MODAL ANALYSIS OF MICROWAVE AND MILLIMETER-WAVE WAVEGUIDES WITH COMPLEX MEDIA

Paulo Delgado

Instituto Superior Técnico
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail: {p.d.delgado} @gmail.com

ABSTRACT

This work addresses the problem of guided-wave propagation in complex media waveguides. The basic equations describing the electromagnetic propagation in each type of media are reviewed. Here, the use of anisotropic media, ferrite, chiral and omega media in common waveguides is considered. The derivation of the modal equations and the respective dispersion diagrams are presented.

Index Terms – Modal Analysis, Complex Media, Anisotropic Media, Ferrite, Chiral Media, Ω Media.

1. INTRODUCTION

In recent years, complex media have received considerable attention in the literature. In fact, the use of complex materials, e.g., anisotropic or nonlinear properties, can be useful in the design of certain features for microwave devices [2]. For instance, to built a directional device, nonreciprocal media can be used in order to explore the nonreciprocal propagation of the electromagnetic waves, Nonreciprocity can be obtained with a ferrite, for example, by applying a static magnetic field [1]. Chiral media are commonly known as materials that have optical activity. The optical activity was first discovered by Arago [5]. Recently, the electromagnetic chirality and chiral materials have been extensively investigated due to a large number of applications particularly in the field of microwave and optical systems.

Saadoun and Engheta [8] have suggested in 1992, for the first time, the Ω-media. Although the omega media are not optically active, they have received much attention due its special characteristics lead to new applications such as phase shifter, phase transformer and waveguides. [8]. Both these, are bianisotropic media (chiral and omega) with magneto-electric coupling . The main difference is how this coupling is induced.

In this paper, we present new and interesting modal characteristics of waveguides involving complex media, namely, anisotropic, ferrite, chiral and omega media. These properties can be explored and applied in the development of microwave devices.

2. ANISOTROPIC WAVEGUIDES

In an anisotropic medium, unlike the case of isotropic media, the electromagnetic properties of the medium are strongly dependent on the specific space direction considered [3].

For the anisotropic media, one may write

\[ \Delta_j = \varepsilon_{ij} \mathbf{E}_j \]  

where \( \varepsilon_{ij} \) are the elements of a second order tensor.

In this paper, one considers the case of planar waveguides containing anisotropic media (Fig. 2.1), specially uniaxial crystals, being the tensor defined for:

\[ \varepsilon = \varepsilon_{||} \mathbf{\hat{x}} \mathbf{\hat{x}} + \varepsilon_{\perp} (\mathbf{\hat{y}} \mathbf{\hat{y}} + \mathbf{\hat{z}} \mathbf{\hat{z}}) \]  

It is assumed that the propagation takes place along the \( z \) axis, in the form \( e^{-j\beta z} \) and that the waveguide has an infinite structure along \( y \).

Figure 2.1 – Uniaxial dielectric slab
The constitutive relations of the media are:

\[ \mathbf{B} = \mu \varepsilon \mathbf{E} \quad (2.3) \]
\[ \mathbf{D} = \varepsilon \varepsilon_0 \mathbf{E} \quad (2.4) \]

in which \( \mathbf{B} \) represents the magnetic induction and \( \mathbf{D} \) the electrical displacement. On the other hand, it is known that:

\[
\begin{bmatrix}
\hat{x} & \hat{y} & \hat{z}
\end{bmatrix}
\begin{bmatrix}
\partial_x & 0 & -j k
\end{bmatrix}
\begin{bmatrix}
jk A_x \hat{x} - \left( jk A_y + \frac{\partial A_x}{\partial x} \right) \hat{y} + \frac{\partial A_z}{\partial x} \hat{z}
\end{bmatrix}
\]

By applying this formula to the electric field and from Maxwell's equations, we obtain the following set of equations:

\[ jk E_x = -j \omega \mu_0 \varepsilon_0 (x) H_x \quad (2.6) \]
\[ jk E_y + \frac{\partial E_x}{\partial x} = j \omega \mu_0 \varepsilon_0 (x) H_y \quad (2.7) \]
\[ \frac{\partial E_y}{\partial x} = -j \omega \mu_0 \varepsilon_0 (x) H_z \quad (2.8) \]

The other components of the fields can be obtained by replacement of \( E \) by \( H \).

