Extended Abstract

Stochastic Diferential Equations

in Financial Models

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Abstract

The pricing of derivatives is a useful tool for anyone involved in the derivatives market. Thus, good tools for the valuation of these financial instruments are crucial. The classic model for this valuation is the Black-Scholes model. However, due to instability of the market, new models had to arise in the financial landscape, as it is the case of models with jumps and models with stochastic volatility. While the new models allow fewer inconsistencies than the Black-Scholes model, they also require a careful calibration of their parameters. Thus, using models proposed by Merton and Heston, a calibration is performed against the values observed in the market. After performing the calibration, the parameters will then be applied in the numerical aproximation, aiming to find an option value as close to the current market as possible. Computational simulations are presented to illustrate the method.

1 Introduction

The purpose of this study is to develop numerical methods for the recovery of a specific type of derivative, the options, where the prices of underlying assets can follow continuous diffusion models or diffusion models with jumps or stochastic volatility models, calibrated to the market.

2 Stochastic Diferential Equation on Financial Models

2.1 Continuous Difusion Models

A Brownian Motion (BM) is a stochastic process$\{B_t\}_{t\geq 0}$ such that $B_0 = 0$, it has independent increments and $B_t - B_s$ is a normal random variable with expectation 0 and variance $t - s$. Finally, the function $t \mapsto B_t$ is continuous. It is also important to note that the BM is a martingale. By 1900, Bachelier discovered that the prices of financial assets can be described by a BM. However, because the BM has a normal distribution, it may reach negative values. Since we are talking about financial assets this situation should not occur. Thus, Black, Scholes and Merton suggested a new process (Black and Scholes (1973), see also Mikosch (1998)), based on the Geometric Brownian Motion (GBM). This new process should satisfy the stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$

Thus, the price of the asset will not follow a normal distribution but a log-normal one.

Since we know we can model the prices of financial assets through functions of one or more BM, then suppose that the price of a particular asset is given by the function $f(t, B_t)$. Using the formal expansion in Taylor series in two variables, assuming f is a regular C^2 function and knowing $dB_t \times dB_t \approx dt$, $dt \times dt \approx 0$ and $dt \times dB_t \approx 0$, we obtain the following formula, known as Itô formula:

$$df(t, B_t) \approx \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial B_t} dB_t + \frac{1}{2} \frac{\partial^2 f}{\partial B_t^2} dt$$

If we consider $f(t, B_t) = e^{(\mu - \frac{1}{2}\sigma^2) + \sigma B_t}$ and we use the Itô formula we get the GBM. Suppose S_t follows a BM and that there is a function G dependent of S_t and t. Applying Itô's formula to this function we get:

$$dG = (\frac{\partial G}{\partial t} + \mu S_t \frac{\partial G}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 G}{\partial S_t^2}) dt + \sigma S_t \frac{\partial G}{\partial S_t} dB_t$$

Again, the GBM is an example to this derivation.

A stochastic differential equation (SDE), in $\mathbb{R}$ is defined as:

$$\begin{cases} dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dB_t \\ S_0(w) = Y(w) \end{cases} \tag{1}$$

where it is added a random term to a deterministic initial differential equation. $\mu(t, S_t)$ and $\sigma(t, S_t)$ are deterministic functions and, any solution S_t will be a stochastic process. The randomness of this solution stems from two factors: the randomness of the initial condition and the noise introduced by the BM. Existing solutions can still be of two types: strong or weak. A strong solution to an Itô's SDE must satisfy the following conditions: S_t should be a function of a BM; the integrals present in the differential equation in question are well defined as Riemann and Itô integrals, respectively. S_t must also satisfy (1). To solve this type of stochastic differential equations we apply the Itô formula and, in the case where μ_t and σ_t are constants and $S_0 = 1$, it can be proved that GBM is the unique strong solution for a homogeneous linear SDE (Mikosch (1998)).

