Electric boat modelling:
energy sources, energy storage system and electric motor control.

Tiago M. Freire, MSc. degree, IST

Abstract—This thesis is an initial study with a scope focused on electric propelled ships.

The electric vehicles are nowadays one of the most important ways to overcome the modern energy paradigm. There are a continuous growing number of electrical propelled ships available on market mainly due to legally imposed restrictions, environment concerns and increased on-board comfort. This opens new challenges on engineering, side-by-side with electric automotive applications, but with another level of needs.

Battery modelling is one of the cores of this work; it’s one of the most concerning bottlenecks on this scope, turning difficult improvements on dynamics and performance in electric vehicles. In this work were used Ni-MH and Lead-Acid batteries.

Energy converters, its control and propulsion motor’s speed control were also focused on this thesis.

It was made a great effort to optimize the dynamic response of the entire energy chain, its control and the models of the batteries.

The simulations performed on Matlab Simulink software seems promising and could contribute to further development in this topic.

Index Terms—Electric boat, ni-mh battery, lead-acid battery, state of charge, variable structure control, sliding mode, current-mode control, quasi-linear converter.

I. INTRODUCTION

Since the beginning of Time the mankind felt the need for transportation, creating a strong value chain capable of generate economic progress and development.

The stress around fossil fuels availability strikes against the lifestyle within modern societies and its long term sustainability. Urges a social and technological revolution capable of bringing a new, stable and durable balance between two heavy interests: progress versus environmental costs.

The goals and the advantages of the electric power are well-known and established within modern societies, improving energy efficiency and promoting its use on a growing number of systems almost in exclusive holding of primary energy sources, some of them highly harmful to the environment.

The electric vehicle is taking some terrain on this scope and, slowly, also the electric boat. The electric boat shows an easy way for conversion to electric propulsion and may use some infrastructures already available, tax free and without the nowadays problems of chaotic traffic on main cities. It is a growing tendency to use this kind of transportation in urban and tourism context.

Besides that, the new developments on electric power sources like fuel cells – hydrogen and other biosustainable fuels – together with the new generation of electric boats, it’s expected to see an improvement on this transportation technology and find solutions to its engineering and economical challenges.

There is a highly potential source of social development and a good way to find a solution to the modern energetic paradigm.

II. SUMMARY

This paper is a short version of the main document and its structure may be summarized as follows:

Introduction: The first section is dedicated to a compact introduction, exploring the scope of this thesis’s work.

Vessel model: The derivation for a resistance force model (vessel) and lumped-parameter dynamic model of a propeller is taken on Section III.

Ni-MH model: Section IV is dedicated to Ni-MH battery modelling and experimental work for determining the parameters of a RCL+E circuit model, including estimation of SOC and discharge evolution modelled by electromotive force (open circuit voltage).

Lead-acid model: Lead-acid modelling is presented on Section V where is available a model for SOC estimation and typical full discharge dynamics.

DC/DC converter: On Section VI is proposed two quasi-linear DC/DC converters with current-mode PWM control (CMC) scheme based on IC Maxim MAX668. One converter is buck-boost and the other is a boost type one.

VSS control: The propulsion motor speed control is composed by a Variable Structure control System (VSS) based on a first approach also shown in Section VII. Was developed a Simulink model to proof the concept and show the advantages of this type of controller.

Conclusions: Section VIII is the final one, where are conceived the conclusions of this entire work. Clearly, there is some appendix not included in this paper and they are available on the main document.

III. VESSEL MODEL

One of the main goals of this thesis is modelling the boat’s vessel. It was taken a hybrid approach from a algebraic model plus empirical model based on.

Then, the total resistance is based on two main sources: friction and residual, functions of Reynolds and Froude number, respectively.

The forward total resistance is composed on three longitudinal components:

• Skin effect ($R_{\text{skin}}$);
• Wave (residual) drag (RW);
• Air drag (RVair).

Then:

$$RT = RV_{skin} + RW + RV_{air}$$  \hspace{1cm} (1)

A. Water viscous drag

$$RV_{skin} = C_{friction} \cdot A_{w} \cdot \rho \cdot V_{x} \cdot |V_{x}|$$  \hspace{1cm} (2)

With $A_{w}$ the submerged area in the platform [$m^2$], $\rho$ fluid’s density [$kg/m^3$], $V_{x}$ tangential speed [$m/s$] and $C_{friction}$ friction coefficient of the vessel defined by [3]:

$$C_{friction} = \frac{0.075}{(log(Re) - 2)^2}$$  \hspace{1cm} (3)

With $Re$ Reynolds number [4]:

$$Re = \frac{V_{x} \cdot L_{w}}{\nu}$$  \hspace{1cm} (4)

And $L_{w}$ the length of line-of-water [meters], $\nu$ fluid kinematic viscosity given in $m^2/s$.

B. Residual resistance (wave)

The wave resistance is given by an empirical model based on [8]:

$$RW = c_1 \cdot c_2 \cdot c_5 \cdot \nabla \cdot \rho \cdot g \cdot exp \left( m_1 \cdot F_n^3 + m_2 \cdot cos(\lambda \cdot F_n^2) \right)$$  \hspace{1cm} (5)

Where $F_n$ is the Froude number:

$$F_n = \frac{V_{x}}{\sqrt{g \cdot L_{w}}}$$  \hspace{1cm} (6)

C. Aerodynamic drag

This force is proportional to the air stream exposed area. It may be, in a first approach, given by [7]:

$$RV_{air} = \frac{1}{2} \cdot C_{air} \cdot \rho_{air} \cdot S_{air} \cdot V_{air}^2$$  \hspace{1cm} (7)

Where $C_{air}$ is the air friction coefficient, $\rho_{air}$ the air density [$kg/m^3$], $S_{air}$ the vessel surface exposed to the air stream [$m^2$] and $V_{air}$ the air velocity relative to the vessel’s one [$m/s$].