Assuming \( \phi = E_y, H_y \), one can easily write

\[ \left( \frac{\partial^2}{\partial x^2} + k_x^2 \right) \phi(x) = 0 \quad (2.9) \]

The complete field solutions in both media are derived from the support component \( E_y \) and \( H_y \), taking always into consideration the fact that there is no propagation below \( x = 0 \), because of the perfect electric conductor.

Enforcing the boundary conditions (BC’s) at the media interface, that is, \( H_y (x = d) \) and \( E_z (x = d) \), one obtains:

\[
\begin{bmatrix}
\cos(hd) & -e^{-a(x-d)} \\
\sin(hd) & e^{-a(x-d)}
\end{bmatrix}
\begin{bmatrix}
C_2 \\
B_2
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix} \quad (2.10)
\]

In order to avoid trivial solutions for this equation, one must set the matrix determinant of (2.10) equal to zero.

With the introduction of normalized variables \( u = \frac{hd}{\lambda} \) and \( w = \frac{ad}{\lambda} \), the modal equation takes the following final form

\[ u = \varepsilon_0 \omega \cot(u) \quad (2.11) \]

The Fig. 2.2 presents the dispersion diagrams of the TE and TM modes.

It can be seen from Fig.2.2-i), that the dispersion curve of the first mode begins at a lower frequency when compared with the curve of the first mode of Fig. 2.2-ii). This is due to the fact that, in the former case, the fields are related to roots of functions with different parities.

Each curve represents a certain propagation mode, or a particular configuration of fields, with well-defined values of cut-off frequency.

Fig. 2.2 – (i) Dispersion diagrams of TM modes; (ii) Dispersion diagrams of TE-odd modes.

3. FERRITE WAVEGUIDES
The ferrites are important materials with high electrical resistivity, good magnetic properties (including anisotropy magnetic) and several technological applications. Therefore, they have attracted much attention in recent decades [4]. In order to satisfy to the demands of modern equipments for generating microwaves, the interest in the characterization and in the development of techniques of manufacture of these materials has been renewed.

3.1 Transversal Magnetic Field

Assuming a transversal magnetic field and a media exhibiting magnetic anisotropy, the magnetic permeability tensor has the following form:

\[
\mathbf{\mu} = \begin{pmatrix}
\mu_\perp & 0 & j\mu_z \\
0 & \mu_\parallel & 0 \\
-j\mu_z & 0 & \mu_\perp
\end{pmatrix}
\] (3.1)

In this case, the constitutive relations are the same of the previous session.

Following the same procedure of the previous session, the equation for the TM mode is easily obtained:

\[
\frac{\partial^2 H_y}{\partial x^2} + k_z^2 H_y = 0
\] (3.2)

For \( x > t \), one has \( B = 0 \Rightarrow H_y = Ae^{-\alpha x} \) indicating a exponentially decreasing wave.

For \( x < t \), the mode will have two components in transversal resonance. The support component defines the parity of the modes. Moreover, for \( E_z(x = 0) = 0 \Rightarrow D = 0 \), one obtains only solutions that characterize the transverse magnetic modes (TM).

According to the above, one has

\[
H_y = \begin{cases}
R \cos(kx), & 0 < x < t \\
Se^{-\alpha(x-t)}, & x > t
\end{cases}
\] (3.3)

After some manipulation, taking into account the media properties, and enforcing the boundary conditions at \( H_y(x = t) \) e \( E_z(x = t) \), one can easily obtain

\[
\begin{bmatrix}
\cos(ht) & -e^{-\alpha(x-t)} \\
\varepsilon_z h \sin(ht) & -\varepsilon_z \alpha e^{-\alpha(x-t)}
\end{bmatrix}
\begin{bmatrix}
S \\
R
\end{bmatrix} = 0
\] (3.4)

