Transport in the vortex state of unconventional superconductors

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Two distinct models, one for a vortex state d-wave superconductor and another one for heavy fermions systems in the context of their superconducting phase are studied. In particular, the electrical, spin and thermal conductivities for vortex state d-wave superconductors are determined in terms of Bogoliubov-de Gennes solutions. In what concerns the conductivities, we calculate the first two term from the vertex correction series which already gives the correction term also obtained from the Boltzmann equation.

The heavy fermions system is studied through the Anderson lattice model with $U = \infty$ which can be achieved after introducing the slave boson formalism. Thermal conductivity is obtained in a similar way as in d-wave superconductors, using the energy density of the hamiltonian. In particular, the ability to write the Green's functions in partial fractions is fundamental since we can use the usual frequency sums of many particles physics.

INTRODUCTION

Superconductivity is one of the most important physics discoveries of the 20th century. Several early attempts to understand this phenomenon were carried out but they were mainly phenomenological even though they could also explain some of the main properties of the superconductors, like London theory and Ginzburg-Landau theory. Bardeen, Cooper and Schrieffer finally arrived to a very successful theory in 1957, now called BCS theory allowing the explanation of the microscopic properties of the material by introducing the concept of the pairing mechanism in these systems.

In the 1980s superconductivity was thought to be a very well understood subject in Physics. However, the experimental discovery of cuprates with a superconducting phase above 30 K [1] opened a whole new chapter because BCS theory couldn't explain the mechanism of some of these high temperature superconductors (or HTSC). And the discussion continues until today.

Some of the properties of the HTSC are known. It was discovered they are type-II superconductors and contain nodes in the gap, leading to a finite density of fermionic excitations at low energies. Apparently, one of their characteristics is the applicability of the the d-wave BCS based phenomenology to a large range of the quasiparticle properties in the superconducting state [2]. Although some issues are still not resolved and are subject to many discussion, like the normal phase of these materials, there is experimental evidence of nodal quasiparticles with Dirac-like dispersion governing the low energy properties of cuprate superconductors from spectroscopic [3] and transport measurements [4].

It has also been argued that for a d-wave superconductor, the addition of impurities generates a finite density of quasiparticle states down to zero energy. Then, if we increase the impurity density in the superconductor, the density of quasiparticles also increases and the quasiparticles lifetime is reduced. Durst and Lee [5] verified the alterations in the electric, spin and thermal conductivity due to vertex corrections, introduced because of the impurities, and Fermi-liquid corrections. The authors identified constant values for $\Omega \to 0$ and $T \to 0$ (or the universal-limit) for the "bare bubble" (meaning, without impurity disorder) conductivities, without a magnetic field. Also, vertex corrections modify the electric conductivity and Fermi-liquid corrections renormalized the charge and spin conductivities while only the thermal one kept the value of the universal-limit.

One of the first interests of the theory was the formation of a Dirac Landau level quantization picture which explains some properties of unconventional superconductors. A singular gauge transformation devised by Franz and Tesanovic [6] allowed them to show that the natural low-energy quasiparticle states were Bloch waves of massless Dirac fermions and not Landau levels for a superconductor in the vortex state. Following this first work, study on the quasiparticle energy spectrum for the vortex state with different pairings has been done [7]. Vafek also demonstrated the electronic density of stated for d-wave superconductors in a vortex lattice scaled with the root mean square vortex displacement $\mathbf{u}_{rms}$ as $\sqrt{H} f(\mathbf{u}_{rms}/\xi^2)$ [8]. This allowed him to also conclude thermal conductivity is independent from the magnetic field which agrees with the experimental results for $YBa_2Cu_3O_7$ obtained by Hill *et al* [9]. They also realized quasiparticles scattering off vortices can't be neglected.

Recently the density of states of a vortex state superconductor has been calculated for vortices distributed randomly with s and d-wave pairing [10]. Strongly peaked low-energy states near vortex cores were observed and both the impurity disorder and vortices due to the presence of a magnetic field fills the density of states at low-energies. However, impurities are present in most systems and the density of states for the same model, with impurities and vortices randomly distributed but with impurities near vortices, was also calculated [11] showing the dominant effect in DOS is due to the vortex scattering.

Transport properties can be very useful to understand unconventional superconductors and some concern regarding the Wiedermann-Franz law has been taken. Near quantum critical points [12, 13] and high-temperature superconductors [14] a violation of this law has been found. The same was verified theoretically in Luttinger liquids [15] and in a d-wave superconductor model with impurities [5]. Large violations of Wiedermann-Franz law usually indicate a big change in the energy spectrum due to the opening of a gap.

This summary continues the line of thought introduced by Vafek *et al.* [16], where they have obtained spin and thermal conductivities for the vortex state d-wave superconductor and more transport properties in heavy-fermions systems are also computed. In the following section, we introduce a basic theory to understand the main calculations done in this thesis. We start by understanding Linear Response theory and the corresponding application to electric and consequently spin conductivities. Thermal conductivity requires a more careful treatment and it is explained. A detailed study of transport properties in a d-wave superconductor with magnetic field: electric, spin and thermal conductivities is the next step where followed by the study of a vortex state d-wave superconductor with the addition of impurity disorder where we recalculate the conductivities. The study of heavy fermions in the superconductive phase is in the final section and the corresponding thermal conductivity is obtained, as a continuation of the work done in previous chapters.

DEFINITION OF CONDUCTIVITIES: LINEAR RESPONSE THEORY

Experiments in condensed matter physics, generally measure quantities which are a response to an external perturbation. For example, when we apply a magnetic field to a spin system, we may want to measure the corresponding linear response, which is the magnetization. If we insert an electrical field, the electrical current is an interesting property to measure. Notice that the linear response approximation is generally valid for a small amplitude of the perturbative field.

In a many-particle system, Kubo Formula is the general name for a correlation function which measures the linear response. There are several of them, depending on the quantity we want. Several kinds of currents exist but we will only focus on the flow of charged particles through a system as well as the spin changes which also occurs and also how heat is transported. Conductivities (and consequently resistivities) are however more complex quantitities to obtain, specially in a many particle system, and linear response theory is an easy method to apply.

We define the correlation function in momentum space

$$\begin{aligned} D_{\mu\nu}(\mathbf{q}, i\Omega) &= -\int_0^\beta d\tau\ e^{i\tau\Omega} < \mathsf{T}_\tau\ j^\dagger_\mu(\mathbf{q},\tau) j_\nu(\mathbf{q},0) > \\ &= -\int_0^\beta d\tau\ e^{i\tau\Omega} < \mathsf{T}_\tau\ j_\mu(-\mathbf{q},\tau) j_\nu(\mathbf{q},0) >, \end{aligned} \tag{1}$$

where j_μ is the current density. In real space it can be written as

$$D_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, i\Omega) = -\int_0^\beta d\tau\ e^{i\tau\Omega} < \mathsf{T}_\tau\ j_\mu(\mathbf{r}_1,\tau) j_\nu(\mathbf{r}_2,0) >, \tag{2}$$

which can easily be obtained from a Fourier transform.

