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Optimization of a Batteries' Distribution Route Planning with Pick ups and Deliveries

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Abstract

Logistics plays a key role in our society. Motivated by the need to sustain their market position and with the increasing pressure by clients to have the goods in the right place and at the right moment, an excellent logistics management is needed to achieve a competitive advantage.

In the present article, it is evaluated a Portuguese battery manufacturer, named A. A. Silva, holder of the Autosil brand. The main goal of this work is to understand how the organization can benefit from the use of a decision making tool that optimizes the route planning process and the use of the transportation resources.

The characteristics of vehicle routing problem, its variants and solution techniques are reviewed. A vehicle routing model is developed and adapted to the case-study under analysis. The working data that has been collected is presented and is included the mathematical formulation of the model.

To simplify the problem, the case-study analysis is focused on one specific delivery region, the Lisbon district and in a particular month, which is January of 2008. The problem is divided into small sub-problems, using the mathematical model p-median. Then the mathematical model developed is applied to each sub-problem.

The results suggest that the application of the mathematical model based on a vehicle routing problem with pick ups and deliveries (VRPPD) and a heterogeneous fleet leads to the reduction of the routes length, to a shorter fleet and less time to perform the distribution plan obtained.

Keywords: Logistics, Vehicle routing problem, Pick ups and deliveries, Fleet optimization

1 – Introduction

Nowadays companies suffer with high competitiveness in the market where they perform. So, the management has been forced to search and implement innovative strategies to guarantee competitive advantages and profitability. Logistics plays a key role in improving companies' efficiency, whereas it is the part of the supply chain management that plans, implements and controls the forward and reverse flow of goods, services and information between the origin and consume points, in order to meet the customers' requirements.

Logistics main focus is the transportation activity, which is the most relevant part of the logistics total costs.

The vehicle routing problem (VRP) has been intensively studied since the experts saw its great applicability in real life situations. As an example, Tarantilis and Kiranoudis [1] analyzed a way to improve fresh milk distribution, as well as Amponsah and Salhi [2] and Kima, B. et al. [3], who studied the recovery and waste management. Shen, Z. et al. [4] also studied routing problems related with huge catastrophes like natural disasters and terrorist attacks, in which a lot of medicines had to be delivered in a very short time period and in dispersed areas. In this way, developing mathematical models to solve these problems is extremely important for the society, in general.

Given a fleet of vehicles with the same capacity, a common distribution center and different customers that represent demand points, VRP tries to find the routes that minimizes the total cost and simultaneously, satisfies all the customers' demand (Tavares, J. et al, [5]). The goal of this problem is to minimize the total transportation cost, the number of vehicles employed and the routing distance.

One of the VRP variants is VRPPD, that is a vehicle routing problem with pick ups and deliveries. This problem was published for the first time in literature by Min, H., in 1989 [6]. In his work, he studied the books distribution and recovery between a distribution center and 21 libraries in Ohio. The increasing concern with the environmental problems has lead to significant changes in companies' processes, either in production level, packaging reuse, as well as at distribution level. According to Dethloff, J. [7] these alterations lead to the incorporation of the goods' reverse flow at the supply chain.

Therefore, route planning includes not only goods distribution but also the recovery of others for recycling and reusing processes. In this way, VRPPD is a type of planning problem in which customers require a certain amount of products to be delivered and another to be recovered (Hoff, A. et al., [8]). In this problem, the delivery and recovery of the products in each client can only be made by one vehicle in a certain instant, so the client can be visited only once a day. It is necessary to guarantee that the vehicle capacity is not exceeded. The objective is to define the set of routes for an homogeneous fleet that minimizes the total distance travelled, so every request can be satisfied, having in mind the available capacity of each vehicle (Berbeglia et al., [9]).