Again, by annulling the determinant of the matrix in (3.4), and introducing normalizes variables, the final modal equation is obtained for the TM modes:

\[
w \frac{E_z}{\varepsilon_z} = u \tan(u)
\] (3.5)

3.2 – Longitudinal Magnetic Field

We will now consider a rectangular guide involving a ferrite with a longitudinally applied magnetic field. In this case, all the propagation modes will be hybrid. Again, one assumes
that the propagation takes place along the \( z \) axis. In this case, the magnetic permeability tensor has the following form:

\[
\mathbf{\mu} = \begin{bmatrix}
\mu & j\mu_x & 0 \\
0 & \mu_x & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  
(3.6)

The Maxwell's equations can be unfolded as follows:

\[
\nabla \times \mathbf{E} = -j\omega \mathbf{B} \implies \nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} \]  
(3.7)

\[
\nabla \times \mathbf{H} = j\omega \mathbf{D} \implies \nabla \times \mathbf{H} = j\omega \varepsilon \mathbf{E} \]  
(3.8)

Applying (2.5) to the Maxwell's equations, we obtain, by substitution,

\[
j\beta E_y = -j\omega \mu_0 (\mu H_x + j\mu_x H_y) \]  
(3.9)

\[
-\left( j\beta E_x + \frac{\partial E_y}{\partial x} \right) = j\omega \mu_0 (-j\mu_x H_x + \mu H_y) \]  
(3.10)

\[
\frac{\partial E_y}{\partial x} = -j\omega \mu_0 H_z \]  
(3.11)

From the previous equations, and introducing normalized variables, we obtain

\[
\frac{\partial^2 H_y}{\partial x^2} = -\frac{b}{a^2} \frac{\partial^2 E_y}{\partial x^2} + (ab - a'b')E_y = 0 \]  
(3.12)

Expressing \( E_y(x) \) as a function of \( \phi_A(x) \) e \( \phi_B(x) \), one has

\[
E_y(x) = A\phi_A(x) + B\phi_B(x) \]  
(3.13)

Applying the second order derivatives to (3.13) and using the normalized variables, one obtains:

\[
H_y(x) = \Gamma_A\phi_A(x) + \Gamma_B\phi_B(x) \]  
(3.14)

Applying the boundary conditions, i.e., the continuity at the interfaces of longitudinal electromagnetic field components, one has

\[
E_i(x = 0) = 0, \quad E_j(x = t) = 0, \quad E_z(x = 0) = 0 \quad e \quad E_j(x = t) = 0. \]

Finally, one obtains

\[
\left[ 1 + \left( \frac{h_i\Gamma_A}{h_j\Gamma_B} \right)^2 \right] \text{sen}(h_i t) \text{sen}(h_j t) = 2 \frac{h_i\Gamma_A}{h_j\Gamma_B} \cos(h_i t) \cos(h_j t) - 1 \]  
(3.15)

Fig. 3.3 presents the dispersion diagrams of a parallel-plate waveguide filled with a longitudinal biased ferrite.

For spacing smaller than \( \lambda/2 \), the dependence of \( \beta \) with \( \mu_x / \mu \) is brought near to a circular line.

\[
\left( \frac{H_z}{\mu} \right)^2 + \beta^2 = 1. \]

This result is useful in the design of new devices as it relates, in a simple way, the propagation constant and the material anisotropy. For larger spacing (e.g., \( D/\lambda = 1.6 \)) this trend is reversed.

**4. CHIROWAVEGUIDES**
Chiral materials are macroscopic continuous media, composed of equivalent chiral objects, uniformly distributed and orientated in a random form [14]. A chiral material is a handedness structure, resulting from a set of particles or molecules, winding around like helices. The direction of winding can be turned left or right.

The electromagnetic consequence of this is that a left-hand circular polarized wave travels with different speed and absorption than a right-hand circular polarized wave.