The dc conductivity is obtained for the limit $\mathbf{q} \to 0$ and after that the limit when $\Omega \to 0$. The $\mathbf{q} \to 0$ is no problem for the correlation function. Even in the real case, we can just make an average in every position to get the same result. However, the Ω limit must have a careful approach because it isn't usually trivial for the correlation function. The real part of the conductivity is

$$\text{Re}\ \sigma_{\mu\nu} = -\lim_{\Omega\to 0} \frac{1}{\Omega} \text{Im}\ D^R_{\mu\nu}(\Omega), \tag{3}$$

where $D^R_{\mu\nu}(\Omega)$ is the retarded function of $D_{\mu\nu}(\Omega)$ which can be easily obtained by an analytic continuation, changing $i\Omega \to \Omega + i\delta$. This equation is valid for the charge and spin conductivities.

The thermal conductivity requires a more careful treatment. Assuming the particle flow is zero, then the heat current is equal to the energy current. The conductivity can also be written using a Kubo formula [17]

$$\frac{\sigma^T_{\mu\nu}(T)}{T} = -\frac{1}{T^2} \lim_{\Omega\to 0} \frac{1}{\Omega} \text{Im}\ D^{R,T}_{\mu\nu}(\Omega) \tag{4}$$

and current-current correlation function is

$$D^T_{\mu\nu}(\Omega) = -\int_0^\beta d\tau e^{i\Omega\tau} < j_{h,\mu}(\tau) j_{h,\nu}(0) >, \tag{5}$$

where j_h is the heat current.

TRANSPORT PROPERTIES IN D-WAVE CLEAN SUPERCONDUCTORS

Franz and Z. Tešanović [6] introduced a new singular gauge transformation, now called Franz-Tesanovic Transformation, allowing the study of quasiparticles properties in a vortex lattice for these superconductors. Durst and Lee [5] determined transport properties for a d-wave superconductor in zero magnetic field with linear approximation around nodes and low temperatures when impurties exist or not (besides other corrections). O. Vafek *et al.* [16] in 2001 determined quasiparticle spin and thermal conductivities introducing a magnetic field leading to a vortex state for a d-wave superconductor. Here, we will recalculate the spin conductivity for the same situation but with a different approach for the thermal conductivity. The electric conductivity is also computed.

The hamiltonian for a 2D superconductor with a magnetic field applied and d-wave pairing is, from the usual BCS hamiltonian,

$$H = \int d\mathbf{x} \Psi^\dagger \hat{H}_0 \Psi(\mathbf{x}), \tag{6}$$

with

$$\hat{H}_0 = \begin{pmatrix} \hat{h} & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{h}^* \end{pmatrix}, \tag{7}$$

and

$$\hat{\Delta} \;=\; \Delta_0 \sum_{\boldsymbol{\delta}=\pm\hat{x},\pm\hat{y}} e^{i\phi(\mathbf{x})/2} \eta_\delta e^{i\phi(\mathbf{x})/2} \tag{8}$$

$$\hat{h} \;=\; -t \sum_{\boldsymbol{\delta}} e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\boldsymbol{\delta}} \mathbf{A}(\mathbf{r})\cdot dl} \overrightarrow{s_\delta} - \mu. \tag{9}$$

In the above equation, we introduced the translation operator $\psi^\dagger(\mathbf{r})\overrightarrow{s_{\boldsymbol{\delta}}}\psi(\mathbf{r}) = \psi^\dagger(\mathbf{r})\psi(\mathbf{r}+\boldsymbol{\delta})$ and

$$\hat{\eta_\delta} = \begin{cases} \overrightarrow{s_\delta} & \text{if } \boldsymbol{\delta} = \pm\hat{x} \\ -\overrightarrow{s_\delta} & \text{if } \boldsymbol{\delta} = \pm\hat{y} \end{cases}. \tag{10}$$

From the Bogoliubov Transformations,

$$\psi_\uparrow(\mathbf{r},\tau) = \sum_n \left(u_n(\mathbf{r})\gamma_{n\uparrow}(\tau) - v_n^*(\mathbf{r})\gamma_{n\downarrow}^\dagger(\tau) \right) \tag{11}$$

$$\psi_\downarrow(\mathbf{r},\tau) = \sum_n \left(u_n(\mathbf{r})\gamma_{n\downarrow}(\tau) + v_n^*(\mathbf{r})\gamma_{n\uparrow}^\dagger(\tau) \right), \tag{12}$$

we can derive the usual Bogoliubov-de Gennes equations

$$\hat{H}_0 \Phi_n = \epsilon_n \Phi_n. \tag{13}$$

Green's functions

In the superconductor case, the corresponding Green's functions were a long time ago determined by Gor'kov [18, 19]. However, in the case we are studying, some subtleties exist due to the magnetic field and the vortex lattice. The easiest way to solve this problem, is to use functions defined in real space. We define a 2x2 Green Function matrix to which we apply Bogoliubov transformations to every operator in this function. After this, our initial Green's Function depends on Bogoliubov quasiparticles propagators that we'll define as

$$g_{11,clean}(\mathbf{r}_1,\mathbf{r}_2,\tau) \;=\; -< \mathsf{T}_\tau \gamma_\uparrow(\mathbf{r}_1,\tau)\gamma_\uparrow^\dagger(\mathbf{r}_2,0) > \tag{14}$$

$$g_{22,clean}(\mathbf{r}_1,\mathbf{r}_2,\tau) \;=\; -< \mathsf{T}_\tau \gamma_\downarrow^\dagger(\mathbf{r}_1,\tau)\gamma_\downarrow(\mathbf{r}_2,0) > \tag{15}$$

Since we know that, for a clean superconductor these diagonalized Green's functions are

$$g_{nn,clean}(i\omega) = (i\omega\mathbb{1} - \epsilon_n\sigma_z)^{-1} \tag{16}$$

without any anomalous terms, $\mathcal{G}$ becomes, after applying a Fourier Transform in time,

$$\mathcal{G}_{11}(\mathbf{r}_1,\mathbf{r}_2,\tau) = -< \mathsf{T}_\tau \psi_\uparrow(\mathbf{r}_1,\tau)\psi_\uparrow^\dagger(\mathbf{r}_2), 0 >= = \frac{1}{\beta}\sum_{n,i\omega}\left[\frac{u_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)}{i\omega-\epsilon_n} + \frac{v_n^*(\mathbf{r}_1)v_n(\mathbf{r}_2)}{i\omega+\epsilon_n}\right] e^{-i\omega\tau} \tag{17}$$

$$\mathcal{G}_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau) = -< \mathsf{T}_\tau \psi_\uparrow(\mathbf{r}_1,\tau)\psi_\downarrow(\mathbf{r}_2), 0 >= = \frac{1}{\beta}\sum_{n,i\omega}\left[\frac{u_n(\mathbf{r}_1)v_n^*(\mathbf{r}_2)}{i\omega-\epsilon_n} - \frac{v_n^*(\mathbf{r}_1)u_n(\mathbf{r}_2)}{i\omega+\epsilon_n}\right] e^{-i\omega\tau} \tag{18}$$