VRP is considered a problem NP – Hard and during the past decades several methods in attempting to find the best possible solution had been developed. For example, the Br*anch and Bound* studied by Fisher [10], the *Savings* Algorithm from Clark and Wright, developed in 1964 [11], the *Multi-Route Improvement Heuristics* from Kinderwater and Savelsbergh [12], the genetic algorithms (Ho, W., et al., [13]) and the *simulated annealing* that has been introduced by Kirkpatrick, S. et al., in 1983 [14]

In this paper, a VRPPD model is applied to a real case of a Portuguese company that manufactures and distributes batteries. The aim is to optimize the routes and the company fleet so that distribution costs are minimum. It is needed to consider all the restrictions and guarantee service quality in the delivery process to the customer, very characteristic of this company. To solve this problem, initially it is necessary to divide the problem into smaller problems and then develop an optimization model using a mathematical programming language in GAMS software and apply it to each sub-problem obtained.

2 – Case Study

A. A. Silva company is the holder of Autosil brand, one of the most important batteries manufacturers in Portugal.

The distribution network counts with 13 facilities: the factory that is located in Paço de Arcos and the warehouses that are spread throughout the country, particularly in Porto, Tondela, Viseu, Coimbra, Santarém, Lisbon, Almada, Setúbal, Beja, Sines and Almancil. It also has three licensed dealers in Vila Real, Aveiro and Castelo Branco. All the warehouses are provided with all kinds of products for a quicker response to the needs of their customers and each customer is supplied by a single warehouse, which is the closest.

The batteries distribution structure can be divided in two distinct parts. The primary distribution consists on the transport of goods from the central warehouse (or plant) to the regional warehouses, as well as to the customers with direct deliveries. The exchange of products, not very common, among the regional warehouses (transhipment) is also included here. The secondary distribution includes the distribution of regional warehouses to all sorts of customers. The recovery of batteries to recycle or to recharge is also part of Autosil service. As well as the distribution, the recovery process is also divided into primary and secondary recoveries.

Customers are visited not only for delivery and collection of batteries, but also to make the conference of the stock on consignment, the collection of claims, for auto sales and to provide technical assistance.

The routes are planned on a daily basis by geographical areas, considering the distribution center to which each client belongs. Each route is associated to a specific driver and his assigned vehicle.

Typically, delivery time to customers does not exceed 24 hours after their request, which implies a good coverage of the national territory in order to satisfy the demand.

Currently transportation is provided by internal means in secondary distribution and in its majority in the primary distribution as well. The company's fleet is heterogeneous, each regional warehouse has a number of vehicles with different transportation capacities.

3 – Problem definition

The distribution operation is particularly problematic in Lisbon due to the difficulty of parking nearby the costumers. Most of the clients are garages which are located in narrow streets with difficult access. This restricts the size of vehicles for the delivery and collection of batteries in the area, and so, this work will only focus on this district.

Furthermore, the distribution operation should follow the distribution center schedule. Although not very rigid, the distribution usually starts at 9am and ends at 6.30 pm, with a lunch break from 1 pm to 2.30 pm. Thus each day distribution has a 8 hours duration.

A. A. Silva company has a very extensive list of customers in this district and it is quite variable over the years. Considering the 2008 year, the company has a total of 2332 costumers. It should be noted that the quantities of batteries ordered by each customer during this year ranged from a single battery to 2437 and the collected batteries varied from none to 5090. To simplify the problem, the model was applied only to the month of January of 2008, because it was the month in which there was the highest demand throughout the year.

The fleet of Lisbon warehouse consists of eight vehicles, five of which have the capacity to 31 batteries and the other three have to 82. Each vehicle is assigned to a single driver and the costs of travel, per km, are different for each type of vehicle.

Its product range consists of seven different types, each one with its characteristics and applications. The company produces starter batteries for light and heavy vehicles, industrial batteries with different series depending on their application. However, in the present study the batteries in general are considered, using only the distinction between deliveries and collections, because there are no major differences in their transport.

In this case study, all routes begin and finish at the distribution center and the clients can be visited only once per day and just by one vehicle.

The time spent in each point depends on the reason of the visit. The time to unload and load is dependent on the number of batteries to deliver and collect. Two different scenarios are tested for the time needed to the other visit requests (to make collections, to collect complaints or to provide technical assistance). In the first scenario it is considered that the average time of these visits is approximately 10 minutes and the second assumes the value of 20 minutes. Then it is determinated which one best fits the reality.

