Lepton Flavour Violation in the Supersymmetric seesaw type-I

António J. R. Figueiredo
(Dated: 15th September of 2009)

Oscillation experiments demand an avoidable extension to the Standard Model (SM) of particle physics. One of the simplest extensions is to introduce 3 heavy right-handed Majorana neutrinos (seesaw type-I). On the theoretical side, the hierarchy problem constitutes a solid hint for some more fundamental theory emerging at a scale \( \lesssim 3 \text{ TeV} \). For this, one of the most well motivated solutions is provided by Supersymmetry. In this work we follow these two \textit{a priori} separate extensions to the SM. Since the smallness of neutrino masses is further justified by extremely heavy RH neutrinos, these will decouple from the low energy theory. Unsatisfactorily, all that could be known about their existence is just what we already know: neutrino masses. This general statement is no longer valid when seesaw is embedded in some more fundamental model with which it can communicate. This is exactly what happens in the supersymmetric seesaw.

In this work we will study the lepton flavour violation (LFV) processes that originate from the presence of these right-handed neutrinos in the context of the minimal supersymmetric standard model (MSSM) with mSUGRA (minimal supergravity) boundary conditions.

There are already interesting bounds \([1]\) on LFV rates, especially in the radiative decay \( \mu \rightarrow e \gamma \), specifically, \( BR(\mu \rightarrow e \gamma) \lesssim 1.2 \times 10^{-11} \), constraining simultaneously the MSSM parameter space and the seesaw parameters.

Keywords: Supersymmetry, Minimal Supersymmetric Standard Model, Neutrino Oscillations, Minimal Supergravity, Seesaw Type-I, Lepton Flavour Violation.

I. INTRODUCTION

It is well known that lepton flavour violating processes in the minimal version of the Standard Model - that is, vanishing neutrino masses - are completely absent. However, there are well established evidences that neutrinos are massive \([2, 3]\). In what concerns the SM, and from the strict point of view of the low energy phenomenology, whether we implement the trivial extension or a seesaw type extension is irrelevant as long as we consider a seesaw with \( Y^\nu \sim O(1) \), i.e., following its primary motivation\(^1\).

It turns out that LFV processes embodied in this extension of the SM are almost negligible and certainly beyond experimental reach when taking into account the smallness of neutrino masses.

The leading order (LO) decay width of a general flavour changing process \( f_i \rightarrow f_j + \gamma \) is proportional to \( \left( \frac{m_f}{m_i} \right)^2 \), where \( m_f \) and \( m_i \) are the masses of the fermions and bosons that run in the loops. In particular, for this simple extension of the SM one arrives at the expression:

\[
BR(\mu \rightarrow e \gamma) = \frac{3\alpha}{32\pi M^4_W} \left| \sum_k \lambda_k^{\mu e} m_k^2 \right|^2 < 10^{-53}, \quad (1)
\]

where \( \lambda_k^{\mu e} \equiv (U_{PMNS})_{\mu k}^* (U_{PMNS})_{e k} \) and \( m_k \) is the neutrino mass of the eigenstate \( k \). In here, \( U_{PMNS} \) is the Pontecorvo-Maki-Nakagawa-Sakata matrix that we parametrize as in \([1]\). Hence, a LFV signal would univocally mean “new physics beyond the SM and/or the \( \nu\text{MSM}\)\([4]\); justifying the present and future efforts devoted to experimental work in this field.

In Table \( \text{I} \) we summarize the current upper bounds on selected flavour violating processes: \( l_i \rightarrow l_j \bar{\nu}_j \gamma \) (radiative decays) and \( l_i \rightarrow l_j \bar{l}_j l_j \) (3-body decays).

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratio (at 90% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu \rightarrow e \gamma )</td>
<td>( &lt; 1.2 \times 10^{-11} )</td>
</tr>
<tr>
<td>( \tau \rightarrow e \gamma )</td>
<td>( &lt; 1.1 \times 10^{-7} )</td>
</tr>
<tr>
<td>( \tau \rightarrow \mu \gamma )</td>
<td>( &lt; 4.5 \times 10^{-8} )</td>
</tr>
<tr>
<td>( \mu \rightarrow e e e )</td>
<td>( &lt; 1.0 \times 10^{-12} )</td>
</tr>
<tr>
<td>( \tau \rightarrow e e e )</td>
<td>( &lt; 3.6 \times 10^{-8} )</td>
</tr>
<tr>
<td>( \tau \rightarrow \mu \mu \mu )</td>
<td>( &lt; 3.2 \times 10^{-8} )</td>
</tr>
</tbody>
</table>

Table I: Experimental upper bounds on LFV radiative decays \( l_i \rightarrow l_j \gamma \) and \( l_i \rightarrow l_j \bar{l}_j l_j \). Values taken from \([1]\).

In the MSSM with a general soft-breaking sector the amount of LFV rates largely exceed the experimental upper bounds for a slepton mass spectrum not unnaturally heavy. This motivates us to consider that, whatever the SUSY-breaking mechanism is, the soft-breaking terms are communicated from the hidden sector to the visible sector as flavour conserving terms and, in a stronger version, as universal flavour blind terms. In particular, we consider the minimal supergravity (mSUGRA) unification scenario where the soft SUSY-breaking scalar masses, gaugino masses and trilinear couplings are universal and flavour diagonal,

\[
\begin{align*}
m^2_L & = m^2_{1R} = m^2_Q = m^2_{uR} = m^2_{dR} = 1 m_0^2, \\
m^2\tilde{H}_u & = m^2\tilde{H}_d = m_0^2, \\
M_1 & = M_2 = M_3 = m_{1/2}. \quad A^{u,d,l} = A_0 Y^{u,d,l},
\end{align*}
\]

at some SUSY-breaking energy scale. Being this scale unknown, we follow the “unification hint” and take it as the GUT energy scale \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \), where gauge couplings unify. With the additional input of the two parameters: \( \text{sign}(\mu) \) and \( \tan \beta \), one completely sets up the pure MSSM part of the theory. Moreover, in the most part of this work, and when not referred otherwise, we will set the mSUGRA parameters to SPS1a’ \([3]\).

In here we consider the MSSM extended by 3 gauge singlet chiral superfields, the RH neutrinos and sneutrinos,
through the super-renormalizable Lagrangian density
\[ \mathcal{L}_{\text{susy}} = \int d^2 \theta \left[ \frac{1}{2} M_{\nu} \bar{\nu}_{i} \tilde{N}_{i} + \epsilon_{ab} Y_{ij} Y_{jk} \tilde{N}_{i} \tilde{N}_{j} h_{b}^c + \text{h.c.} \right], \] (5)
in a basis where \( \mathcal{M} \) and the charged lepton Yukawa couplings are diagonal. We have introduced the total antisymmetric tensor \( \epsilon_{ab} = (i \sigma_2)_{ab} \). Moreover, in the spirit of the soft SUSY breaking terms we have the correspondent RH neutrino soft-breaking terms:
\[ \mathcal{L}_{\text{soft}} = -\tilde{M}_{\nu}^2 \tilde{\nu}_{i} \tilde{N}_{i} - \epsilon_{ab} A^c_{ij} \tilde{N}_{i} \tilde{N}_{j} H_{b} + \text{h.c.}. \] (6)
We extend the mSUGRA boundary conditions to include seesaw soft-breaking parameters, through
\[ \tilde{M}^2 = 1 m_0^2, \quad A^c = A_0 Y^c. \] (7)
Seesaw mediating particles, such as the RH Majorana neutrinos for type-I seesaw, radiatively generate flavour violating entries in the soft-susy sector, giving rise to LFV processes whose rates further depend on the seesaw realisation and its parameters. In turn, the seesaw parameters are related to the low energy neutrino parameters: masses and mixing angles. Thus, seesaw realisations of the MSSM provide a promising window into the high energy model from low energy observables.

