Development of a Hill-Type Muscle Model With Fatigue for the Calculation of the Redundant Muscle Forces using Multibody Dynamics

André Ferro Pereira
Instituto Superior Técnico
Universidade Técnica de Lisboa
andre.ferro.pereira@ist.utl.pt

Abstract

The aim of this work is to develop a versatile muscle model and robustly implement it in an existent multibody system dynamics code with natural coordinates. Two different models are included: the first is a Hill-type muscle model that simulates the functioning of the contractile structures both in forward and in inverse dynamic analysis; the second is a dynamic muscular fatigue model that considers the force production history of each muscle and estimates its fitness level using a three-compartment theory approach and a physiological muscle recruitment hierarchy.

The existent equations of motion formulation is rearranged to include the referred models using the Newton's method approach. This allows, in a forward dynamics perspective, for the calculation of the system's motion that results from a pattern of given muscle activations, or, in an inverse dynamics perspective, for the computation of the muscle activations that are required to produce a prescribed articular movement. In both perspectives the system presents a redundant nature that is overcome in the latter case using an SQP optimization algorithm.

The methodologies and models are applied to several case studies to evaluate their robustness and accuracy. An upper extremity model with seven muscles is designed to evidence the effectiveness of the implementation of a muscle fatigue model in a multibody system. A second model, encompassing the lower extremity musculoskeletal apparatus with twelve muscles, is proposed for the exclusive calculation of muscle activations. The results are presented and conclusions are discussed. The work concludes with a perspective of possible future developments.

1 Introduction

The field of biomechanics, *i.e.*, the scientific domain that concerns to the mechanics of living systems, has suffered dramatic developments in terms of its relevant applications. This is a corollary of the rapid development that computer machines have exhibit in the last few decades. Multibody dynamics systems had therefore emerged, in pair with its adaptation to biomechanical systems, with significance for the musculoskeletal complex of the human body [1].

In the human body, this terminology can be adapted to the kinetics of body articulations with the existence of bone stress, joint reactions or even muscle forces. To implement such a template, some existent mathematical models that rule the behaviour of muscle activation and contraction are practicably employable. These have the potential of calculating the force output of the contractile structures of a determined muscle, knowing some physiological and geometrical parameters. In the same way, other models predict the fatigue state of a muscle, for a given developed force history and can be included in a multibody system. The applications of such a tool are undeniably appealing, regarding its competence for calculating, in a non-invasive manner, muscle efforts, neural motor activations or even how physically exhausted is a person after a established activity. This type of formulation has a major importance for medical applications in rehabilitation prosthetics [2, 3], gait disorders [4], sports [5], study of muscle

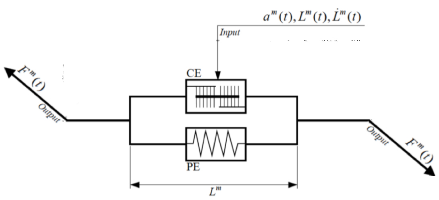


Figure 1: The used mathematical Hill-type model [1].

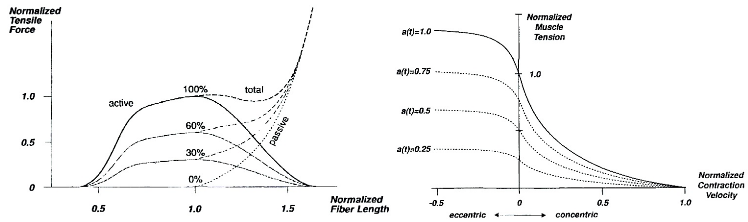


Figure 2: Activation scaling evidence: Force-length and force-velocity relationships for different levels of muscle activation $a(t)$ [10].

diseases [6] and equipment ergonomics [7] where its application can be specifically adapted to a certain subject by adjustment of several anthropometrical and physiological parameters.

The aim of this work is to develop a versatile muscle model and robustly implement it the existent multibody system dynamics code with natural APOLLO [1], in the interest of allowing the inclusion of muscle actuators and incorporate its characteristics to the mechanical system by implementing a muscle model accountable for the inherent contraction and fatigue dynamics. The final scheme must be capable of calculating either the muscle activations for a given kinematics or the dynamics response of the system when the muscle activations are prescribed, with the option of considering muscle fatigue. Therefore, it is desired to use a versatile formulation, that will allow the operation in both forward and inverse dynamics paradigms. An important point of any inverse dynamics system that deals with redundant muscle forces is the optimization process, that will select one of the infinite available combinations of muscle forces that justify a certain kinematic condition.

2 Dynamics of the Muscle Tissue

In what muscle tissue modelling is concerned, two major types of dynamics are usually identified: Activation Dynamics and Contraction dynamics. The former describes the conversion of a CNS neural signal $u(t)$ into a muscle tissue activation state $a(t)$. The latter correlates $a(t)$ with muscle force development. A novel model to multibody methodologies is introduced: a muscle fatigue dynamics model.