An option is a derivative. This derivative is a contract between a buyer and a seller giving the buyer the right, but not the obligation, to buy or sell a particular asset on the maturity date for a price, called strike price. An option may be classified by their type: may be a call option, that gives the buyer the right to buy the underlying asset, or a put option, that gives the buyer the right to sell the underlying asset. They also may be distinguished regarding their exercise date: an option it says European can only be exercised when their maturity date is reached and an American one can be exercised at any time before the maturity (Wilmott (2006)).

Suppose we have an European call option. Since there is no obligation to exercise this option this should only happen when the holder can profit from the transaction, when the asset value at maturity (S_T) is greater than the strike price (K), and the option value at maturity:

$$V(S_T, T) = max\{S_T - K, 0\}$$

We want to reach the known formula of Black-Scholes. Applying the Itô formula to $V(S_t, t)$:

$$dV = (\mu S_t \frac{\partial V}{\partial S_t} + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2}) dt + \sigma S_t \frac{\partial V}{\partial S_t} dB_t \tag{2}$$

Now, we create a portfolio of one shorted option and a certain number of shares of stock:

$$\begin{aligned}\Pi &= -V + S_t\frac{\partial V}{\partial S_t} \\ \Leftrightarrow d\Pi &= -dV + \frac{\partial V}{\partial S_t}dS_t\end{aligned} \tag{3}$$

Knowing that our portfolio evolves according to a risk-free asset

$$d\Pi = r\Pi dt \tag{4}$$

and if we replace (2) in (3) and equal to (4) we obtain the equation ofe Black-Scholes:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 V}{\partial S_t^2} + rS_t\frac{\partial V}{\partial S_t} - rV = 0$$

Thus, the price of an option according to the theory of Black-Scholes is:

$$\begin{array}{ll} C = S_0N(d1) - Ke^{-rT}N(d2) & P = Ke^{-rT}N(-d2) - S_0N(-d1) \\ d1 = \frac{Ln(S_0/K)+(r+\frac{\sigma^2}{2})}{\sigma\sqrt{T}} & d2 = \frac{Ln(S_0/K)+(r-\frac{\sigma^2}{2})}{\sigma\sqrt{T}} \end{array} \tag{5}$$

The derivation of these equations can be found, for instance, in Hull (2006).

2.2 Jump Diffusion Models

The financial markets are far away from the perfection demanded by the model of Black-Scholes, once they suffer constant oscillations, that can provoke discontinuities, called jumps, in the asset prices(Wilmott (2006)). The new process is composed by two different parts: one modeled by a GBM and the another that introduces the referred jumps in the process . The simplest form to represent these jumps is to consider a Poisson process where the probability of occurrence of an event is defined as:

$$dq = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ 1, & \text{with probability } \lambda dt \end{cases}$$

In the case where $dq = 1$ this is the occurrence of a jump of amplitude J. This amplitude will be represented by a random variable and it describes the impact that each jump prints to the assets. J is independent of both the Poisson process and the GBM. Thus, the differential equation representing this model is:

$$dS = \mu Sdt + \sigma SdB_t + (J-1)Sdq$$

In the case of MBG, $\mu = r$, where r represents the interest rate and also the instantaneous expected return for a risk-free asset, $E[\frac{dS_t}{S_t}] = rdt$. To have this equality verified in the model with jumps we should consider $\mu = r - \lambda k$, where $k = E[J-1]$. The λk tries to correct the bias created by the non null mean value, allowing the Poisson process to be a martingale.