Considering universal scalar masses at GUT \( m_{H_u}^2 = \tilde{M}_{kk}^2 = m_0^2 \), and moreover \( A^c = A_0 Y^c \), it has been first shown in [7, 8] that at leading log approximation (LLA) the flavour violation is imprinted in the slepton mass matrix and in the charged slepton trilinear soft-breaking couplings:
\[ m_{\tilde{L}_{ij}}^2 \approx m_0^2 + \frac{1}{8 \pi^2} (3 m_0^2 + A_0^2) Y_{ik} Y_{jk}^\dagger, \] (8)
\[ A_{ij}^c \approx \delta_{ij} A_0 t_k + \frac{3}{16 \pi^2} A_0 Y_{ik} Y_{jk}^\dagger, \] (9)
where \( t_k \equiv \ln \left( \frac{M_{\tilde{M}_{kk}}}{M_{\tilde{M}_{ij}}} \right) \). Notice that we are working in a basis with diagonal charged lepton Yukawa couplings.

In here we will study the correlations between the seesaw parameters and the LFV radiative decays \( l_i \rightarrow l_j \gamma \) (\( i \neq j \)), 3-body decays \( l_i \rightarrow l_j l_j l_j \) (\( i \neq j \)), and the tree-level LFV decays of the heaviest stau \( \tilde{\tau}_2 \rightarrow l_i \tilde{\chi}_3^0 \) (\( l_i \neq \tau \)) in context of the seesaw type-I extended MSSM with mSUGRA boundary conditions.

The SUSY diagrams contributing to the radiative LFV decay processes at LO are depicted in Fig.1.

![Diagram of leading order diagrams for the radiative LFV decays](image)

Figure 1: Leading order diagrams for the radiative LFV decays \( l_i \rightarrow l_j \gamma \) from neutralino (left) and chargino (right) channels.

Moreover, one can show that in most part of the constrained MSSM parameter space, and even in the case of the Higgs coupling enhancement through large top mass, one has:
\[ BR(l_i \rightarrow l_j \gamma) \approx \frac{\alpha}{3 \pi} \left( \ln \frac{m_0^2}{m_j^2} - \frac{11}{4} \right) BR(l_i \rightarrow l_j \gamma), \] (10)
due to the dominance of the photon-penguin diagrams over the \( Z \) and \( H \) penguins, and the dominance of the penguin diagrams over the box diagrams.

II. NEUTRINO YUKAWA COUPLINGS RECONSTRUCTION

To reconstruct the high energy Yukawa couplings which satisfy the low energy constraints, namely, the light neutrino mass splittings and mixing angles, we apply the procedure outlined in [10]. Then, the neutrino Yukawa couplings can be written as:
\[ Y^{\nu T} = \frac{1}{v_u} \sqrt{M} R \sqrt{M^\nu U_{\text{PMNS}}} \] (11)
where \( v_u \equiv \langle H_u^0 \rangle, \quad M^\nu = U_{\text{PMNS}}^T M^\nu U_{\text{PMNS}} \) is the diagonal neutrino mass matrix and \( R \) is a general complex orthogonal matrix which we parametrize as
\[ R = \left( \begin{array}{ccc} c_2 c_3 & -c_1 s_2 & s_1 \bar{s}_2 c_3 \\ c_1 c_3 & s_1 s_2 & -s_1 c_3 \bar{s}_2 \\ s_2 & s_1 c_2 & c_1 c_2 \end{array} \right), \] (12)
being \( \theta_i \) complex angles. Thus, the complex matrix \( Y^{\nu} \) is determined by \((3 \times 2)_R + (3 + 3)_{\text{PMNS}}\) or \(3_{\text{m}} + 3_{\mathcal{M}} = 18\) continuous parameters.

A. Assumptions on \( 3_{\mathcal{M}} + (3 + 3)_R \) parameters

We will focus on scenarios which are phenomenologically more interesting by (i) providing a potential explanation for the BAU via thermal leptogenesis[11–13] and (ii) having observable LFV branching ratios at near future experimental sensitivities, namely, at a level of \( BR(\mu \rightarrow e \gamma) \gtrsim 10^{-13} \).

Thus, for the \( 3_{\mathcal{M}} \) parameters we will transform them into \( 1+(+1) \) continuous parameter(s) plus 1 hierarchy type as shown in Table II and assume the RH neutrino masses in the range:
\[ 10^{10} \text{ GeV} \leq M_i \leq 10^{15} \text{ GeV}. \] (13)

<table>
<thead>
<tr>
<th>Hierarchy type</th>
<th>Mass spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEG</td>
<td>( M_R = M_1 = M_2 = M_3 )</td>
</tr>
<tr>
<td>HIE</td>
<td>( M_R \equiv M_3 \gg M \equiv M_2 = M_1 )</td>
</tr>
</tbody>
</table>

Table II: Right-handed neutrino hierarchy types and mass spectra. The acronyms are formed by: DEG = Degenerated, HIE = Hierarchical.

3 In general only the heavier will have a relevant role.
Moreover, a successful BAU via thermal leptogenesis requires an imaginary $R$-matrix, [9]. Hence, by requiring that $Y^\nu$ stays perturbative and inspired by the BAU via leptogenesis, we take the $R$-matrix angles in the following range:

$$0 \leq |\theta_i| \leq 3, \quad |\arg \theta_i| \leq \pi.$$  

(14)

B. Assumptions on $(3+3)\nu_{PMNS} + 3m_\nu$ parameters

In this work we will set the $U_{PMNS}$ angles at GUT to TBM with the exception of the reactor angle, $\theta_{13}$, which remains free within experimental bounds of $0 \leq s_{13}^2 < (s_{13}^2)^{max} = 0.05$, [1]. Moreover, we will set the neutrino mass matrix eigenvalues at high energy scale (hereafter called neutrino masses at GUT, for brevity) so that we reproduce the correct low energy mass splittings. For this we choose the best fitting point values (b.f.p.) [15]:

$$\text{b.f.p.:} \begin{cases} 
\Delta m_{\odot}^2 = 7.6 \times 10^{-5} \text{ eV}^2, \\
\Delta m_{\text{atm}}^2 = 2.4 \times 10^{-3} \text{ eV}^2.
\end{cases}$$

The ansatz for the high scale mixing angles must be taken with care because the experimental values for the mixing angles are, by definition, measured at low scale. As a support for this top-down approach we note that it is well known [19] that neutrino mixing angles run very little and the effect is only manifest in the solar angle, $\theta_{12}$, and especially for the QD-type light neutrino hierarchies.

