta)$ and γ, is in good agreement with the experimental value. From $|V_0^d|$, we clearly see that the flavour-changing neutral currents (FCNC) are naturally suppressed in this model.

Finally, note that the fine scan of the parameter space allowed us to find a region in which the mass of the vector-like quark is between approximately 450 and 650 GeV, as can be seen in the figure 2.

4 Conclusion

In this paper, we have investigated the pattern of quark masses and mixings in an extension of the SM with just one vector-like quark, in the framework of USY. We have studied a specific model with the quark content of the SM, plus just a down-type vector-like isosinglet quark. We assumed that the Yukawa couplings between the SM quarks are in the framework of USY, but allowed the remaining couplings involving the vector-like down quark to be general complex numbers.

We have analysed three different ansatz-schemes, which differ in the submatrix of USY form, and ended with one ansatz which, as in the pure USY case, reproduces the phenomenological CKM matrix as a function of the quark masses, but in addition gives significant contributions to CP violation. Furthermore, the flavour-changing neutral currents (FCNC) for this ansatz are naturally suppressed, as expected.

Finally, we performed a fine scan of the input parameter space around the values previously found and we were able to find a region where the computed physical quantities are all in excellent agreement with their experimental values. Additionally, we estimated that the mass of the vector-like quark is between 450 and 650 GeV. This shows that it is possible to formulate a consistent model with only one additional vector-like quark of mass below the TeV range. In Ref. [8], Higuchi, Senami and Yamamoto stated that it would be necessary at least two additional down-type quarks with masses in the TeV range. Note that this is possible because the postulated ansätze have significant differences.

In conclusion, we have proposed an ansatz in the framework of USY for the flavour structure of the quark sector, with an additional vector-like quark, where all the experimental data on quark masses and mixings are reproduced, thus suggesting that significant CP violation effects could come from new Physics beyond the Standard Model.

This work also provides motivation to address the question of finding a symmetry principle which may lead to universal Yukawa couplings and give a theoretical foundation for this hypothesis. Furthermore, since specific extensions of the Standard Model predict extra exotic fermions, such as grand unified theories and extra dimensions, this work showed that, at least at low energies, where only one more quark produces visible effects, those theories can be accommodated within the current experimental limits.

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