The solution of the stochastic differential equation on the diffusion model with jumps can not be calculated the same way as for the continuous diffusion model, due to the new term introduced in the process. This new term is no more than the sum of the discontinuities randomly provoked in the asset price, such that the associated integral will be interpreted as the sum of those discontinuities. Thus, applying the Ito formula to the continuous part of the differential and the equivalent theorem for the Poisson process and considering again that $f(S_t) = Ln(S_t)$ we get:

$$dS_t = (\mu - \frac{1}{2}\sigma^2)S_t dt + \sigma S_t dB_t + Ln(J)S_t dq$$

which is the version of Ito's lemma for jump diffusion processes. The solution of this differential is:

$$S_t = e^{(\mu - \frac{1}{2}\sigma^2)dt + \sigma B^t} J^{dq}$$

Like we did for the Black-Scholes model, we will get the expression for the variation of a risk-free portfolio. Let us assume that our portfolio consists of an option $V(t, S_t)$ and by an amount $-a_t$ of assets. We are only interested in studing the case where jumps occur, because if they do not occur we return to Black-Scholes model. Thus, if $dq \neq 0$, $a_t = \frac{\partial V}{\partial S_t}$ does not completely eliminate the risk. However, Merton, when he proposed this model, argued that the risk associated with the jumping can not be regarded as systematic, so that its existence should not be taken into account (Merton (1976)). Thus, assuming $a_t = \frac{\partial V}{\partial S_t}$ and following the rules applied to the risk-free assets we get the following partial differential equation:

$$-rV + rS_t \frac{\partial V}{\partial S_t} + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2} - \frac{\partial V}{\partial t} S_t E[J-1]\lambda = 0$$

The obtained formula for pricing European options is:

$$\sum_{i=0}^{\infty} \frac{e^{-\lambda(1+k)(T-t)}}{i} (\lambda(1+k)(T-t))^i V_{BS}(S_t, T-t, \sigma_i, r_i)$$

onde $\sigma_i^2 = \sigma^2 + i\frac{(\sigma')^2}{T-t}$ e $r_i = r - \lambda k + \frac{iLn(1+k)}{T-t}$

2.3 Stochastic Volatility Models

The volatility is a measure of risk of the price of an asset, associated to market instabilities. The classic models, like Black-Scholes, use a constant volatility, being common to accomplish an estimate of the volatility to use. The so-called historical volatility can be calculated as the standard deviation of a series of asset prices, measured at regular intervals (Hull (2006)). We can also reverse the pricing model to determine the implied volatility, but this process is not trivial at all(Hull (2006)). With the evolution of markets, volatility tends to have strong variations over time, being a stochastic process. Thus, the models that use stochastic volatility have proven to be more accurate and are intended to correct errors that have been committed previously.

The Heston model is a model that uses stochastic volatility for assets valuation. As the volatility is itself a stochastic process, this model has two correlated stochastic differential, with a correlation factor ρ. Then we have:

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{V_t} S_t dB_t^1 \\ dV_t &= k(\theta - V_t)dt + \sigma\sqrt{V_t} d_t^2 \\ dB_t^1 dB_t^2 &= \rho dt \end{aligned}$$

V_t represents the variance (the square of volatility) of the price of the asset. k represents the reversion speed of the model to the average of the variance. θ it is the long term average of the variance and σ is the volatility of volatility (Moodley (2005)).

In this model there are two sources of risk and neither one can be ignored. Until now, we only need an asset to eliminate the risk involved, but now we need another one (Wilmott (2006), Gatheral (2006)). Thus, our portfolio will consist of an option F, an amount $-a_t$ of assets S_t and an amount $-b_t$ of G options:

$$\Pi = F - a_t S_t - b_t G$$

F and G depend on both the price of the asset and its variance, so we need to apply the multidimensional Itô's lemma. The variation in our portfolio will be:

$$\begin{aligned} d\Pi &= [\frac{\partial F}{\partial t} + \frac{\partial^2 F}{\partial S_t \partial V_t} V_t S_t \sigma \rho + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} V_t S_t^2 + \frac{1}{2}\frac{\partial^2 F}{\partial V_t^2}\sigma^2 V_t]dt \\ &\quad -b_t[\frac{\partial G}{\partial t} + \frac{\partial^2 G}{\partial S_t \partial V_t} V_t S_t \sigma \rho + \frac{1}{2}\frac{\partial^2 G}{\partial S_t^2} V_t S_t^2 + \frac{1}{2}\frac{\partial^2 G}{\partial V_t^2}\sigma^2 V_t]dt + [\frac{\partial F}{\partial S_t} - a_t - b_t \frac{\partial G}{\partial S_t}]dS_t + [\frac{\partial F}{\partial V_t} - b_t \frac{\partial G}{\partial V_t}]dV_t \end{aligned}$$

