NATURAL FRACTURE NETWORKS

- MODELING MIDDLE-EAST RESERVOIR -

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Abstract: *The necessity in predicting an accurate image of the subsurface has lead to the creation of methods that allow the optimization of the knowledge originated from exploration. Modeling using Discrete Fracture Network (DFN) has come with the advantage of similarity to reality. Instead of characterizing the fractured context as a continuum variable it allows the individualization of each fracture as a discrete element. The fracture goes from being some value in a spectrum to a whole entity described by many characteristics bringing the need to create methodologies to describe every one of them. A study has been made, using the DFN modeling software Fracman, about the characteristics size, shape, intensity, orientation and fracture count in order to predict the possible inconsistencies that could happen in a modeling project. This study was made using stochastic generation by Monte Carlo method categorizing groups of simulations with specific differences in the description of each characteristic. This allowed recognizing misleading situations and methods to correct it. These methods were applied in the real case of Middle-East reservoir where using scarce data it was possible to do a satisfactory model of fracture spatial dispersion by using numeric and geostatistical treatment of the available variables. The fracture intensity characteristic of the Middle-East reservoir is defined by a mathematical relationship between permeability and porosity and the equivalence of cumulative distribution functions of various estimations made in the course of this study.*

Keywords: *Fracture system, Oil Reservoir, Modeling, Discrete Fracture Network, Simulation, Geostatistics*

I.INTRODUCTION

Even with the amazing progress made in fracture detection methods in the subsurface, usually recurring to seismology, the resolution obtained rarely gives an accurate image of the fracture network. The difficulty in detecting sweet spots on oil reservoir has led to an increasing specialization on numerical treatment concerning data originated from exploration. Mathematical relations resulting in better conditioning gives new dimension to former, directly applied, variables. The consequence was the application of discrete element methods (static) to classic simulation which brought more detailed pictures of fluid flow dynamics, clusters or storability. The discrete fracture network (DFN) approach explicitly incorporates the geometry and properties of discrete features as a central component controlling flow and transport[1]. Much controversy has arisen from the concept of representative elementary volume (REV) which states that there is a volume at which heterogeneities and discrete features can be ignored due to a process of averaging to produce continuum effective data. Still the advantages of the method have contributed to its use for the last 30 years. The characterization of fracture network can be done in several manners, none of them truly accurate but efficient to different objectives.

II.THEORETICAL STUDY

Fracture origin, propagation and spatial dispersion depend greatly on the context of the rock mass where they reside. Most fractures on oil reservoir are MODE I (dilating fractures or joints) and MODE II (shearing fractures or faults). This concerns with geological phenomena that occurred in the location and it varies from place to place giving different characteristics to fractures with the same temporal and spatial origin. Applying fracture theory to petroleum reservoir demands the optimization of interpretation of remote-sensing methods since these areas usually do not have visible surface expression. In oil reservoir, fractures are mainly important because of permeability rather than porosity due to low storability that can quickly get exhausted in initial production[2]. As so fractures provide permeable pathways in the transport of hydrocarbons. The knowledge of these highly permeable locations is important information for anyone trying to get a spot on oil industry. Being impossible to be sure, the result of a fracture network model is as precise as the better its conditioning.

II.1 – STATISTICALLY CONDITIONED MODELING

A DFN model needs to describe every physical parameter of a fracture in order to produce a good outcome. The workable data is always scarce and maximizing its usefulness is a priority for every good investigator. The parameters regarding spatial aspects are size, orientation, intensity, shape and count. The bigger the fractures, the better the chances of intercepting a well. Experience has shown that most **fracture size** distributions are either *Power-Law* or *Log-Normal*[3]. When it comes to *Power-Law* usage usually the *Pareto* version is the better adequate. The probability density function (pdf) for it and probably the most widely used in DFN modeling[4] (being the case of fracture modeling software *Fracman*) is:

$$f(x; x_{min}, \alpha) = \alpha \frac{x_{min}^{\alpha}}{x^{\alpha-1}} \quad , \quad x \geq x_{min} > 0$$

where x_{min} is the minimum fracture size. Notice that being α and x_{min} two constants the *Pareto* distribution is an ever monotonously decreasing function pretty similar with a *Log-Normal* when the standard deviation (σ) is a high number. The *Log-Normal* distribution is one whose variable is normally distributed while logarithmic. The pdf is given by (being μ the mean):

$$f(x; \mu, \sigma) = \frac{1}{2x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$$