The chosen methodology to solve the problem was to initially aggregate the customers in the borough to which they belong. Then divide the problem into smaller problems by applying the p-median model and finally apply the optimization model developed to each sub-problem obtained.

3.1 – P-median problem

The first mathematical formulation of p-median problem was presented by Hakimi in 1964 [15] and actually is known as a NP-Hard problem.

Assuming a given number of *p* infrastructures to be installed in a network and a set of n demand points or clients, p-median problem can be mathematically defined by the following way.

- **Element Sets**

- i, j – places to be visited.

- **Parameters**

- d_{ij} – driving distance from place i to place j.
- p – total medians number.

- **Decision Variable**

- x_{ij} = binary variable that is equal to 1 when the place *i* is associated to the median in local j, otherwise it will assume the value 0

- **Objective Function**

Minimize: $$\sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij} \cdot x_{ij} \qquad (1)$$

The objective function (1) tries to minimize the sum of the total distances between each median and all the associated points.

- **Restrictions**

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j \qquad (2)$$

$$\sum_{i=1}^{n} x_{ii} = p \qquad (3)$$

$$x_{ij} \le x_{ii}, \quad \forall j \qquad (4)$$

$$x_{ij} \in \{0,1\}, \quad \forall i, j \qquad (5)$$

Restriction (2) ensures that each point j will be associated to only one point i, that is a median. On the other hand, equation (3) guaranties that number p corresponds to the quantity of medians to be considered. Restriction (4) guaranties that the points will only be affected to points considered as median. The equations (2) and (4) are responsible for each point j to be allocated to only one point i, which should be a median.

Equation (5) ensures that x is a binary variable, assuming only the value 0 or 1.

3.2 – Application of p-median model

In order to calculate the quantity of medians that should be chosen for each day, the model VRPPD was tested for different quantities of visit points and the computational results obtained are presented on table 1.

Table 1 – Variation of computational results with the number of boroughs

Nº Boroughs	Nº Variables	Nº Restrictions	Execution Time (s)
3	153	439,0	0,765
5	313	14.763,0	1,735
7	537	89.855,0	4,36
9	825	342.739	5,390
10	993	598.253,0	12,188
11	1177	988.071,0	19,578
12	1377	1.559.425,0	47,875
13	1593	2.369.531,0	4540,844

When the number of boroughs is inferior to 9, the results are obtained with a short execution time. On the other hand, it is possible to observe that there is a complexity increase when the number of visit points is 13. In this way, the numbers that should be considered are 9, 10, 11 or 12.

P-median model is applied to all days, because the quantity of boroughs to be visited in each day is variable. However sometimes it is needed to apply this model again to the groups that still have a large number of boroughs.

Thus, it is possible to simplify the problem and then apply the model developed.

4 – Model Formulation

Assuming a set n of points (clusters) that have to be visited on a given day, each one has a quantity q_i of batteries to be delivered and another r_i to be collected.

The vehicle fleet has *m* vehicles with different characteristics. The vehicles are differentiated by the number of batteries they can carry.

For a vehicle moving from place *i*, $i \in \{0 \ldots n\}$ to place $j \in \{0 \ldots n\}$ and $j \neq i$, there is a travelling time t_viagem_{ij} and a driving distance d_viagem_{ij}. Place *i=0* is attributed to the distribution center.

Each route will be associated to only one vehicle v, $v \in \{1 \ldots m\}$, with a maximum capacity $capv_v$ and it will visit a group of clusters in order to satisfy all their requests.

Each vehicle has as starting point on the distribution center and returns to it after completing the respective route.

The distribution time consists of two parts, the fixed time t_f, which is consumed on administrative issues and the variable time t_v which depends on the visit purpose of each cluster.

Now, it is possible to define the element sets, the parameters, the variables and all the functions.