$$\mathcal{G}_{21}(\mathbf{r}_1,\mathbf{r}_2,\tau) = -< \mathsf{T}_\tau \psi_\downarrow^\dagger(\mathbf{r}_1,\tau)\psi_\uparrow^\dagger(\mathbf{r}_2, 0) >= = \frac{1}{\beta}\sum_{n,i\omega}\left[-\frac{u_n^*(\mathbf{r}_1)v_n(\mathbf{r}_2)}{i\omega+\epsilon_n} + \frac{v_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)}{i\omega-\epsilon_n}\right] e^{-i\omega\tau} \tag{19}$$

$$\mathcal{G}_{22}(\mathbf{r}_1,\mathbf{r}_2,\tau) = -< \mathsf{T}_\tau \gamma_\downarrow^\dagger(\mathbf{r}_1,\tau)\gamma_\downarrow(\mathbf{r_2}, 0) >= = \frac{1}{\beta}\sum_{n,i\omega}\left[\frac{u_n^*(\mathbf{r}_1)u_n(\mathbf{r}_2)}{i\omega+\epsilon_n} + \frac{v_n(\mathbf{r}_1)v_n^*(\mathbf{r}_2)}{i\omega-\epsilon_n}\right] e^{-i\omega\tau} \tag{20}$$

where u_n and v_n are solutions of Bogoliubov-deGennes equation 13.

Currents

With the model defined, a standard method can be used to obtain the electrical, spin and thermal conductivities. From equation 3 we know we have to calculate the correlation function. To obtain it we must derive a current for each case through the continuity equation. Through the charge, spin and heat densities derivatives in time, we are able to obtain the corresponding current. Since we are working on a tight binding lattice, it is useful to use the following discrete continuity equation

$$\dot{\rho}^\alpha \;=\; -\left(\frac{j^\alpha(x+\delta_x,y) - j^\alpha(x-\delta_x,y)}{\delta_x} + \right. \left. + \frac{j^\alpha(x,y+\delta_y) - j^\alpha(x,y-\delta_y)}{\delta_y}\right) \tag{21}$$

where $\alpha = e, s, T$ (electric, spin and thermal). From it we compare with the expanded sums in $\boldsymbol{\delta}$ of the different densities derivatives.

The charge density is

$$\rho_e = e\left(\psi_\uparrow^\dagger\psi_\uparrow + \psi_\downarrow^\dagger\psi_\downarrow\right). \tag{22}$$

Since charge is not a conserved quantity due to the mixture of particle and hole character of the quasiparticles the charge current will not contain pairing terms in its expression. In fact, the current is, in Nambu notation

$$j^e_{\delta_\mu}(\mathbf{r}) = \frac{e}{\hbar}\Psi^\dagger(\mathbf{r})\hat{V}^e_\mu(\mathbf{r},\delta_\mu)\Psi(\mathbf{r}), \tag{23}$$

with the Nambu spinor defined by $\Psi^\dagger(\mathbf{r}) = (\psi_\uparrow^\dagger(\mathbf{r})\ \psi_\downarrow(\mathbf{r}))$. The velocity matrix in the current expression is a 2x2 matrix

$$\hat{V}^e_{11,\mu}(\mathbf{r},\delta_\mu) = it\delta_\mu(\overleftarrow{s_{\delta_\mu}} e^{\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl} - -e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}\overrightarrow{s_{\delta_\mu}}) \quad (24)$$

$$\hat{V}^e_{22,\mu}(\mathbf{r},\delta_\mu) = it\delta_\mu(\overleftarrow{s_{\delta_\mu}} e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl} - -e^{\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}\overrightarrow{s_{\delta_\mu}}) \quad (25)$$

$$\hat{V}^e_{12,\mu}(\mathbf{r},\delta_\mu) = \hat{V}^e_{21,\mu}(\mathbf{r},\delta_\mu) = 0 \quad (26)$$

where we defined a new translation operator which is applied to the function in its left $\psi^\dagger(\mathbf{r})\overleftarrow{s_\delta}\psi(\mathbf{r}) = \psi^\dagger(\mathbf{r}+\delta)\psi(\mathbf{r})$.

For the spin case, starting from the corresponding density

$$\rho_s = \frac{\hbar}{2}\left(\psi^\dagger_\uparrow\psi_\uparrow - \psi^\dagger_\downarrow\psi_\downarrow\right), \quad (27)$$

we compute the following current

$$j^s_{\delta_\mu}(\mathbf{r}) = \frac{1}{2}\Psi^\dagger(\mathbf{r})\hat{V}^s_\mu(\mathbf{r},\delta_\mu)\Psi(\mathbf{r}) \quad (28)$$

where $\mu = x, y$ and the velocity matrix is

$$\hat{V}^s_{11,\mu}(\mathbf{r},\delta_\mu) = \overleftarrow{s_{\delta_\mu}} e^{\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl} - -e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}\overrightarrow{s_{\delta_\mu}}) \quad (29)$$

$$\hat{V}^s_{12,\mu}(\mathbf{r},\delta_\mu) = i\Delta_0\delta_\mu e^{i\phi(\mathbf{r})/2}\eta_\delta(\overrightarrow{s_{\delta_\mu}} - -\overleftarrow{s_{\delta_\mu}})e^{i\phi(\mathbf{r})/2} \quad (30)$$

$$\hat{V}^s_{21,\mu}(\mathbf{r},\delta_\mu) = i\Delta_0\delta_\mu e^{-i\phi(\mathbf{r})/2}\eta_\delta(\overrightarrow{s_{\delta_\mu}} - -\overleftarrow{s_{\delta_\mu}})e^{-i\phi(\mathbf{r})/2} \quad (31)$$

$$\hat{V}^s_{22,\mu}(\mathbf{r},\delta_\mu) = it\delta_\mu(e^{\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}\overrightarrow{s_{\delta_\mu}} - -\overleftarrow{s_{\delta_\mu}} e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}) \quad (32)$$

For the thermal current, we follow the same method Durst and Lee [5] used to obtain transport properties in a d-wave superconductor without magnetic field, which is similar to what we have done for the spin and charge currents. Vafek et al. [16] have also determined the thermal current for a vortex state d-wave superconductor. However, they introduced a pseudo-gravitational potential following work done by Luttinger [17] and Snrčka and Středa [20]. This method, allowed Vafek, Melikyan and Tešanović to understand that the transverse thermal current was not only given by Kubo formula but had corrections due to magnetization.

