MONITORING TECHNIQUES AND INTEGRITY IN GPS

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ABSTRACT

The aim of this work is to study the operation of GPS system, the factors that influence the positioning accuracy obtained by a receiver and the accuracy of information provided by each satellite. Emphasis will be given to algorithms for autonomous monitoring of data integrity for data provided by each satellite, as the resolution of the equation of navigation is compromised if one or more satellites provide incorrect data (time or position).

Keywords
Monitoring Integrity, GPS, RAIM, DGPS, GBAS, SBAS, WAAS, EGNOS.

1. INTRODUCTION

GPS is a radio satellite navigation system developed and operated by the Departments of Defense and Transportation of the United States of America which provides to the user who has proper equipment, accurate coordinates of three-dimensional positioning/navigation information and the time in various situations. Its design allows a user (civil or military) anywhere on the earth's surface or near at any time, at least four visible satellites enabling navigation in adverse weather conditions.

Currently, the status of the satellites is permanently monitored, but can take several minutes until the receiver is informed of any problems. Therefore, the state system should be checked locally by the receiver, making it necessary that the receivers have internal mechanisms to detect the presence of anomalous GPS signals and possibly its identification and elimination from the navigation solution. The methods used to perform locally (at the user level) the detection and identification of anomalous situations are called RAIM (Receiver Autonomous Integrity Monitoring). Two RAIM algorithms are implemented in MATLAB. The residual method (Least-squares-residuals method) is used to check a possible defective satellite and perform the subsequent elimination of the equation of navigation. For comparison, the Range Comparison Method is also implemented.

With the implementation of RAIM algorithms, we can identify inconsistencies in data received in the shortest amount of time to avoid the inclusion of erroneous data in the solution of the navigation equation. It should be noted that in some applications, such as in aviation, an incorrect solution would be catastrophic.

2. BASIC CONCEPTS

The basic elements of GPS are the three segments:

- Space Segment

The space segment consists of all GPS satellites orbiting the Earth. In elliptical orbits with four satellites per orbit separated by 90 degrees, the orbital planes are inclined 55 degrees from the equator. The satellites continuously send information from their positions and GPS time. Originally it was foreseen to 24 satellites at 20,200 km altitude and orbital period of 11 hours and 58 minutes. Each satellite was deployed at 20,200 Km altitude and orbital period of 11 hours and 58 minutes.

- Control Segment

The control segment consists of a main control station and five monitoring stations distributed around the globe: Kwajalein and Hawaii (Pacific Ocean), Diego Garcia (Indian Ocean), Ascension (Atlantic Ocean), and Colorado Springs (USA).

- User Segment

Includes all who have a GPS equipment (portable or fixed receivers) in operating conditions and are at any point in space between the surface and the orbits of satellites.
3. CALCULATING THE POSITION OF THE USER

Imagine that we see four satellites with:

![Diagram of four satellites and a receiver](image)

Figure 1: Principle of positioning.

In order to determine the user position in three dimensions and the receiver’s clock offset, pseudo-range measurements are made to 4 satellites resulting in the system of equations

\[
\begin{align*}
P_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} - cT \\
P_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} - cT \\
P_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} - cT \\
P_4 &= \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} - cT
\end{align*}
\]

where \((x_i, y_i, z_i)\) is the position of satellite \(i\), \((x_u, y_u, z_u)\) is the receiver position, and \(cT\) is an error due to the receiver’s clock bias.

These nonlinear equations can be solved for the unknowns by employing either: the direct method, like the Bancroft algorithm, the linearization least-squares method or Kalman filtering.

4. INTEGRITY MONITORING

Integrity Monitoring is the system's ability to provide flawless data and alerts in time when the system or a particular satellite should not be used. In addition to providing information about the position of the user and time, the GPS must be able to provide warnings about the state of the system, whether it should or not be used. The anomalies of the integrity although rare, occurring only a few times a year, can be critical mistakes leading to important damages and be tragic especially in air navigation, since users are traveling at high speeds and can quickly deviate from the intended route. This function of integrity becomes especially critical if GPS is used as the main navigation system.