To eliminate the risk, we will consider:

$$a_t = \tfrac{\partial F}{\partial S_t} - \tfrac{\partial G}{\partial S_t}\tfrac{\partial F}{\partial V_t}/\tfrac{\partial G}{\partial V_t} \quad b_t = \tfrac{\partial F}{\partial V_t}/\tfrac{\partial G}{\partial V_t} \tag{6}$$

so we get the partial differential equation:

$$\begin{aligned} &[\frac{\partial F}{\partial t} + \frac{\partial^2 F}{\partial S_t \partial V_t} V_t S_t \sigma \rho + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} V_t S_t^2 + \frac{1}{2}\frac{\partial^2 F}{\partial V_t^2}\sigma V_t - rF + rS_t\frac{\partial F}{\partial S_t}]/\frac{\partial F}{\partial V_t} \\ = \; &[\frac{\partial G}{\partial t} + \frac{\partial^2 G}{\partial S_t \partial V_t} V_t S_t \sigma \rho + \frac{1}{2}\frac{\partial^2 G}{\partial S_t^2} V_t S_t^2 + \frac{1}{2}\frac{\partial^2 G}{\partial V_t^2}\sigma V_t - rG + rS_t\frac{\partial G}{\partial S_t}]/\frac{\partial G}{\partial V_t} \end{aligned}$$

This equation will be satisfied only if both sides are equal to an equation $-x(t, S_t, V_t)$, which usually is defined as $x(t, S_t, V_t) = \kappa(\theta - V_t) - \sigma\sqrt{V_t}\Lambda(t, S_t, V_t)$, where Λ represents the price of the volatility risk (Gatheral (2006))and its value is impossible to estimate, but it is proportional to the variance (Heston (1993)):

$$\Lambda\sigma\sqrt{V_t} = \lambda V_t$$

Thus, the value of any option F must satisfy:

$$\frac{\partial F}{\partial t} + \frac{\partial^2 F}{\partial S_t \partial V_t} V_t S_t \sigma \rho + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} V_t S_t^2 + \frac{1}{2}\frac{\partial^2 F}{\partial V_t^2}\sigma V_t - rF + rS_t\frac{\partial F}{\partial S_t} + (\kappa(\theta - V_t) - \lambda V_t)\frac{\partial F}{\partial V_t} = 0$$

As we need $E[\frac{\partial S_t}{S_t}] = rdt$ the alterations needed to verify this condition lead to dV_t which is

$$dV_t = \kappa^*(\theta^* - V_t)dt + \sigma\sqrt{V_t}d\widetilde{B}_t^2$$

where $\kappa^* = \kappa + \lambda$, $\theta^* = \frac{\kappa\theta}{\kappa+\lambda}$, $d\widetilde{B}_t^2 = dB_t^2 + \Lambda$ and $\mu = r$.

Heston (1993) solves this differential equation for European call options, obtaining the expression:

$$C(t, S_t, V_t) = S_t P_1 - Ke^{-r(T-t)}P_2$$

where,

$$\begin{aligned}
P_j &= \frac{1}{2} + \frac{1}{\Pi}\int_0^\infty Re[\frac{e^{-i\phi ln(K)} f_j(ln(S_t), V_t, T, \phi)}{i\phi}]d\phi \\
f_j(ln(S_t), V_t, T, \phi) &= e^{C(T-t,\phi)+D(T-t,\phi)V_0+i\phi ln(S_0)} \\
C(T-t,\phi) &= r\phi i(T-t) + \frac{\kappa\theta}{\sigma^2}[(b_j - \rho\sigma\phi i + d)(T-t) - 2ln(\frac{1-ge^{d(T-t)}}{1-g})] \\
D(T-t,\phi) &= \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d(T-t)}}{1-ge^{d(T-t)}}] \\
g &= \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d} \\
d &= \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)} \\
& \quad u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa - \rho\theta, b_2 = \kappa
\end{aligned}$$