Fracture orientation follows a more complex kind of distribution meaning a probability distribution on the $\mathbb{R}^3$ dimension sphere. Usually defined by one or more dispersion or concentration parameter (*k*) and a mean direction (μ) it is only correctly observed in a stereographic projection. Usually fracture networks are organized in either clusters or tendency axis. When function mean is constant the cluster effect can be produced by a *Fisher* distribution (also known as *Von Mises-Fisher* distribution) due to its circular dispersion (which is as bigger as *k* is smaller[5]) *over* the stereographic projection. The *Bingham* distribution produces an axis like tendency using two means and concentration parameters like a bivariate function. Bivariate functions like bivariate *Fisher* or bivariate *Normal* allow the user to insert two focuses (two means) on the stereographic projection in the case of *Fisher* with a hard candy effect and *Normal* with a funnel like effect. The mean in directional statistics, especially in modeling science, has two sections (if in a spherical coordinate system, otherwise λ, β, and ν if in Cartesian with $\mathbb{R}^3$ euclidian space), trend (angle in the plane) and dip (angle between plane and vertical axis). Note that different authors may use other nomenclature. The retrieval of trend (ω) and dip (θ) from Cartesian $\mathbb{R}^3$ can be made with the following transformation:

$$\omega = \arctan\left(\frac{\beta}{\lambda}\right)$$

$$\theta = \arccos\left(\frac{\nu}{\sqrt{\lambda^2 + \beta^2 + \nu^2}}\right)$$

But only for strictly planar $\mathbb{R}^2$ symmetric fractures since:

$$\arctan\left(\frac{\beta}{\lambda}\right) = \arctan\left(\frac{-\beta}{-\lambda}\right)$$

Trend and dip are two individual variables in spherical distributions and can have their own individual distributions (usually bi-dimensional distributions). The final composite of functions for a pdf on the sphere is:

$$D\left(\mu\left[\omega\left(\frac{\bar{x}}{\sigma}\right); \theta\left(\frac{\bar{x}}{\sigma}\right)\right]\Big| k\right)$$

being $\bar{x}$, the mean, and σ, the standard deviation, in the most typical pdf functions (but not all, *Power-Law* uses "x_{min}, α" parameters for example). This composition of functions can result in a different outcome one expected for the previously commented $\mathbb{R}^3$ dimension sphere functions. The third parameter is **fracture intensity** which relates to the spatial dispersion of fractures. This only has meaning if fracture generation is stochastic and needs a function to correspond. This is given by either a pdf or a direct relationship. Direct relationship can be given by grid variables or indexes like P_{10} (fractures per unit of length) or P_{33} (fractures per unit of volume). Indexes tend to be biased to reality since they correspond to no variation in the generation volume but if the region considered is small it's worth the try. Grid variable relationship, like a seismic map fracture equivalent variable, is the better option in stochastic modeling since there´s a correlation in every location of the generation region. Obviously the quality of this generation will be proportional to quality of the mathematical relation between fracture number and causation variable. If none direct relationship is possible, the equivalence of a pdf (even if constant) is the remaining option. There are many ways of describing fracture intensity with n-dimensional functions using algebraic treatment or derivation from one or two dimension processes. *Box Dimension Analyses* is a method in which, for different interval sizes in the same sample (like an image

log), the number of intervals is counted and plotted on a log-log plot of the number of intervals with cumulative fracture count vs. the interval size. The subinterval size is then plotted against the number of filled intervals[3]. In a log-log scale graphic if the result (for many interval steps) is a line then the relation between the numbers of filled intervals is a *Power-law*. If its exponent is less than 1 the spatial model is described by a fractal pattern, if near 1 the spatial model is *Poissonian* (random spatial pattern[6]). There are other well known functions to describe fractal patterns like *Levy-Lee* model (fractal fracture model) based on the *Levy Flight* process for which the length, L, of each step is given by:

$$P_{L_s}[L > L_s] = L_s^{-D}$$

where D is fractal mass dimension of the point field of fracture centers and L_s the distance from one fracture to the next. If $D = 0$, the distribution of the step length is uniform and consequently there´s no clustering of discrete elements being similar to *Baecher* model where the fracture centers are located uniformly in space[7]. **Fracture shape** relates to the plane figure fractures have in a model. It is usually defined with a geometric shape (polygonal) and an axis ratio (since one may exceed the other extending the figure to either side). Its importance grows with fracture size or orientation. Small fractures (bellow REV) don´t really have a major contribution to the model dynamics so its shape is rather unimportant. On the other hand if the size is significant fractures may occur in unexpected areas especially if orientation promotes the major axis towards that area. The equivalent radius of a fracture is the radius of a circular disk with the same area of this fracture, therefore given by the formula [3]:

$$R_e = \sqrt{\frac{A_f}{\pi}}$$

Fracture count relates to the total number of fractures the fracture network being modeled has. This information is rarely available and its importance falls mainly in the adaptation of the

fracture model to real verified data. Contrary to fracture size its behavior is highly influenced by intensity function since, in some cases, more fractures in the system could actually mean less fractures in some given location (meaning fracture count curve may have a change in monotony reaching a certain point).

II.2 – VARIABLE CONDITIONED MODELING

Of the five parameters considered in this study fracture size, orientation and intensity are the most variable dependable. Using a variable to condition statistical distribution is effective since, if done right, not only will the distribution coincide with the estimation but there's a spatial localization of the pretended effect. For example if we apply a *Normal* distribution to size, the modeling software will not recognize a specific place where to apply specific value intervals of that distribution even if in reality, there are places where the size is higher and other places where the size is lower. The result is a random process of big and small where should only be big or small. A variable has already a spatial dispersion. Should we be able to calculate an accurate relation between our fracture parameter, the variable and the parameter distribution, and we'll have the right characteristic in the right place.

II.2.1 – FRACTURE SIZE

Fracture size can be studied from trace length maps (bi-dimensional maps with lines representing fractures at some section of the studied volume) with careful discretion. There are some biases affecting the estimation with trace length[2]: orientation bias (if fracture is orthogonal to trace map it will be represented as a smaller fracture than actually is), length bias (smaller fractures have lower probability of showing in trace map usually being underrepresented) and censoring (the sample area is limited and some fracture will too). Even if the distribution is well described, there is a

need for it to be local. Size variable relationships are uncommon and may depend on the phenomena that gave origin to the fracture system. In some cases fracture size relate to stress zones. If the fracture is oriented in the correct position the stress induced by the neighboring rock will augment its size. If this is a difficult variable to calculate near surface, on deep surface is some work of art. Nevertheless if the reservoirs lithology (mud logging) is spatially well described it may be a possibility to predict some notion (at least to some degree) of stress direction and strength by relating with rock formation displacement. Acknowledge that if the generation region is big enough it could be an advantage to make one or more estimation (like krigage) to fracture size (you could make different sets of fractures for different intervals of fracture sizes) creating a new size dependable variable. If the problem of characterizing size distribution and location in a model is severe then it should be made to be adequate to well fracture intersection. When significant, size behavior in modeling is generally linear and easier than fracture count, for example, to adequate.

II.2.2 – FRACTURE ORIENTATION

Different phenomena may originate different orientation sets so instead of trying to understand its origins (and even if you do there's no real proof that constructed geological models apply to your case) you should try to make a mathematical relation to a known spatial variable which is also or only defined by orientation. This could be made with reservoir orientation or bedding (usually described in grid blocks orientation). In Figure 1 there's a fictional example of two blocks grid:
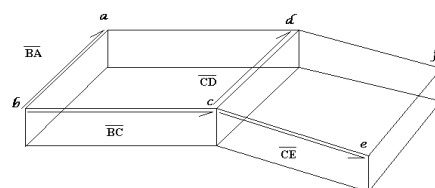

Figure 1 - Fictional example of two blocks from a grid.