It is reported what happened on January 1, 2004 when satellite operators at Schriever Air Force Base, Colorado, found that the positioning errors exceeded the threshold of 30 meters reached 280,000 meters [1]. Only about three hours later the faulty satellite (SVN 23) was identified. This event could have been catastrophic, and marked the second time that a signal has reached such a big error. Please note that only two and a half years earlier, in July 2001, SVN 22 had sent a signal with positioning error of 200 km. This shows that it is extremely important to monitor the integrity.

Six years have passed since the 2004 event and a similar incident did not happen again, because many lessons were learned and GPS users are being better served. Now the operator immediately removes the anomalous satellite service and then investigates the cause. Furthermore, the operations of the control segment have improved with increasing number of monitoring stations and other methods.

The three methods used in monitoring the integrity of GNSS systems are: GBAS (LAAS), SBAS (WAAS, EGNOS, MSAS), RAIM.

4.1. GBAS (Ground Based Augmentation Systems)

An ambitious initiative promoted by the civil aviation FAA consists of developing a GBAS system for use in precision approaches to airports in Categories I / III, including automatic landing. This extension of the GPS is called Local Area Augmentation System (LAAS).

![Diagram of GBAS System](image)

Figure 2: GBAS System

The purpose of LAAS integrity monitoring is to protect against HMI (Hazardously Misleading Information), which is defined as a Navigation System Error (NSE) that exceeds the alert limit while no alert is given within the time to alarm [2]. The requirement is that the probability of such occurrences, called missed detections, is very small: less
than $10^{-7}$ per CAT I approach. The LAAS integrity monitoring system must provide upper bounds on the lateral and vertical position estimation errors, called Lateral protection Level (LPL) and Vertical Protection Level (VPL), each of which must satisfy this requirement. In other words,

- $\text{Prob} \{\text{Vertical Position Error} > \text{VAL} | \text{VPL} < \text{VAL} \} < 10^{-7}$
- $\text{Prob} \{\text{Lateral Position Error} > \text{LAL} | \text{LPL} < \text{LAL} \} < 10^{-7}$

where VAL and LAL represent Vertical Alert Limit and Lateral Alert Limit, respectively.

The LAAS integrity requirements are defined in terms of alert limits, time to alarm and probability of missed detection. Table 1 and Table 2 show the vertical and lateral requirements for CAT I through CAT III precision approaches [2]. It is immediately obvious that the vertical requirements are much more stringent than the lateral requirements.

<table>
<thead>
<tr>
<th>Performance Category</th>
<th>VAL (meters)</th>
<th>Time to Alarm (seconds)</th>
<th>$P_v$(HMI) (per approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAT I</td>
<td>10.2</td>
<td>6</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT II</td>
<td>5.3</td>
<td>1</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT III</td>
<td>4.5</td>
<td>1</td>
<td>$10^{-7}$</td>
</tr>
</tbody>
</table>

Table 1 – Vertical integrity requirements

<table>
<thead>
<tr>
<th>Performance Category</th>
<th>LAL (meters)</th>
<th>Time to Alarm (seconds)</th>
<th>$P_l$(HMI) (per approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAT I</td>
<td>36.5</td>
<td>10</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT II</td>
<td>17.3</td>
<td>1</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT III</td>
<td>15.5</td>
<td>1</td>
<td>$10^{-7}$</td>
</tr>
</tbody>
</table>

Table 2 – Lateral integrity requirements

4.2. SBAS (Satellite Based Augmentation Systems)

SBAS seeks to correct errors in the positioning provided by GPS, thereby improving the positioning accuracy achieved with the system. It consists of a network of reference stations, master stations, telecommunications stations and geostationary satellites. This system has significant applications in operations that require high positioning accuracy (land, sea or air). But as we can see in figure 3 they are not available in all the Earth.

![Figure 3: Availability of SBAS systems](image3)

4.2.1. WAAS (wide area augmentation system)

WAAS corrects the GPS signal errors caused by ionospheric disturbances, the satellite orbit errors and provides vital information about the health of each GPS satellite [3].