Contraction Dynamics

Archibald Hill introduced an adaptation of the Kelvin model, including an additional contractile element [8] that is able to simulate the active macroscopic action of the cross-brige cycle theory of skeletal muscle production described by Huxley [9]. Hill-type models are widely used in the biomechanics field to reproduce both contractile and passive behaviour of muscle tissue, since these have accessible parameters and they are computationally tractable for systems with several muscles. The model used in this work consists in an arrangement of Hill's classic model. It is composed by a passive element (PE) that takes the non-linear passive elastic properties of the muscle tissue and a contractile element (CE) that accounts for both the contractile structures and the viscous force produced by intercellular and intercellular fluids enclosed in the muscle [1]. It is illustrated in Figure 2 and the mathematical expression of the exerted muscle force will straightforwardly be

$$F^m = F^m_{CE} + F^m_{PE} = F^m_{PE} + \hat{F}^m_{CE} a^m \tag{1}$$

Where F^m_{CE} and F^m_{PE} are respectively the forces developed by the contractile and passive elements. $\hat{F}^m_{CE}$ corresponds to the available contractile force and a^m to the muscle activation (the rate of fiber in contraction). There are three key properties of muscles. The first one states that it can only produce tensile forces. Even in the possibility of muscles being able to push, tendons would buckle, cancelling that effect [10]. The other two properties concern the evolution of muscle forces with fiber length and fiber velocity. The contractile force depends the these two properties, *i.e.*, it will enclose in its expression dependencies of length $L^m(t)$ and velocity $\dot{L}^m(t)$.

$$F^m_{CE}(a(t), L^m(t), \dot{L}^m(t)) = \frac{F^m_L(L^m(t))F^m_{\dot{L}}(\dot{L}^m(t))}{F^m_0} a(t) \tag{2}$$

A passive force emerges from the length evolution of muscle fibers. It is though [11] that this force is due to intrafiber elasticity. For muscle lengths larger than the resting length, this tensions become apparent, reaching values larger than F^m_0 – Figure 2 For the analytical expressions and further details about this implementation, please refer to the following references: [1, 12, 13].

Fatigue dynamics

Muscle fatigue is physiological understood by the consumption of intramuscular glycogen (where glucose is stored) and ATP [14]. In addition, the conditions of neural signal transmission are also altered after extended muscle action narrowing its output [14]. Initial empirical models were developed by Rohmert [15], later overpowered by new theoretical ones that introduced a collection of physiological parameters and fatigue dynamic laws, such as the ones presented in Ding el al. [16]. Despite the precision of these models, their complexity and numerous parameters lead them to be computationally disadvantageous when analysing multiple muscles.

New theoretical models derive the muscle force function from simple biophysical principles. This work uses an example of these new theoretical models, adapted from the work by Xia and Law [17]. The principle of this model derives from the MU-based fatigue model proposed by Liu et al. [18] that considers the fatigue fitness level of a muscle as a dynamic phenomenon. Muscle units are considered to be ideally activated, ideally resting of ideally fatigued. Physiologically it is never expectable to have a MU either fully fatigued or fully comprised in one of the other states. However, this model gives us the whole muscle fitness state by the mixture of the set of MUs that form it and therefore mathematically it will be the sum of the compartment proportions.

Muscle units exerting maximum force will fall in the activated compartment M_A and the ones with zero tension will be denoted to the fatigued compartment M_F. The compartment of the resting MUs is symbolised as M_R – Figure 3. This concept is known as compartment theory, which as been applied in a diverse set of scientific areas, namely substance transport modelling and chemical reaction phenomena modelling [17].

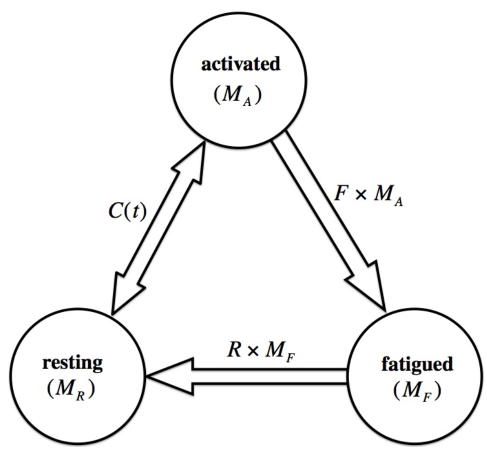


Figure 3: Three-compartment theory flowchart [17].