III. SUPERSYMMETRIC SEESAW AND ITS CONSEQUENCES FOR LOW ENERGY PHENOMENOLOGY: LFV AND EDM

Assuming that the charged slepton left-right mixing is negligible, moreover, that the slepton soft breaking mass matrix is diagonally dominant with non-degenerate entries, the LH charged slepton mass matrix is, to a good approximation, diagonalized by the rotation matrix (following [17]):

$$R^\dagger \simeq \begin{pmatrix}
1 & \delta_{12} & \delta_{13} \\
-\delta_{12} & 1 & \delta_{23} \\
-\delta_{13} & -\delta_{23} & 1
\end{pmatrix}, \quad \delta_{ij} = \frac{\Delta m_{L(ij)}^2}{m_{L(i)}^2 - m_{L(jj)}^2},$$

(16)

and likewise for the LH sneutrino mass matrix.

The RGE induced mixing in the slepton mass matrix leads to two low energy phenomena which depend on the quantity $Y^\nu TY^{\nu d}$ (with $T_{kk'} = \delta_{kk'}l_{kk'}$):

1. Lepton Flavour Violation

Since $Y^\nu$ may be of order $\mathcal{O}(1)$, the off-diagonal terms in slepton rotation matrix, [16], can be significant and, as a consequence, the lepton flavour violating processes, as the LFV radiative decay $\mu \rightarrow e \gamma$, can get important contributions from loops with LH sleptons, changing abruptly the panorama of what one would expect from the simple seesaw-type realizations of the SM. We will explore this further in following sections.

2. Electric Dipole Moment of leptons

The electric dipole moment (EDM) of the charged lepton $i$ is the coefficient $d_i$, of the effective dimension-5 operator:

$$\mathcal{L}_{\text{EDM}} = -i \frac{1}{2} d_i \bar{\psi}_i \sigma_{\mu\nu} \gamma_5 \psi_i F^{\mu\nu},$$

(17)

where $F^{\mu\nu}$ is the electromagnetic energy-momentum tensor.

Assuming mSUGRA boundary conditions, the CPV phases will only appear in the off-diagonal elements of the slepton mass matrix, via $Y^\nu TY^{\nu d}$. Therefore, in this scenario, lepton EDMs are related to LFV rates and are typically bounded by the experimental upper bounds on LFV radiative decays $\mu \rightarrow l \gamma$.

Indeed, in the work developed we have found EDMs in the following range$^5$:

$$\begin{align*}
-1.9 \times 10^{-34} &< |d_e| < 8.4 \times 10^{-34}, \\
-1.6 \times 10^{-30} &< |d_{\mu}| < 3.8 \times 10^{-30}, \\
-5.1 \times 10^{-29} &< |d_{\tau}| < 3.4 \times 10^{-29},
\end{align*}$$

(18)

by applying the bounds on LFV radiative decays $BR(l_i \rightarrow l_j \gamma)$ shown in Table I. As expected, these values were well within the present experimental bounds [1].

IV. NUMERICAL PROCEDURE

All the numerical results presented in this work were made using the public code PHENO [19]. On top of PHENO we used a program that iteratively varied the two higher neutrino masses at GUT, that were then fed into the neutrino Yukawa couplings (parametrized as in [11] and sent as an input to PHENO to obtain the low energy neutrino masses) to obtain the b.f.p. values for the neutrino mass squared differences [15]. For this we used a quick minimization algorithm complemented with MINUIT [20]. Note that the lightest neutrino mass was kept free.

To discriminate among different hierarchy types we determined the mass eigenstates $X$ and $Y$ (which were ordered $m_1 < m_2 < m_3$) with maximal and minimal content of $\nu_e$, respectively. If $X = 1 \wedge Y = 3$ we were in the normal hierarchy and if $X = 2 \wedge Y = 1$ we were in the inverted hierarchy. This was especially important for fitting QD-type hierarchies because the fitting procedure would easily scan values which changed the hierarchy type at low energy scale.

For the fitting procedure initial values we determined the two higher eigenvalues from applying the solar and atmospheric splittings together with the input of the lightest eigenvalue. One fitting run was composed of three steps. For the normal hierarchy we took the reasonable

---

$^5$ These extremes occurred for SPS1a', TBM mixing angles except $\theta_{13}^0$, SNH light neutrinos, hierarchical RH neutrinos (with $M_R = 10^{12}$ GeV and $M_1 = M_2 = 10^{10}$ GeV) and a general $R$-matrix - see section VIII B. As noted in [18], the case of non-degenerate RH neutrinos enhances significantly the EDMs. For degenerate RH neutrinos we have found $|d_e| \lesssim 10^{-35}$ e cm, $|d_\mu| \lesssim 10^{-35}$ e cm and $|d_\tau| \lesssim 10^{-36}$ e cm.
### Fitting steps

<table>
<thead>
<tr>
<th>Step</th>
<th>Normal hierarchy</th>
<th>Inverted hierarchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>( m_{32}^0 )</td>
<td>( m_{32}^0 )</td>
</tr>
<tr>
<td></td>
<td>( \Delta m_{sol}^2 )</td>
<td>( \Delta m_{sol}^2 )</td>
</tr>
<tr>
<td>#2</td>
<td>( m_{32}^0 )</td>
<td>( m_{32}^0 )</td>
</tr>
<tr>
<td></td>
<td>( \Delta m_{atm}^2 )</td>
<td>( \Delta m_{atm}^2 )</td>
</tr>
<tr>
<td>#3</td>
<td>( m_{32}^0 )</td>
<td>( m_{32}^0 )</td>
</tr>
<tr>
<td></td>
<td>( \Delta m_{sol}^2 )</td>
<td>( \Delta m_{sol}^2 )</td>
</tr>
</tbody>
</table>

Table III: Fitting steps of the solar and atmospheric splittings for normal and inverted light neutrino hierarchies.

The assumption of \( m_{32}^0 - m_{23}^0 \) stability under \( m_{32}^0 \) variations. In turn, for the inverted hierarchy we assumed the stability of \( m_{32}^0 - m_{23}^0 \) under \( m_{32}^0 \) variations. The fitting steps are summarized in Table III.

When one fitting run failed we returned to the first step and used MINUIT for convergence. With MINUIT we realized a multi-dimensional fitting in each of the steps.

The use of MINUIT was mandatory to obtain convergence in the following situations: (i) strict inverted hierarchies; (ii) QD-type hierarchies; and (iii) strict hierarchies with a general complex R-matrix.

### V. MSUGRA Parameters Influence on LFV

A rough estimate for the LFV decay widths is made in the context of the mass insertion approximation (MIA)

\[
BR(l_i \rightarrow l_j \gamma) \approx \left( \frac{1}{c_{2\beta}^2 s_{\beta}} \right) \left( \frac{3m_0^2 + A_0^2}{(m_0^2 + 0.5m_{1/2}^2)^2} \right)^2 |\delta_{ij}'|^2 \left( \begin{array}{c} 6.36 \times 10^{-10} \\ 3.58 \times 10^{-9} \end{array} \right), \quad \text{for } i = \tau, \quad \text{for } i = \mu, \quad (19)
\]

with \( \delta_{ij}' \equiv v_u^2 \left[ Y_u^T Y_u^r \right]_{ij} \).