As described in [13] and [12] to a metallic chirowaveguide, the longitudinal components of the fields $E_z$ and $H_z$ can be expressed as the functions $U_+$ and $U_-$ as:

$$E_z = p_+ U_+ + p_- U_- \tag{4.3}$$
$$H_z = q_+ U_+ + q_- U_- \tag{4.4}$$
$$\nabla^2 U_+ + p_+ U_+ = 0 \tag{4.5}$$
$$\nabla^2 U_- + p_- U_- = 0 \tag{4.6}$$

where

$$p_+ = (k^2 - \beta^2) \tag{4.7}$$
$$p_- = (k^2 - \beta^2) \tag{4.8}$$
$$q_+ = \frac{(k_+^2 - k_-^2) p_+}{4 j \omega^2 \mu^2 \xi_c} \tag{4.9}$$
$$q_- = \frac{(k_+^2 - k_-^2) p_-}{4 j \omega^2 \mu^2 \xi_c} \tag{4.10}$$

$$k_\pm = \pm \omega \mu \xi_c + \sqrt{k^2 + (\omega \mu \xi_c)^2} \tag{4.11}$$

where $k_+$ and $k_-$ are the wavenumbers of two characteristics waves with right and left circular polarization, respectively.

To describe the behaviour of the functions $U_+$ and $U_-$ inside the waveguide, i.e., for $\rho < R$, it is better to use cylindrical coordinates ($\rho, \phi, z$).

It is assumed that the electrical conductor is perfect. Thus:

$$D = \varepsilon_0 \varepsilon \varepsilon_0 \mu_0 H$$
$$B = \mu_0 \mu H - i \chi \varepsilon_0 \mu_0 \varepsilon E$$

where $\chi$ is a parameter that represents the medium chirality. Chirality is so the generalization of a most familiar phenomenon – the optical activity.
\[ U_+ = A_1 J_n \left( \sqrt{p + \rho} \right) e^{in\phi} \] (4.12)
\[ U_- = A_2 J_n \left( \sqrt{p - \rho} \right) e^{in\phi} \] (4.13)

where \( J_n \) represents the Bessel function of order \( n \), \( A_1 \) and \( A_2 \) are constants to be determined.

Hence, enforcing the boundary conditions at \( \rho = R \), it comes:
\[ 0 = \rho \left( E_\rho + E_\phi \phi + E_z z \right)_{\rho=R} = (E_\phi z - E_z \phi)_{\rho=R} \] (4.14)

From the expressions of \( E_\phi \) depending on \( E_z \) and \( H_z \) together with the equations (4.1) to (4.14), and after some manipulations, one reaches a set of equations, which can be reduced to the following matrix form:
\[ \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} Q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \] (4.15)

with
\[ Q = \begin{bmatrix} p J_n \left( \sqrt{p+R} \right) & p J_n \left( \sqrt{p+R} \right) \\ \sqrt{p J_n \left( \sqrt{p+R} \right) \left( j a q + p q \right) R} & \sqrt{p J_n \left( \sqrt{p+R} \right) \left( j a p + p q \right) R} \end{bmatrix} + \begin{bmatrix} j n J_n \left( \sqrt{p-R} \right) \left( j a q + p q \right) R & j n J_n \left( \sqrt{p-R} \right) \left( j a p + p q \right) R \end{bmatrix} + \begin{bmatrix} j n J_n \left( \sqrt{p-R} \right) \left( j b q + q p \right) R & j n J_n \left( \sqrt{p-R} \right) \left( j b p + q p \right) R \end{bmatrix} \]
(4.16)

where \( a, b, p, q \) are constants [14].

To obtain non trivial solutions, it is necessary to impose:
\[ \det[A] = 0 \] (4.17)

where, \( A \) is the coefficient matrix.

The results for the dispersion diagrams of the circular metallic chirowaveguide are shown in Fig. 4.2.

One should note that, for \( n = 0 \), there is only a branch. Another important feature found in these waveguides is that the chiral medium restricts the propagation of modes whose polarization is opposite to the medium handedness.

As the operation frequency increases or the order of modes decreases, the dispersion characteristics are more easily influenced by the waveguide core than the border. The reason for this is that with increasing operating frequency, the fields are more concentrated in the central area.