![Figure 4: WAAS](image4)

The figure shows the scheme of operation of the WAAS system, where:
1. GPS and GLONASS satellites transmit navigation signals;
2. Users receive navigation signals;
3. The networks of monitoring stations receive the same navigation signals;
4. The central processing station receives signals from the monitoring stations and processes the data to determine the integrity of constellation, that is, if a given satellite should or not be used;
5. Navigation Earth Stations (NES) receive messages from the navigation and information if the system can be used. The navigation signal with spread spectrum is synchronized with the reference time and sent to geostationary satellites;
6. The transmission of the signal and the navigation message from the NES to the geostationary satellite is performed using the C-band;
7. INMARSAT satellite navigation where the signal is heterodyned and relayed to the user;
8. Users receiver the INMARSAT satellite signals at frequency L1;
9. The NES receiver using the C-band;
10. NES secondary station, a priori, is in stand-by. If an abnormality is detected in the primary stations the NES automatically goes into operation.

WAAS allows the use of GPS as primary means of navigation from takeoff until landing in the case of aviation. Other users also benefit from improved accuracy, availability and integrity that WAAS can provide, but it is noteworthy that the WAAS allows only warn the pilot of the aircraft in case of aviation in the range of 6 seconds after the beginning of the anomaly on the satellite which can be catastrophic if the aircraft is in the phase of landing. It is therefore necessary that system integrity is verified locally at the receiver.

### 4.2.2. EGNOS (European Geostationary Navigation Overlay Service)

The EGNOS system was declared operational on 1 October 2009 by the European Commission. it was developed to increase accuracy, availability and integrity of GPS in Europe, also to use the Glonass and Galileo in the future.

EGNOS signals are broadcast by two Inmarsat-3, one over the eastern Atlantic, the other over the Indian Ocean and the third is the ESA Artemis satellite over Africa. There are 40 ground reference stations (RIMS) that monitor the integrity of GPS transmit signals and four main stations generate the control signal with data integrity and corrections of the type WADGPS for Europe and 6 stations for up-link which are distributed throughout Europe. Three geostationary satellites transmit the signal used to model the date that has characteristics similar to the GPS signal. The satellites send back the signals to users. Users can thus benefit from increased availability without the need to purchase a different receiver. The accuracy achieved is about 1m across Europe and this is independent of the distance from the user ground stations.

The structure of the message sent by the EGNOS message is different from the DGPS system since it has yet to integrate information integrity. EGNOS uses the GPS L1 frequency and similar codes. There are 16 types of data messages defined for sending data integrity and corrections WADGPS that have a cycle of 6 seconds in order to prioritize the parameter alarm six seconds integrity and to minimize the startup time of EGNOS.

Information integrity is given at two levels. The First is the indication for using or not using of satellites due to their workload.In the second, two parameters are available \( \sigma_{UDRE}^2 \) and \( \sigma_{UIVE}^2 \), which are statistical estimates of the errors of satellite and air respectively, after application of corrections WADGPS. These parameters are used to obtain an estimate of the position error. It is also noteworthy that EGNOS integrity that allows alerts to warn you within six seconds (the pilot of the aircraft in case of aviation) for problems that are occurring. As stated previously, these errors can be catastrophic if the aircraft is in the act of landing, making it necessary for system integrity to be verified locally in the receiver (Receiver Autonomous Integrity Monitoring).
4.2.3. RAIM

The RAIM is a software function built in GPS receiver designed to provide integrity by detecting failure of a satellite. In this technique the receiver uses the redundancy of measurements from satellites, to make a finding of self-consistency between the measurements available, thereby determining the existence of a fault condition that can lead to errors in position outside the specified limits.

The RAIM may offer several modes of operation:

1. Fault Detection (FD)

Compares the position of the receiver and the clock delay resulting from the combination of four satellites in a series of at least five satellites. Thus, a faulty satellite can be detected (but not identified) and the user has a warning that the system must not longer be used for navigation. There are generally two types of RAIM. One type indicates that there are not enough satellites to provide integrity monitoring and the other indicates that the RAIM integrity monitoring has detected a potential error that exceeds the limit. Without RAIM capability, the user is not sure of the accuracy of the GPS satellites.