We have checked explicitly that this approximation is always within less than an order of magnitude from the values of a full numerical evaluation as long as we have moderate \( |A_0| \lesssim 500 \text{ GeV} \) and \( \tan \beta \gtrsim 3 \). In the low \( \tan \beta \lesssim 3 \) and high \( |A_0| \gtrsim 1 \text{ TeV} \) regime the deviation can amount to 2 orders of magnitude.

The branching ratios dependence has been conveniently separated into three types of contributions: (i) seesaw parameters \( |\delta_{ij}'|^2 \), (ii) dimensionless mSUGRA parameters \( c_{2\beta}^2 s_{\beta}^{-4} \) and (iii) dimensionful mSUGRA parameters \( \left( 3m_0^2 + |A_0|^2 \right)^2 \), with \( m_0^2 \approx m_0^2 + 0.5m_{1/2}^2 \).

We have confirmed that the LFV branching ratios depend strongly on three mSUGRA parameters: \( m_0, m_{1/2} \) and \( |A_0| \); and slightly on the sign of both \( \mu \) and \( A_0 \). Moreover, this transversal is among flavours and specific of the dynamical structure of the process.

Concretely, one can take an excellent approximation in the ratio that the ratios,

\[
BR(\tilde{\tau} \rightarrow l_i \tilde{\chi}^0_1) \approx f(m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)),
\]

\[
BR(\tilde{\tau} \rightarrow l_j \gamma) \approx g(m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)),
\]

for \( l_i \neq \tau \), depend uniquely on the mSUGRA point and are seesaw independent. For SPS1’a \( (m_{1/2} \approx 97 \text{ GeV}, m_0 \approx 194 \text{ GeV}) \), we find:

\[
f(\text{SPS1'a}) \approx 1.3 \times 10^6, \quad g(\text{SPS1'a}) \approx 8 \times 10^4, \quad (21)
\]

being, as expected, reasonably accurate throughout the seesaw parameters explored in this work.

A joint measure of two LFV decays with the same flavour transition can shed some light on the mSUGRA dimensionful parameters and also \( \tan \beta \).

The branching ratios dependence is set the relative size between branching ratios with the same dynamical structure. Specifically, the ratios

\[
BR(\tilde{\tau} \rightarrow l_i \tilde{\chi}^0_1) \approx \left| \frac{|\delta_{ij}'|^2}{|\delta_{ij}'|^2} \right|, \quad \text{for } l_i, l_j \neq \tau, \quad (22)
\]

are, to an excellent approximation, mSUGRA independent. Thus, to determine the LFV rates of the model one needs to study the quantities \( f, g \) (and alike mSUGRA functions) and \( \delta_{ij}' \).

To end this section we note the approximate relations among the radiative LFV decays:

\[
BR(\mu \rightarrow e \gamma) \approx 5.63 \times \left| \frac{|\delta_{12}'|^2}{|\delta_{13}'|^2} \right|^2, \quad BR(\tau \rightarrow e \gamma) \approx \left| \frac{|\delta_{13}'|^2}{|\delta_{23}'|^2} \right|^2,
\]

where the leading order amplitude is proportional to \( |\delta_{ij}'|^2 \) (\( i \neq j \), recall equation (16) of the small angle approximation) which comes from the lepton-(LH slepton) flavour transition \( i \rightarrow j \).

For the radiative LFV decays \( l_i \rightarrow l_j \gamma \), with an on-shell photon, the initial and final leptons have opposite chiralities, therefore, the transition amplitudes are proportional to the masses of the fermions which flip chirality: (i) \( m_i \), (ii) \( m_j \) and (iii) \( m_{\text{neutralino}} \) or \( \text{chargino}. \)

Moreover, and referring to the leading log approximation (LLA), the amplitude dominant contribution comes from loops with LH sleptons, consequently, is described by: an incoming RH (LH) lepton will couple to a LH slepton through Yukawa-type (gaugino-type) coupling and the outgoing LH (RH) lepton will couple to the LH slepton through gaugino-type (Yukawa-type) coupling, while the neutralino or chargino flips chirality. Moreover, as \( \hat{g}_{ij} > \hat{g}_{ji} \), the dominant process is indeed: an incoming RH lepton will couple to a LH slepton through Yukawa-type coupling and the outgoing LH lepton will couple to the LH slepton through gaugino-type coupling, while the neutralino or chargino flips chirality.

We find for a general \( l_i \rightarrow l_j \gamma \) branching ratio:

\[
\frac{BR(\tilde{\tau} \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1)}{BR(\tau \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1)} \approx \left| \frac{|\delta_{ij}'|^2}{|\delta_{ij}'|^2} \right|, \quad \text{for } l_i, l_j \neq \tau.
\]

For SPS1’a \( (m_{1/2} \approx 97 \text{ GeV}, m_0 \approx 194 \text{ GeV}) \), we find:

\[
f(\text{SPS1'a}) \approx 1.3 \times 10^6, \quad g(\text{SPS1'a}) \approx 8 \times 10^4, \quad (21)
\]

being, as expected, reasonably accurate throughout the seesaw parameters explored in this work.

A joint measure of two LFV decays with the same flavour transition can shed some light on the mSUGRA dimensionful parameters and also \( \tan \beta \).

In turn, the seesaw parameters influence will set the relative size between branching ratios with the same dynamical structure. Specifically, the ratios

\[
BR(\tilde{\tau} \rightarrow l_i \tilde{\chi}^0_1) \approx \left| \frac{|\delta_{ij}'|^2}{|\delta_{ij}'|^2} \right|, \quad \text{for } l_i, l_j \neq \tau, \quad (22)
\]

are, to an excellent approximation, mSUGRA independent. Thus, to determine the LFV rates of the model one needs to study the quantities \( f, g \) (and alike mSUGRA functions) and \( \delta_{ij}' \).

To end this section we note the approximate relations among the radiative LFV decays:

\[
BR(\mu \rightarrow e \gamma) \approx 5.63 \times \left| \frac{|\delta_{12}'|^2}{|\delta_{13}'|^2} \right|^2, \quad BR(\tau \rightarrow e \gamma) \approx \left| \frac{|\delta_{13}'|^2}{|\delta_{23}'|^2} \right|^2,
\]

where the leading order amplitude is proportional to \( |\delta_{ij}'|^2 \) (\( i \neq j \), recall equation (16) of the small angle approximation) which comes from the lepton-(LH slepton) flavour transition \( i \rightarrow j \).
where $\Gamma_\mu$ and $\Gamma_\tau$ are the total decay widths of the $\mu$ and $\tau$, respectively, and experimental values were taken on both of them. We verified numerically that the proportionality factor between $\mu \to e \gamma$ and $\tau \to e \gamma$ branching ratios was more accurately given by

$$\frac{BR(\mu \to e \gamma)}{BR(\tau \to e \gamma)} \approx 4.69 \times \frac{|\delta_{12}|^2}{|\delta_{13}|^2}.$$  \hspace{1cm} (23)

VI. SEESAW PARAMETERS IMPACT ON LFV PROCESSES

The dominant RH neutrino mass parameter, $M_R$, has the role of scaling LFV branching ratios proportionally\(^6\), in particular, one can extract heuristically that for every 1 order of magnitude increment in $M_R$ the LFV branching ratios grow 2 orders of magnitude. This can be seen from the dependence:

$$|\delta_{ij}| \propto M_R \ln \frac{M_R}{M_{\text{GUT}}} \Rightarrow \frac{|\delta_{ij}(M_R = 10^5)|}{|\delta_{ij}(M_R = 10^9)|} \sim 10^{x-y},$$  \hspace{1cm} (24)

where $m \equiv \log(M_{\text{GUT}})$ (base-10 logarithm) and, obviously, $m > x, y$.