- **Element sets**

i – Visited place
j – Place to be visited
v - Vehicle

- **Parameters**

- q_i = Quantity of batteries to be delivered at place i.
- r_i = Quantity of batteries to be collected at place i.

- $maqr_i$ = maximum absolute value between the batteries to be delivered and those to be collected at each cluster.
- *$capv_v$* = maximum capacity of each vehicle, in quantity of batteries they can transport.
- $cdist_v$ = cost of travel one km with each vehicle, in €/km.
- tv_i = Variable time needed at place i, in minutes.
- d_viagem_{ij} = travel distance between place i and j, in km.
- t_viagem_{ij} = travel time between place i and j, in minutes.
- t_f = fixed time to deal with administrative issues, in minutes.
- ttotal = daily time available to the distribution operation, in minutes.

- **Variables**

Decision Variables:

- X_{ijv} = binary variable that assumes value 1 when the vehicle v visits cluster j after visiting cluster i, otherwise it assumes value 0.
- vf_v = binary variable that assumes value 1 if vehicle v executes a route, otherwise it assumes value 0.

Positive Variables:

- $distotal_v$ = variable necessary to obtain the total distance traveled by each vehicle.
- $tetotal_v$ = variable necessary to keep the total time needed to complet each route.

- **Objective Function**

Minimize:

$$\sum_{v}^{m} \sum_{i}^{n} \sum_{j}^{n} x_{ijv} \cdot d\_viagem_{ij} \cdot cdist_v \qquad (6)$$

The objective of this model is to minimize the total transportation cost. As the cost of transportation depends upon the driving distance, function (6) minimizes the total distance traveled by the distribution vehicles between delivery clusters in order to define optimal distribution routes. This is obtained through the multiplication of the distance between each two points, the cost to travel that distance and the variable that indicates if any vehicle does that route.

- **Restrictions**

$$\sum_{i>1} x_{jiv} = vf_v, \text{ j = distribution center} \wedge \forall v \qquad (7)$$

$$\sum_{i>1} x_{ijv} = vf_v, \text{ j = distribution center} \wedge \forall v \qquad (8)$$

$$\sum_{i \neq j} \sum_{v} x_{ijv} = 1, \quad \forall j > 1 \qquad (9)$$

$$\sum_{i \neq j} \sum_{v} x_{jiv} = 1 \quad , \quad \forall j > 1 \qquad (10)$$

$$\sum_{j} x_{jiv} = \sum_{j} x_{ijv} \quad , \quad \forall i \wedge \forall v \qquad (11)$$

$$\sum_{i>1}\sum_{j\neq i} maqr_i \cdot x_{ijv} \leq capv_v \quad , \ \forall v \tag{12}$$

$$x_{ijv} \leq vf_v \quad , \quad \forall i \wedge \forall j \wedge \forall v \tag{13}$$

$$\sum_{i>1}\ \sum_{j>1}(t\_viagem_{ij} + tv_i + tf)x_{ijv} \leq ttotal \quad , \ \forall v \tag{14}$$

$$\sum_{v_i \in S}\ \sum_{v_j \in S} x_{ijv} \leq |S| - 1 \quad , \forall S \subseteq V \setminus \{0\};\ S \neq \varnothing;\ v = 1,\ldots,m \tag{15}$$

$$distotal_v = \sum_i \sum_j \left(x_{ijv} \cdot d\_viagem_{ij}\right) \ , \ \forall v \tag{16}$$

$$tetotal_v = \sum_i \sum_j \left[x_{ijv} \cdot \left(t\_viagem_{ij} + tv_i + tf\right)\right] \ , \ \forall v \tag{17}$$

$$X_{ijv} = \{0,1\} \qquad i, j = 1,\ldots, n;\ v = 1,\ldots,m \tag{18}$$

Equation (7) guarantees that all routes start at the distribution center. On the other hand, equation (8) ensures that all routes end at the distribution center. Equation (9) guarantees that the arrival to each delivery cluster j is made only by a vehicle. Equation (10) ensures that the departure of each cluster j is made only by one vehicle. In this way, constraints 9 and 10 guarantee that each cluster is visited only once a day, by one vehicle.