This drag force may be neglected, as air limited density reduces the weight of this force between the others applied to the vessel. This is only valid if the air stream is not strong enough to increase its force to a significant level.

D. Dynamic propeller model

In this subsection is presented a lumped-parameter dynamical model for the propeller to install on the boat. Most small-to-medium-sized over-water vehicles are powered by electric motors driving propellers mounted in ducts. In general, the propeller is mounted in a duct or shroud which increases the static and dynamic efficiency of the thruster, like the setup shown on Fig. [1].

The derivation of a lumped-parameter dynamical hydrodynamic model for propellers operating in incompressible fluids found in most introductory fluid dynamics texts [9][12], take us to Eq. [8]:

$$T_{inst} = (\rho \cdot l \cdot \gamma) \cdot v_p + (\Delta \beta \cdot \rho \cdot a) \cdot v_p \cdot |v_p|$$  \hspace{1cm} (8)

With the control volume ($\rho \cdot l \cdot \gamma$), $\gamma$ and $\Delta \beta$ two empirically determined added mass and flux coefficients. $T_{inst}$ is the thrust generated and $v_p$ the axial fluid velocity at the propeller.

It is well known, exempli gratia [9][12], that under bollard-pull conditions a symmetrical propeller’s steady-state axial thrust, $T$, is proportional to the square of the propeller’s rotational velocity, $n_p$.

$$T_{inst} = \rho \cdot A \cdot r^2 \cdot \eta_p^2 \cdot \tan(p) \cdot (n_p)^2$$  \hspace{1cm} (9)

where $\eta_p$ is the propeller efficiency coefficient, $p$ is the propeller pitch (rad), $A$ is the propeller area ($m^2$), $\rho$ is the fluid density ($kg/m^3$) and $r$ the radius of propeller (m).

Then, we employ an energy balance analysis equating axial power expended at the propeller disk ($T \cdot v_p$) to power expended on the propeller shaft ($Q_p \cdot n_p$):

$$T \cdot v_p = Q_p \cdot n_p$$  \hspace{1cm} (10)

Applying this assumption directly to Eq. [8] we obtain a differential equation of motion with independent variable of propeller angular velocity and with an input – the shaft torque:

$$n_p = \frac{1}{(\eta_p)^2 \cdot r^2 \cdot \tan(p)^2 \cdot \rho \cdot \gamma \cdot Q_p - \eta_p \cdot r \cdot \tan(p) \cdot A \cdot (n_p)^2}$$  \hspace{1cm} (11)

This approach omits transient momentum balance terms to write instantaneous propeller thrust [10] using the steady-state equation [9].
IV. Ni-MH modelling

On this study is proposed a Thévenin circuit for modelling the dynamics from milliseconds to several hours time scale. It is based on a RLC+E equivalent circuit, as shown on Fig. 2. The state-of-charge estimated through the open circuit voltage (OCV) (V<sub>oc</sub>) of the battery, we must show the relation that opens a way to estimate E<sub>m</sub> through the battery output voltage (V<sub>out</sub>), modelled by the circuit on Fig. 2.

![Equivalent circuit proposed for the Ni-MH battery.](image)

U<sub>Bat</sub> and I<sub>Bat</sub> are the voltage and current at the battery terminals, E<sub>m</sub> the open circuit voltage (V<sub>oc</sub>), R<sub>i</sub> the internal resistance (terminals, electrodes and electrolyte), R<sub>D</sub> and C<sub>D</sub> represent the effects on the surface of electrodes (double layer capacity) and R<sub>K</sub> and C<sub>K</sub> describes internal phenomena due to diffusion processes on electrolyte core. The E<sub>m</sub> source is a controlled one, depending on the dynamics between OCV and discharging of the battery.

The passive (R<sub>i</sub>) and reactive (L e C<sub>i</sub>) parameters are taken as constants throughout all discharging process, while V<sub>oc</sub> is defined by Eq. 12:

\[
V_{oc} = [V_H(I_{Bat}) + V_\phi(SOC, T)]
\]

Where V<sub>H</sub>(I<sub>Bat</sub>) represents the voltage hysteresis – functional of discharge current of the battery (and not studied on this document) – and V<sub>\phi</sub>(SOC, T) the part based on Nernst modified equation [32]:

\[
V_\phi(SOC, T) = N \cdot \left[ U^0 + \frac{R_g \cdot T_{ref}}{n_e \cdot F} \cdot \ln \left( \frac{SOC - \Xi}{100 - SOC} \right) \right] - \gamma \cdot SOC + (T - T_{ref}) \cdot \frac{\Delta S}{n_e \cdot F}
\]

Knowing the expression that gives the OCV (V<sub>oc</sub>), we must show the relation that opens a way to estimate E<sub>m</sub> through the battery output voltage (V<sub>out</sub>), modelled by the circuit on Fig. 2.

\[
V_{out}(t) = E_m(t) - I_{out} \cdot R_i + R_L \cdot e^{-\alpha_L \cdot t} + R_D \cdot \left( 1 - e^{-\alpha_D \cdot t} \right) + R_K \cdot \left( 1 - e^{-\alpha_K \cdot t} \right)
\]

Trivially,

\[
V_{oc} = E_m \cdot I_{out} = 0 \text{ and } t \rightarrow \infty
\]

So,

\[
E_m = \left\{ \begin{array}{ll}
V_{out}(t) - I_{out} \cdot [R_i + R_L + e^{-\alpha_L \cdot t}] + R_D \cdot \left( 1 - e^{-\alpha_D \cdot t} \right) + R_K \cdot \left( 1 - e^{-\alpha_K \cdot t} \right)]

N \cdot \left[ U^0 + \frac{R_g \cdot T_{ref}}{n_e \cdot F} \cdot \ln \left( \frac{SOC - \Xi}{100 - SOC} \right) \right] - \gamma \cdot SOC + (T - T_{ref}) \cdot \frac{\Delta S}{n_e \cdot F}
\end{array} \right.
\]

A. Algorithm for SOC estimation

The estimation of SOC is composed by: Coulomb-Accumulation method (CAM) and by Open Circuit Voltage method (OCVM). The first one is based on the historical evolution of output current – including self-discharging and lifetime decay (both not studied here) – and the second one follows the relationship between OCV and SOC evolution.