However, for larger values of $M_R$ this scaling behaviour is altered since one has sizable flavour violating terms in the slepton mass matrix. Thus, the small angle approximation fails and the fully diagonalization procedure must be accounted for.

Furthermore, we observed that $BR(\bar{\tau}_2 \to \mu \bar{\chi}_0^0)$ can reach $\sim 10^{-4}$ for SNH and $10^{-2}$ for SIH. The reason for this distinction has to do with the lower $\mu \to e \gamma$ branching ratio for SIH light neutrinos - see Fig.2- for the same settlement of parameters. Finally, we confirm that the ratio between $BR(l_i \to l_j \gamma)$ and $BR(l_i \to l_j l_j l_j)$ is constant as in (10), a correlation that was verified throughout the parameter space explored in this work.

A. On the impact of subdominant RH neutrino masses

We have argued above that the LFV processes depend largely on the heaviest RH neutrino and very little on the other RH neutrino masses. However, this is not entirely true for some characteristic choices of parameters. Indeed, the smallness of the reactor angle can be an important suppression factor in the case of hierarchical RH neutrinos. This suppression could be further enhanced in the case of strictly hierarchical light neutrinos. In the SIH light neutrinos case the $M_R$ parameter is doubly suppressed by $m_{\nu_{\mu}}^0$ and $s_{13}^0$. For SNH light neutrinos the suppression is only set by the reactor angle because $m_{\nu_{\mu}}^0$ is the heaviest LH neutrino. One can understand what is happening by looking at (40).

For SIH light neutrinos there is a direction $M_1 = M_2$ in the subdominant mass space where a significant increase in $BR(\tau \to \mu \gamma)$ can be achieved while keeping $BR(\mu \to e \gamma)$ low. This situation is analogous to the case of degenerate RH neutrinos, a subject to be discussed in section [VIIIA].

Another subdominant situation comes from shifting $M_3$ with $M_1 - \text{achieved by a R-matrix of the form of (31)}$ which we call $R = \text{dominant}_1$ - for SNH light neutrinos. In this case, the dominant mass scale will be suppressed by the lightest neutrino mass $m_{\nu_{\mu}}^0$ and the dominant RH masses will be $M_1$ for $\tau-\mu$ transitions and $M_2$ for $\tau-e$ and $\mu-e$ transitions.

Concluding remarks

For hierarchical RH neutrinos the reactor angle suppression can only discriminate among LFV branching ratios - and be distinguishable from the case of absent $M_R$ - in the following cases for the light neutrino hierarchies: SNH, QDNH and QDIH.

Moreover, the scenario of strong hierarchical RH neutrinos ($M_R \gg M_2 \simeq M_1$) can resemble the case of degenerate RH neutrinos if and only if we have strictly hierarchical light neutrinos. This occurs "naturally" ($R = 1$) for SIH light neutrinos and can also occur for SNH with $R = \text{dominant}_1$.

B. Organizing note

So the relative size of the branching ratios of the LFV radiative decays obeys

$$BR(\mu \to e \gamma) > BR(\tau \to e \gamma) \simeq BR(\tau \to \mu \gamma),$$  \hspace{1cm} (25)

corresponding to $|\delta_{12}|$ quantities of roughly the same size - this is the natural scenario. However, the flavour information carried by $s_{13}^0$ (the square root of the amount of electron-muon in the neutrino3 mass eigenstate) can change the panorama and distinguish flavour transitions involving the electron-flavour from those of no electron-flavour. This is what we have confirmed, with $\tau-\mu$ flavour transitions dominating both $\tau-e$ and $\mu-e$ transitions, being related to the discriminative role of $s_{13}^0$ suppression for $s_{13}^0 \sim 0$ and as long as no double suppression occurs from the $m_{\nu_{\mu}}^0$ mass. Note that we are restricting ourselves to cases where $R^T M R^* = \text{diagonal}$. In the previous section we have identified the situations where this discriminative $s_{13}^0$ suppression can occur. We will see below that with degenerate RH neutrinos the reactor angle can also change the natural order [25], and this includes the degenerate-like hierarchical cases: SIH light neutrinos with $R = 1$ and SNH with $R = \text{dominant}_1$.

Moreover, $\tau-e$ and $\mu-e$ flavour transitions have roughly the same size being any distinction among them played simultaneously by the reactor angle and the Dirac phase, $s_{13}^0 e^{-i\theta}$. This is even valid for an arbitrary $R$-matrix. Thus, the relative size

$$BR(\mu \to e \gamma) > BR(\tau \to e \gamma),$$  \hspace{1cm} (26)

is a strong natural scenario.

In the following sections we will discuss the case of degenerate RH neutrinos with $R = 1$ (which, for the low energy LFV processes, is equivalent to real $R$) and hierarchical RH neutrinos with $R = 1$. Then, we will consider cases with $R \neq 1$ but still real (we will talk

\(^6\) This is not completely true in the case of dominant RH neutrinos, because not all the LFV branching ratios depend upon the dominant mass scale, $M_R$.}
about $R$-matrices set to dominant$_1$ and dominant$_2$) and end with the case of a completely general complex $R$-matrix. We will care to analyse the impact of (i) the reactor angle, (ii) the light neutrino hierarchies and (iii) the Dirac phase on the relative size of the LFV branching ratios.

VII. REFERENCE CASE: $R = 1$

The $R = 1$ case belongs to a broader class of $R$-matrices which guarantee that the mixed light neutrino mass terms $\sqrt{m_im_j}$ ($i \neq j$) vanish in $Y^{ijT}Y^{ji}$. In this class of $R$-matrices the dependence on Majorana phases drops out and the LFV seesaw parameters are reduced to 6 continuous parameters: $s_{13}^D$, $\delta^D$, $M_1$ and $m_\nu$ (the lightest neutrino mass); plus the hierarchy of the light neutrinos.

$R$-matrices of this class display a more pronounced sensitiveness to the light neutrino hierarchies and the reactor angle. We postpone for section VIIIA the discussion about this class.

A. Degenerate right-handed neutrinos

In spite of not being the preferable scenario for a successful BAU via thermal leptogenesis, one can establish a correspondence between hierarchical scenarios that display a degenerate-like behaviour. This can happen as long as the light neutrinos are strictly hierarchical. For instance, with SNH light neutrinos the case of degenerate RH neutrinos is similar to the $R = 1$ hierarchical case of $M_1 \ll M_2 \approx M_3$ due to $m_\nu_1$ suppression. Similarly for SIH light neutrinos with $R = \text{dominant}_1$, see (31), due to $m_\nu_3$ suppression.