Restriction (11) ensures the flux of vehicles within the network by imposing that when a vehicle arrives in a delivery point has to leave from it.

Equation (12) guarantees that the capacity of the vehicle is not exceeded. The capacity limit assumes the value of the maximum between the batteries to deliver and to collect. Thus vehicle only visits the number of clusters that its capacity allows. Restriction (13) defines if a vehicle is necessary or not for a certain distribution operation allowing that it may stay in the warehouse. If a vehicle is used, only one route is allowed. Equation (14) guarantees that the time-in-transit between each pair of two points and the time needed to complete all the distribution, including the fix and the variable time at each cluster, respect the available daily time for this operation. Being S a sub-set of clients, restriction (15) eliminates the creation of sub-routes,

Equation (16) calculates the total travel distance for each vehicle. Equation (17) calculates the total time needed to complete the distribution plan obtained with the model. Restriction (18) defines the domain of the binary variable x_{ijv}.

5- Results

In January of 2008, the distribution operation was carried out in 22 days. The model is applied 110 times to each scenario, because each day is divided into groups. The computational results of the model application to both scenarios are described in Table 2. The results presented are only from the first distribution day in January, because it would be too extensive to present the results for all days. On this day there were 40 boroughs to be visited. So it was necessary to apply p-median model with 4 medians. However, the first group still had 21 points to be visited, so the model was applied again, only to this group. Thus, for this day, the boroughs were divided into 5 different groups. Then, the developed model was applied to each one.

Table 2 – Computational results

		Nº Variables	Nº Restrictions	Objective Function (€)	Gap (%)	Execution Time (s)
Group 1.1	Scenario 1	1.177	988.071	25,297	2,926	25,297
	Scenario 2	1.177	988.071	23,864	2,204	24,407
Group 1.2	Scenario 1	825	342.739	35,466	0	15,078
	Scenario 2	825	342.739	35,466	0	28,406
Group 2	Scenario 1	537	89.855	65,93	0	4,031
	Scenario 2	537	89.855	65,93	0	1,687
Group 3	Scenario 1	825	342.739	26,714	0	12,391
	Scenario 2	825	342.739	26,714	0	5,641
Group 4	Scenario 1	537	89.855	96,254	0	4,625
	Scenario 2	537	89.855	96,254	0	1,968

The resulting MILP model was solved by GAMS/CPLEX (built 22.8), in a Intel (R) Core ™ 2 Duo CPU, 3.00 GB, 2.26 GHz.

5.1 – Vehicle fleet

In figures 1 and 2 are presented the results obtained for the number of vehicles required for each day in both scenarios and in real case.

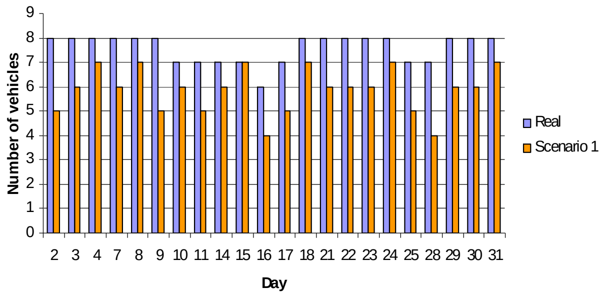

Figure 1 - Number of vehicles required for each day, for scenario 1.

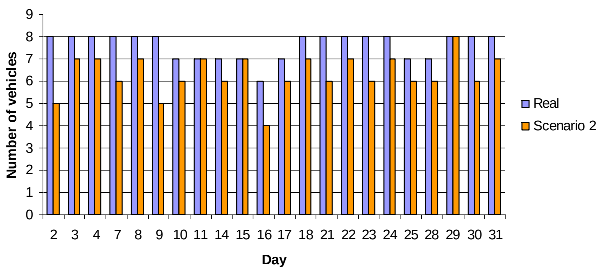

Figure 2 - Number of vehicles required for each day, for scenario 2.