3 Numerical Results

3.1 Monte Carlo Method

Monte Carlo simulations are used essentially to approximate numerically great complexity integrals. Thus several observations are generated from a probability distribution and the sample is used to calculate the desired approximation. This method is easy to use and a more accurate approximation can be obtained by doing more simulations, which may also lead to slow algorithms (Wilmott (2006)). According to Black-Scholes' theory, the value of an option on its maturity date is their *payoff*. Therefore various values of assets should be simulated and their *payoff*. Next the expected value of the options *payoff* are calculated. To calculate the initial value of an option we simply multiply by the discount factor,which depends on the total time and the interest rate. Asset prices are obtained from discretizations of the models already presented using Euler's method.

3.1.1 Simpson's Rule applied to improper integrals

Since we needed to calculate an improper integral, we also needed a numerical method for that approach. FFor that purpose we adapted a Simpson Rule. Simple Simpson's rule approximates the integral in an interval (a,b):

$$\int_a^b f(x)dx \approx \frac{h}{3}(f(a) + 4f(\frac{b-a}{2}) + f(b))$$

to which an error formula is associated:

$$Error_f = -\frac{h^5}{90} f^{(4)}(\xi), \xi \in [a, b]$$

Thus we need to truncate the limits of the integral and prove tthat this approximation is neglectable. The convergence is slower than desired, but values of the integral converge. The upper and lower limits used are, respectively, 100 and $\frac{1}{100}$.

3.1.2 Numerical Aproximation Results

To obtain the desired approximations we used two approaches: we will increase the number of simulations in order to increase the accuracy of the calculated expected values, and we will increase the number of sub-intervals of time considered for the calculation of each BM discretization. In the Figures (1),(2) and (3), we show how, if we increase the number of simulations, this converges to the theoretical result obtained by the pricing equations. The first two models return very close approaches of the theoretical results, while Heston's model presents a small divergence relatively to the analytical solution. This discrepancy can be explained by the existence of two random parameters in the construction of the model making the convergence of this slower.

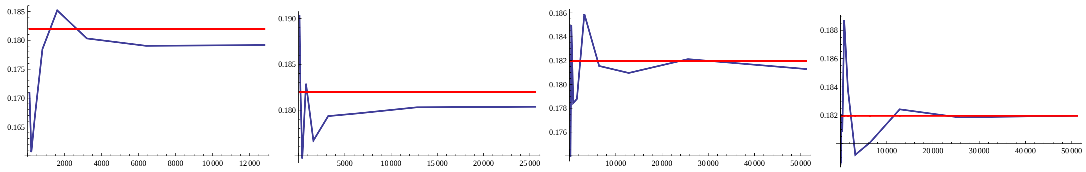

Figure 1: Graphs representing the numerical approximation for continuous diffusion model. Respectively, from left to right and from top to bottom represent the results for 25 ,50, 100 and 200 subintervals

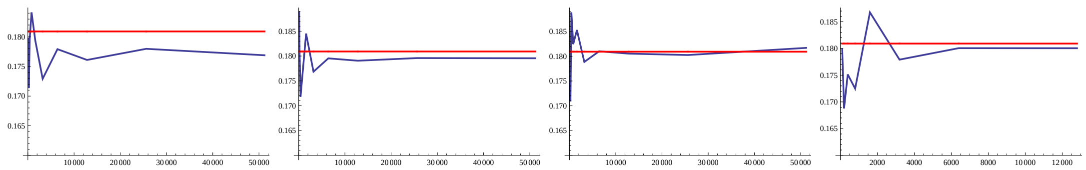

Figure 2: Graphs representing the numerical approximation for jump diffusion model. Respectively, from left to right and from top to bottom represent the results for 25 , 50, 100 and 200 subintervals