The output of this two methods are named as SOC<sub>c</sub> e SOC<sub>v</sub>, respectively, and both are combined on a single value defined by Eq. 16 [30] [32]:

\[
SOC = w \cdot (SOC_v) + (1 - w) \cdot (SOC_c)
\]

Where w is a weighting factor.

B. Initial SOC calculation

As initial condition to estimate SOC we must define SOC<sub>0</sub> as [30]:

\[
SOC_0 = \left\{ \begin{array}{ll}
SOC_{last} & 0 \leq t < 86400 \\
SOC_{last} - SOC_{selfdischarge} & t \geq 86400
\end{array} \right. \text{ [s]}
\]

With SOC<sub>last</sub> the last value recorded from state-of-charge and SOC<sub>selfdischarge</sub> the self-discharging lost SOC Both not taken on this study approach.

C. SOC<sub>c</sub> estimated algorithm

The CAM contribution (SOC<sub>c</sub>) is determined through the following Eq. 18:

\[
SOC_c = SOC_0 - \frac{\int_0^t \theta(\tau) d\tau}{C_{nom}(I_{out_{avg}})} \cdot 100
\]

With SOC<sub>0</sub> the initial state-of-charge, \( \theta(\tau) \) the evolution of battery output current (positive convention), \( t \) the integration lapse and \( C_{nom} \) the nominal capacity considered.

D. SOC<sub>v</sub> estimated algorithm

The state-of-charge estimated through the open circuit voltage is based on the Nernst modified equation:

\[
V_{oc}(SOC_v, T) = N \cdot \left[ U^0 + \frac{R_g \cdot T_{ref}}{n_e \cdot F} \cdot \ln \left( \frac{SOC_v - \Xi}{100 - SOC_v} \right) \right] - \gamma \cdot SOC_v + (T - T_{ref}) \cdot \frac{\Delta S}{n_e \cdot F}
\]

\[
V_{oc} \text{ is determined taking into consideration the circuit laws and Eq. 19. Solving Eq. 19 in order to } SOC_v \text{ is possible to estimate it.}
\]

E. The weighting factor w

The SOC of the battery is jointly determined by SOC<sub>v</sub> e SOC<sub>c</sub>, so the weighting factor w should be calculated carefully. Considering that the battery is a dynamic and time-varying system, if we can get an accurate current measurement from the battery parameter sample system, the weighting factor of SOC<sub>c</sub> should dominate the composite SOC of the battery. In this case, w is given by:
\[ 0 \leq w \leq 0.5 \]

When the OCV is under the steady-state, the \( SOC_v \) has a higher accuracy. Therefore, we should consider both the value of OCV and the time of \( V_{oc} \) goes to steady, and then calculate the weighting factor as below.

Considering the range of \( V_{oc} \) like \([V_{\alpha}, V_{\beta}]\), we assume that the voltage corresponding to the \( SOC_v \) has a higher accuracy near both lower and higher end of that range. Therefore, we divide it into 100 sections, each one weighting value \( w_1 \) calculated as Eq. 20:

\[
w_1 = 2 \cdot |k - 50| \cdot 0.5/100, \quad 1 \leq k \leq 100 \quad (20)
\]

On the other hand, the average time of \( V_{oc} \) going to steady-state needs about 1800 sec according to the experiments performed and \[31\].

Then, if the time between two sample points is larger than 1800 sec, since \( V_{oc} \) goes to steady state:

\[ w = w_1 \]

For less than 1800 sec:

\[ w = w_1 \cdot t/1800 \]

So, \( w \) is determined by Eq. 21 \[30\]:

\[
w = \begin{cases} 
2 \cdot |k - 50| \cdot 0.5/100 & \text{if } 1 \leq k \leq 100 \text{ & } t \geq 1800[s] \\
(2 \cdot |k - 50| \cdot 0.5/100) \cdot t/1800 & \text{if } 1 \leq k \leq 100 \text{ & } t < 1800[s] 
\end{cases} \quad (21)
\]

Were performed several tests on lab and Simulink model. The lab part is composed by ON/OFF tests, done at \( SOC \) equal to 50\% and 100\%. The idea was to proof the invariance of battery dynamics for repetitive and sequencial ON/OFF states, different discharge currents and state-of-charge. For example, in Fig. 3 are represented six essays and overlayed by one simulation with the same parameters extracted by curve fitting, using \textit{cftool()} in Matlab software.

The results shows that in the thesis model it is possible to simulate with high accuracy both dynamics for 50\% and 100\% for the range of discharge currents imposed to the battery.

E.1 Simulation of \( SOC \) evolution for standard discharges

After determining the parameters of Eq. 19 through curve fitting methods (\textit{cftool()} in Matlab software), were done several discharge simulations to find the evolution of \( SOC \) by solving Eq. 19 (\( SOC_v \)) and through CAM. Was also found a relationship between the average discharge...
current and the nominal capacity of the battery, as (22).

$$C_{nom}(I_{out_{avg}}) = 928 \cdot (I_{out_{avg}})^2 - 4400 \cdot I_{out_{avg}} + 14200 \ [C] \ (22)$$

The unavailable technical data from manufacturer was partially solved by determining the output currents for standard discharge rates, like $C_2$, $C_4$ and $C_6$. This is illustrated in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$C_2$</th>
<th>$C_4$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp$_{avg}$</td>
<td>26,10</td>
<td>24,50</td>
<td>24,15</td>
</tr>
<tr>
<td>$[C]$</td>
<td>1,50</td>
<td>0,80</td>
<td>0,47</td>
</tr>
<tr>
<td>I$_{avg}$</td>
<td>1,50</td>
<td>3,40</td>
<td>6,25</td>
</tr>
<tr>
<td>Exp. time</td>
<td>9688</td>
<td>11275</td>
<td>12355</td>
</tr>
</tbody>
</table>

As an example from one of the discharges simulated on Simulink model, on Fig 5 is shown the evolution of a $C_4$ discharge.