As it can be seen, there is a clear reduction in the quantity of vehicles needed to complete the distribution on most of the days. In the first scenario only for day 15 is needed the same vehicles than in the real case and in the second scenario this happens in 11, 15 and 29 of January.

In the first scenario, the model application allowed a reduction of 38 vehicles when compared to the real case which means an average of, approximately, 2 vehicles for each day. In the second scenario the model enabled a reduction of 28 vehicles, which means an average of 1 vehicle per day.

5.2 – Total travel distance

In figures 3 and 4, the total distance travelled in the two scenarios and the real case study can be compared.

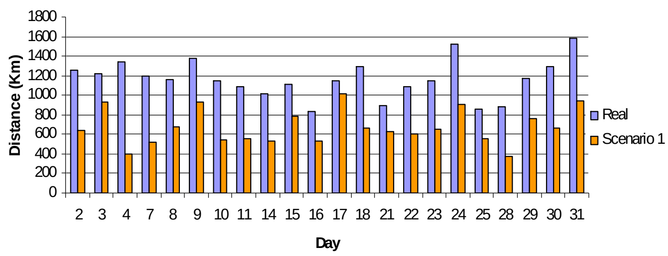

Figure 3 - Total distance travelled, for scenario 1.

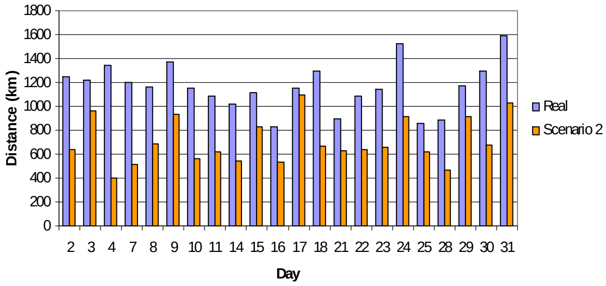

Figure 4 - Total distance travelled, for scenario 2.

As it was expected the total travelled distance in the real case is far superior when compared to the situations modeled. This happens not only through the routes optimization, but also because the number of vehicles has decreased. Comparing the two scenarios, sometimes the distance traveled in the second scenario is higher, mainly due to the fact that are used more vehicles.

5.3 – Total operation time

The results for the average duration of each route in the two scenarios are shown in Figures 5 and 6.

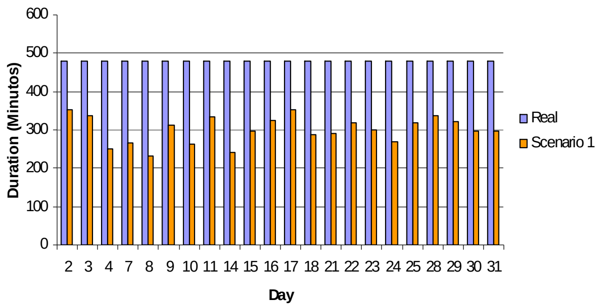

Figure 5 – Operation time average per day, for scenario 1.

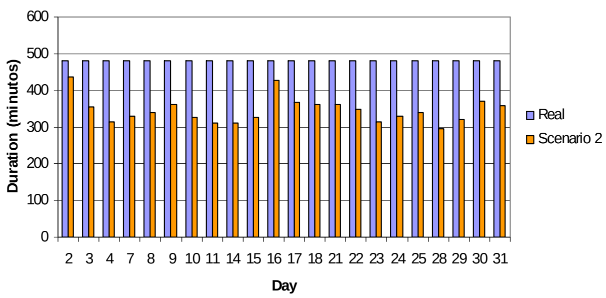

Figure 6 – Operation time average per day, for scenario 2.

Comparing the real case with the first scenario, there is a wide disparity in the average routes duration. In the first one, the operation time corresponds to the drivers working hours, 8 hours a day. In the second, the average of all routes time, the duration has a value of 297.4 minutes, or approximately 5 hours. These different values may be due to the fact that the variable time in each borough considered is too low, because in each one there are several clients to be satisfied and the time required may be higher. In scenario 2, there is a reduction, not as pronounced because it is needed more time in each borough. The average of all day's routes duration has a value of 343 minutes, or approximately 5 hours and 43 minutes.

