Massive Neutrinos and Leptogenesis

David Seixas
Under supervision of Gustavo C. Branco

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Abstract

We review here the recent developments in the lepton sector since neutrino masses and leptonic mixings have been found. We will give a brief account of neutrino oscillations which are a consequence of neutrino mass and we will present ways to give mass to the neutrinos in the Standard Model and the Left-Right Symmetric Model. We will then address the basics of generating the baryon asymmetry of the universe through these new couplings in the neutrino sector, a mechanism which is called leptogenesis.

1 Introduction

The quest for the origin of matter has been in our thought long before the dawn of modern physics. However, since the discovery of universal expansion and of the microwave background as supporting for the Big Bang theory, the quest for how the present complex universe evolved from an initial singularity became crucial. The main issue is basically to explain how the present universe which is mainly composed of matter and of virtualy no anti-matter may have evolved from an initially symmetric universe. The conditions for a dynamical generation of this asymmetry were laid down by Sakharov in the 1960's. They are baryon number violation, C and CP violation and fallout of equilibrium dynamics. It may be quite shocking at first to realise that the current Standard Model of fundamental interactions (SM) contains all these ingredients: non-perturbative processes called sphalerons violate $B + L$, C and CP violation have been measured in the quark sector, and the inflation model provides the universe with enough energy which then decays during a phase called reheating which happens at very high temperatures at which the sphaleron processes are significant. Unfortunately, the only problem with electroweak baryogenesis without any intervention from supersymmetry or other variations, is that it simply fails to generate a sufficient Baryon Asymmetry of the Universe (BAU) (which is known nowadays with quite some accuracy). We will come back to this problem latter in our discussion.

In the couple of decades several experiments have struggled with the Solar Neutrino Problem (SNP) where the electron neutrino flux predicted by the Standard Solar Model (SSM) are measured with lesser values than expected. The SNO collaboration has done an incredible work in testing the hypothesis that the massless neutrinos of the SM are actually lightly massive and – just like quarks –

they mix, because the mass eigenstates and the electroweak eigenstates are in different bases, and oscillate along they propagation to earth reducing the flux of electron neutrinos that reach our planet.

1.1 Neutrino oscillations

The phenomenon of neutrino oscillations is purely quantum mechanical. It comes from the fact that

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \tag{1}$$

where $\alpha = e, \mu, \tau$ are the weak eigenstates and $i = 1, 2, 3$ are the mass eigenstates. Suppose a pion decay produces an electron and a neutrino $\pi^+ \to e^+ \nu_e$. The probability of finding an electron neutrino (survival probability) after a time t (or distance x) from the vertex is[1]

$$\begin{aligned} < P_{ee} > &= | < \nu_e | \nu_e(t) > |^2 \\ &= | < \nu_e | \sum_i U_{ei} e^{-iE_i t} | \nu_i > |^2 \\ &= \sum_i |U_{ei}|^2 |U_{ei}|^2 + \sum_{i \neq j} U_{ei} U^*_{ej} U^*_{ej} U_{ej} \cos[(E_i - E_j)t] \\ &= \sum_i |U_{ei}|^2 |U_{ei}|^2 + \sum_{i \neq j} U_{ei} U^*_{ej} U^*_{ej} U_{ej} \cos\left(\frac{2\pi x}{l_{ij}}\right) \end{aligned} \tag{2}$$

where

$$l_{ij} = \frac{2\pi}{E_i - E_j} \simeq \frac{4\pi p}{|m_i^2 - m_j^2|} \approx 2.5\text{m}\left[\frac{p(\text{MeV})}{\Delta m^2(\text{eV})^2}\right] \tag{3}$$

for $p \gg m_i$. The effect of neutrino oscillation was used to explain the SNP. Formula eq.2 is only valid for the vacuum. It could be that the solar flux' deficiency in electron neutrinos measured is mostly due to vaccum oscillations from the Sun to the Earth. This has however already been ruled out by SNO obervations REF. The clue lies in the enhancing effect caused by matter which was realized by Smirnov and Mikheyev REF. In this case the evolution happens with an effective Hamiltonian which accounts for charged current (CC) elastic $\nu_e e$ scattering and neutral current (NC) interaction, and in the the two-flavour approximation is

$$i\frac{d}{dt}\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \tilde{H}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{4}$$

with

$$\tilde{H} = E + \frac{m_1^2 + m_2^2}{4E} - \frac{1}{\sqrt{2}} G_F n_n + \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2} G_F n_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \tag{5}$$

where $n_{e,\mu}$ are the electron and muon densities, $E = |\vec{p}|$, G_F is the Fermi constant and θ is given by

$$\tan 2\theta = \frac{2(UHU^\dagger)_{12}}{(UHU^\dagger)_{22} - (UHU^\dagger)_{11}} \tag{6}$$