The energy density is

$$h(\mathbf{r}) = \psi^\dagger_\uparrow(\mathbf{r})\hat{h}(\mathbf{r})\psi_\uparrow(\mathbf{r}) + \psi_\downarrow(\mathbf{r})\hat{\Delta}^*(\mathbf{r})\psi_\uparrow(\mathbf{r}) + +\psi^\dagger_\uparrow(\mathbf{r})\hat{\Delta}(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r}) - \psi_\downarrow(\mathbf{r})\hat{h}^*(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r}) \quad (33)$$

Calculating the derivative in time and using the discrete continuity equation we obtain

$$j^T_{\delta_\mu}(\mathbf{r}) = \frac{i}{\hbar}\Psi^\dagger(\mathbf{r})\hat{V}^{(T1)}_\mu(\mathbf{r},\delta_\mu)\Psi(\mathbf{r}) \quad (34)$$

and the velocity matrix $\hat{V}^{(T1)}_\mu$ elements are

$$\hat{V}^{(T1)}_{11,\mu} = \sum_{\gamma=\pm\hat{x},\pm\hat{y}} \xi^*(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu+\gamma}} - -\overleftarrow{s_\gamma}\eta(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu}} \quad (35)$$

$$\hat{V}^{(T1)}_{12,\mu} = \sum_{\gamma=\pm\hat{x},\pm\hat{y}} \overleftarrow{s_\gamma}y(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu}} + +z(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu+\gamma}} \quad (36)$$

$$\hat{V}^{(T1)}_{21,\mu} = \sum_{\gamma=\pm\hat{x},\pm\hat{y}} -\overleftarrow{s_\gamma}y^*(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu}} - -z^*(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu+\gamma}} \quad (37)$$

$$\hat{V}^{(T1)}_{22,\mu} = \sum_{\gamma=\pm\hat{x},\pm\hat{y}} \xi(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu+\gamma}} - -\overleftarrow{s_\gamma}\eta^*(\mathbf{r},\delta_\mu,\gamma)\overrightarrow{s_{\delta_\mu}} \quad (38)$$

In this matrix we have also defined the following functions

$$\eta(\mathbf{r},\delta_\mu,\gamma) = \delta_\mu e^{-i\phi(\mathbf{r}+\delta_\mu)/2}\eta_\gamma\eta_{\delta_\mu}e^{i\phi(\mathbf{r}+\gamma)/2}\Delta_0^2 + +\delta_\mu t^2 e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}+\gamma}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl} \quad (39)$$

$$\xi(\mathbf{r},\delta_\mu,\gamma)\delta_\mu e^{-i\phi(\mathbf{r})/2}\eta_\gamma\eta_{\delta_\mu}e^{i\phi(\mathbf{r}+\delta_\mu+\gamma)/2}\Delta_0^2 + +\delta_\mu t^2 e^{\frac{ie}{\hbar c}\left(\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl + \int_{\mathbf{r}+\delta_\mu}^{\mathbf{r}+\delta_\mu+\gamma}\mathbf{A}(\mathbf{r}+\delta_\mu)\cdot dl\right)} \quad (40)$$

$$w(\mathbf{r},\delta_\mu,\gamma) = = \delta_\mu t e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}e^{-i\phi(\mathbf{r})/2}\eta_\gamma e^{-i\phi(\mathbf{r}+\gamma)/2}\Delta_0 - -\delta_\mu t e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\gamma}\mathbf{A}(\mathbf{r})\cdot dl}e^{-i\phi(\mathbf{r})/2}\eta_{\delta_\mu}e^{-i\phi(\mathbf{r}+\delta_\mu)/2}\Delta_0 \quad (41)$$

$$z(\mathbf{r},\delta_\mu,\gamma) = = \delta_\mu t e^{\frac{ie}{\hbar c}\int_{\mathbf{r}+\delta_\mu}^{\mathbf{r}+\delta_\mu+\gamma}\mathbf{A}(\mathbf{r}+\delta_\mu)\cdot dl}e^{i\phi(\mathbf{r})/2}\eta_{\delta_\mu}e^{i\phi(\mathbf{r}+\delta_\mu)/2}\Delta_0 - -\delta_\mu t e^{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}+\delta_\mu}\mathbf{A}(\mathbf{r})\cdot dl}e^{i\phi(\mathbf{r}+\delta_\mu)/2}\eta_\gamma e^{i\phi(\mathbf{r}+\delta_\mu+\gamma)/2}\Delta_0 \quad (42)$$

Conductivities

The conductivities calculations require initially the correlation function 2, where we introduce the currents. We can now apply the Wick's theorem to every term, expressing averages of products of operators in terms of products of contractions of two operators, in all possible ways. In this case, two possibilities arise: one correspondes to two densitites multiplied and will not be considered here and the other one is an open bubble with 2 vertices in time τ and 0 and analytically corresponds to two propagators multiplied. Inserting the Green's functions 17-20 into the correlation function we can use the Fourier transform to obtain a Dirac delta. Since there is a sum in frequencies, one of four possibilites arises. The first two are

$$S_{\alpha\alpha}(i\Omega,\epsilon_1,\epsilon_2) = \frac{1}{\beta}\sum_{i\omega}\frac{1}{i\omega\ \alpha\ \epsilon_1}\frac{1}{i\omega + i\Omega\ \alpha\ \epsilon_2} = = \frac{n_F(\epsilon_1) - n_F(\epsilon_2)}{i\Omega + \epsilon_1 - \epsilon_2}, \quad (43)$$

where $\alpha = \pm$ and n_F is the Fermi function. The final two are

$$S_{\alpha\gamma}(i\Omega, \epsilon_1, \epsilon_2) = \frac{1}{\beta}\sum_{i\omega} \frac{1}{i\omega \ \alpha \ \epsilon_1} \frac{1}{i\omega - i\Omega \ \gamma \ \epsilon_2} = \\ = \frac{\gamma(1 - n_F(\epsilon_1) - n_F(\epsilon_2))}{i\Omega \ \alpha \ \epsilon_1 \ \alpha \ \epsilon_2}, \quad (44)$$

where $\alpha, \gamma = \pm$ and $\alpha \neq \gamma$. In the electric, spin and thermal cases, terms multiplied with S_{-+} will cancel with S_{+-}. The terms left can also be simplified because$S_{++}(i\Omega, \epsilon_1, \epsilon_2) = S_{--}(i\Omega, \epsilon_1, \epsilon_2)$. Now it is only a matter of taking the analytic continuation (with the change $i\Omega \to \Omega + i\delta$) and the imaginary part of the correlation function. After taking the limit when $\Omega \to 0$ we obtain the conductivity. The only part of the correlation function which depends on Ω is the one resulting from the frequency sum. From the conductivity definition Vafek *et al.* [16] uses, we obtain

$$\sigma^e_{\mu\nu} = \lim_{\Omega \to 0} \frac{i}{\Omega}\left(D^R_{\mu\nu}(\Omega) - D^R_{\mu\nu}(0)\right) \propto \\ \propto \lim_{\Omega \to 0} \frac{i}{\Omega}\left(\frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{\Omega + \epsilon_m - \epsilon_n + i\delta} - \frac{n_F(\epsilon_m) - n_F \epsilon_n}{\epsilon_m - \epsilon_n + i\delta}\right) = \\ = -i\frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{(\epsilon_m - \epsilon_n + i\delta)^2}. \quad (45)$$

The final conductivity for the electric current is

$$\sigma^e_{\mu\nu} = \frac{2e^2 i}{\hbar^2}\sum_{n,m}\sum_{\mathbf{r}_1,\mathbf{r}_2} \Phi^\dagger_m(\mathbf{r}_1)\hat{V}^e_\mu(\mathbf{r}_1, \delta_\mu)\Phi_n(\mathbf{r}_1) \cdot \\ \cdot \quad \Phi^\dagger_n(\mathbf{r}_2)\hat{V}^e_\nu(\mathbf{r}_2, \delta_\nu)\Phi_m(\mathbf{r}_2)\frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{(\epsilon_m - \epsilon_n + i\delta)^2}, \quad (46)$$

where $\Phi^T_n(\mathbf{r}) = (u_n(\mathbf{r}) \ v_n(\mathbf{r}))$ Bogoliubov-de Gennes solutions and $\mu, \nu = x, y$.The velocity matrix $\hat{V}^e_\mu$ was previously defined in equations 24-26.