If the blocks are quadrilateral then it´s possible to describe two unit vectors (normalized vectors) and consequently the upper (any other could be used) plane of the block using the following equations to describe the first block (Cartesian $\mathbb{R}^3$ coordinates):

$$\overline{u} = \frac{b_1 - a_1, b_2 - a_2, b_3 - a_3}{\sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}}$$

$$\overline{v} = \frac{b_1 - c_1, b_2 - c_2, b_3 - c_3}{\sqrt{(b_1 - c_1)^2 + (b_2 - c_2)^2 + (b_3 - c_3)^2}}$$

$$\overline{BA} = u \ and \ \overline{BC} = v$$

The upper plane can now be defined with both vectors (only orientation has been imprinted since the new vectors are defined by a point conditioned to origin and that´s all we need) by a pole (orthogonal to the plane) vector using the following cross product equation:

$$u \times v = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

Now we have a perpendicular direction to the plane given by the coordinates:

$$\lambda = (u_2 v_3 - u_3 v_2)$$

$$\beta = (u_3 v_1 - u_1 v_3)$$

$$\nu = (u_1 v_2 - u_2 v_1)$$

Since the final orientation is given by dip and trend we may need, depending on the modeling software, two different distributions from the same pole vector given by the former commented retrieval formula:

$$trend = \omega = \arctan\left(\frac{\beta}{\lambda}\right)$$

$$dip = \theta = \arccos\left(\frac{\nu}{\sqrt{\lambda^2 + \beta^2 + \nu^2}}\right)$$

This mathematical process would have to be applied in all blocks of the grid in order to give a spatial character to the new variable (pole vector of bedding). The variable has been created and now we need the factual relation that fracture orientation has with bedding orientation. The easiest way to determine this relation is watching a stereographic projection

with the newly created bedding and statistically conditioned fracture orientation. In Figure 2 we have a fictional example of bedding with an approximate average of $\theta = 0^\circ$ and $\omega = 0^\circ$ (center of the stereonet) and fracture with $\theta = 0,90^\circ$ and $\omega = 90,0^\circ$ (edges of the stereonet):
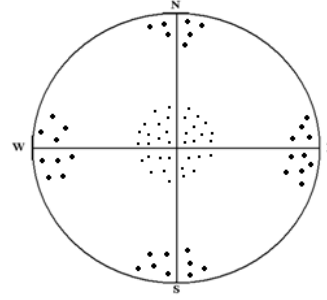

Figure 2 - Stereonet of bedding (center) and fracture (edges) orientation.

This fictional case seems to follow the Stearns model since fracture orientation is orthogonal with bedding orientation with respect to two different sets. We need to foresee this bias between fracture and bedding orientation when we´re going to relate both of them so our final distributions will be:

$$trend = \omega = \arctan\left(\frac{\beta}{\lambda}\right) + trend\ bias$$

$$dip = \theta = \arccos\left(\frac{\nu}{\sqrt{\lambda^2 + \beta^2 + \nu^2}}\right) + dip\ bias$$

Together with pdf on $\mathbb{R}^3$ dimension sphere we would have a final variable dependable distribution with a more or less random factor depending on *k* (concentration parameter).

II.2.3 – FRACTURE INTENSITY

Fracture intensity follows the same logic that fracture size but usually the variables of dependence (VOD) are more reliable and abundant. The better option in creating a satisfactory mathematical relation between number of fractures and VOD is the use of dispersion graphic. Dispersion graphic (scatterplot) locates points in the plane using both variables as axis. This usually results in a

cloud with a more or less degree of tendency which could either be described by a linear or curved function (henceforth called ponderer when the result is the number of fractures). If the tendency is linear then a linear equation ($y = \theta_1 x + \theta_2$) will describe the ponderer. If not other approximations like polynomial, potential, logarithmic or exponential functions need to be considered. This could be done with curve fitting methods but acknowledge that the idea is not having the best-values curve fitting model but a smooth standard curve able to interpolate unknown values. This can be done with linear, multi-linear and nonlinear regression (depending on the case) without much concern about what model to use[8]. Simple linear regression:

$$f(x) = \theta_1 x + \theta_2$$

, polynomial regression:

$$f(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_m x^{m-1}$$

, or low order exponentials models:

$$f(t) = A_1 e^{(K_1 t)} + A_2 e^{(K_2 t)} + \cdots + A_n e^{(K_n t)} + C$$

can be used[9] but with some precautions since some of the results may not be strictly monotonously giving chance for miscalculation. Usually the relation between VOD and number of fractures has always the same monotonous behavior. Fitting our curve to the scatterplot gives us our ponderer but it may not be true to all generation region. If so it needs to be inhibited by henceforth called inhibiter. An inhibiter is a weighted coefficient to the ponderer. If it's known that when some variable is high the number of fractures is lesser then, to correct the ponderer influence in the generation region, we should use it as an inhibiter. The strength of this inhibiter could be given by an exterior constant (coefficient of inhibition) like in a direct multiplication or potentiating the inhibiter. The right strength has to be studied by real data validation in the model or previously known by a direct relation between ponderer and inhibiter. If the number of fractures is as lower as the inhibiter variable

is high than a possible weight to the ponderer could be (merely an example):

$$Inhibiter = I(\eta) = \frac{1}{R(\eta)^\alpha}$$

where $R(\eta)$ is the relation number of fractures vs. η variable and α, the coefficient of inhibition. Applying it to the initial fracture ponderer we get:

$$IFI = \varphi(\alpha; \delta, \eta) = \frac{P(\delta)}{R(\eta)^\alpha}$$

where $P(\delta)$ is the initial ponderer to fracture number. Evidently the result of this operation won't be a fracture number but an intensity fracture index (*IFI* or φ). The *IFI* needs to be converted to fracture number by function equivalency. Both *IFI* and fracture number (from real data) have cumulative distribution functions (cdf) that can be made to converge by using the probability result from the cdf of *IFI* and inserting it in the inverse cdf of fracture number.

$$Fracture\ number = \ cdf(F)^{-1}\big(cdf(IFI)\big)$$

being *F*, fracture number taken from real data. This is only an option if the inverse is calculated by numerical algorithm (analytical inversion of cdf is impossible) which, usually, modeling software's don't have. Other method is using a QQ-plot which is a graphical method of comparing two probability distributions. If so another curve fitting will have to be done to the QQ-plot. The more similar the functions the closer to linear relation both pdf will have. If the final result is not satisfactory (the QQ-plot is anything but linear) the previous steps should be revised. Even if the relation is good and a nice curve is found there is still the need to validate it with real data since we don't know if the quantiles (points taken at regular intervals from the cdf) are proportional. If they are we have our final ponderer of fracture number:

$$P_f(\alpha; \delta, \eta) = CFF\left(\frac{P(\delta)}{R(\eta)^\alpha}\right)$$

being CFF the curve fitting function.

III – MIDDLE-EAST RESERVOIR CASE STUDY

Middle-East (ME) is a layered heterogeneous limestone and dolomite unit with lateral and vertical lithological changes. The porosity was mainly created by dolomitization in the subsurface where eleven lithofacies and eight dominant rock types are identified[10]. In ME reservoir most of the fractures appear in the central corridor with two different sets of major fractures, namely N033E and N073E (referring to fracture orientation)[11]. It´s assumed that the major sets of fractures have a size within [500, 1000] meters.