![Figure 5](image)

**Figure 5**

**Evolution of SOC$_c$, SOC and SOC$_v$ during a $C_4$ discharge rate on Simulink model.**

We could see that the error between the SOC and SOC$_c$ is lower than 1%, from the beginning to the end of simulation at $C_4$. The SOC$_v$ also is very similar to the other ones but around SOC equal to 50%, it shows some ripple inherent to the solving method applied to (19). Although, SOC$_v$ shows a saturation zone, near SOC equal to 0 %, from a limiting block submited on V$_{oc}$ value to avoid an overflow during the simulation.

V. Lead-Acid Modelling

This work was also focused on lead-acid battery modelling. In the section is presented a RC+E model, based on [70]. The intent is to modelling the battery dynamics during complete discharges taken from dozens of minutes to several hours. The model is shown on Fig. 6.

![Figure 6](image)

**Figure 6**

**Electric model of Lead-Acid battery on study.**

$U_{Bat}$ and $I_{Bat}$ are the output voltage and current of the battery. $E_m$ is the open-circuit voltage ($V_{oc}$), $R_0$ is the internal resistance (terminals, electrodes and electrolyte) and $R_1$ e $C_1$ represents the effects of the diffusion dynamics on electrolyte core and at electrodes surface. The $E_m$ voltage source is a controlled one by the evolution of the battery state-of-charge, along of its discharge process.

$R_j$ and $C_j$ as passive and reactive electrical elements, respectively, are taken as variable parameters throughout the discharge [70]:

$$R_0 = R_{00} \cdot [1 + A_0 \cdot (1 - SOC)]$$
$$R_1 = -R_{10} \cdot \ln(DOC)$$
$$C_1 = \frac{\tau_1}{R_1} \ (23)$$

While $R_{00}$, $A_0$ and $R_{10}$ are empirical parameters. $\tau_1$ is the time constant of RC circuit loop.

$$E_m = E_{m0} - K_E \cdot (273 + \Theta) \cdot (1 - SOC) \ (24)$$

A. State-of-charge estimation

The block used to model the thermal dynamic of electrolyte, is based on a function of Joule losses ($P_{Joule}$), ambient temperature and battery thermal properties. All of this is done with a first order differential model, whose parameters are the thermal resistance ($R_\Theta$) and thermal capacity ($C_\Theta$) – see Eq. 25.

$$C_\Theta \left( \frac{\partial \Theta}{\partial t} \right) = \frac{\Theta - \Theta_{c}}{R_\Theta} + P_{Joule}$$

$\Theta_{initial}$, $\Theta_{bat}$, $\Theta_{amb}$ are, respectively, the initial measured temperature, instantaneous temperature and ambient one.

The SOC block outputs the state-of-charge and depth-of-charge (DOC) of battery based on the electric charge.
extracted \((Q_e)\), electrolyte temperature \((\text{Eq. 25})\), and two other capacities determined with \((\text{Eq. 26} [70])\).

\[
C(I_{bat}; \Theta)_{\tau=\text{const}} = \frac{K_c \cdot C_{0r} \cdot \left(1 + \frac{\Theta}{\Theta_f}\right)^\varepsilon}{1 + (K_c - 1) \cdot \left(\frac{I_{bat}}{I_{ref}}\right)^\delta}
\]  

(26)

And,

\[
SOC = 1 - \frac{Q_e}{C(I_{bat}; \Theta)}
\]

\[
DOC = 1 - \frac{Q_e}{C(I_{avg}; \Theta)}
\]

(27)

Where \(K_c\), \(\varepsilon\) \(\delta\) are fitting parameters, \(\Theta_f\) is the melting point of electrolyte, \(I_{ref}\) is a reference current for the fitting parameters and \(I_{avg}\) is the average current at the terminals of battery. So:

\[
Q_e(t) = \int_0^t I_{bat}(\tau) \, d\tau
\]

(28)

With positive convention of current flowing out of battery.

\(C_{0r}\) is a parameter determined through \((\text{Eq. 29})\):

\[
C(I_{ref}; \Theta_{ref}) = C_{0r} \cdot \left(1 + \frac{\Theta}{\Theta_f}\right)^\varepsilon
\]

\(\varepsilon = \alpha \cdot (\Theta_{ref} - \Theta_f)\)

(29)

with \(\alpha\) taken as a constant, as \((\text{Eq. 70})\):

\[
\alpha = \frac{1}{C} \cdot \left(\frac{\partial C}{\partial \Theta}\right)
\]

B. Simulation of SOC evolution for standard discharges

To find the SOC evolution were done several discharge simulations through CAM. Was also found a relationship between the average discharge current and the nominal capacity of the battery, as \((\text{Eq. 30})\):

\[
C_{\text{nom}}(I_{out_{avg}}) = 36993 \cdot (I_{out_{avg}})^2 - 45971 \cdot I_{out_{avg}} + 19695 \quad [\text{C}]
\]

(30)

This is a regression based on data collected for the range \(I_{out_{avg}} = [0.40; 0.76]\) Ampere, with a coefficient of determination \(R^2 = 1\).