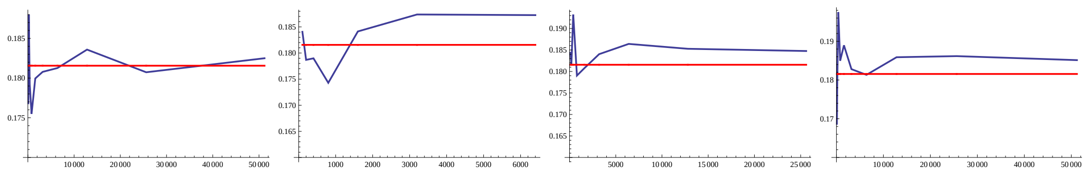

Figure 3: Graphs representing the numerical approximation for stochastic volatility model. Respectively, from left to right and from top to bottom represent the results for 25,50, 100 and 200 subintervals

3.2 Calibration of the Model

The calibration of the models is a very important tool because it allows us to adjust the parameters of the theoretical models to the values we observed in the market. To accomplish the calibration of the theoretical models an algorithm which consists of a probabilistic search, Simulated Annealing, was used. This is a local search algorithm whose main goal is to find the best solution among a finite number of neighboring solutions. The Simulated Annealing replaces the current solution by a randomly chosen solution in the neighborhood. The new solution will be chosen by an objective function, for which the result should converge to 0. This is a quite versatile algorithm and can be applied to a large number of problems. However, it may take more than desired.

Tests were performed for two ranges of data. The first ones correspond to August 23, 1995 and they were taken from Kahalé (2005). The seconds concern the index S&P500 of August 30, 2004 . Accordingly, adjustments to the calibration of each of the theoretical models were made for each of the ranges of data. In both cases, the model that best results presented was Heston, resulting in both cases in a lower value for the objective function. Once carried out the calibrations, the parameters used to determine the minimum of each objective function are used to determine the numerical approximations of the respective models.

For data taken from Kahalé (2005), we can observe that for very low maturity dates the approaches present some divergence on the foreseen results. These results can only indicate the need for a greater number of simulations in order to obtain a more accurate result. As for the behavior of models in particular, the Heston model showed improvements as the growth of the maturity date. The jump model has the best for the case of options out-of-the-money. We should note that there were exceptions to the two previous events: in the case of the Heston model are the options out-of-the-money; for the jump model these were very low maturities. The Figures (4),(5) and (6) are representative of these situations. In red we have the market values, in green the numeric approaches and in blue the analytical solutions.

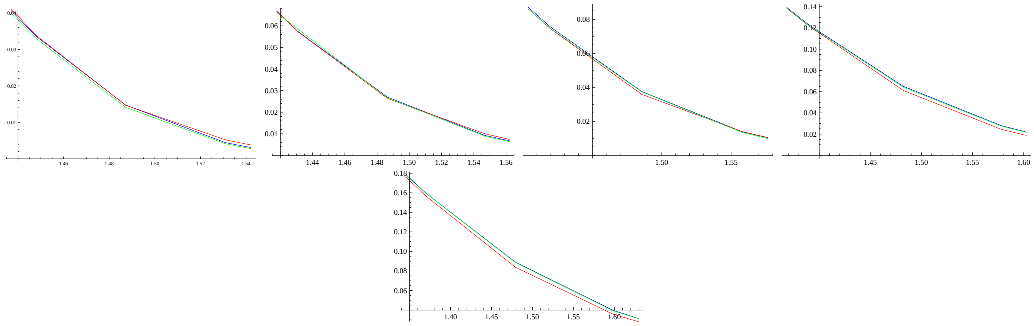

Figure 4: Strike vs. Value of the option using continuous diffusion models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.5, T=0.75