III.1 – AVAILABLE and PRODUCED DATA

The data available for this study is a group of point data and a coinciding grid (assumedly with the shape of the reservoir). The point data have three available variables being DRT (quality of the rock), permeability and porosity. The original grid was used in the modeling parameters study and consequential method creation (described in the former sections of this article) and a new grid was created due to the need of having all blocks with the same volume (therefore called regular grid), which the original grid didn't have (compromising the results and due to inexistence of available software to compute it). Also available is real borehole data specified in a merger between petrel well trace and log ASCII standard (LAS) file format. The LAS files were organized with intervals (accordingly to depth) of DRT, permeability and porosity. Later, fictional boreholes were inserted in different versions of the model to study fracture occurrence oscillation due to changes in the modeling parameters. The upscaling of the point data to the grids were made using the arithmetical mean operator in the case of porosity and permeability (henceforth mean porosity and mean permeability), the maximum value for block operator for permeability (max permeability) and median operator (because of its similarity to mode or the most frequent value to appear in a sample) to DRT. DRT upscaling was poor, therefore used with necessary discretion in the ongoing of the study. The main grid variables are mean porosity and max permeability (it was assumed the higher values of permeability were, in fact, due to fracture existence so it´s an important variable to relate to fracture occurrence).

III.2 – FRACTURE OCCURRENCE in REAL DATA BOREHOLE

There are no file testimonies (which was the only available information) of real fractures occurring in the mud logs but permeability, contrary to porosity and DRT, has an enormous variance due to the existence of high kurtosis (variance is due to infrequent extreme deviations). These peaks were assumed to be due to fracture existence. In Figure 3 there´s a graphic of depth (m) vs. permeability (mD) from a real borehole where this behavior is easily observed.
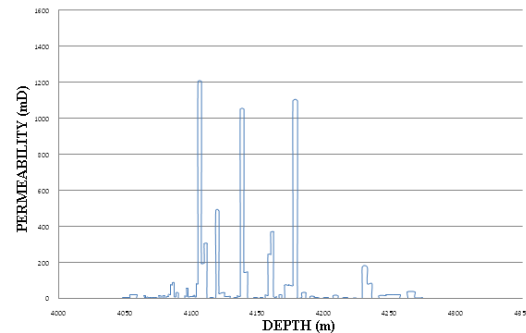

Figure 3 - Depth (m) vs. permeability (mD) graphic from a real borehole.

For every well the number of peaks together with the maximum permeability achieved have been registered with the corresponding bi-dimensional localization (x and y coordinates). In order to have some initial notion of how fracture occurrence behaves in space estimation by krigage was done to this data in a bi-dimensional grid. Since only the fictional borehole have the I, J coordinates (grid coordinates) a rotation and adequateness (by

visual comparison) was made to real borehole localization (due to different orientation and distortion towards north). This is an important step to refer since bias exists and may have had some inference in the outcome results (slight displacement from real location but probably enough to invalidate the comparison between real data and pondered maps). The formula used to rotate the data was:

$$C_x^2 = \left(\sqrt{(C_x^1)^2 + (C_y^1)^2} \right) * \cos \left(tan_{rad}^{-1}(C_x^1; C_y^1) + r^* \right)$$

$$C_y^2 = \left(\sqrt{(C_x^1)^2 + (C_y^1)^2} \right) * sen \left(tan_{rad}^{-1}(C_x^1; C_y^1) + r^* \right)$$

where:

$C_x^1; C_y^1$ - Original borehole coordinates.

$C_x^2; C_y^2$- Final borehole coordinates.

r^*- Angle ($c\pi, c = constant$) of rotation.

$tan_{rad}^{-1}(slope)$ – Equivalence slope/angle.

The final borehole coordinates were later made adequate to I, J grid coordinates using similar locations between real and fictional well. Using the new coordinate the estimation was made and superimposed by upper reservoir topography as can be seen in Figure 4.
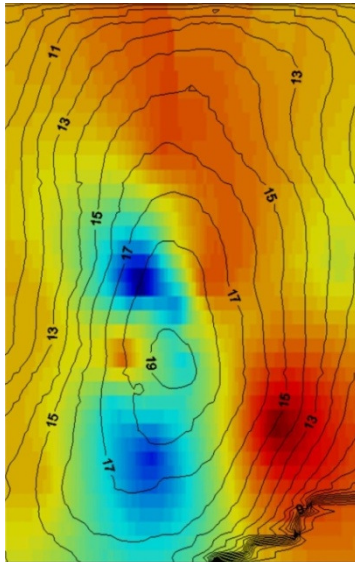

Figure 4 - Krigage of number of fractures with real borehole superimposed by upper reservoir topography.