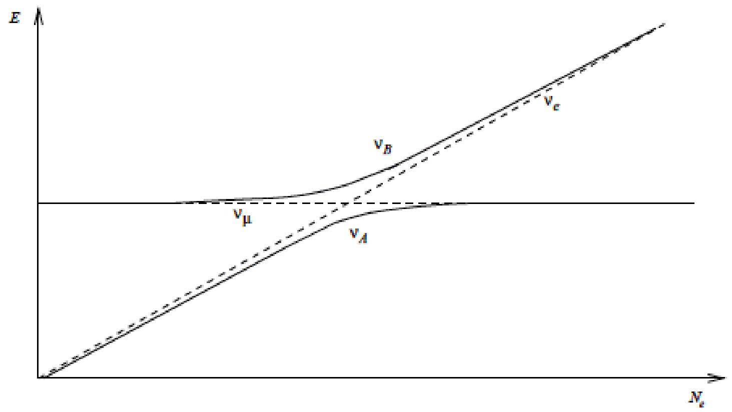

Figure 1: Neutrino energy levels versus electron density in the medium, where the dashed line stands for the pure states and the solid line stands for the mixed states.

H being the diagonal Hamiltonian matrix for the mass eigenstates in the vacuum. We define $A = 2\sqrt{2}G_F n_e E$ for the sake of simplicity. The idea of Mekheyev and Smirnov was the following: imagine a ν_e produced in a region of high density like the core of the Sun. Let's assume for now that θ is small. Fig.1.1 shows that for $A \gg \Delta m^2 \cos 2\theta$, the heavier state is almost purely ν_e and the lighter one almost purely ν_μ, whereas in the vacuum $A = 0$ one finds the exact opposite situation. If the transistion between these two limiting cases is done slowly (adiabatically) in respect to the oscillation length then we will have transformed (for the heavier state) a beam of almost pure ν_e into almost pure ν_μ. However, this adiabaticty condition doesn't allways hold and there is a probability of one flavour jumping into another which we will call P_{LZS}. One can show that for n jumps the along its path (like for a neutrino produced in the far half of the Sun) the survival probability is

$$P_{ee} = \frac{1}{2}\left[1 + (1 - P_{LZS})^n \cos 2\tilde{\theta} \cos 2\theta\right] \tag{7}$$

where $\tilde{\theta}$ is an effective mixing angle whose formula can be found in the thesis, as well as the one for P_{LZS}. This treatment of neutrino oscillations in non-uniform matter turned out to be crucial to explain the solar neutrino data and is one major success of the massive neutrino hypothesis. Combining solar, atmospheric and reactor neutrino data, one may say without further ado that neutrinos are massive with masses in the sub-eV scale.

1.2 neutrino mass models

One must now think of a way to extend the SM in order to give mass to the neutrinos. This isn't much of a challenge at first sight. In fact, the neutrinos are only massless because we made them so by

[1]assuming that all neutrinos in the beam are produced with the same momentum p and energy eigenvalues $E_i^2 = p^2 + m_i^2$

having them be left-handed, unlike all other fermions which have both left and right chiralities. So the most trivial extension of the SM would be to include right handed neutrinos. It may not strike you a first, but this simple action has more to it than meets the eye. All fermions are Dirac particles which means they have four spinoral degrees of freedom and there is no ambiguity in choosing between two different components with same chirality because they have charge. However, the neutrino states are neutral, so there is no way to distinguish between a left handed state of positive energy and a right handed state of negative energy. This ambiguity leads to the definition of a Majorana spinor, which has only two components. But its term violates lepton number conservation by two units because it combines (like the dirac mass term) a left and a right handed component, which in this case a the neutrino and the anti-neutrino. Nevertheless, the lepton and baryon number conservation at lagrangian level in the standard model is only an accidental symmetry. The inclusion of right-handed neutrinos with both Dirac and Majorana mass terms without imposing any new symmetries (such as lepton number conservation) leads to the mass Lagrangian (omitting the flavour indices for simplicity)

$$\begin{aligned} & M_D \ell_{iL} N_R \phi_i^* + M_R \bar{\nu_R} N_R + \text{h.c.} \\ &= \frac{1}{2} \begin{pmatrix} \bar{\nu}_L \\ \bar{N}_L^c \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{h.c.} \end{aligned} \tag{8}$$

where the i is an $SU(2)$ component index. In the one flavour case, one sees that upon diagonalization with an orthogonal matrix the mass eigenvalues are

$$m_{1,2} = \frac{1}{2}(\sqrt{M_R^2 + 4M_D}) \mp M_R) \tag{9}$$

Now, if the $M_R \gg M_D$ then $m_1 \sim M_D^2/M_R \ll M_D$ meaning that the light neutrino masses are very much suppressed. And the high scale of M_R only comes naturally when one expects to link it to some higher energy scale at which grand unification happens, especially since the right handed neutrino is the only missing particle to complete the $SO(10)$ fundamental representation.