From [20] with $\delta_{ij}M_R \ln \frac{m_\nu}{m_{GUT}} \equiv \delta_{ij}^\prime$, one finds:

$$
\delta_{21} = \frac{s_{13}c_{13}}{\sqrt{2}} e^{i\delta} \left( \Delta m_{32} + \frac{2}{3} \Delta m_{21} \right) + \frac{c_{13}}{3} \Delta m_{21},
$$

$$
\delta_{31} = \frac{s_{13}c_{13}}{\sqrt{2}} e^{i\delta} \left( \Delta m_{32} + \frac{2}{3} \Delta m_{21} \right) - \frac{c_{13}}{3} \Delta m_{21},
$$

$$
\delta_{32} = \frac{c_{13}^2}{2} \left( \Delta m_{32} + \frac{2}{3} \Delta m_{21} \right) - \frac{1}{3} \left( \frac{1}{2} + i\sqrt{2} s_{13} \sin \delta \right) \Delta m_{21},
$$

where $\Delta m_{ij} \equiv m_i - m_j$. An effective GIM-like cancellation mechanism clearly appears for degenerate RH neutrinos, since in the limit of high degeneracy between light neutrino masses one expects rather low values for $|\delta_{21}|$, $|\delta_{31}|$ and $|\delta_{32}|$, hence, for the LFV observables involving $\mu$-$\tau$, $\tau$-e and $\tau$-$\mu$ flavour transitions. If one drops the degeneracy condition, the cancellation mechanism disappears, as it happens for hierarchical RH neutrinos (excluding degenerate-like hierarchical cases), and the LFV branching ratios will grow proportionally with the lightest neutrino mass, that is, with neutrino degeneracy.

For degenerate RH neutrinos with $R = 1$ the relative size of the LFV branching ratios is highly sensitive to paired variations arranged in the following two cases:

1. the light neutrino hierarchies vs $s_{13}^D$: setting the relative size of $\tau$-$\mu$ vs $\mu$-$e$ flavour transitions;
2. $s_{13}^D$ vs $\delta^D$ for SNH light neutrinos: setting the relative size of $\mu$-$e$ vs $\tau$-$e$ flavour transitions.

The reactor angle affects significantly the LFV branching ratios involving $\mu$-$e$ and $\tau$-$e$ flavour transitions and is completely negligible for $\tau$-$\mu$. The Dirac phase can shift the $s_{13}^D$ role, a shift whose amount is set by the size of $|a|^{-1} = \frac{\Delta m_{21}}{\Delta m_{31}}$.

The ordering of the branching ratios of the LFV radiative decays differ from the natural ordering of (25), specifically:

$$
BR(\tau \to \mu \gamma) > BR(\mu \to e \gamma) > BR(\tau \to e \gamma),
$$

(27)
a consequence of the $s_{13}^0$ discriminative role. When $|a|^{-1}$ is sizable, as it happens for SNH and QD-type light neutrinos, the ordering of (25) can be further altered when the Dirac phase is large, $|\theta| > 3\pi/4$,

$$BR(\mu \to e \gamma) < BR(\tau \to e \gamma),$$

for moderate $s_{13}^0$ (for e.g. $0.025 \lesssim s_{13}^0 \lesssim 0.15$ for QD-type and $0.075 \lesssim s_{13}^0 \lesssim (s_{13}^0)_{\text{max}}$ for SIH light neutrinos, with $|\theta| = \pi$). Moreover, in the limit of high $s_{13}$ and very large $|a|^{-1}$, as in SNH light neutrinos, one can get

$$BR(\tau \to \mu \gamma) \approx BR(\mu \to e \gamma),$$

for $|\delta| < \pi/4$. The following case is excluded: $BR(\tau \to e \gamma) \gtrsim BR(\tau \to \mu \gamma)$.

Independently of determining the mSUGRA point one can say that if the rate of a $\mu$-$e$ flavour transition is larger than that of a $\tau$-$\mu$ flavour transition for the same type of process, then high $\delta^0$ is favoured and SIH light neutrinos is disfavoured. If additionally one determines the neutrino mass scale and the hierarchy type then we would set a favoured range for $s_{13}^0$, or the other way around: from determining $s_{13}^0$ and guessing the hierarchy type or even the neutrino mass scale.

If the mSUGRA point is known and it has a light neutrino mass scale is already constrained from above by the experimental bounds on radiative LFV decays. If one is able to measure $BR(\mu \to e \gamma)$ and other process involving a $\tau$-$\mu$ flavour transition, a hint on both the light neutrino hierarchy and the mSUGRA point can be determined.

**VIII. R-MATRIX ANALYSIS**

In this section we will analyse the impact of a general complex R-matrix upon the LFV rates and establish a comparison to the phenomenology of the specific R-matrix scenarios we have been discussing. Due to the complex nature of the R-matrix angles one can generally say that a deviation from identity can be characterized by three types of impacts on the $|\delta_{ij}|$ ($i \neq j$) matrix elements:

1. Moderate sinusoidal influence when all the phases or the absolute values are small;
2. Large enhancement proportional to $\cosh^2 |\theta|$, which can represent a maximum shift of about $\sim 3$ orders of magnitude for $|\theta| \sim 3$;
3. Large reduction due to the cancellation between terms.

We will explore the case 1 in the limit of a real R-matrix and for the case 2 we will consider the most general form of a complex R-matrix. We will discard the third case because the cancellations are unstable under small variations of parameters, such as the light neutrino masses.

**A. Case 1: real R**

The study of an arbitrary real R-matrix is more easily done by selecting limiting cases, such that any arbitrary real R-matrix can be envisioned as a qualitative linear combination of these limiting cases. Thus, we chose for these limiting cases the class of real R-matrices which guarantee that the product $R^T M R^*$ is diagonal\(^7\). One of such cases has already being studied, $R = 1$. The remaining cases are the permutations of elements of the diagonal matrix $M$.

Two of these permutations are $M_1 \leftrightarrow M_3$ (dominant1) and $M_2 \leftrightarrow M_3$ (dominant2), concretely:

- dominant1: $\theta_2 = \frac{\pi}{2}$, $\theta_1 = 0$, $\theta_3 = 0$;
- dominant2: $\theta_2 = \frac{\pi}{2}$, $\theta_1 = 0$, $\theta_3 = 0$.

The permutation involving the subdominant RH neutrino masses, $M_1 \leftrightarrow M_2$, and R-matrices formed by a composition of permutations, can be reduced to the cases we have considered.

With the exception of SNH light neutrinos for $R = \text{dominant1}$, we have:

\begin{align*}
\text{dominant1: } & \left\{ \begin{array}{l}
\delta_{21} \simeq \frac{m_3 c_{13} s_{12}}{\sqrt{2}} e^{i\delta}, \\
\delta_{31} \simeq m_3 \frac{c_{13}}{\sqrt{2}} s_{12} e^{i\delta}, \\
\delta_{32} \simeq m_3 \frac{c_{13}}{2},
\end{array} \right. \\
\text{dominant2: } & \left\{ \begin{array}{l}
\delta_{21} \simeq \frac{m_3 s_{12}}{\sqrt{2}} (1 + \sqrt{2} s_{13} c_{13} e^{i\delta}), \\
\delta_{31} \simeq \frac{m_3 c_{13}}{\sqrt{2}} (1 - \sqrt{2} s_{13} e^{i\delta}), \\
\delta_{32} \simeq \frac{m_3}{2} (1 - 2i \sqrt{2} s_{13} \sin \delta - 2 s_{13}^2),
\end{array} \right.
\end{align*}

None of these matrix elements show the type of discriminative suppression as in the $R = 1$ hierarchical case. Therefore, we expect the branching ratios to follow the natural ordering of $\delta_{ij}$, with a very small influence of the reactor angle and the Dirac phase.