Liu's model is based in simple biophysical principles and solely considers three parameters: a fatigue factor F, a recovery factor R and the total number of motor units in the muscle; and an input factor describing the brain effort [18]. The BE variable plays an important role in this model, since it is analogous to the muscle activation a^m previously mentioned. Note that this model becomes appropriate for fitting experimental data, due to the few parameters that are involved. However this model fails to consider conditions of non-constant muscle efforts, which is a major disadvantage in our work, as it involves a non-linear application of muscle forces to a multibody system. Xia and Law [17], in response to this downside, developed a model that considers peripheral muscle fatigue for both plain and complex exercises for fluctuating muscle force intensities. The same compartment theory model of Liu's was taken into consideration. For model flexibility purposes [17], the quantity of muscle fibers set in a particular compartment are given in percent of maximum voluntary contraction (%MVC). In the beginning of the exercise, it is considered that every muscle fiber is resting, *i.e.*, it is assumed that $M_R = 100\%$. The available contractile force will then be limited to the fitness state of the whole muscle. The MUs that are available for force production will be the ones laying in the activated and resting compartments, *i.e.*, the muscle available contractile force $\hat{F}^f_{CE}$ will be the product of $\hat{F}^m_{CE}$ with the residual capacity RC and the muscle activation $a(t)$

$$\hat{F}^f_{CE} = RC(t) \times \hat{F}^m_{CE} \tag{3}$$
$$RC(t) = M_A + M_R = 100\% - M_F. \tag{4}$$

The flow of MUs between compartments used by Xia and Law, *i.e.*, the dynamic process of muscle fatigue, depends on the same paramenters from Liu's model: the fatigue coefficient F and the recovery coefficient R, as indicated in Equations 5 to 7. These differential equations dictate the percentage of muscle fibers that are transferred between compartments and used when a force solicitation is made, updating the fitness state of the muscle structure. Accordingly:

$$\frac{dM_R}{dt} = -C(t) + R \times M_F(t) \tag{5}$$
$$\frac{dM_A}{dt} = C(t) - F \times M_A(t) \tag{6}$$
$$\frac{dM_F}{dt} = F \times M_A(t) - R \times M_F(t) \tag{7}$$

where the muscle activation–deactivation driving controller $C(t)$ is the term that gives this model competence to process the dynamics of muscle fatigue with variable efforts. The rate of resting MUs in Equation (5) is reduced by $C(t)$ and increased by a multiplication factor between the recovery coefficient R and the amount of fatigued MUs M_F, a term that expresses the amount of fatigued fibers that recovered in the course of the muscle effort history. Equation 6 dictates the evolution of active muscle fibers and has a similar behaviour to the previous one. This term will increase with the diving controller $C(t)$ and decrease with the number of freshly fatigued fibers (the multiplication between the fatigue coefficient F and the active fibers M_A). The rate of fatigued units, as indicated in Equation (7), is prescribed by the difference between the amount of recently fatigued fibers and the the amount of fatigued fibers that just recovered.

To define the driving controller $C(t)$, stated in Equation 8, Xia and Law added two additional parameters L_D and L_R, respectively the muscle force development and relaxation factors, whose values are not of major relevance since these were added merely to ensure good system behaviour [17]. In Xia and Law's model, $C(t)$ is a bounded controller that depends on the relation between the the compartments' state and a target load TL, *i.e.*, the force that the muscle is required to exert. In this work the target load is referred as the instantaneous solicited contractile muscle force F^m_{CE}, since Xia and Law assume that the neuromuscular system can produce the required force [17], *i.e.*, the TL is limited by the available contractile force. Moreover, the model used in this work assumes that the passive element force contribution does not play a part in the fatigue process, therefore it is not considered for the calculation of $C(t)$.

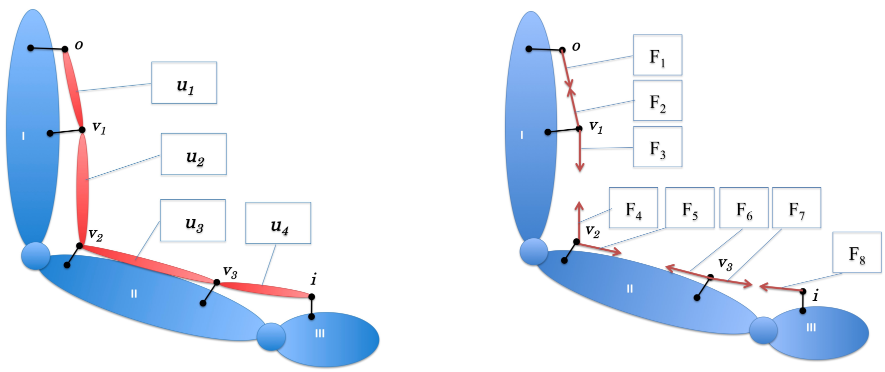
$$C(t) = \begin{cases} L_R \times (F^m_{CE} - M_A \times \hat{F}^m_{CE}) & F^m \leq M_A \hat{F}^m_{CE} \\ L_D \times (F^m_{CE} - M_A \times \hat{F}^m_{CE}) & M_A \hat{F}^m_{CE} < F^m \leq (M_A + M_R)\hat{F}^m_{CE} \\ L_D \times M_R \times \hat{F}^m_{CE} & (M_A + M_R)\hat{F}^m_{CE} < F^m \end{cases} \tag{8}$$

Since F, R, L_D and L_R are the only parameters under consideration, then it becomes accessible to infer expressions that express these factors by experimental data fitting, granting a relevant flexibility and applicability to this model.