The unavailable technical data from manufacturer was partially solved by determining the output currents for standard discharge rates, like \(C_2\), \(C_4\) and \(C_6\). This is illustrated in Table \(\text{II}\).

In Table \(\text{III}\) is written the result of curve fitting and parameter estimation for the ones on Fig. \(\text{I}\) (\(\text{Eq. 23, Eq. 24 and Eq. 26})\).

B.1 Conclusion notes

It is important to emphasize that the work presented gives a complementary approach to the usually ones found in recommended literature focused on this scope, especially on:

- Obtaining voltage curves for constant current discharge for standard times, like 2 hours, 4 hours and 6 hours \((C_2, C_4\) and \(C_6)\);
- Estimation of the parameters for the equivalent circuit, based on curve fitting methods;
- Thermal model constructed on Matlab Simulink for re-enforcing side-by-side the electric model;
- Electric model with variable parameters, function of state-of-charge \(\text{SOC}\);
- Possibility for estimation of nominal capacity, within a discharge current rated between \(C_2\) and \(C_6\) (\(\text{Eq. 30})\).

VI. DC/DC converter

The main limitations of nowadays DC/DC converters are, namely, optimization of its reactive filters (LC) to try approaching a zero ripple value at the output, small dimensions, reducing power losses and high performance in load and source transients.

Besides that, boost converters and buck-boost DC/DC converters suffer from slow dynamic response due to the presence of a characteristic right-half-plane zero \((36)\), whose location shifts with the operation point. The consequence is the evolution of output states tending, on initial moments, in the opposite direction of the final value. This forces the designers to limit the overall closed-loop bandwidth \((27)\), making difficult its control with a closed-loop.

\[
\begin{array}{cccccc}
\text{Parameter} & \text{Mean value} & \text{Unit} \\
\text{Temp}_{\text{avg}} & 26.05 & \text{Volts} & 25.00 & 25.40 & 23.70 \\
I_{\text{avg}} & 0.76 & \text{Ampere} & 0.76 & 0.46 & 0.40 \\
\text{Exp. time} & 2:15 & \text{hours} & 2:27 & 3:51 & 6:20 \\
\text{Nominal capacity} & 6124 & \text{Coulomb} & 6694 & 6376 & 7226 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Parameter} & \text{Mean value} & \text{Unit} \\
E_m & 6.337 & \text{Volt} \\
K_c & 1.566E-3 & \text{Volt} \cdot K^{-1} \\
R_{\text{ref}} & 1.264E-1 & \text{Ohm} \\
A_0 & 0.4209 & \text{Adim} \\
R_{\text{ref}} & 0.0370 & \text{Ohm} \\
\tau_1 & 1.281 & \text{s}^{-1} \\
\end{array}
\]
The main goal of the approach proposed in this work is to implement a system with the following advantages, for achieving a high performance and high power converters:

- Non-linear voltage control with current mode control \( \text{CMC} \) with high impedance output [37]. The \( \text{CMC} \) also solves the problem of resonance between the main inductor and the capacitor at the output [37], minimizing electric oscillations.
- Unlike some approaches using high capacitance at the output to stabilize the voltage and reducing ripple, this solution applies [24]: An output linear voltage regulator stage that minimizes load voltage ripple without the use of large output capacitance (cost reduction);
  - An independent current injection stage to limit variations on output voltage that eliminates voltage deviation during fast transient response. Also, this stage suppress the right-half-plane zero influence in the boost and buck-boost converters, thus significantly improving the transient response bandwidth.

On Fig. 7 are shown both quasi-linear converters, boost and buck-boost ones.

MOSFETs \( Q_2 \) and \( Q_3 \) and resistor \( R_s \) are added to a conventional boost converter, along with a tapped inductor \( L_1 \) and an auxiliary storage capacitor \( C_{aux} \). \( Q_2 \) is controlled using an op-amp \( \text{OP1} \) to realize a voltage follower function, with \( V_{ref} \) being the output reference voltage. Under steady state conditions, \( Q_2 \) operates in the linear zone with the op-amp modulating the gate voltage to keep the output virtually ripple free.

If the load current decreases suddenly, the output voltage increases suddenly as the converter control attempts to reduce the inductor current. This forces the voltage across \( Q_2 \) to be higher, moving it into a saturation zone, wherein \( Q_2 \) sees higher dissipation momentarily, but here the output voltage remain constant and equal to \( V_{ref} \).

Under a step increasing in load current, the output voltage would decrease. This would in turn cause the controller to increase the duty cycle, allowing the inductor current to build up. If the system operates in a continuous mode under full load with low current ripple, the current increment could take 3 to 10 cycles [24], causing a significant transient in the output voltage. Smaller values of capacitance exacerbate the problem of voltage deviation under this conditions.

The issue of large voltage deviation during step change in load is addressed through a current injection stage. This is done with \( Q_3 \) ON/OFF state – is also operated as a voltage follower using op-amp \( \text{OP2} \). Under normal operating conditions, the diode \( D_3 \) is reverse biased but, within a sudden load transient, the boost converter output drops, forcing \( D_3 \) to be forward biased and clamping converter output voltage using a current injection from the capacitor \( C_{aux} \) into the output capacitor \( C \).

\[ \text{Figure 7} \]

**A. Current-Mode Control**

A current-mode control was implemented in the Matlab Simulink model for these converters, which is represented in Fig. 8.

This method regulates the output current and, with infinite loop gain, the output is a high-impedance source [37]. In \( \text{CMC} \) the current loop is nested with a voltage loop, as shown in Fig. 8, a ramp is generated by the slope of the inductor current (\( V_{sense} \)) and compared with the error signal (\( V_{error} \)). So, when the output voltage sags, the \( \text{CMC} \) supplies more current to the load.

In \( \text{CMC} \), the duty cycle is determined by the number of times which inductor current reaches the maximum limit defined by the voltage control loop signal (see Fig. 9).