But there are other ways to generate nuetrino masses. The product of a lepton singlet and a lepton doublet indeed is a doublet, requiring a doublet Higgs to make it gauge invariant. But one can also forsake the addition of a singlet neutrino and make products of left handed or right handed neutrinos, which require a doubly charged scalar or triplet, or even a triplet fermion. The addition of a triplet scalar is called the type-II seesaw. When the triplet Higgs acquires a VEV it fills in the first entry in the matrix in (8). The light neutrino condition implies for this model that the triplet's VEV v_3 is much smaller than the doublet VEV v_2. Current measurements of the $W^\pm$, the Z^0 and θ_W imply that $v_3 < 0.07 v_2$, so the type-II seesaw may also be a natural way of explaning the smallness of neutrino masses. There is still a type-III seesaw which involves the addition of triplet fermions which yields a formula similar to the type-I seesaw.

These are all possible extensions within the $SU(2)_L \times U(1)_Y$ SM. If takes the $SO(10)$ GUT as a serious motivation for the type-I seesaw then – since $SU(5)$ has already been ruled out – there must be a scale at which $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ appears. Anyway, the idea of parity being a symmetry of the Lagrangian which is spontaneously broken by the vacuum is undoubtfully attractive from a theoretical point of view. In this model, called the Left-Right Symmetric Model (LRSM), each chirality has its own weak isospin interaction, *e.g.*, the RH electron is still a singlet of the SM weak isospin group but is a doublet of another $SU(2)_R$ group under which all LH fermions are singlets. One interesting feature which was not realized right from the first proposal of the model, is that the $U(1)$ symmetry has a corresponding number which is $(B-L)/2$. This symmetry must of course be broken somehow. It was first proposed that two doublets were enough to break the LRSM into the SM. But such a mechanism only leads to Dirac masses for all fermions including neutrinos. The model we're interested in makes use of one neutral bidoublet scalar Φ, $(\mathbf{2}, \mathbf{2}, 0)$ under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and two triplets Δ_L and Δ_R, $(\mathbf{3}, \mathbf{1}, 2)$ and $(\mathbf{1}, \mathbf{3}, 2)$ respectively. When these Higgses develop a VEV, k and k' for the bidoublet and v_R for the "RH" triplets, they generate Dirac and RH Majorana masses for the neutrinos. A type-I seesaw mechanism is thus put in place very much like the one we introduce earlier for the minimal extensions of the SM:

$$m^{\text{light}} = -\frac{1}{v_R} m_D f^{-1} m_D^T \tag{10}$$

where $f_{ij} v_R$ is the right handed Majorana mass matrix and m_D is the Dirac mass matrix. This however leads to the formula

$$m\nu_\ell \approx \frac{m_\ell^2}{M_N} \tag{11}$$

Assuming that comparatively the RHνmass is generation independent, the light neutrino mass differences will be proportional to the difference of the squared masses of the leptons, making the spectrum obviously unreasonably spanned in regard to the available experimental data. Furthermore, the same reasoning leads to the conclusion that a LRSM at TeV scale where majorana masses arise at Lagrangian level is ruled out by present data.

2 leptogenesis

So now that we have established a source of lepton number violation at lagrangian level let's reanalyse the problem of the origin of matter. The principle of leptogenesis lies on the decay process of the heavy neutrinos into a charged lepton and a Higgs. The standard leptogenesis simplifies this by assuming that any asymmetry produced by the two heavier singlet fermions is erased by the later N_1 leptogenesis through inverse decays and scatterings. The final asymmetry will depend on several different parameters:

- During thermalization RH$\nu$are created thanks to inverse decays $\ell_j\bar{\phi} \to N_k$ and $N_k \to \bar{\ell}_j\phi$. A lepton asymmetry is generated
$$Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} \tag{12}$$
where Y_L is the comoving number density. At this stage it is $\sim -\epsilon/CY_N$ where
$$\epsilon_{kj} \equiv \frac{\Gamma(N \to \ell_j\bar{\phi}) - \Gamma(N \to \bar{\ell}_j\phi)}{\Gamma(N \to \ell_j\bar{\phi}) + \Gamma(N \to \bar{\ell}_j\phi)} \tag{13}$$
and C is a wash-out factor greater than unity accounting for the partial depletion of the lepton asymmetry during thermalization. The equilibrium density of RHνat the end of the thermal epoch is roughly $Y_N \sim 10^{-3}$.