3 Multibody

The formulation presented in this work employs fully Cartesian (or natural) coordinates as introduced by Jalon and Bayo [19] and applied in Silva [1]. The system is defined as the set of Cartesian coordinates of the points and vectors that define the body elements, without making use of angular variables.

In order to insert a muscle model into the existing multibody routines, its data management must be carefully defined and implemented. Muscle structures are spatially defined by the coordinates of their origin (subscript o), insertion (subscript i) and eventual via-points (subscripts v_1, v_2, ... , v_{vp}, with vp as the number of via-ponts). This representation, for a muscle 3 via-points applying a force F^m, is shown in Figure 4 (a). The direction vectors defined by the muscle segments ($\mathbf{u}_1$, $\mathbf{u}_2$, ... , $\mathbf{u}_d$) define the orientation of the forces applied by the muscle structure, *i.e.*, for a certain application point p $\mathbf{F}_p^m = \mathbf{u}_{d'} F^m$. The correspondent arrangement is illustrated in Figure 4 (b). Note that some important geometrical assumptions are made, that do not correspond to the muscle anatomy of humans: muscles have rectilinear orientations, constant cross-sectional area and no wrap around structures in its via-points.



(a) Model direction for 3 via-points. (b) Correspondent muscle force representation.

Figure 4: Muscle representation for a biomechanical system.

To adapt these physical quantities to the numerical nature of the multibody system, the matrix $\mathbf{C}_p$ must be considered. Matrix $\mathbf{C}_p$ is responsible to express the Cartesian coordinates of any given point p, belonging to a rigid body e, as a linear combination of the generalised coordinates used to describe that element.

$$\mathbf{r}_p = \mathbf{C}_p \mathbf{q}_e. \tag{9}$$

This transformation matrix can be applied in the domain of concentrated forces. Therefore the expression for the transformation of a Cartesian representation of forces $\mathbf{F}_p^m$ in its generalised configuration $\mathbf{g}_e^{F^m}$:

$$\mathbf{g}_e^{F^m} = \mathbf{C}_p^T \mathbf{F}_p^m = \mathbf{C}_p^T \mathbf{u}_{d'} F^m. \tag{10}$$

Since each generalised force vector $\mathbf{g}_e$ is specific for a certain body (the body containing the point where the force is applied), then a particular muscle framed in nb bodies will have nb different $\mathbf{g}_e$ to be added to the equations of motion. The exemplifying muscle in Figure 4 is arranged by three different bodies I, II and III, and therefore will have three independent $\mathbf{g}_e$ force vectors:

$$\mathbf{g}_e^I = \left[(\mathbf{C}_o^T - \mathbf{C}_{v_1}^T)\mathbf{u}_{ov_1} + \mathbf{C}_{v_1}^T \mathbf{u}_{v_1 v_2}\right] F^m \tag{11}$$

$$\mathbf{g}_e^{II} = \left[-\mathbf{C}_{v_2}^T \mathbf{u}_{v_1 v_2} + (\mathbf{C}_{v_2}^T - \mathbf{C}_{v_3}^T)\mathbf{u}_{v_2 v_3} + \mathbf{C}_{v_3}^T \mathbf{u}_{v_3 i}\right] F^m \tag{12}$$

$$\mathbf{g}_e^{III} = \left[-\mathbf{C}_i^T \mathbf{u}_{v_3 i}\right] F^m \tag{13}$$

Having a numerical representation of all the forces of the muscle, it is now possible to assemble $\mathbf{g}_e^I$, $\mathbf{g}_e^{II}$ and $\mathbf{g}_e^{III}$ in the vectors of generalised muscle forces expressed in the general coordinates of the system $\mathbf{g}^I$, $\mathbf{g}^{II}$ and $\mathbf{g}^{III}$. Since these are all not specific to the bodies, but to the multibody system, they are summable, and therefore the assembling of all muscle forces in terms of the system general coordinates $\mathbf{g}^{F^m}$ is given as

$$\mathbf{g}^{F^m} = \mathbf{g}_I + \mathbf{g}_{II} + \mathbf{g}_{III} \tag{14}$$