The spin conductivity is

$$\sigma^s_{\mu\nu} = -\frac{i}{2}\sum_{n,m}\sum_{\mathbf{r}_1,\mathbf{r}_2} \Phi^\dagger_m(\mathbf{r}_1)\hat{V}^s_\mu(\mathbf{r}_1, \delta_\mu)\Phi_n(\mathbf{r}_1) \cdot \\ \cdot\Phi^\dagger_n(\mathbf{r}_2)\hat{V}^s_\nu(\mathbf{r}_2, \delta_\nu)\Phi_m(\mathbf{r}_2)\frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{(\epsilon_m - \epsilon_n + i\delta)^2}, \quad (47)$$

and in the thermal case

$$\frac{\sigma^T_{\mu\nu}}{T} = -\frac{i}{\hbar^2 T^2}\sum_{n,m}\sum_{\mathbf{r}_1,\mathbf{r}_2}\Bigg[\Phi^\dagger_m(\mathbf{r}_1)\hat{V}^{(T1)}_\mu(\mathbf{r}_1, \delta_\mu)\Phi_n(\mathbf{r}_1) \cdot \\ \cdot\Phi^\dagger_n(\mathbf{r}_2)\hat{V}^{(T1)}_\nu(\mathbf{r}_2, \delta_\nu)\Phi_m(\mathbf{r}_2) + \\ +\Phi^\dagger_m(\mathbf{r}_1)\hat{V}^{(T2)}_\mu(\mathbf{r}_1, \delta\mu)\Phi_n(\mathbf{r}_1) \cdot \\ \cdot\Phi^\dagger_n(\mathbf{r}_2)\hat{V}^{(T2)}_\nu(\mathbf{r}_2, \delta_\nu)\Phi_m(\mathbf{r}_2)\Bigg]\frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{(\epsilon_m - \epsilon_n + i\delta)^2}. \quad (48)$$

In this quantity, we also introduced a new matrix $\hat{V}^{(T2)}$ and its elements are

$$\hat{V}^{(T2)}_{11,\mu}(\mathbf{r}, \delta_\mu) = \sum_{\boldsymbol{\gamma}=\pm\hat{x},\pm\hat{y}} \overleftarrow{s_{\delta_\mu}}\eta^*(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma})\overrightarrow{s_{\boldsymbol{\gamma}}} - \\ -\overleftarrow{s_{\boldsymbol{\gamma}+\delta_\mu}}\xi(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma}) \quad (49)$$

$$\hat{V}^{(T2)}_{12,\mu}(\mathbf{r}, \delta_\mu) = \sum_{\boldsymbol{\gamma}=\pm\hat{x},\pm\hat{y}} \overleftarrow{s_{\delta_\mu}}y(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma})\overrightarrow{s_{\boldsymbol{\gamma}}} + \\ +\overleftarrow{s_{\boldsymbol{\gamma}+\delta_\mu}}z(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma}) \quad (50)$$

$$\hat{V}^{(T2)}_{21,\mu}(\mathbf{r}, \delta_\mu) = \sum_{\boldsymbol{\gamma}=\pm\hat{x},\pm\hat{y}} -\overleftarrow{s_{\delta_\mu}}y^*(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma})\overrightarrow{s_{\boldsymbol{\gamma}}} - \\ -\overleftarrow{s_{\boldsymbol{\gamma}+\delta_\mu}}z^*(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma}) \quad (51)$$

$$\hat{V}^{(T2)}_{22,\mu}(\mathbf{r}, \delta_\mu) = \sum_{\boldsymbol{\gamma}=\pm\hat{x},\pm\hat{y}} \overleftarrow{s_{\delta_\mu}}\eta(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma})\overrightarrow{s_{\boldsymbol{\gamma}}} - \\ -\overleftarrow{s_{\boldsymbol{\gamma}+\delta_\mu}}\xi^*(\mathbf{r}, \delta_\mu, \boldsymbol{\gamma}) \quad (52)$$

It is important to notice that this matrix shares several similarities with $V^{(T1)}_\mu$. We can conclude that $iV^{(T2)}_\mu = (iV^{(T1)}_\mu)^\dagger$.

TRANSPORT PROPERTIES IN IMPURITY DISORDERED D-WAVE SUPERCONDUCTORS

This section mainly consists in obtaining the transport properties from previous section when spinless impurities are inserted. Our choice here depends on some approximations and considerations usual when treating systems with impurities. One of them is only considering the imaginary part of the impurity self energy, neglecting the anomalous term, and considering it is a constant Σ.

We will start by representing the conductivities 46, 47 and 48 we obtained in section 3, in spectral representation. We will then, replace the original Green's functions, by the corresponding altered ones with impurities and also obtain the first term of the vertex correction series. In summary, we will obtain the corresponding quantities of diagrams in figure 1 and 2.

The Matsubara Green's functions can be written differently, in real space, in the so called Spectral or Lehmann Representation

$$\mathcal{G}(\mathbf{x}, \mathbf{y}, \omega) = \frac{1}{2\pi}\int d\epsilon \frac{A(\mathbf{x}, \mathbf{y}, \epsilon)}{i\omega - \epsilon}, \quad (53)$$