The FD technique is exemplified for a set of five satellites \( \{A, B, C, D, E\} \). Suppose that there are five satellites, the number of combinations 4 to 4 is \( C_5^4 = 5 \), and the possible combinations are

\[ A, B, C, D \]
\[ A, B, C, E \]
\[ A, B, D, E \]
\[ A, C, D, E \]
\[ B, C, D, E \]

Solving the equation of navigation for each of these combinations there is a failure of such consistency. In fact, the only combination that leads to an approximately correct result is the last (the one that excludes the satellite anomaly).

The receiver can not, however, determine which of the five solutions is correct, this RAIM technique allows detecting the existence of a satellite anomaly, but can not identify it.

2. Fault Detection and Exclusion (FDE)

The FDE technique uses six or more visible satellites, not only to detect a satellite failure, but also to remove it from of the navigation solution and continue to provide a navigation solution with the remaining satellites. The FDE algorithm consists of two functions: fault detection and fault exclusion. The purpose of fault detection is to verify the information resulting from the combination of five satellites of a set of at least six satellites. Generally, only one satellite, the most likely cause of error would be identified and eliminated (fault exclusion). However it’s possible that more than one satellite may be the source of failure.

The FDE technique is exemplified, for a set of six satellites \( \{A, B, C, D, E, F\} \). Suppose that all satellites are in normal operating mode, except that satellite \( A \) presents an anomaly. Since there are 6 satellites, six subgroups of 5 satellites are formed; that is, the number of combinations of satellites is \( C_6^5 = 6 \), in which the possible combinations are

\[ A, B, C, D, E \]
\[ A, B, C, D, F \]
\[ A, B, C, E, F \]
\[ A, B, D, E, F \]
\[ A, C, D, E, F \]
\[ B, C, D, E, F \]

It then examines each of these subsets in terms of consistency of the navigation solution. For example, consider the first of the sub sets \( \{A, B, C, D, E\} \) shows that the various solutions obtained with four satellites are not consistent. Thus, the solution obtained with the satellites \( \{A, B, C, D\} \) will be different from the solution that uses satellites \( \{B, C, D, E\} \), etc. The same behavior occurs for the following four subsets of five satellites. Having a satellite anomaly, only the last of the subsets, this is, \( \{B, C, D, E, F\} \) shows consistency. Thus, the satellite to be deleted is not present in this subset satellite (satellite \( A \)).

We will describe next two methods of RAIM [4]: the residual method proposed in [5] and the range comparison method suggested in [6].
4.2.3.1. METHOD OF RESIDUALS

Imagine that we see six satellites. The navigation problem is to solve a system of six equations and four unknowns. Now suppose that we solve the equations by the least squares and obtain a solution that satisfies the six equations as the solution of linear equation of GPS measurements:

$$\Delta P = H \Delta x + \varepsilon$$

where $H$ is a matrix of six rows and four columns and $\varepsilon$ is the vector of measurement errors that can contain both deterministic and random terms. The solution of least squares is:

$$\Delta \hat{x}_{ls} = (H^T H)^{-1} H^T \Delta P$$

Now use the solution of least squares to obtain an estimate of prediction $\Delta P$ according to:

$$\Delta \hat{P}_{ls} = H \Delta \hat{x}_{ls}$$

The differences between the pseudo-distances and estimated incremental steps are the vector of residuals [4] (see Figure 7)

$$w = \Delta P - \Delta \hat{P}_{ls}$$

Figure 7: Determination of the vector of residuals

The sum of squared residuals can be obtained through

$$SSE = w^T w$$

where SSE is a scalar nonnegative. This method can only be used with $n \geq 5$ visible satellites.