The current-mode scheme has several advantages [33]...
over the conventional voltage-mode scheme (VMC):
1 Several switching converters can be operated in parallel without a load-sharing problem because all the switching converters receive the same PWM control signal from the feedback circuit and carrying the same current.
2 During current-mode operation, the average inductor current follows a reference voltage. As such, the inductor acts as a current source. Thus, the inductor behaves as a voltage-controlled current source that supplies the output capacitor and the load, thereby reducing the order of the system by one. This simplifies its feedback compensation considerably.

The major drawback of the CMC is its instability. An oscillation generally occurs whenever the duty cycle exceeds 50%, regardless of the type of switching converter. However, this instability can be eliminated by the addition of a cyclic artificial ramp either to the sample of the inductor current ($V_{\text{sense}}$) or to the voltage control signal ($V_{\text{error}}$).

This control scheme was implemented on Matlab Simulink based on the Maxim MAX668 integrated circuit.

The model is an approach for fixed frequency PWM cycle-by-cycle current model control. Additional information about this implementation can be seen in [34].

A.1 Step response to RL load type

The quasi-linear converter model and its control were submitted to a limited number of steps on its input references (voltage and current), for two different types of loads: pure resistance (R) and series resistance+inductance (RL), as shown in Fig. 11 and Fig. 12.

On figures above was applied a step with 25 Volt and 25 Ampere slope, for both voltage and current input references. It is visible on output voltage evolution that the settling time is 45 ms and 32.5 ms, for start-up and step up dynamics, respectively. This times are equal to a resistive load only (R) too. The settling time for steeping down and start-up are 75 ms e 45 ms, respectively, like in the R type load situation. No overshoot was observed on anyone of the simulations done.

The error on steady condition is contained between 1% and 2% of reference values for output voltage and current.
Figure 12
Output voltage ($V_{out}$) evolution for a step down on voltage and current input references, to a series $R = 1 \ \Omega$ and $L = 1 \ \text{mH}$ load.

It is also important to notice that the response dynamics (time and evolution) seems to be independent of load type for the parameters submitted in simulation.

A.2 Current injection stage

On figures below is shown the evolution of output voltage of a buck-boost quasi-linear converter, for a $R = 1 \ \Omega$ type load. It was submitted with a step up with slope of 25 Ampere for the current reference ($t_{on} = 350\text{ms}$).

The Fig. 13 illustrates the transient response on $V_{out}$ for a step up in reference current.

On Fig. 14 is illustrated the dynamic of the current injected by its own stage, in the same conditions of Fig 13.

This current flows by a pulsing way, so that the output voltage ($V_{out}$) stays constrained to a *priori* defined minimum limit, as a turn-on parameter of this stage. The amplitude of this current varies in a range from 160 Ampere to 133 Ampere. This is also due to the fast overshoot visible on $V_{out}$ with origin on the dynamic imposed by the inductive load.

There are evident the advantages of this stage: faster response (rising time about half on off-line state) and lower voltage dip. The influence of right-hand-plane zero is almost suppressed on voltage load.
on semiconductors and others minor approximations.

**VII. VSS Control**

DC motor drives have been widely used in industry for speed control because of their excellent control characteristics. One of the commonly used techniques for controlling a separately excited DC motor is closed-loop integral control, where the speed is controlled by varying the voltage applied either to the armature terminals or to the field terminals. It is capable of taking the steady-state error at zero, e.g., on motor speed, however it shows a week dynamic performance clearly visible in prominent overshoots or very long settling time.

The Variable Structure control System with Sliding Mode (VSS-SM) was developed on mid 50’s of 20th century in USSR by the hand of Emelyanov [60] and followed by others [58] [59] [63]. This pioneer researches tried to approach 2nd order linear systems control by phase-shifting. Since then, the VSS have been developed as a general application method, applied in a variety of other types of non-linear, discrete and stochastic systems. Moreover, the main goals of this control method is its robustness and it could be insensitive to uncertainty on plant parameters and external disturbances.

### A. DC motor drive model

Consider a separately excited DC motor, as shown, in Fig. 17. The equations describing the dynamic behaviour of the motor are as following:

\[
\begin{align*}
V_a &= R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + K_f \cdot i_f \cdot \omega \\
V_f &= R_f \cdot i_f + L_f \cdot \frac{di_f}{dt} \\
\frac{d\omega}{dt} &= \frac{K_f \cdot i_f \cdot i_a - T_{load}}{J} - \frac{J}{J} \cdot \omega
\end{align*}
\]

Where \( K_f \) is a constant.

**Figure 17**

*Equivalent DC motor circuit.* [38].

Linearising Eq. [31] about the operating point \( x_0 \) including the integral controller, we obtain the linearised state equation of the DC motor drive system as [32]:

\[
\dot{x} = A \cdot x + B \cdot u + \Gamma \cdot z \\
x(0) = 0
\]

(32)

Which the state vector, \( x \), is:

\[
x = [x_1 \ x_2 \ x_3]^T
\]

(33)

Where:
\[ x_1 = \int (\Delta \omega_{ref} - \Delta \omega) \cdot dt \]
\[ x_2 = \Delta \omega \]
\[ x_3 = \Delta i_a \]

And:
\[ u = [0 \ 0 \ \Delta V_a] \]
\[ z = [\Delta \omega_{ref} \ \Delta T_{load} \ 0] \]

The system matrix:
\[ A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -f/J & \frac{K_f I_a}{J} \\ 0 & -\frac{K_f I_a}{J_a} & -\frac{R_a}{J_a} \end{bmatrix} \] (34)
\[ B = [0 \ 0 \ \frac{1}{L_a}]^T \] (35)
\[ \Gamma = [1 \ -1/J \ 0]^T \] (36)

Quod Erat Demonstrandum.