This term can be divided in the muscle model components, in the same manner as in Equation (1):

$$\mathbf{g}^{F^m} = \mathbf{g}_{PE}^{F^m} + \mathbf{g}_{CE}^{F^m} = \mathbf{g}_{PE}^{F^m} + \hat{\mathbf{g}}_{CE}^{F^m} a \tag{15}$$

where $\mathbf{g}_{CE}^{F^m}$ and $\mathbf{g}_{PE}^{F^m}$ are respectively the generalised force vectors for the contractile and passive elements contributions, and $\hat{\mathbf{g}}_{CE}^{F^m}$ corresponds to the generalised force vector of the maximum available contractile force, *i.e.*, the generalised representation of $\hat{F}_{CE}^{m}$. Considering now a musculoskeletal multibody system with nm muscles, it comes that the generalised external forces $\mathbf{g}$ is giver by:

$$\mathbf{g} = \mathbf{g}^{ext} + \mathbf{g}^{M_1} + \mathbf{g}^{M_2} + \ldots + \mathbf{g}^{M_{nm}} \tag{16}$$

where $\mathbf{g}^{ext}$ is the remaining external forces (excluding muscle forces). Including this expression in the equations of motion of a multibody dynamics system [1, 19] comes that

$$\begin{gathered} \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \boldsymbol{\Phi}_{\boldsymbol{q}}^{\boldsymbol{T}}\boldsymbol{\lambda} = 0 \\ \mathbf{M}\ddot{\mathbf{q}} - (\mathbf{g}^{\mathbf{ext}} + \mathbf{g}^{\mathbf{M_1}} + \mathbf{g}^{\mathbf{M_2}} + \ldots + \mathbf{g}^{\mathbf{M_{nm}}}) + \boldsymbol{\Phi}_{\boldsymbol{q}}^{\boldsymbol{T}}\boldsymbol{\lambda} = 0 \end{gathered} \tag{17}$$

Taking the equations of motion into the paradigm of inverse dynamics, the analysis will be performed knowing the anthropometric data (specified by the mass matrix $\mathbf{M}$ and the Jacobian $\boldsymbol{\Phi}_q$), the system motion (given by the vector of generalised accelerations $\ddot{\mathbf{q}}$), the muscle passive and available contraction forces (respectively $\mathbf{g}_{PE}^{m}$ and $\hat{\mathbf{g}}_{CE}^{m}$) and the remaining external forces and velocity-dependent inertial forces (available in $\mathbf{g}^{ext}$). The unknowns for this analysis, and therefore our output, are the Lagrange multipliers column vector $\boldsymbol{\lambda}$, that will give us the forces associated with each degree-of-freedom of the system, and the muscle activations a^{M_1} to $a^{M_{nm}}$. According to Equation (15) the equations of motion can be rearranged in the following manner, using the Newton's method [1, 19]:

$$\begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{q}}^{T} & -\hat{\mathbf{g}}_{CE}^{M_1} & \cdots & -\hat{\mathbf{g}}_{CE}^{M_{nm}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\lambda} \\ a^{M_1} \\ \cdots \\ a^{M_{nm}} \end{Bmatrix} = \mathbf{g}^{ext} + \mathbf{g}_{PE}^{M_1} + \ldots + \mathbf{g}_{PE}^{M_{nm}} - \mathbf{M}\ddot{\mathbf{q}} \tag{18}$$

or in a more compact fashion

$$\begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{q}}^{T} & -\boldsymbol{\chi}^{T} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix} = \mathbf{g}^{ext} + \mathbf{g}_{PE}^{M_1} + \ldots + \mathbf{g}_{PE}^{M_{nm}} - \mathbf{M}\ddot{\mathbf{q}} \tag{19}$$

with

$$\chi = \begin{bmatrix} \hat{\mathbf{g}}_{CE}^{M_1} \\ \cdots \\ \hat{\mathbf{g}}_{CE}^{M_{nm}} \end{bmatrix} \text{ and } \mathbf{a} = \begin{bmatrix} a^{M_1} \\ \cdots \\ a^{M_{nm}} \end{bmatrix} \tag{20}$$

The final configuration of the EOM is an indeterminate system, *i.e.*, it has an infinite set of solutions. This fact is physiologically understood by existence of muscular redundancy – there are an infinite number of muscle force combinations that will result in a specific motion, but the central nervous system will only chose one. Optimization procedures are used to solve the system, by minimising a cost function $\mathcal{F}_0$ that describes the muscle system energy depletion. The used cost function, proposed by Crowninshield and Brand [20], was

$$\mathcal{F}_0 = \sum_{m=1}^{nm} \left(\sigma_{CE}^{m}\right)^3 \tag{21}$$

An important detail of our implementation is assigned to the boundaries of control variables in the optimization problem. When muscles are not implemented and a motion is prescribed, the Lagrange multipliers associated with the kinematic drivers (that prescribe the motion of system's points and joints) hold the forces contribution that induces the movement. When muscles are considered in the model, then it is desired to eliminate the contribution from those Lagrange multipliers that are related to the drivers of joints spanned by muscles and convey this contribution to the muscles' activations. The optimizer used in our program was the DNCONG routine, the double precision version of the NCONG routine. It is a FORTRAN routine available in the IMSL Library developed by Visual Numerics, Inc. [21].