- When the temperature of the universe becomes $\lesssim M_{RH\nu}$ the heavy neutrinos decouple from the thermal bath and decay giving way to a lepton asymmetry $\sim \epsilon Y_N$. The total asymmetry is
$$Y_L \approx \underbrace{\epsilon\left(1 - \frac{1}{C}\right)}_{\eta} Y_N \tag{14}$$
In thermal leptogenesis, $\eta \sim 0.1$.

- Finally, the baryon asymmetry is owed to the work of sphalerons which partly convert the lepton-asymmetry into a baryon-asymmetry. This factor is of about 1/2. Hence, in thermal leptogenesis $Y_B \sim 10^{-5}\epsilon$ which requires $\epsilon \sim 10^{-6} - 10^{-7}$ in order to mimic the observed BAU of $Y_B \approx 8.7 \times 10^{-11}$

Another simplification can be made by disregarding the flavour of the product leptons. The basic principles of unflavoured leptogenesis are:

- A thermalization phase where RHν's are produced by scatterings and inverse decays of SM particles. This process evolves until equilibrium is reached. In a strong washout regime (SWR), RH$\nu$are thermalised fast and equilibrium is reached at high temperatures; conversely, in the weak washout regime (WWR), equilibrium is only reached at lower energies. Anyhow, both cases produce a lepton asymmetry which is $\propto Y_N$.

- In the next phase the RHνdensity decreases. In the SWR Y_N tracks closely its equilibrium value due to fast decays while in the WWR the decays come later, when Y_N has already left its equilibrium value. Another asymmetry of opposite sign is created which threatens to cancel out the first one.

- Eventually, temperatures decrease and the RHν's become too diluted freezing out the processes that involve them. A small residual asymmetry survives. In the SWR, the resulting lepton asymmetry is accurately given by

$$Y_L \simeq \varepsilon_1 \left(\frac{0.4}{K_1^{1.16}} \right) Y_N^{eq}(z_{in}) \tag{15}$$

$$\eta_s = 0.4/K_1^{1.16} \tag{16}$$

  in the WWR, the resulting lepton asymmetry is

$$Y_L \simeq 1.3\varepsilon_1 K_1^2 Y_N^{eq}(z_{in}) \qquad \eta_w = 1.3K_1^2 \tag{17}$$

- The lepton asymmetry is then partially converted into a baryon asymmetry via sphaleron processes according to

$$Y_B = -\frac{28}{51} Y_L \approx -1.38 \times 10^{-3} \varepsilon_1 \eta \tag{18}$$

However this cannot always be done. When flavour becomes relevant (which means that the Boltzmann equations that describe its evolution become decoherent in flavour space), it can enhance leptogenesis. For example, some models have $\varepsilon_1 = \sum_\alpha \varepsilon_{1\alpha} = 0$ with each $\varepsilon_{1\alpha}$ being however non-zero. In this case, the asymmetry may be preserved within some flavours without cancelling out. Other issues such as the role of heavier singlet neutrinos must also be considered, or the possibility of other mass models of neutrino mass. Their range of relevance is the following:

- The asymmetry generated by N_2 is important when $T_{reheat} > M_2$. In this case either the N_1 washout factors are too weak to erase the N_2 asymmetry or the its couplings are strong enough but don't work in a flavour direction where N_2-generated asymmetry is produced.

- The mass difference betwenn N_1 and N_2 is of the order of the decay rate. In this case the self-energy component of the CP asymmetry is greatly enhanced.

- leptogenesis with type-II seesaw alone cannot work, but a normal highly hierarchical mass spectrum allows Dirac neutrino leptogenesis to work thanks to the tiny couplings. This flexibility nevertheless poses a problem when one tries to consider ways to test the theory.

3 Conclusion

Even though many data on neutrino masses and mixings are about to come from various experiments, there is little hope that leptogenesis, the most astonishing window opened by the discovery of neutrinos masses, will ever be tested directly, or probably even indirectly. However, the CERN LHC might help to unveil the neutrino mass mechanism by discovering a triplet Higgs, along with a positive signal in the already active quest for neutrinoless double-beta decay, adding motivation for the basics of leptogenesis. Ruling out other models such as supersymmetric $SU(5)$ GUT or electroweak baryogenesis at LHC might also strengthen the case of leptogenesis by killing its competitors. The discovery of a flavour theory and a better understanding of the symmetry breaking mechanism might also help to establish leptogenesis as the standard model of matter generation much as once nucleosynthesis was once established for the creation of light elements in the universe. Without any doubt, the future neutrino experiments' results are eagerly awaited.

Keywords/Palavras-chave: Neutrino mass models, seesaw mechanisms, neutrino oscillations, leptogenesis.

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