The retarded Green's function allows a definition relating it with the spectral function

$$A(\mathbf{x}, \mathbf{y}, \omega) = -2\text{Im } G^{ret}. \quad (54)$$

Applying to the Green's functions 17-20 and after the analytic continuation $i\Omega \to \Omega + i\delta$, we get the following spectral functions

$$
\begin{aligned}
A_{11}(\mathbf{r}_1,\mathbf{r}_2;\omega) &= -2\sum_{n,i\omega}\left[\frac{u_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)}{(\omega-\epsilon_n)^2+\Gamma^2}+\right. \\
&\quad \left.+\frac{v_n^*(\mathbf{r}_1)v_n(\mathbf{r}_2)}{(\omega+\epsilon_n)^2+\Gamma^2}\right]\Gamma \qquad (55)\\
A_{22}(\mathbf{r}_1,\mathbf{r}_2;\omega) &= -2\sum_{n,i\omega}\left[\frac{u_n^*(\mathbf{r}_1)u_n(\mathbf{r}_2)}{(\omega+\epsilon_n)^2+\Gamma^2}+\right. \\
&\quad \left.+\frac{v_n(\mathbf{r}_1)v_n^*(\mathbf{r}_2)}{(\omega-\epsilon_n)^2+\Gamma^2}\right]\Gamma \qquad (56)\\
A_{12}(\mathbf{r}_1,\mathbf{r}_2;\omega) &= -2\sum_{n,i\omega}\left[\frac{u_n(\mathbf{r}_1)v_n^*(\mathbf{r}_2)}{(\omega-\epsilon_n)^2+\Gamma^2}-\right. \\
&\quad \left.-\frac{v_n^*(\mathbf{r}_1)u_n(\mathbf{r}_2)}{(\omega+\epsilon_n)^2+\Gamma^2}\right]\Gamma \qquad (57)\\
A_{21}(\mathbf{r}_1,\mathbf{r}_2;\omega) &= -2\sum_{n,i\omega}\left[-\frac{u_n^*(\mathbf{r}_1)v_n(\mathbf{r}_2)}{(\omega+\epsilon_n)^2+\Gamma^2}+\right. \\
&\quad \left.+\frac{v_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)}{(\omega-\epsilon_n)^2+\Gamma^2}\right]\Gamma. \qquad (58)
\end{aligned}
$$

The easiest way to replace the Green's function, as it was obtained for the clean state, by the spectral functions defined is by first introducing a generic current-current correlation function

$$D^{\alpha}_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2,i\Omega) = k_\alpha\int_0^\beta d\tau\; e^{i\Omega\tau} d^{\alpha}_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2,\tau) \qquad (59)$$

and

$$
\begin{aligned}
d^{\alpha}_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2,\tau) &= \Big[\mathsf{Tr}\Big(m_{\alpha,1}(\mathbf{r}_1,\delta_\mu).\mathcal{G}\left(\mathbf{r}_1,\mathbf{r}_2+\delta_\nu,\tau\right).m_{\alpha,2}(\mathbf{r}_2,\delta_\nu).\mathcal{G}\left(\mathbf{r}_2,\mathbf{r}_1+\delta_\mu,-\tau\right)\Big)+\\
&+\mathsf{Tr}\Big(m^D_{\alpha,2}(\mathbf{r}_1,\delta_\mu).\mathcal{G}\left(\mathbf{r}_1+\delta_\mu,\mathbf{r}_2+\delta_\nu,\tau\right).m_{\alpha,2}(\mathbf{r}_2,\delta_\nu).\mathcal{G}\left(\mathbf{r}_2,\mathbf{r}_1,-\tau\right)\Big)+\\
&+\mathsf{Tr}\Big(m_{\alpha,1}(\mathbf{r}_1,\delta_\mu).\mathcal{G}\left(\mathbf{r}_1,\mathbf{r}_2,\tau\right).m^D_{\alpha,1}(\mathbf{r}_2,\delta_\nu).\mathcal{G}\left(\mathbf{r}_2+\delta_\nu,\mathbf{r}_1+\delta_\mu,-\tau\right)\Big)+\\
&+\mathsf{Tr}\Big(m^D_{\alpha,2}(\mathbf{r}_1,\delta_\mu).\mathcal{G}\left(\mathbf{r}_1+\delta_\mu,\mathbf{r}_2,\tau\right).m^D_{1\alpha,1}(\mathbf{r}_2,\delta_\nu).\mathcal{G}\left(\mathbf{r}_2+\delta_\nu,\mathbf{r}_1,-\tau\right)\Big)\Big], \qquad (60)
\end{aligned}
$$

where $\alpha = e,s,T$, identifying electrical, spin and thermal cases. Also notice that $m_{\alpha,1}(\mathbf{x},\delta_\mu)$ $m_{\alpha,2}(\mathbf{x},\delta_\mu)$ are 2x2 matrices containing the Pierls and pairing terms and are different in each case. Finally, k_α constants are

$$k_\alpha = \begin{cases} \dfrac{e^2}{\hbar^2} & \text{, if } \alpha = e\\ \dfrac{1}{4} & \text{, if } \alpha = s\\ \dfrac{1}{\hbar^2 T} & \text{, if } \alpha = T \end{cases} \qquad (61)$$

The first term we calculated was the bubble as represented in figure 1.

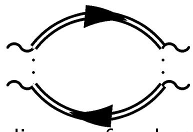

Figure 1: Bubble diagram for electrical, spin and thermal conductivities with dressed propagators

In this case, the conductivity becomes

$$
\begin{aligned}
\sigma^{\alpha,(0)}_{\mu\nu} &= -\frac{1}{4\pi}k_\alpha\sum_{\mathbf{r}_1,\mathbf{r}_2}\int d\epsilon\Big[\mathsf{Tr}\Big(m_{\alpha,1}(\mathbf{r}_1,\delta_\mu).A\left(\mathbf{r}_1,\mathbf{r}_2+\delta_\nu,\epsilon\right).m_{\alpha,2}(\mathbf{r}_2,\delta_\nu).A\left(\mathbf{r}_2,\mathbf{r}_1+\delta_\mu,\epsilon\right)\Big)+\\
&+\mathsf{Tr}\Big(m^D_{\alpha,2}(\mathbf{r}_1,\delta_\mu).A\left(\mathbf{r}_1+\delta_\mu,\mathbf{r}_2+\delta_\nu,\epsilon\right).m_{\alpha,2}(\mathbf{r}_2,\delta_\nu).A\left(\mathbf{r}_2,\mathbf{r}_1,\epsilon\right)\Big)+\\
&+\mathsf{Tr}\Big(m_{\alpha,1}(\mathbf{r}_1,\delta_\mu).A\left(\mathbf{r}_1,\mathbf{r}_2,\epsilon\right).m^D_{\alpha,1}(\mathbf{r}_2,\delta_\nu).A\left(\mathbf{r}_2+\delta_\nu,\mathbf{r}_1+\delta_\mu,\epsilon\right)\Big)+\\
&+\mathsf{Tr}\Big(m^D_{\alpha,2}(\mathbf{r_1},\delta_\mu).A\left(\mathbf{r}_1+\delta_\mu,\mathbf{r}_2,\epsilon\right).m^D_{\alpha,1}(\mathbf{r}_2,\delta_\nu).A\left(\mathbf{r}_2+\delta_\nu,\mathbf{r}_1,\epsilon\right)\Big)\Big]\frac{dn_f(\epsilon)}{d\epsilon}. \qquad (62)
\end{aligned}
$$

Finally, the first order term of the vertex correction series (figure 2) is much more complex, due to a more difficult frequency sum, and the result is not shown here.