Another important feature is that the axial ratio of polarization tends to 1 as the chiral admittance \( \xi_c \) increases.

In a high frequencies, for \( \xi_c > 0 \) the dispersion constant approaches \( k_+ \) (PCD) while for \( \xi_c < 0 \) the dispersion constant approaches \( k_- \) (PCE), this, in the limited chirowaveguide where \( k_+ / k_- \approx 1.445 \).

For the results herein presented it is always assumed a positive chiral admittance. However, if there is a change of the admittance sign, all the characteristics related to the HE branch will also affect the EH branch and vice versa.

5. OMEGA WAVEGUIDES
Among the many complex media, some have already studied in previous session, there are the omega media. The omega media have been proposed in 1991 by N. Engheta and M. Saadoun [8] as a variant of chiral media. This new type of material can be obtained by doping a host isotropic medium with $\Omega$-shaped conducting microstructures where both the loop and stamps lie in the same plane. As a result the electric field induces not only electric but also magnetic polarizations.

This session addresses the problem of electromagnetic wave propagation in a nonradiative dielectric waveguide where the isotropic slab is replaced by a pseudochiral $\Omega$-slab - the $\Omega$-NRD waveguide. It is shown that this type of waveguide supports longitudinal-section electric (LSE) and longitudinal-section magnetic (LSM) hybrid modes. The modal equations for the LSE and LSM modes are derived, and the dispersion curves and the corresponding operational diagrams are presented. The effect of including $\Omega$-shaped microstructures is analyzed. The omega media share the same constitutive relations that the chiral media, treating bianisotropic media.

The nonradiative dielectric (NRD) waveguide technology is very promising for microwave and millimeter-wave integrated circuits [16].

3.2 – Field Equations

The structure proposed for consideration consists of a pseudochiral rectangular $\Omega$-strip located between two parallel metal plates, as depicted in Fig. 1.

![Figure 5.1 – Geometry of an NRD waveguide with a pseudochiral media. The $\Omega$-NRD-guide. The surrounding medium is the air.]

Now, the relative electric permittivity and relative magnetic permeability tensors have the following uniaxial form:

$$\varepsilon = \varepsilon_|| \hat{x} \hat{x} + \varepsilon_\perp \left( \hat{y} \hat{y} + \hat{z} \hat{z} \right)$$  \hspace{1cm} (5.1)

$$\mu = \mu_\parallel \hat{x} \hat{x} + \mu_\perp \left( \hat{y} \hat{y} + \hat{z} \hat{z} \right)$$  \hspace{1cm} (5.2)

and the dimensionless magneto-electric tensors $\zeta$ and $\xi$ have the following dyadic representation [18]:

$$\xi = j\Omega (\hat{y} \hat{z} - \hat{z} \hat{y})$$  \hspace{1cm} (5.3)

$$\zeta = j\Omega (\hat{y} \hat{z} - \hat{z} \hat{y})$$  \hspace{1cm} (5.4)

Being a reciprocal media, it happens in these conditions, $\zeta = -\xi^T$. The pseudochiral dimensionless coefficient related with the pseudochiral admittance [8] according to the following expression:

$$\Omega = \frac{\mu_\perp \varepsilon_\parallel \varepsilon_\perp}{\varepsilon_\parallel}$$  \hspace{1cm} (5.5)

Cartesian coordinates is considered and it is assumed that the dispersion place along the axis zz of the form $\exp(-j\beta z')$. For the gradient operator is has:

$$\nabla = \partial_x \hat{x} + \partial_y \hat{y} + j\beta \hat{z}$$  \hspace{1cm} (5.6)