The result looks a bit displaced from the topographical map but there seems to be some coherence. The zones with higher slope seem to have higher fracture occurrence than neighbor blocks exception made with the upper left corner of the map. This could be due to uncertainty since most wells are placed in the center corridor of the map. The variance map calculated from thirty Gaussian simulations with the same variography of krigage applied confirms high variance in the upper left corner as can be seen in Figure 5.
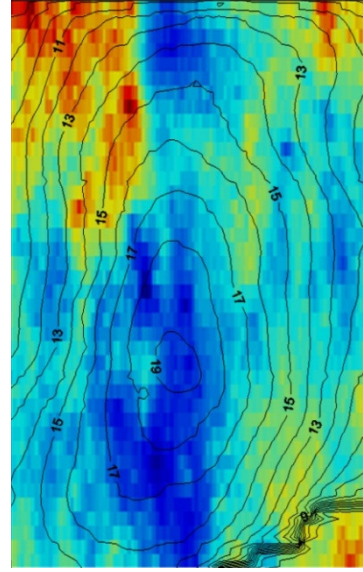

Figure 5 - Variance map calculated from thirty Gaussian simulations with real borehole superimposed by upper reservoir topography.

Nevertheless, it´s easily observable the strong heterogeneous character of fracture dispersion in space.

III.3 – MECHANICAL STRESS in MIDDLE-EAST RESERVOIR

The analysis of upper reservoir topography suggests that there are areas where the slope is greater, probably due to horizontally orientated mechanical stress. Hypothetically this should concord with fracture orientation sets. Figure 6 shows a topographical map with specified zones of greater slope and potential stress direction.
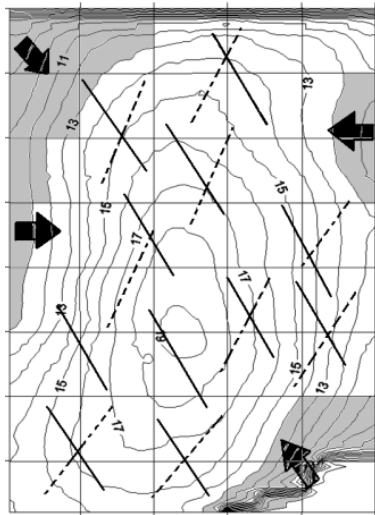
Figure 6 - Topographical map with specified zones of greater slope and potential stress direction.

III.4 – RELATION BETWEEN FRACTURE NUMBER and MAXIMUM PERMEABILITY

Three attempts of relating fracture number with reservoir variables have been made (mean porosity "ϕ", mean permeability, and max permeability) but the best relation was given by max permeability (K_2). The scatterplot revealed that the relation is not linear and the number of fractures (F) decreases as max permeability increases. The Figure 8 shows the resulting scatterplot.

Figure 8 - Scatterplot of max permeability vs. number of fractures fitted with a potential function.

Of the two specified orientation sets only one seems to concord with mechanical stress directions. An analysis on the stereographic projection, provided by the ME Team report, on fracture orientation taken from real borehole data, shows that only the N033E set is an actual orientation focus as can be seen on Figure 7.
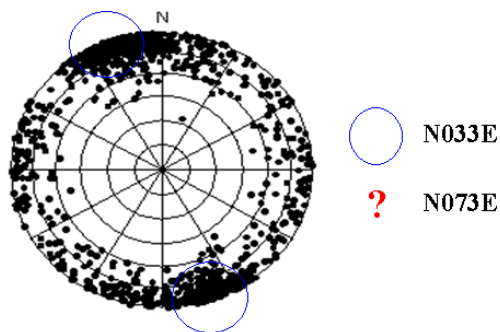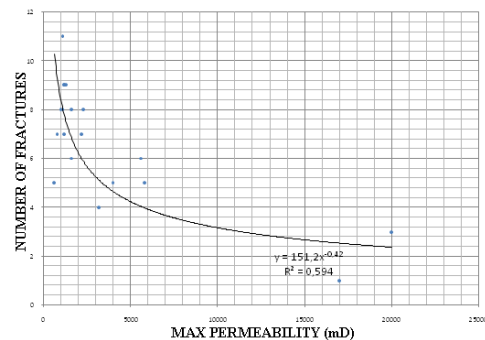
The curve fitted in the scatterplot is a potential given by the formula:

$$F = 151{,}2K_2^{-0{,}42}$$

Figure 7 - Stereonet of fracture orientation showing that only N033E is an actual orientation focus.