To test statistic it is convenient to use another scalar variable related to SSE [4]:

$$t = \sqrt{\frac{SSE}{n - 4}}$$

The fault detection is based on a hypothesis test in which the decision variable $t$ is tested for an alert threshold $\lambda$. The decision criterion is

$$\begin{cases} t \geq \lambda \rightarrow \text{fault} \\ t < \lambda \rightarrow \text{no-fault} \end{cases}$$

The test performance is characterized by the probability of false alarm ($P_{fa}$) and the probability of missed detection ($P_{md}$) “missed detection”. The probability of false alarm is given by

$$P_{fa} = \int_{\lambda}^{\infty} p_t(r) \cdot dr = 1 - \int_{0}^{\lambda} p_t(r) \cdot dr$$

Where $p_t$ is the probability density function of $t$.

Table 3 shows the normalized threshold $\lambda/\sigma$ for $P_{fa} = 1/15000$.

<table>
<thead>
<tr>
<th>number of satellites</th>
<th>degrees of freedom</th>
<th>$\lambda/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>3.99</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3.10</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>2.71</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>2.47</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>2.31</td>
</tr>
</tbody>
</table>

Table 3 – Threshold values normalized

Thus, for example, for $n = 6$ satellites and $\sigma = 8$ meters, we obtain $\lambda = 3.99 \sigma \approx 32$ meters.

Figure 8: Probability density functions $p_t(r)$ (no-fault) and $\tilde{p}_t(r)$ (fault)

The probability of failing to detect a situation with anomaly is given by
\[ P_{md} = \int_{0}^{2} \tilde{p}_i(r) \cdot dr \]

Where \( \tilde{p}_i \) is the probability density function of \( t \) when there is an anomalous satellite.

### 4.2.3.2. RANGE COMPARISON METHOD

Imagine that we are viewing \( n \geq 5 \) satellites. So we would have 4 equations and four unknowns. Suppose we solve the first 4 equations and obtain a solution that satisfies these equations. The outcome of the four equations can be used to predict the remaining measurements of pseudo-ranges, and the predicted values can then be compared with results measured currently.

Suppose that the subset of satellites used to solve the equation of navigation is

\((A, B, C, D)\)

Once the navigation solution is obtained, we determine the pseudo-ranges for the two remaining satellites \((E, F)\). The residuals vector \((w_1, w_2)\) will probably be a point outside the decision boundary.

The integral of \( p_w (w_1, w_2) \) over an area \( A \) bounded by the ellipse

\[
\left( \frac{w_1}{\sigma_1} \right)^2 - \frac{2Pw_1w_2}{\sigma_1\sigma_2} + \left( \frac{w_2}{\sigma_2} \right)^2 = \alpha^2 (1 - \rho^2)
\]

Is given by [3]

\[
\iint_A p_w (w_1, w_2) dw_1 dw_2 = 1 - \exp \left( -\frac{\alpha^2}{2} \right)
\]

We look for a decision rule that divides the plane into two distinct regions: one corresponding to the “no failure” hypothesis and the other corresponding to the “failure”

![Regions of decision for six visible satellites](image)

Figure 9: Regions of decision for six visible satellites [9]

Under these conditions the presence of an anomaly in one of the six satellites is detected but the faulty satellite cannot be immediately identified.

Next step consists of generating all combinations of five satellites and apply to each combination of five satellites the method of range comparison. Accordingly, only the combination of satellites \((B, C, D, E, F)\) produces a result indicating absence of failure, which identifies the satellite A as the satellite being damaged.

The probability of false alarm is

\[
P_{fa} = 1 - \iint_A p_w (w_1, w_2) dw_1 dw_2 = \exp \left( -\frac{\alpha^2}{2} \right)
\]

where \( p_w (w_1, w_2) \) is joint probability density function of vector \((w_1, w_2)\) in the absence of a satellite anomaly.

The probability of failing to detect the anomaly is given by:

\[
P_{md} = \iint_A p_w (w_1, w_2) dw_1 dw_2
\]

where \( p_w (w_1, w_2) \) is the probability density function of vector \((w_1, w_2)\) in the presence of a defective satellite.