In Fig. we show a compilation of the branching ratios of the LFV radiative decays for each RH neutrino scenario we have been discussing, arranged into four groups of light neutrino hierarchies. We see that (i) in non

\(^7\) Note that the interesting case is that of hierarchical RH neutrinos, otherwise any real R-matrix is equivalent to any other since $R$ is orthogonal.
Figure 3: Branching ratios of the LFV radiative decays for LH neutrino hierarchies (SNH, SIH, QDNH and QDIH) and RH neutrino hierarchies (DEG, HIE, DOM1 and DOM2 - see text). The branching ratio range shown for each pair \{LH hierarchy, RH hierarchy\} comprehends a variation of \(0 \leq s_{13}^0 \leq (s_{13}^0)_{\text{max}}\) for 4 different values of \(\delta^0 = 0, \pi/4, \pi/2, \pi\). Points with \(\delta^0 = \pi/2\) are shown as blue triangles, red circles and green circles. Parameters were set to: SPS1a', TBM mixing angles except \(s_{13}^0\) and \(\delta^0\), RH neutrino masses \(M_R = 10^{12}\) GeV and \(M = 10^{10}\) GeV. The horizontal red line in the left panel is the experimental upper bound on \(BR(\mu \to e\gamma)\).

B. Case 2: general \(R\)

In a first step we studied, via the LFV radiative decays, correlations between \(\tau-\mu\), \(\mu-e\) and \(\tau-e\) flavour transitions, by spawning randomly the 6-dimensional \(R\)-matrix parameter space within the bounds referred in [14].

We observed that the correlation between \(\mu-e\) and \(\tau-e\) flavour transitions is strong, since (i) the mean value of \(BR(\mu \to e\gamma)/BR(\tau \to e\gamma)\) is stable under \(s_{13}^0\) variations and (ii) the spread around the mean value is small in comparison to \(BR(\tau \to \mu\gamma)/BR(\mu \to e\gamma)\) and \(BR(\tau \to \mu\gamma)/BR(\tau \to e\gamma)\). Indeed, the ratio follows closely the natural ordering \(BR(\mu \to e\gamma)/BR(\tau \to e\gamma)\sim 4.69\), see expression (22).

In Table IV we list the mean values and the extremes corresponding to 1σ deviations, above and below the mean, for 3 values of \(s_{13}^0\) and limiting cases of \(\delta^0\).

Moreover, the correlations for the case of hierarchical RH neutrinos is very similar to that of degenerate RH neutrinos. The only distinction between them is the higher spread of the former in the limit of very small \(s_{13}^0\). This is related to what we have seen previously: the higher separation between \(\tau-\mu\) and \(\mu-e\) flavour transitions is achieved in scenarios with hierarchical RH neutrinos and \(R = 1\), especially for small \(s_{13}^0\).

For small \(s_{13}^0\) the preferred ordering of branching ratios is that of (27), concretely, \(BR(\tau \to \mu\gamma) \sim 3 BR(\mu \to e\gamma)\) and \(BR(\tau \to \mu\gamma) \sim 14 BR(\tau \to e\gamma)\). This separation between \(\tau-\mu\) vs \(\mu-e\) flavour transitions is lowered for larger \(s_{13}^0\). When the reactor angle is close to the experimental bound we have

\[
BR(\tau \to \mu\gamma) \approx BR(\mu \to e\gamma). \tag{35}
\]

Recall that we have been analysing the case with SNH light neutrinos. For SIH light neutrinos the panorama changes since the \(s_{13}^0\) discriminative role is removed due to the suppression by the lightest neutrino mass \(m_{S2}\). Concretely, taking 300 random points in the \(R\)-matrix parameter space and setting \(s_{13}^0 = 1.74 \times 10^{-3}\) we found:

\[
\begin{align*}
\langle BR(\tau \to \mu\gamma) \rangle &= 0.22^{+0.47}_{-0.15}, \\
\langle BR(\mu \to e\gamma) \rangle &= 4.69^{+0.13}_{-0.13}, \\
\langle BR(\tau \to e\gamma) \rangle &= 1.03^{+2.18}_{-0.70},
\end{align*}
\]

for degenerate RH neutrinos and SIH light neutrinos. This agrees with the natural ordering (23) with all the quantities \(|\delta_{ij}| \ (i \neq j)\) roughly of the same size. Similar results were obtained for hierarchical RH neutrinos. Moreover, we verified that these correlations are stable under \(s_{13}^0\) variations.

In a second step we determined the set of parameters that lead to extreme cases of separation between the LFV rates. For this we used MINUIT to look for these extremes while requiring that no unnatural cancellation is at work. The implemented criterion to avoid unnatural cancellation was \(BR(l_i \to l_j\gamma) \gtrsim BR(l_i \to l_j\gamma)|_{R=1}\). We found...
that the extreme cases occurred for variable real \( \theta_1 \), high |
\( \theta_2 \)| with arg \( \theta_2 = \pi/2 \) and arg \( \theta_1 = \theta_0 = 0 \).

We observed that a greater separation is achieved for larger \( \theta_2 \) while the value of \( \theta_1 \) controls the “sign” of the
separation, that is, whether \( BR(\tau \rightarrow \mu \gamma) / BR(\mu \rightarrow e \gamma) \)
increase or decrease in comparison to the \( R = 1 \)
case. Moreover, we have checked that (i) the separations increase with increasing \( \theta_2 \) and (ii) no unstable cancel-
lation is at work. For a separation of \( |\theta_2| \gtrsim 1 \) magnitude one must have \( |\theta_2| \gtrsim 0.6 \). Moreover, the Dirac phase has
no role in the very small \( s_{13}^0 \) regime, as expected, and for high \( s_{13}^0 \) the Dirac phase acts shifter of the \( R_{3232} \)
peaks in the \( \theta_1 \) space. This is manifestly evident in the comparison between the (a) and (b) panels of Fig[4].