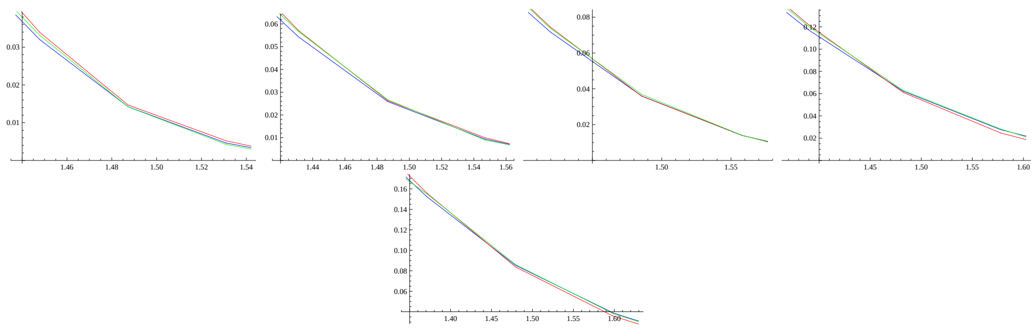

Figure 5: Strike vs. Value of the option using jump diffusion models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.5, T=0.75

For the second range of data first it must be noted that the results obtained by the jump diffusion are very close to those obtained by the continuous diffusion model, because the parameters obtained in the calibration mean that the probability of a jump is almost null. In this example the Heston model is the most accurate in almost all situations. The few exceptions occur in out-of-the-money options or in options whose maturity date is very low. These results reinforce the conclusions drawn in the previous example and may not have been clear enough at the time. The Figures (7), (8) and (9) are representative of these results.

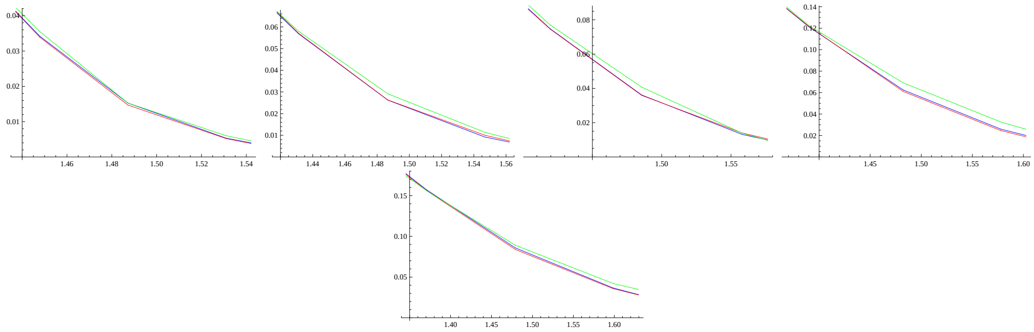

Figure 6: Strike vs. Value of the option using stochastic volatility models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.5, T=0.75

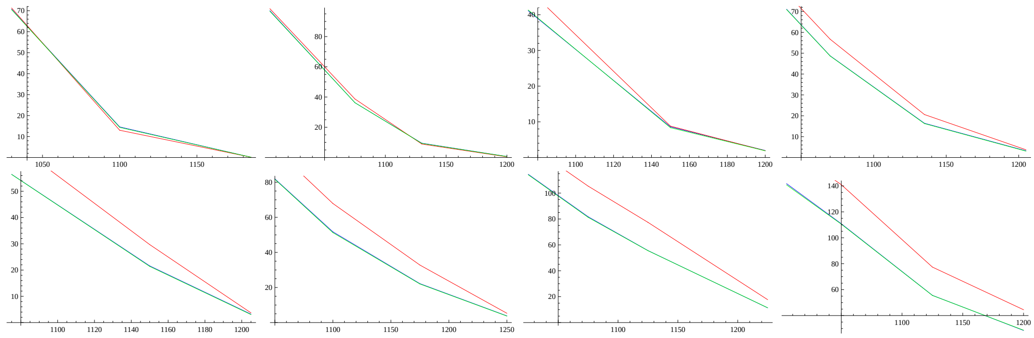

Figure 7: Strike vs. Value of the option using continuous diffusion models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.3333, T=0.5833, T=0.8333, T=1.3333 e T=1.8333