On Fig. 18 is illustrated the block diagram of the control system above presented. It has a 3\textsuperscript{rd} order dynamics, with a single integrator at the output of speed error, \( \Delta \omega_e \). The block SMC includes the controller described below.

\[ U_v = -\rho \cdot \frac{c^T \cdot x}{|c^T \cdot x| + \delta}, \quad \rho > 0 \quad e \quad \delta > 0 \] (38)

With \( L \) is a linear state feedback matrix:
\[ L = [l_1 \ l_2 \ l_3] \] (39)
and \( c \) is the switching vector defined as:
\[ c = [c_1 \ c_2 \ 1]^T \] (40)

The elements of \( L \) are defined as \[63\] \[67\]:
\[ l_1 = c_1 \cdot L_a \cdot \Phi^* \] (41)
\[ l_2 = L_a \cdot \left[ c_1 + c_2 \cdot (\Phi^* + \frac{f}{J}) \right] + K_m \] (42)
\[ l_3 = L_a \cdot \left[ \frac{1}{T_a} + \Phi^* - c_2 \cdot \frac{K_m}{J} \right] \] (43)

Where \( \Phi^* \) is a non positive scalar \[69\].

The function \( U_l \) should always be able to bring the system state trajectory from anywhere in state space into the manifold in which the sliding mode occurs. The unit vector control function \( U_v \) switches dynamically to force the trajectory to remain in the manifold and slide towards the origin of the state space. On the other hand, the magnitude of switching control function \( U_v \) is relatively small \[67\], distinguishing this sliding mode control from the bang-bang one.

Additionally, the smoothing factor, \( \delta \), in Eq. 38 and discussed on \[68\] \[69\] lowers the ripple on the system evolution during sliding mode, as also the empirical parameter \( \rho \) in 38.

B.2 Robustness of the drive to a step load disturbance

Considering the ideal trajectory in sliding mode is expressed by:
\[ c_1 \cdot x_1 + c_2 \cdot x_2 + c_3 \cdot x_3 = 0 \] (44)

The transfer function of the drive speed deviation to a step load disturbance, during sliding mode, is given by \[32\] and \[44\]:
\[ \frac{\Delta \omega_r(s)}{\Delta \omega_{ref}(s)} = \frac{-c_1 \cdot K_m}{s^2 + (c_2 \cdot \frac{K_m}{J} + \frac{f}{J}) \cdot s - c_1 \cdot \frac{K_m}{J}} \] (45)

Under sliding mode, the drive speed response to a step load disturbance is reduced from a 3\textsuperscript{rd} order to a 2\textsuperscript{nd} order impulse-type response. Re-writing (45) in a well-known canonical 2\textsuperscript{nd}-order form \[46\]:
\[ W(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2} \] (46)

it is possible to write \( c_1 \) and \( c_2 \) in function of \( \omega_n \) and \( \zeta \) parameters \[66\].
\[ c_1 = -\frac{\omega_n^2 \cdot J}{K_m} \]  \hspace{1cm} (47) \\
\[ c_2 = \frac{2 \cdot \xi \cdot \omega_n \cdot J - f}{K_m} \]  \hspace{1cm} (48)

And the following condition must hold \textbf{63}:

\[ \xi > \sqrt{\frac{1}{2} \left( \frac{f^2}{2 \cdot J^2 \cdot \omega_n^2} + 1 \right)} \]  \hspace{1cm} (49)

In \textbf{63} was proved that are needed high values of \( \omega_n \) and \( \xi \) to lowering the steady-state error in angular velocity and settling time, during step-load disturbances, on this VSS controller. This is unusual, namely in conventional design principle based on linear control theory; there is always a compromise between \( \omega_n \) and \( \xi \), due to the intrinsic conflict to achieve a good balance on large overshoot \textit{versus} long settling time. The region of \( \xi > 1 \) is almost always abandoned because sluggish response will result. This situation is very favourable for the sliding mode control system, because the strategy of two-stage state motion (\( U_l \) and \( U_u \)) is very effective for resolving this conflict \textbf{67}. Moreover, the independent selection of the values of \( \xi \) and \( \omega_n \) based on the freedom in the selection of the switching vector \( c \) provides a new level of performance, capable of achieving both fast response and robust performance, impossible in approaches based on linear control theory.

\textbf{B.3 Simulations tests – Start-up}

On Fig. 19 and Fig. 20 are illustrated the time evolutions of DC motor speed (\( \Delta \omega_r \)) and its voltage control signal (\( u = \Delta V_a \)).

For the curves 1 to 4 the dynamic of those does not exhibit overshoot and has high range of settling times, varying from slow to high speed evolution performance. \( \Delta V_a \) has a smooth evolution, without a switching evolution typical of Bang-Bang VSS controllers, as proved in the main document.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Curva} & \textbf{\( \omega_n \)} & \textbf{\( \xi \)} & \textbf{\( t_{\text{estab}} \)} & \textbf{\( c_1 \)} & \textbf{\( c_2 \)} \\
\hline
1 & 2,438 & 1,34 & 3,30 & -1 & 1,672 \\
2 & 5,358 & 1,80 & 1,96 & -4,577 & 3,251 \\
3 & 7,377 & 2,20 & 1,79 & -9,432 & 5,534 \\
4 & 10,971 & 3,00 & 1,66 & -25,854 & 11,317 \\
5 & 90 & 1,00 & 0,130 & -1,410 & 31,258 \\
6 & 300 & 0,71 & 0,108 & -15675 & 74,163 \\
\hline
\end{tabular}
\caption{Table with the parameters submitted on simulation (Fig. 19 and Fig. 20). \textbf{66}.}
\end{table}

\textbf{B.4 Evolution on output parameters of the model}

To obtain the evolutions of the output parameters on this controller were done multiple tests with this simulation model. All the tests were done with \( \omega_n = 90 \text{rad/s} \) e \( \xi = 1 \).