4 Results

4.1 Fatigue

To test the fatigue model functionality, a simple right upper extremity model considering some of the most important muscles of the elbow joint was designed with an immobilised vertically standing humerus. A graphic representation of the model is shown in Figure 5. Notice that a concentrated force $P = 150N$ was applied at $30cm$ from the elbow joint and, in addition, the whole biomechanical system is subjected to a constant gravitational force $\vec{g} = -9.81\vec{e_z}$. The model is defined by three rigid bodies: torso, arm and a forearm-hand complex; and seven different muscles: *biceps brachii* long and short heads, *brachialis*, *brachioradialis*, and *triceps brachii* lateral, long and medial heads. The remaining muscles that cross the elbow joint, such as the *extensor carpi radialis longus* or the *pronator teres*, are not considered in the model, since these muscle's functions are not related to the action in analysis. For the geometrical and inertial properties of both used muscles and rigid bodies, please refer to [13]. The used fatigue parameters for these muscles is displayed in Table 1. These have no correlation with any experimental results or literature values and are only considered for example purposes.

Table 1: Fatigue parameters used for all the muscles in the upper extremity with elbow muscles model. These have no correlation with experimental results. Values are given in units of $1/s$.

F	R	L_D	L_R
0.01	0.002	10	10

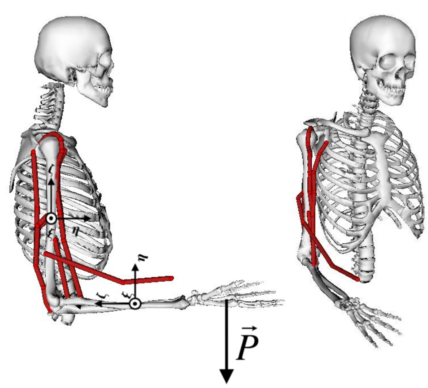


Figure 5: Simple upper extremity model with 7 elbow muscles and constant force **P** applied. This image conceived using the OpenSim software [22].

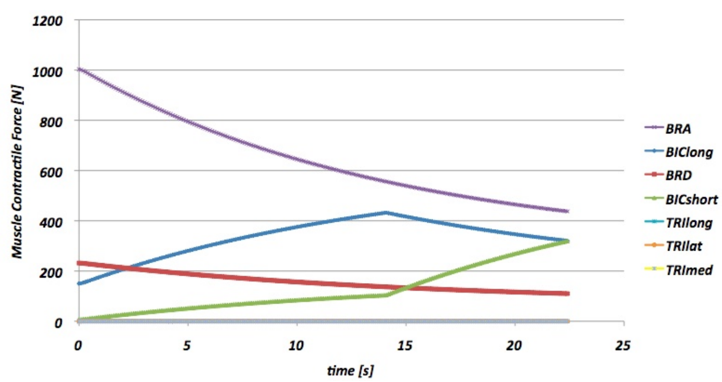


Figure 6: Resultant muscle forces of the contractile element F^m_{CE} of the biomechanical model of the upper extremity, when susceptible to fatigue dynamics.

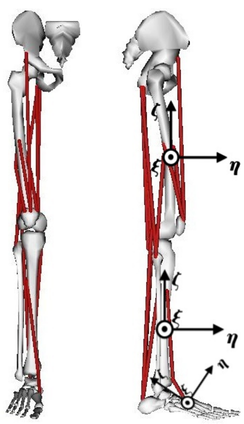
This model is tested in a Inverse Dynamics analysis perspective where a kinematic driver for the elbow joint is prescribed, imposing a constant angle of 90°, *i.e.*, the humerus is immobilised in a vertical position and the muscles in the biomechanical model maintain the body that represents the complex radius-ulna-hand horizontal. The analysis was performed till the time instant when the fatigue state of the muscles causes the system to be incapable to hold the load P and sustain a horizontal orientation of the forearm. The calculated muscle forces of the contractile element F^m_{CE} for the considered muscles are illustrated in Figure 6.

The optimizer computed that, in the first instants, the position is sustained by full activation of the *brachialis* and *brachioradialis* muscles and a partial activation of the long head of the *biceps brachii.* The first two loose the capacity of maintaining the tonus, since they are using the full capacity of the muscle, becoming importantly fatigued. To compensate the loss angular momentum competence of these muscles, both long and short heads of the *biceps brachii* increase the force output, and therefore their muscle activations. Note that, around the $14s$ of analysis, the long head reaches its maximum level of activation, leading to an increase of the rate of short head recruitment. At $time = 22.4s$ the analysis terminates, since the horizontal position of the forearm cannot be supported by the muscle framework. The activation of the three heads of the *triceps brachii* muscle is 0 for the whole analysis, since these work as antagonist muscles, not being able to perform a practical action towards the bearing of load P.

4.2 Human gait

A simple model of the right lower extremity with 12 muscles is conceived with the purpose of testing both the practicability and accuracy of the formulation described in Section 2. The model consists in four rigid bodies: torso, thigh, shank and foot, however the torso inertial properties are again not considered for this analysis, since it is only included for the insertion of muscle origins. The model is illustrated in Figure 7.