In Fig[4] we show the branching ratios of the LFV radiative decays, with \( M_R \) set to saturate the experimental
bounds, as a function of \( \theta_1 \) with \( |\theta_2| = 3 \). We observed that in the region of maximum \( \tau \rightarrow e \) flavour transitions one has:

\[
BR(\tau \rightarrow e \gamma) \gg BR(\tau \rightarrow \mu \gamma) \approx BR(\mu \rightarrow e \gamma),
\]
occuring for \( \theta_1 \approx 5\pi/8 \) and being stable under \( s_{13}^0 \) and \( \delta^0 \) varitions. The same stability under both \( s_{13}^0 \) and \( \delta^0 \)
variations occurs for the region where \( BR(\mu \rightarrow e \gamma) \) is the higher branching ratio,

\[
BR(\mu \rightarrow e \gamma) \gg BR(\tau \rightarrow \mu \gamma) \approx BR(\tau \rightarrow e \gamma),
\]
for \( \theta_1 \approx 3\pi/8 \). The size of the \( \tau \rightarrow \mu \) flavour transition is the most sensist to \( s_{13}^0 \) and \( \delta^0 \) varitions. In the low
\( s_{13}^0 \) regime \( BR(\tau \rightarrow \mu \gamma) \) is maximum for \( \theta_1 = 0 \), while for \( s_{13}^0 = (\delta^0)_{\max} \) its maximum is shifted to \( \theta_1 \approx 3\pi/4 \)
for \( \delta^0 = 0 \) and \( \theta_1 \approx \pi/4 \) for \( \delta^0 = \pi \).

Concluding remarks

We have seen that, even for a completely general complex
\( R \)-matrix, there is a fundamental distinc on
between SNH and SNH light neutrinos in the rela size of the LFV rates. Specifically, for the latter the natural or-
dering is favoured while for the former the favoured ordering follows [25]. Very small \( s_{13}^0 \) and SNH light neu-
trinos is clearly favoured to achieve higher separations between \( \tau \rightarrow \mu \) and \( \tau \rightarrow e \) flavour transisions. On the other hand, no fundamental distinction exists between degener-
ate RH neutrinos and hierarchical RH neutrinos for a completely general \( R \)-matrix, that is, with \( f_{ij}, g_{ij} \neq 0 \) for \( i \neq j \). Moreover, the size of \( \mu \rightarrow e \) and \( \tau \rightarrow e \) flavour transisions are highly correlated for any of the light neutrino hierarchies.

We have also established that there is always a wide region where the relative size between any two of the
three types of flavour transisions is at least of \( \pm 1 \) order of magnitude. Nevertheless, the two following cases for

<table>
<thead>
<tr>
<th>( s_{13}^0 )</th>
<th>( \delta^0 )</th>
<th>( \text{slope} )</th>
<th>( \text{BR}(\tau \rightarrow e \gamma) / \text{BR}(\mu \rightarrow e \gamma) )</th>
<th>( \text{slope} )</th>
<th>( \text{BR}(\tau \rightarrow \mu \gamma) / \text{BR}(\mu \rightarrow e \gamma) )</th>
<th>( \text{slope} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.74 \times 10^{-7} )</td>
<td>( 0 )</td>
<td>( 2.86 \times 10^{-2} )</td>
<td>( 4.70 \times 10^{-2} )</td>
<td>( 14.40 \times 10^{-2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3 \times 10^{-2} )</td>
<td>( 0 )</td>
<td>( 2.75 \times 10^{-5} )</td>
<td>( 5.04 \times 10^{-5} )</td>
<td>( 13.85 \times 10^{-5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_{(13)} ) = ( 3 )</td>
<td>( 0 )</td>
<td>( 1.09 \times 10^{-5} )</td>
<td>( 5.36 \times 10^{-6} )</td>
<td>( 5.84 \times 10^{-5} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table IV: Average slopes taken from datasets with 3000 points in the \( R \)-matrix parameter space (\( |\theta_1| \leq 3 \) and \( |\arg \theta_2| \leq \pi \)). 1σ deviation extremes, above and below the mean, are shown. Each dataset corresponds to a choice of \( \{ s_{13}^0, \delta^0, \text{RH hierarchy} \} \). Parameters were set to: SPS1a, TBM mixing angles except \( s_{13}^0 \) and \( \delta^0 \), SNH light neutrinos with \( m_\nu \approx 10^{-6} \) eV and RH neutrino masses \( M_R = 10^{12} \) GeV and \( M_1 = M_2 = 10^{10} \) GeV.

IX. CONCLUSIONS

We have seen how lepton flavour violation arises in the
MSSM extended with right-handed Majorana neutrinos and
how it depends on two subsets of parameters: (i) the pure MSSM and (ii) the seesaw. Concerning the MSSM
part we have taken the opportunely motivated simplified
assumption of mSUGRA boundary conditions, while for
the seesaw parameters we applied the currently available
constraints from neutrino physics.

In this context we have shown that it is possible, to an
excellent approximation, to factorize the LFV rates into
a mSUGRA function and a seesaw function, studying
in particular the case of LFV radiative decays \( \ell_i \rightarrow \ell_j \gamma \).
We concluded that the LFV rates depend strongly on the
mSUGRA parameters, a dependence which is roughly the
same for the same type of process and does not distin-
guish the flavour being violated.

We argued that, while a low mSUGRA mass spectrum
could easily push the LFV radiative decays to its cur-
rent experimental upper bounds, the interesting feature of
this mSUGRA dependence was to relate different types of
LFV processes, namely, both those involving known
particles (as for the LFV radiative decays) and those in-
volving sparticles. This is further motivated because the
LFV rates largely depend upon the RH neutrino masses,
which can only be directly constrained by the LFV rates.
Thus, specific knowledge of the LFV branching ratios
would ultimately set the RH neutrino mass scale.

On the other hand, the seesaw functions distinguish
the type of flavour being violated while being the same
for every type of process. Thus, apart from the RH
neutrino mass scale, the interesting way to probe the seesaw
sector is to study the relative size of the different flavour
transitions. We have taken this approach in the second
half of the preceding chapter.

General conclusions can be drawn which do not rely
on specific R-matrix assumptions: (i) a larger separation
between \( \tau \rightarrow \mu \) and \( \mu \rightarrow e \) flavour transitions is favoured in
scenarios of SNH and QD-type light neutrinos with a very
small reactor angle; (ii) the case of SNH light neutrinos
favours the natural ordering (all flavour transitions are
roughly of the same size) with the exception of degenerate
and degenerate-like RH neutrinos.
Moreover, we established two types of ordering for the branching ratios of the LFV radiative decays, $BR(\mu \to e\gamma) > BR(\tau \to e\gamma)$ (natural) and $BR(\tau \to e\gamma) > BR(\mu \to e\gamma) > BR(\tau \to \mu\gamma)$, that were the most common situations obtained with a real $R$-matrix. We then showed that these could be manifestly changed if one allows the $R$-matrix to be complex, and determined a region in the $R$-matrix parameter space that strongly displayed these different types of ordering: real variable $\theta_1$, large $|\theta_2|$ with $\arg \theta_2 = \pi/2$ and $\arg \theta_1 = \theta_3 = 0$.

Figure 4: $BR(l_i \to l_j \gamma)$ as a function of $\theta_1$ for $\delta^0 = 0$ (a) and $\delta^0 = \pi$ (b) with $s_{13}^0 = \max(s_{13}^0)$, $|\theta_2| = 3$, $\arg \theta_2 = \pi/2$, $\arg \theta_1 = \theta_3 = 0$. The RH neutrino mass scale (black colour) was set to saturate the experimental bounds on $BR(l_i \to l_j \gamma)$. The limitative role was played by $BR(\mu \to e\gamma)$. Parameters were set to: SPS1a', TBM mixing angles except $s_{13}^0$ and $\delta^0$, SNH light neutrinos and degenerate RH neutrinos.