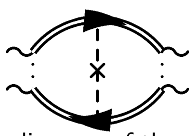

Figure 2: First order diagram of the vertex correction series for electrical, spin and thermal conductivities with dressed propagators

TRANSPORT IN HEAVY FERMIONS IN THE SUPERCONDUCTING PHASE

Heavy fermion materials are characterized by specific heat coefficients that are very large leading to a very large effective mass. Their properties are unconventional but rich since these materials can have different kinds of ordering. In a low temperature regime $UAgCu_4$, UCu_7, U_2Zn_{17} have an antiferromagnetic order but a superconductive state appears for UBe_{13}, $CeCu_2Si_2$ and UPt_3. However, no ordering is also a possibility and it happens for $CeAl_3$, $UAuPt_4$, $CeCu_6$ and UAl_2.

A mixture of states is also a possibility for some heavy fermion materials. It has been shown for UPd_2Al_3 [21–23] and UNi_2Al_3 [24], superconductivity and local momentum antiferromagnetism coexist.

A good description of the heavy fermions basic physics is believed to be given by the Anderson model and its lattice extensions [25]. In this model, the conduction electrons, c, hybridize with local states, f, and the interaction between electrons are such that the Coulomb potential U between two f-electrons is the largest energy scale in the problem. The limit $U=\infty$ is usually taken since many experimental values of U are large. This model also has Fermi-liquid-like properties in the normal nonmagnetic state and, at low temperatures, is able to explain the universality and the large effective masses of these materials.

Following the work done in previous section, we'll obtain here an analytic result for the thermal current and consequently the thermal conductivity.

Anderson lattice model

The Anderson model is described by a hamiltonian for an impurity at each site in a metal containing contribution from electrons of the conduction band, the f orbital and the hibridization which allows the transfer between the electrons in the conduction band with those from the f orbital. It also contains a repulsive energy from the f electrons, similar to the Hubbard term. It should be refered that both c and f operators are fermionic and obey the usual anticommutation rules. Finally, the condition $U=\infty$ is applied using the slave boson formalism, due to Coleman [26]. This method is specially helpful since it prevents the double occupancy of f-states. It mainly consists is replacing the f operators with

$$f_{i\sigma} \rightarrow b_i^\dagger f_{i\sigma} \tag{63}$$
$$f_{i\sigma}^\dagger \rightarrow f_{i\sigma}^\dagger b_i. \tag{64}$$

There are some consequences after applying this formalism. From it, we can renormalize the energy of the f orbital electrons and there is the restriction

$$Q=\sum_\sigma f_\sigma^\dagger f_\sigma + b^\dagger b. \tag{65}$$

The $U=\infty$ is achieved for $Q=1$. Mean field theory is also used and b bosons are assumed to condense (meaning $b \rightarrow < b >= \sqrt{z}$). In this problem, we are using the extended s-wave pairing. Following Araujo *et al.* [27] we write the effective hamiltonian as

$$\begin{aligned}
H_{eff} &= \sum_{i,\sigma} (\epsilon_f-\mu) f_{i\sigma}^\dagger f_{i\sigma} \\
&-t\sum_i \sum_{\boldsymbol{\delta}=\pm\hat{x},\pm\hat{y}} \left(c_{i,\uparrow}^\dagger c_{i+\delta,\uparrow} + c_{i+\delta,\downarrow}^\dagger c_{i,\downarrow}\right) + \\
&+\sqrt{z}V\sum_{i,\sigma}\left(c_{i,\sigma}^\dagger f_{i,\sigma} + f_{i,\sigma}^\dagger c_{i,\sigma}\right) \\
&+\frac{z}{2}\Delta_0 \sum_i \sum_{\boldsymbol{\delta}=\pm\hat{x},\pm\hat{y}} \left(f_{i,\uparrow}^\dagger f_{i+\delta,\downarrow}^\dagger + f_{i,\downarrow} f_{i+\delta,\uparrow}\right) - \\
&-\frac{N_s}{2J}\Delta_0^2 + (\epsilon_f-\epsilon_0)(z-1)N_s.
\end{aligned} \tag{66}$$

Thermal conductivity

The method to obtain the heat current and the thermal conductivity follows the path estabilished to obtain the quantities in the vortex state d-wave superconductor 46, 47 and 48 . From the energy density, directly from the hamiltonian 66 we obtain the following heat current

$$j_{\mu,A}^T(\mathbf{r}) = \frac{i}{\hbar}\Theta^\dagger(\mathbf{r}) V_{\mu,A}^T(\mathbf{r})\Theta(\mathbf{r}), \tag{67}$$

with Θ as the extended Nambu spinor $\Theta^\dagger = (c_\uparrow^\dagger \; c_\downarrow \; f_\uparrow^\dagger \; f_\downarrow^\dagger)$. The velocity matrix for the Anderson model is then defined as

$$V_{\mu,A}^T = \begin{pmatrix} \Gamma_\mu(\mathbf{r}) & 0 & \chi(\mathbf{r}) & 0 \\ 0 & -\Gamma_\mu(\mathbf{r}) & 0 & \chi(\mathbf{r}) \\ 0 & -w_\mu(\mathbf{r}) & z_\mu(\mathbf{r}) & \beta_\mu(\mathbf{r}) \\ w_\mu(\mathbf{r}) & 0 & -\beta_\mu(\mathbf{r}) & z_\mu(\mathbf{r}) \end{pmatrix}. \tag{68}$$

The above matrix has some new functions defined like

$$\Gamma_\mu(\mathbf{r}) = t^2\delta_\mu\sum_{\boldsymbol{\gamma}}\overleftarrow{s_{\boldsymbol{\gamma}}}\overrightarrow{s_{\delta_\mu}} - t^2\delta_\mu\sum_{\boldsymbol{\gamma}}\overrightarrow{s_{\delta_\mu+\boldsymbol{\gamma}}} \quad (69)$$

$$\chi_\mu(\mathbf{r}) = t\delta_\mu\sqrt{z}V\overrightarrow{s_{\delta_\mu}} - t\delta_\mu\sqrt{z}V\overleftarrow{s_{\delta_\mu}} \quad (70)$$

$$\beta_\mu(\mathbf{r}) = (\epsilon_f-\mu)\frac{z}{2}\Delta_0\delta_\mu\overrightarrow{s_{\delta_\mu}} - -(\epsilon_f-\mu)\frac{z}{2}\Delta_0\delta_\mu\overleftarrow{s_{\delta_\mu}} \quad (71)$$

$$w_\mu(\mathbf{r}) = \sqrt{z}V\frac{z}{2}\Delta_0\delta_\mu\overleftarrow{s_{\delta_\mu}} - \sqrt{z}V\frac{z}{2}\Delta_0\delta_\mu\overrightarrow{s_{\delta_\mu}} \quad (72)$$

$$z_\mu(\mathbf{r}) = \left(\frac{z}{2}\Delta_0\right)^2\delta_\mu\sum_{\boldsymbol{\gamma}}\overleftarrow{s_{\boldsymbol{\gamma}}}\overrightarrow{s_{\delta_\mu}} - \left(\frac{z}{2}\Delta_0\right)^2\delta_\mu\sum_{\boldsymbol{\gamma}}\overrightarrow{s_{\delta_\mu+\boldsymbol{\gamma}}}. \quad (73)$$