The anthropometrical data of the alluded rigid bodies, the physiological and geometrical parameters of the considered muscles, and the four different kinematic drivers that are used for motion prescription are available in the work by Pereira [13]. The developed inverse dynamics routines process the model information and the prescribed data, resulting in the muscle force patters of Figure 8. In this figure shows the summation of the total muscle force of each functional group considered.



(a) Anterior view. (b) Lateral view.

Figure 7: Simple leg model with 12 muscles spanning the knee and ankle joints. This image conceived using the OpenSim software [22].

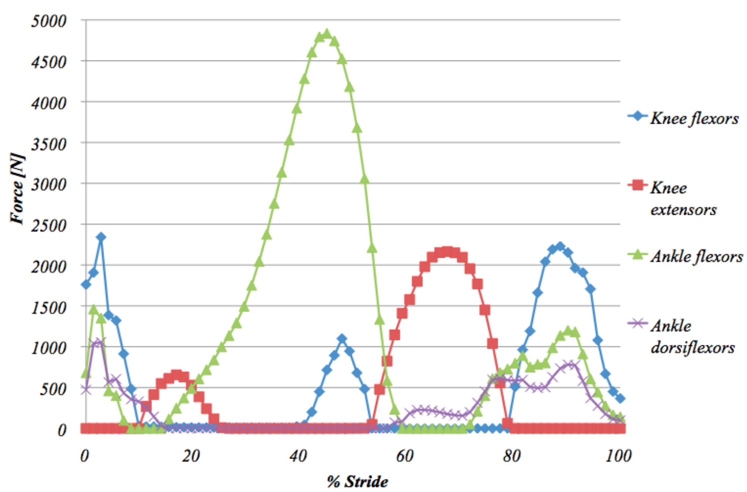


Figure 8: Resultant total muscle forces of each considered muscle group in the lower leg extremity model.

5 Conclusions and Future Developments

It was proven in by Anderson and Pandy [23] and Pereira [13] that the non-inclusion of muscle contraction dynamics, when time-independent performance criterions are used, may not lead to significant result differences when predicting muscle forces, when muscles are required to work below the physiological limits. Such type of model turns to be fundamental when it is intended to acquire muscle activations or when the physiological restrictions of muscles are to be considered.

The used fatigue model is relatively straightforward one, and such characteristic was essential for its choice. The whole formulation of this model proved to be well suited and easily integrated in the multibody system routines, and compatible with the considered contraction model. The study of muscle fatigue using versatile mathematical models in multibody system dynamics methodologies, gains a special motivation due to the fact that these can be easily applied in complex musculoskeletal systems with intricate kinematic patterns are to be analysed. No studies were found to date that included muscle fatigue models in a multibody dynamics system, so the intention of keeping it simple is understandable. Other similar fatigue models coupled with experimental trials, such as the one in Ma et. al. [24], should be considered and the parameters values analysed. An engrossing procedure in the following of this fatigue model, would be to do some experimental trials, in order to test the correlation of the fatigue parameters with actual physiological situations, and infer the model's validation.

Static optimization itself only accounts for instantaneous performance criteria, that may not simulate in the best way the behaviour of the CNS when choosing a muscle activation set. According to Ackermann [4], these instantaneous functions are unable to accurately describe the key performance criterion: the total energy expenditure. Our inverse dynamics approach consisted in the adaptation of the equations of motion to the muscle formulation using the Newton's method [1, 19]. With this methodology, it is possible to calculate in the same analysis rigid body internal reactions, joint reactions, and muscle and activations, with the nuisance of increasing the complexity of its implementation and solving process. The developed formulation was successfully included in the multibody routines, has observed in the results obtained in Section 4.

The optimization procedure, described in Section 3, proved to solve the developed models in an accurate and efficient manner. However, the prime fragility of the developed FORTRAN routines is most probably the used optimizer. DNCONG is a very fast, however very sensible algorithm. When DNCONG is close to the global solution, it reaches it rapidly and with sharp precision. Nevertheless, it is quite demanding to find the region of the global minima. DNCONG calculates a local-minima and, when far from any solution, the algorithm simply cannot converge. A possible future development, regarding the precision of the optimizer, would be to use a genetic algorithm for the computation of an initial guess and give this guess as an input to DNCONG, letting DNCONG converge to the global minima. Another approach, and this one similar to physiological criteria to a greater extent, would be to use electromyography signals to drive the obtained the muscle activation results, *i.e.*, to use and EMG-driven models to estimate muscle forces. Several authors used this approach [25–28].

References

[1] Miguel P T Silva. Human motion analysis using multibody dynamics and optimization tools. *PhD Thesis, Departamento de Engenharia Mecânica - Instituto Superior Técnico*, pages 1–347, Feb 2004.

[2] J Michael. Modern prosthetic knee mechanisms. *Clinical orthopaedics and related research*, 361:39–47, 1999.