Two steps were applied to the model: \( \Delta \omega_{\text{ref}} = 500 \text{RPM} \) (\( t_{\text{on}} = 1 \text{s} \)) and \( \Delta T_{\text{load}} = 25 \text{N.m} \) (\( t_{\text{on}} = 5 \text{s} \)), shown on Fig. 21.

The motor speed \( \Delta \omega_r \) shows a settling time about 130 ms, without any visible overshoot. At 5 secs of simulation time, was applied a step-up on load torque input of 25 Nm, responding the system with a small down-dip on \( \Delta \omega_r \) lower than 2.97% of its steady-state value. The intrinsic settling time of this disturbance is about 95 ms.

The highest overshoots on \( \Delta V_a \) and \( \Delta i_a \) are visible at the start-up of the simulation, due to the fast dynamic imposed by the controller. This was only done as an example of its capacity to achieve that, however this situation is beyond the ability that a typical DC/DC converter could operate within bearable cost and dimensions.

At the torque step-up disturbance, both electrical quantities shows an overshoot around +4.35%. The system response gives an almost unnoticeable dip on motor speed, due to the fast dynamics of the controller.

Therefore, we can conclude that the system presents a the lower insensibility to step disturbances on load torque.

\textbf{B.5 External disturbances response}

To analyse possible disturbances, was applied one step-up on \( \Delta \omega_{\text{ref}} = 500 \text{RPM} \) (\( t_{\text{on}} = 0 \text{s} \)) and extracted the evo-
Evolution on output current signal $\Delta i_a$.

Evolution on output voltage signal $\Delta V_a$.

Evolution on output motor speed $\Delta \omega_r$.

Figure 21

Evolutions on the mechanical and electrical outputs on the VSS controller.

This VSS controller shows a virtual insensitivity to system parameter disturbances, namely on the armature circuit time constant $T_a$ (Fig. 22(a)). On the $K_m$ situation, the controller can tolerate, without significant impact on its performance until 80% of its nominal value. With $K_m$ at 0.5 p.u., it is visible a small overshoot with a faster rising time, but at the cost of a worst dynamic performance.

Therefore, we can assume a lower insensitivity for external parameters disturbances on this controller performance.

Evolution of $\Delta \omega_r$ for 3 different values of $T_a$.

Evolution of $\Delta \omega_r$ for 3 different values of $K_m$.

Figure 22

Evolution of angular motor speed $\Delta \omega_r$ for some values of $T_a$ and $K_m$.

VIII. Conclusion

This paper shows a first approach to modelling a ship with electrical propulsion and were taken in consideration several models, namely, the vessel propeller turbine torque model, battery modelling (both Lead-Acid and Ni-MH), DC/DC converters and its control system, and speed control for the DC motor model.

The most evident contribution given to this work could be found in battery modelling of Ni-MH one. Is was experimentally confirmed its micro-second dynamic (ON/OFF transients), its slow discharging dynamics ($E_{m}$ evolution modelling) and SOC estimation. In ON/OFF transient, was verified that it could be modelled by a constant parameter model along SOC evolution, for the discharge currents imposed. The curve fitting done to the experimental points proved a high performance modelling within this approach. Moreover, this work accomplished a good basis structure to future studies and could be improved on many ways, mostly due to the $E_{m}$ hysteresis, influence of discharge current on battery output voltage dynamic and other internal/external disturbances.

It is important to emphasize that the work presented on Lead-Acid modelling gives a complementary approach to the usually ones found in recommended literature focused on this scope, especially on:

- Obtaining voltage curves for constant current discharge for standard times, like 2 hours, 4 hours and 6 hours ($C_2$, $C_4$ and $C_6$);
• Estimation of the parameters for the equivalent circuit, based on curve fitting methods;
• Thermal model constructed on Matlab Simulink for re-enforcing side-by-side the electric model;
• Electric model with variable parameters, function of state-of-charge (SOC);
• Possibility for estimation of nominal capacity, within a discharge current rated between C2 and C6 (Eq. 30).

On this work were also implemented models for optimization on DC/DC converters, especially on transient response, for a auxiliary injection current block, reducing then the presence of a characteristic right-half-plane zero for both converters (boost and buck-boost). Was also proposed a ripple rejection model, capable of reducing substantially the capacitance needed at the converter output. This could reduce also the cost, dimensions and wearing on DC motor due to ripple on its output torque. The controller for both converters can be built only with digital components, capable also of PWM control at constant frequency, with current-mode control done cycle-by-cycle. This approach is based on Maxim MAX668 integrated circuit, which is a current-mode controller highly efficient, independent of the load type (R, RL, etc) and with virtually unlimited output voltage range for both converters.

Finally, the motor speed control was approached by a non-classic way. A Variable Structure control System (VSS) with unit vector sliding mode was tested in Matlab Simulink, an easy way to test its performance, based on industrial parameters (ω, ε, ζ), and with wide range and high performance transient response, without difficult control compensation methods for settling time or other critical parameters.
Tiago M. Freire was born in Lisbon, in 1984. He received the MSc. degree in electrotechnical engineering from Instituto Superior Técnico – Universidade Técnica de Lisboa, Lisbon, in 2009.

IX. Acronyms

CAM  Coulomb-Accumulation method
CMC  Current-Mode Control
DC  Direct Current
DOC  Depth-of-Charge
IC  Integrated Circuit
MOSFET  Metal-Oxide Semiconductor Field-Effect Transistor
Ni-MH  Nickel-Metal Hydride
OCV  Open Circuit Voltage
OCVM  Open Circuit Voltage method
PWM  Pulse Width Modulation
SOC  State-of-Charge
VMC  Voltage Mode Control Pulse Width Modulation
VSS  Variable Structure control System
VSS-SM  Variable Structure control System with Sliding Mode