In general, this expression is difficult to evaluate. However, approximate results can be obtained through Monte Carlo Simulation.
5. ANALYSIS OF THE RESULTS

From the study of GPS you can learn how to determine the position of the user involving four unknowns, which are the three physical dimensions (X, Y, Z) and the time shift of the user. In Table 4 we can see that the integrity provided by the GPS system is very bad, since you can spend 1 hour or more, to inform users that the system should not be used. On the other hand illustrates very well the responsiveness when using the GPS with the aid of technical integrity, especially with RAIM. It also shows the improvement in monitoring the integrity when using two global systems, as in the case of GPS + GLONASS, despite being far from the expected values for use in aviation and other civil services. One of the main advantages of using satellite signals in addition to GPS + GLONASS is the increased number of available satellites.

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>FOC</th>
<th>SCOPE</th>
<th>INTEGRITY</th>
<th>HORIZONTAL ACCURACY (95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>1993</td>
<td>Global</td>
<td>1 a 2 horas</td>
<td>&lt; 33 metros</td>
</tr>
<tr>
<td>DDGPS</td>
<td>-</td>
<td>Local</td>
<td>&lt; 5 seg</td>
<td>0.5 – 5 metros</td>
</tr>
<tr>
<td>GPS-WAAS</td>
<td>2003</td>
<td>Regional</td>
<td>&gt; 6 seg</td>
<td>1 – 3 metros</td>
</tr>
<tr>
<td>GPS+EGNOS</td>
<td>2009</td>
<td>Regional</td>
<td>&gt; 6 seg</td>
<td>1.5 metros</td>
</tr>
<tr>
<td>GPS-VMSAS</td>
<td>2009</td>
<td>Regional</td>
<td>&gt; 6 seg</td>
<td>&lt; 5 metros</td>
</tr>
<tr>
<td>GPS+GLONASS</td>
<td>2010</td>
<td>Global</td>
<td>1 hora</td>
<td>8 – 16 metros</td>
</tr>
<tr>
<td>GPS+RAIM</td>
<td>2010</td>
<td>Global</td>
<td>&lt; 1 seg</td>
<td>&lt; 33 metros</td>
</tr>
<tr>
<td>GPSW+EGNOS+RAIM</td>
<td>2010</td>
<td>Regional</td>
<td>&lt; 1 seg</td>
<td>1 – 3 metros</td>
</tr>
<tr>
<td>GALILEO+EGNOS</td>
<td>2013</td>
<td>Regional</td>
<td>&gt; 6 seg</td>
<td>1 metro</td>
</tr>
</tbody>
</table>

Table 4 – Integrity monitoring of navigation systems for satellites [8]

Using only the GPS, the number of visible satellites is between 5 to 11 with an average of about 7 visible satellites. The following chart shows the influence that the number of visible satellites may have. The values of parameters DOP (Dilution of Precision) depend strongly on the geometry and number of visible satellites (see figure 11).

Figura 10: DOP parameters vs Number of Satellites

The greater the number of visible satellites the most likely is to find satellites far apart from each other, which corresponds to low values of the geometric dilution of precision GDOP (good geometry).

Figures 11 and 12 show the need for a careful choice of the threshold $\lambda$, since the objective is to minimize simultaneously the two occurrences: false alarm and missing fault detection. The alarm limit represents the maximum error value permitted in the positioning of the user before an alarm is triggered. This alarm limit depends on assumptions made by the operator, and each user is responsible for his determination to possess the integrity necessary for an operation using the information received from the GPS.
6. SIMULATION RESULTS FOR SIX SATELLITES

- False Alarm

Occurs when the constellation of satellites in use has no defective satellite but the threshold is reached. The choice of threshold depends on the scope and the criteria required, but a very small value reduces the probability of a failure in the constellation not being detected and increases the probability of false alarm. Consider the following: \( n = 6 \) satellites, \( \sigma = 8m \) and \( P_{fa} = 10^{-2} \). The corresponding threshold is:

\[
\lambda = \sigma \sqrt{\ln(100)} = 17.17 \text{m}
\]

and the probability of missed fault detection is

\[
P_{md} = 1 - Q(7.071, 3.035) \approx 0
\]

where \( Q(, \lambda) \) is the Marcum Q function.