[3] T Schmalz, S Blumentritt, and R Jarasch. Energy expenditure and biomechanical characteristics of lower limb amputee gait: The influence of prosthetic alignment and different prosthetic components. *Gait & Posture*, 16(2):255–263, 2002.

[4] Marko Ackermann. Dynamics and energetics of walking with prostheses. *PhD Thesis, Department of Mechanical Engineering, University of Stuttgart*, 2007.

[5] C Raasch, Felix E Zajac, B Ma, and W Levine. Muscle coordination of maximum-speed pedaling. *Journal of Biomechanics*, 30(6):595–602, 1997.

[6] G Venture, K Yamane, and Y Nakamura. Identifying musculo-tendon parameters of human body based on the musculo-skeletal dynamics computation and hill-stroeve muscle model. *IEEE-RAS Int. Conf. on Humanoid Robots*, pages 351–356, 2005.

[7] J Rasmussen, M Damsgaard, ST Christensen, and E Surma. Design optimization with respect to ergonomic properties. *Structural and Multidisciplinary Optimization*, 24:89–97, 2002.

[8] AV Hill. The heat of shortening and the dynamic constants of muscle. *Proceedings of the Royal Society of London. Series B, Biological Sciences*, 126(843):136–195, 1938.

[9] A Huxley. Muscle structure and theories of contraction. *Progress in Biophysics and Biophysical Chemistry*, pages 255–318, 1957.

[10] G Tad Yamaguchi. Dynamic modeling of musculoskeletal motion: A vectorized approach for biomechanical analysis in three dimensions. *Springer*, pages 1–257, 2005.

[11] Felix E Zajac. Muscle and tendon: properties, models, scaling, and application to biomechanics and motor control. *Critical reviews in biomedical engineering*, 17(4):359–411, 1989.

[12] Matthew L Kaplan. Efficient optimal control of large-scale biomechanical systems. *PhD dissertation, Stanford University*, 2000.

[13] André Ferro Pereira. Development of a hill-type muscle model with fatigue for the calculation of the redundant muscle forces using multibody dynamics. *MSc Thesis, Instituto Superior Técnico, Universidade Técnica de Lisboa*, 2009.

[14] Arthur C Guyton and J Edward Hall. Textbook of medical physiology. *Elsevier Saunders*, 2006.

[15] W Rohmert. Determination of the recovery pause for static work of man. *Internationale Zeitschrift für angewandte Physiologie*, 18:123–164, 1960.

[16] J Ding, AS Wexler, and SA Binder-Macleod. A predictive model of fatigue in human skeletal muscles. *Journal of Applied Physiology*, 89(4):1322–1332, 2000.

[17] T Xia and L Frey Law. A theoretical approach for modeling peripheral muscle fatigue and recovery. *Journal of Biomechanics*, 41(14):3046–3052, 2008.

[18] JZ Liu, RW Brown, and GH Yue. A dynamical model of muscle activation, fatigue, and recovery. *Biophysical journal*, 82(5):2344–2359, 2002.

[19] Javier Garcia Jalón and Eduardo Bayo. Kinematic and dynamic simulation of multibody systems: The real time challenge. *Springer-Verlag*, 1994.

[20] D Crowninshield and Richard A Brand. A physiologically based criterion for muscle force predictions on locomotion. *Bulletin- Hospital for Joint Diseases*, 43(2):164–170, 1983.

[21] Imsl fortran subroutines for mathematical applications. *Visual Numerics, Inc.*, 1997.

[22] Scott L Delp, FC Anderson, AS Arnold, P Loan, A Habib, CT John, E Guendelman, and DG Thelen. Opensim: Open-source software to create and analyze dynamic simulations of movement. *IEEE Transactions on Biomedical Engineering BME*, 54(11):1940–50, 2007.

[23] FC Anderson and Marcus G Pandy. Static and dynamic optimization solutions for gait are practically equivalent. *Journal of Biomechanics*, 34(2):153–161, 2001.

[24] Liang Ma, Damien Chablat, Fouad Bennis, and W Zhang. Dynamic muscle fatigue evaluation in virtual working environment. *International Journal of Industrial Ergonomics*, 39(1):211–220, 2009.

[25] SC White and DA Winter. Predicting muscle forces in gait from emg signals and musculotendon kinematics. *Journal of Electromyography and Kinesiology*, 4(1):217–231, 1993.

[26] Maxime Raison. On the quantification of joint and muscle efforts in the human body during motion. *PhD Thesis, École Polytechnique de Louvain, Université catholique de Louvain*, 2009.

[27] DG Lloyd and TF Besier. An emg-driven musculoskeletal model to estimate muscle forces and knee joint moments in vivo. *Journal of Biomechanics*, 36(6):765–776, 2003.

[28] Thomas S Buchanan, Scott L Delp, and JA Solbeck. Muscular resistance to varus and valgus loads at the elbow. *Journal of Biomechanical Engineering*, 120:634–639, 1998.