From Maxwell’s equations and after substitution of the tensors equations (5.3), (5.4), (5.5) and (5.6) in the constitutive relations (5.1) and (5.2), one obtains a set of equations that relates the components of the in the dielectric layer as:

$$-j(\partial_y H_y + j\beta E_y) = \varepsilon_\parallel E_x$$  \hspace{1cm} (5.7)

$$-j(\partial_y H_y - \partial_y H_x) = \varepsilon_\perp E_z - j\Omega H_y$$  \hspace{1cm} (5.8)

$$j(\partial_y E_y + j\beta E_y) = \mu_\parallel H_x$$  \hspace{1cm} (5.9)

$$j(\partial_y H_x + j\beta H_x) = \varepsilon_\perp E_x + j\Omega E_y$$  \hspace{1cm} (5.10)

$$-j(\partial_y E_y + j\beta E_y) = \mu_\perp H_y + j\Omega E_z$$  \hspace{1cm} (5.11)

$$j(\partial_y E_y - \partial_y E_x) = \mu_\perp H_z - j\Omega E_y$$  \hspace{1cm} (5.12)

It can be easily shown that for this type of geometry and with these constitutive relations, the -NRD waveguide only
supports LSE (i.e., with $E_x = 0$) and LSM (i.e., with $H_x = 0$) hybrid modes. Therefore, for the LSM modes (former TM modes), taking $H_x = 0$ in (5.9) and (5.10) and choosing $H_y$ as the supporting field component, the following differential equation can be easily derived:

$$\partial_x^2 H_y + \frac{\varepsilon_{\perp}}{\varepsilon_{||}} \partial_y^2 H_y = -\left( \varepsilon_{\perp} \mu_{\perp} - \Omega^2 - \frac{\varepsilon_{\perp}}{\varepsilon_{||}} \beta^2 \right) H_y$$

(5.13)

### 3.2 – Modal Equations

As is well known from the isotropic case, the LSM mode is the most interesting mode for applications due to its monotonous decrease in wall attenuation with frequency. Writing $H_y$ as a product of two separate-variable functions in the form:

$$H_y = f(x')g(y')\exp(-j\beta z')$$

(5.14)

such as:

$$\partial_x^2 f + \beta_x^2 f(x') = 0$$

(5.15.i)

$$\partial_y^2 g + \beta_y^2 g(y') = 0$$

(5.15.ii)

After substituting into (5.13), the following relations between the normalized wave numbers can be obtained:

$$h^2 + \frac{\varepsilon_{\perp}}{\varepsilon_{||}} \left( \beta_y^2 + \beta_x^2 \right) = \varepsilon_{\perp} \mu_{\perp} - \Omega^2$$

(5.16)

$$-\alpha^2 + \beta_y^2 + \beta_x^2 = 1$$

(5.17)

The index $n$ gives the number of half-waves along $y$. It is possible to check that the electric-field components $E_x$ and $E_z$ are asymmetric with respect to the geometrical symmetry plane of the waveguide—i.e. $x' = 0$.

So one writes:

$$f(x') = \begin{cases} F_1 \exp[\alpha(x'+l')], & x' < -l' \\ F_2 [\cos(hx') + R \sin(hx')], & 0 < x' < l' \\ F_3 \exp[-\alpha(x'-l')], & x > l' \end{cases}$$

(5.22)

To obtain the modal equation, it is necessary to use continuity conditions at both planes: $x' = -l'$ and $x' = l'$. Hence, enforcing the boundary conditions at $x' = -l'$ and $x' = l'$ the modal equation for LSM modes can be easily derived:

$$[h \cot(hl') + \alpha \mu_{\perp}] \left[ h \tan(hl') - \alpha \varepsilon_{\perp} \right] + \Omega^2 = 0$$

(5.23)

By the same process it is obtained for LSE:

$$[h \cot(hl') + \alpha \mu_{\perp}] \left[ h \tan(hl') - \alpha \varepsilon_{\perp} \right] + \Omega^2 = 0$$

(5.24)

### 3.2 – Numerical Results
The solutions of the modal equations (5.22) and (5.23) together with the relations between the transverse wavenumbers given by (5.15) and (5.16) lead to numerical results to be presented and analyzed.