These theoretical results were confirmed by Monte Carlo simulation. We used \( 10^4 \) independent trials having obtained the results in Table 5.

<table>
<thead>
<tr>
<th>Number of runs</th>
<th>( \lambda / \sigma )</th>
<th>Number of false alarms</th>
<th>Experimental ( P_{fa} )</th>
<th>Number of missed detections</th>
<th>Experimental ( P_{md} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>2.146</td>
<td>104</td>
<td>0.0104</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5 – Simulation results using the method of residuals and \( P_{fa} = 0.01 \)

Note that the probability of false alarm approaches the proposed theoretical value (\( P_{fa} = 0.01 \)). The number of false alarms obtained (104) is sufficient to guarantee a good approximation between experimental and theoretical probabilities.

- Detection failure

Occurs when the constellation of satellites in use has a defective satellite but the threshold is not violated. This occurs when the sum of the squared residuals is smaller than the level of protection. Now consider the following values: \( n = 6 \) satellites, \( \sigma = 8m \) and \( P_{md} = 10^{-2} \). It is further assumed that \( \beta / \sigma = 5 \). The following threshold is obtained theoretically:

\[
\lambda = 27.32 \text{m}.
\]

and the associated probability of false alarm is

\[
P_{fa} = 1 - 2 \int_0^{\lambda / \sigma} x \cdot \exp(-x^2) \, dx = \exp\left(-\frac{\lambda^2}{\sigma^2}\right) = 8.6 \times 10^{-6}
\]

Theoretical results were confirmed by Monte Carlo simulation. For independent trials the following results were obtained the results in Table 6. It may be noted that the probability of missed detection approaches the theoretical value proposed (\( P_{md} = 10^{-2} \)).
Table 6 – Simulation results using the method of residuals and $P_{md} = 0.01$

<table>
<thead>
<tr>
<th>Number of runs</th>
<th>$\lambda / \sigma$</th>
<th>Number of missed detections</th>
<th>Experimental $P_{md}$</th>
<th>Number of false alarms</th>
<th>Experimental $P_{ph}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>3.415</td>
<td>95</td>
<td>0.0095</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

The aim of this work was to study the GPS navigation system and analyze of GPS monitoring techniques for signal integrity. Particular emphasis was given to developing a RAIM algorithm. It began with the implementation of an algorithm for determining the GPS satellite constellation, generating the sequence of pseudo-distances, solving the equation of navigation using the method of least squares and finally applying RAIM algorithms, namely the residuals and the range comparison methods.

The method of residuals uses the estimated position obtained from all satellites in view to predict the pseudo-ranges. The residuals are calculated and the sum of squares is used as the test statistic.

The method of range comparison uses four pseudo-ranges to estimate the position, followed by the prediction of pseudo-ranges of the remaining satellites in view. The differences (residuals) between the measured and predicted pseudo-ranges are combined in a statistical test.

The initial goal was achieved and allows you to test and discuss the results provided by the program for determining the status of the GPS system in a period of time compared to GPS and SBAS operational standard.

8. SUGGESTIONS FOR FUTURE WORK

- To study RAIM applications in urban areas where the angles of the mask are of the order of 30 degrees, considering the local effects such as multipath, examining whether these would be the main source of errors and therefore the main threat to the accuracy and integrity. Check if the conventional notion is still valid measurement defective satellite (errors of pseudo-distances) and large measurement error;

- To analyze the problem of integrity combining GPS and Galileo. This topic raises new challenges such as the need to increase the minimum number of visible satellites so that you can still use the RAIM techniques, originally developed for the exclusive use of GPS satellites [10]. In principle, the number of satellites in view has to be increased due to the fact that GPS and Galileo have different frames of time. It is, however, under study the possibility that both GPS and Galileo broadcast the time shift GPS-Galileo called GGTO [11].

BIBLIOGRAPHY