The fitting was done in spreadsheet EXCEL software and since the curve is smooth enough no other attempts have been done. The ponderer seems pretty good in the borehole area (central corridor) but it lacks trueness in the surroundings. There are many high permeability zones in the laterals which coincide with high porosity and that may deteriorate the quality of the ponderer. In order to correct this aspect an inhibiter is needed. To make the inhibiter a new relation has been made between grid mean porosity and fracture number pondered with permeability. The relation has little quality due to almost no real relation between fracture existence and porosity but there´s no need for a good ponderer, since there´s already one, but some equation that weights the permeability effect in the generation region. The inhibiter is, therefore, the following exponential function:

The N073E set has a low concentration and seems to have an axis like tendency. The best statistical functions to describe the two sets are *Fisher* for N033E and *Bingham* for N073E with high k parameters for both (k_1 and k_2 in *Bingham* case). Fracture size could also be somewhat assumed since fractures should be bigger the closer they are to mechanical stress zones, especially if they have a favorable orientation (like N033E).

$$Inhibiter(\phi) = \frac{1}{(15,57e^{-0,04\phi})^\alpha}$$

Notice the coefficient of inhibition (α) comes in the form of potential. Naturally the *IFI* comes in the form of:

$$IFI = \varphi(\alpha; K_2, \phi) = \frac{151,2 * K_2^{-0,42}}{(15,57e^{-0,04\phi})^\alpha}$$

The best α seems to be a number between 1,6 and 2,2 so $\alpha = 1,8$ was chosen as an optimal value (real certainty would only exists with tries on modeling software and its concordance with the real data). Figure 9 shows the result pondered map.
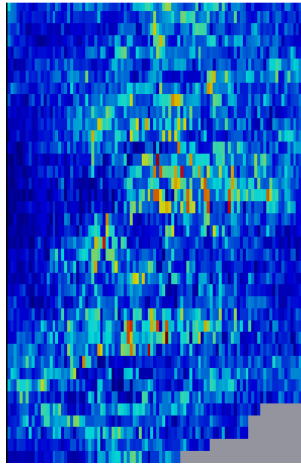

Figure 9 - Result pondered map with inhibition coefficient of $\boldsymbol{\alpha = 1,8}$.

The equivalence of *IFI* to fracture number was made with representative values taken from the QQ-plot and then inserted in statistical software *SimFit* to curve fitting. The curve fitting resulted in the following function:

$$F(IFI) = 13,27\left(1 - e^{(-153,4IFI)}\right) - 4,019$$

which, substituting for the calculated *IFI*, results in:

$$F = 13,27\left(1 - e^{\left(-153,4\frac{151,2*K_2^{-0,42}}{(15,57e^{-0,04\phi})^{1,8}}\right)}\right) - 4,019$$

being *F* the number of fractures in relation to porosity and permeability in a stochastic generation. This equation should be leveled with real data validation in the generated model since there´s no guaranty that the quantiles are proportional in both cdf´s. Nevertheless this is the fracture relation to ME´s variables.

IV – FURTHER DEVELOPMENTS

- Fracture orientation VOD would improve greatly the correct imposition of this parameter in ME fracture model. The VOD can be created from the original grid.
- Fracture size VOD can be made using the slope of the blocks of the original grid and then be assigned as a distribution between 500 and 1000 meters.
- Fracture number in fracture intensity must be leveled in modeling software before optimal usage.
- Count and shape fracture parameters can be used to history match the model without need to modify the former conclusions.

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