The variation of the cutoff parameter \( l/\lambda_c \) with the pseudochiral \( \Omega \) parameter for the first modes \( LSM_m \) is shown in Fig. 5.2 for \( b/\lambda = 0.4 \). Hence, in this case, the cutoff of each mode is determined by the closed waveguide cutoff condition (i.e. \( \beta = 0 \)).

![Figure 5.2](image1)

**Figure 5.2** – Variação do parâmetro de corte \( l/\lambda_c \) com \( \Omega \) para os primeiros modos \( LSM_m \) de um guia \( \Omega \) com \( \epsilon_\parallel = 2, \epsilon_\perp = 3, \mu_\parallel = 1, \mu_\perp = 2 \) e \( b/\lambda = 4.0 \).

The vertical dashed line corresponds to \( h = 0 \), and sets a limiting value for \( \Omega \) above which there is no guided wave propagation. This value is achieved from:

\[
\Omega_c = \sqrt{\epsilon_\perp \mu_\parallel - \epsilon_\parallel \beta^2} \quad (5.25)
\]

where \( \beta \) is given by (5.19) for \( n = 1 \).

The operational diagram for the \( \Omega \)-NRD waveguide is depicted in Fig. 6 for several values of the pseudochiral parameter. The thinner line represents the non-pseudochiral case, i.e. \( \Omega = 0 \) when the medium simply becomes uniaxial anisotropic.

If the LSE modes are not taken into account, the bandwidth for single-mode operation in the -NRD waveguide is limited below by the \( LSM_{01} \)-mode cutoff, and above by either the \( LSM_{11} \)-mode cutoff or the radiation-suppression condition \((b/\lambda < 0.5)\). As one can see, even with the inclusion of \( \Omega \)-shaped microstructures, the fundamental \( LSM_{01} \) mode is the only mode that has no cutoff frequency for \( 5.0/\lambda > 0.4 \).

All the other modes suffer an increase of their cutoff frequency due to the pseudochirality.

In Fig. 4, the variation of the longitudinal wavenumber \( \beta \) with \( b/\lambda \) is depicted for \( \Omega = 0, \Omega = 1 \) and \( b/\lambda = 0.4 \).

![Figure 5.3](image2)

**Figure 5.3** – Operational diagram for the first propagating \( LSM \) modes of an \( \Omega \)-NRD waveguide with \( \epsilon_\parallel = 2, \epsilon_\perp = 3, \mu_\parallel = 1, \mu_\perp = 2 \) for several values of \( \Omega \). There is a critical value \((\Omega_c \approx 2)\).

![Figure 5.4](image3)

**Figure 5.4** – Variation of the longitudinal wavenumber \( \beta \) with \( b/\lambda \), for the first propagating \( LSM \) modes of the -NRD waveguide.
The thin lines again correspond to the non-pseudochiral case. For \( l / \lambda \rightarrow \infty \) (i.e. in the high frequency regime) one has \( h \rightarrow 0 \), so that the longitudinal wavenumber converges to its highest value obtained from:

\[
\beta_{\text{max}} = \sqrt{\frac{\varepsilon_{\|}}{\varepsilon_{\perp}}} (\varepsilon_{\perp} - \mu_{\perp} - \Omega^2 - \beta_y^2) \quad (5.26)
\]

One should note that due to an increased value of the cutoff parameter \( l / \lambda_c \) for the LSM1 mode, the range for monomodal operation is increased compared to the non-pseudochiral case, thus allowing a larger bandwidth for single-mode operation.

Finally, Fig. 9 shows the variation of \( \beta \) with \( \Omega \) for \( b / \lambda = 0.4 \) and \( l / \lambda = 0.5 \). One should note that for any LSM mode, there is an upper bound for the magnitude of the pseudochiral parameter, beyond which the mode is at cutoff.

5. CONCLUSION

In this paper, a modal complete analysis was presented with respect to the propagation of electromagnetic waves in waveguides filled with certain complex media. It examined the use of uniaxial crystals, ferrite, chiral and omega media in waveguides. It is derived the modal equations and is presented their dispersion diagrams. It also highlighted the value of the propagation characteristics of these media in the development of microwave and millimetre-waves devices.

7. REFERENCES