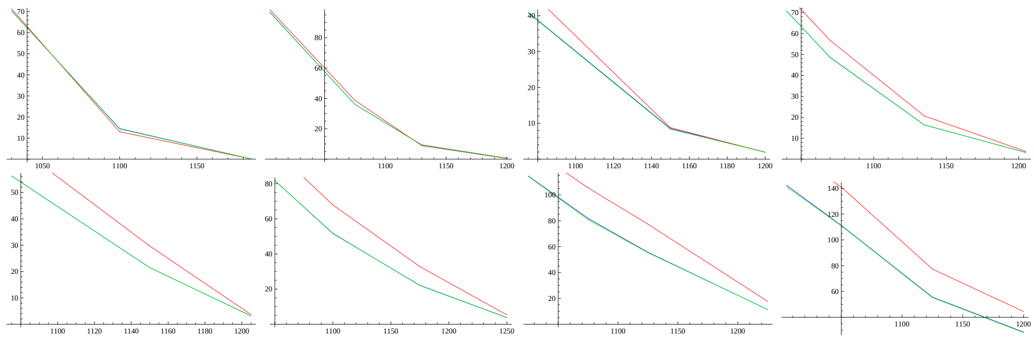

Figure 8: Strike vs. Value of the option using jump diffusion models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.3333, T=0.5833, T=0.8333, T=1.3333 e T=1.8333

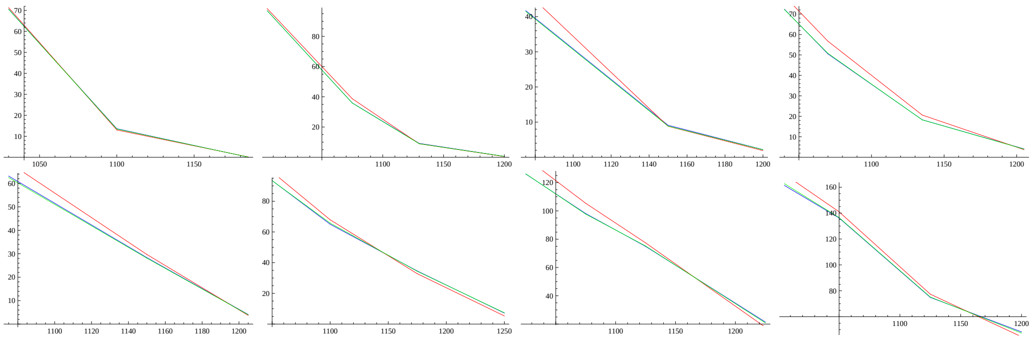

Figure 9: Strike vs. Value of the option using stochastic volatility models. From left to right and from top to bottom T=0.0833, T=0.1667, T=0.25, T=0.3333, T=0.5833, T=0.8333, T=1.3333 e T=1.8333

4 Conclusion

The initial numerical results show us that the presented numerical methods are able to return values of options very close to those presented by theoretical models. After calibration of models, we observed that the model that best approximates market value is the model of Heston. Concerning numerical approximations, Heston's model presents better results for in-the-money options and the model with jumps for out-of-the-money options. The standard Black-Scholes model presents good results in specific situations, in which the relative cases are included to the second example of market data, where the model of Black-Scholes and the one of Merton tends to return very close results, due to the low probability of occurrence of jumps. For future work, it would be interesting to apply the studied methodologies to other derivatives. It would also be interesting to carry out valuation of such derivatives using methods of numerical simulation other than the Monte Carlo simulations.

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