Comparing the two scenarios, in terms of the operation time average, it can be observed a difference of only 43 minutes.

5.4 – Distribution costs

The total distribution cost in the two scenarios modeled and their division for the two types of vehicles that can be used are presented in figures 7 and 8.

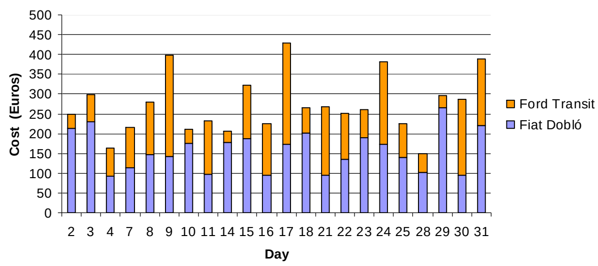

Figure 7 – Total distribution costs, for scenario 1.

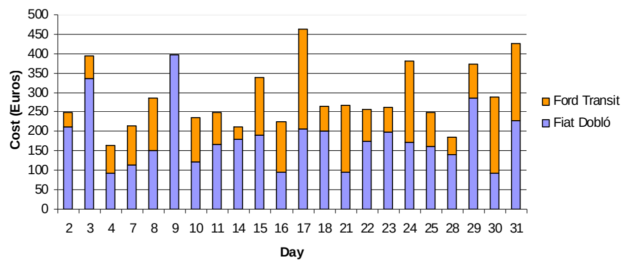

Figure 8 – Total distribution costs, for scenario 2.

Comparing the two scenarios, it is clear that the first leads to lower costs, because there are used fewer vehicles, it is travelled a smaller total distance and less time is needed to carry out the plan of distribution. However, the variable time of 10 minutes on each borough is only verified in those where there are not visited many customers.

Nevertheless, the company A. A. Silva has a vast list of clients, and many of them are nearby of each other, which means that in most of the boroughs there are visited several customers. Thus, it is considered that 10 minutes are insufficient to satisfy the customers in the same borough. So, the second scenario is more representative of reality, however can be sometimes an excessive duration in the case of boroughs that do not have many customers to visit. Even in that case, it suggests significant improvements over what happens in reality.

6 – Conclusions

The vehicle routing problems have been hardly studied due to the importance and appliance transportation has in nowadays' society.

In this context, this article arises with the objective of optimizing the routes and the fleet of a Portuguese company, named A. A. Silva.

In this work a mathematical model was developed in GAMS software to define the optimal set of routes that minimizes the total costs, in terms of the fleet vehicles as well as the total distance travelled by all vehicles, taking into account the capacity of each one.

After reviewing the literature, this problem can be characterized as being a VRPPD with heterogeneous fleet.

Given the high complexity of a VRPPD the clients were aggregated into the boroughs they belong. Then it was applied the p-median model to divide the problem into simpler sub-problems.

The model was applied to two different scenarios, in which the variable time needed to visit each cluster varies. In the first scenario, the time to make the conference of the stock on consignment, the collection of claims, the auto sales and to provide technical assistance is 10 minutes. In the second scenario the variable time is 20 minutes. Then the time to delivery and collect batteries must be added in both scenarios.

Comparing with the real case, there have been major improvements in both tested scenarios. However, it was considered that the second scenario is more appropriate for reality, despite the fact that sometimes its variable time can be excessive in the case of boroughs that do not have many customers to visit.

However, it is necessary to take into account that the aggregation of customers in the boroughs and then in groups through the p-median model may have an impact on results, which could be different if the model was applied only once a day to all clients.

As a future work perspective, A. A. Silva could apply the developed model to the remaining regions of the country in order to obtain the routes and the number of vehicles needed to minimize the total logistic costs. Moreover, the time horizon can also be extended and it would be interesting to analyze other resolution methods, in particular a combination of the mathematical model with some heuristics. It should also be invested more time to solve the same problem with less aggregation, because it may influence the results.

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