For the d-wave superconductor, the entire problem was made in the real space. In this case, since we don't have a magnetic field, we can easily work in the momentum space. Also, we know the Green's function can be written as [27]

$$\mathcal{G}_A(\mathbf{k},i\omega) = -\sum_{i=1,2}\sum_{\alpha=\pm}\frac{u_i^\alpha}{i\omega_n+\alpha E_i}, \quad (74)$$

which is a very similar form as the one for the d-wave vortex state problem. This way of representing the propagators allows us to use the frequency sums 43-**??** and terms multiplying with S_{-+} and S_{+-} will cancel, just like in the previous system. The thermal conductivity for the Anderson lattice model is given by

$$\frac{\sigma^T_{\mu\nu,A}}{T} = -\frac{i\beta}{\hbar^2T^2}\sum_{\mathbf{k}}\cdot$$
$$\cdot\Big\{-\mathrm{Tr}\left[\mathcal{U}^{(-)}(\mathbf{k})\cdot V^T_{\mu,AN}(\mathbf{k})\cdot\mathcal{V}^{(-)}(\mathbf{k})\cdot V^T_{\nu,AN}(\mathbf{k})\right]F_{12} -$$
$$-\mathrm{Tr}\left[\mathcal{U}^{(-)}(\mathbf{k})\cdot V^T_{\nu,AN}(\mathbf{k})\cdot\mathcal{V}^{(-)}(\mathbf{k})\cdot V^T_{\mu,AN}(\mathbf{k})\right]F_{21} -$$
$$-\mathrm{Tr}\left[\mathcal{U}^{(+)}(\mathbf{k})\cdot V^T_{\mu,AN}(\mathbf{k})\cdot\mathcal{V}^{(+)}(\mathbf{k})\cdot V^T_{\nu,AN}(\mathbf{k})\right]F_{21} -$$
$$-\mathrm{Tr}\left[\mathcal{U}^{(+)}(\mathbf{k})\cdot V^T_{\nu,AN}(\mathbf{k})\cdot\mathcal{V}^{(+)}(\mathbf{k})\cdot V^T_{\mu,AN}(\mathbf{k})\right]F_{12}\Big\} \quad (75)$$

where

$$F_{12} = \frac{n_F(E_1)-n_F(E_2)}{(E_1-E_2+i\delta)^2} \quad (76)$$

and $V^T_{\mu,AN}$ is similar with the old velocity matrix 77 but with some changes in the translation operators

$$V^T_{\mu,AN}(\mathbf{k}) = \begin{pmatrix} \Gamma_\mu(\mathbf{k}) & 0 & \chi(\mathbf{k}) & 0 \\ 0 & -\Gamma_\mu(\mathbf{k}) & 0 & \chi(\mathbf{k}) \\ 0 & -w_\mu(\mathbf{k}) & z_\mu(\mathbf{k}) & \beta_\mu(\mathbf{k}) \\ w_\mu(\mathbf{k}) & 0 & -\beta_\mu(\mathbf{k}) & z_\mu(\mathbf{k}) \end{pmatrix}. \quad (77)$$

with functions now in momentum space

$$\Gamma_\mu(\mathbf{k}) = t^2\delta_\mu\sum_{\boldsymbol{\gamma}}e^{-i\mathbf{k}\cdot\boldsymbol{\gamma}}e^{i\mathbf{k}\cdot\delta_\mu} - -t^2\delta_\mu\sum_{\boldsymbol{\gamma}}e^{i\mathbf{k}\cdot(\delta_\mu+\boldsymbol{\gamma})} \quad (78)$$

$$\chi_\mu(\mathbf{k}) = t\delta_\mu\sqrt{z}Ve^{i\mathbf{k}\cdot\delta_\mu} - t\delta_\mu\sqrt{z}Ve^{-i\mathbf{k}\cdot\delta_\mu} \quad (79)$$

$$\beta_\mu(\mathbf{k}) = (\epsilon_f-\mu)\frac{z}{2}\Delta_0\delta_\mu e^{i\mathbf{k}\cdot\delta_\mu} - -(\epsilon_f-\mu)\frac{z}{2}\Delta_0\delta_\mu e^{-i\mathbf{k}\cdot\delta_\mu} \quad (80)$$

$$w_\mu(\mathbf{k}) = \sqrt{z}V\frac{z}{2}\Delta_0\delta_\mu e^{-i\mathbf{k}\cdot\delta_\mu} - -\sqrt{z}V\frac{z}{2}\Delta_0\delta_\mu e^{i\mathbf{k}\cdot\delta_\mu} \quad (81)$$

$$z_\mu(\mathbf{k}) = \left(\frac{z}{2}\Delta_0\right)^2\delta_\mu\sum_{\boldsymbol{\gamma}}e^{-i\mathbf{k}\cdot\boldsymbol{\gamma}}e^{i\mathbf{k}\cdot\delta_\mu} - -\left(\frac{z}{2}\Delta_0\right)^2\delta_\mu\sum_{\boldsymbol{\gamma}}e^{i\mathbf{k}\cdot(\delta_\mu+\boldsymbol{\gamma})}. \quad (82)$$

Besides V we also defined new matrices $\mathcal{U}^{(+)}$, $\mathcal{U}^{(-)}$, $\mathcal{V}^{(+)}$ and $\mathcal{V}^{(-)}$ which contain every coherence factor u_i^α originated in the Green's function 74.

CONCLUSIONS

Transport properties in unconventional superconductors, specially with a magnetic field, is a vast subject. In this thesis, we have focused primarily in two distinct models, one for a vortex state d-wave superconductor and another one for heavy fermions systems in the context of their superconducting phase.

For a d-wave superconductor with a magnetic field, a formulation using a vortex lattice was used following previous work [6, 7, 16]. Our main interest was to calculate electric, spin and thermal conductivities in this situation. We were able to verify our equation for spin conductivity follows what was obtained by Vafek *et. al*. This specific quantity has several important properties. Spin conductivity vanishes for half-filling ($\mu = 0$) and is quantized in units of $1/(16\pi\hbar^2)$. The thermal conductivity result followed the same method Durst and Lee [5] have used by just picking the energy density from the hamiltonian and not the one by Vafek *et. al* [16] where the authors introduced a pseudo-gravitational field into the original Hamiltonian.

We expanded the results introducing impurities in the original problem and recalculated the new conductivities. However, some thought was put into the modified propagators because self consistent equations and the inverse of sums are usually some of the problems to obtain them. To deal with impurity disorder, we used the standard vertex correction approach to the first order. We immediatly verified that explicit calculation of further terms of the ladder series leads to an increasing complexity. The reader should notice the first term already gives the same correction as the one which appears through Boltzmann equation. We should also remember Durst and Lee [5] verified that in the absence of magnetic field, the vertex correction series only modify electrical conductivity and this result should be compared with the new conductivities we've obtained in the 4th section, after the corresponding numerical analysis.

In the final section of this thesis we applied the line of thought for d-wave superconductors as in previous sections but for the superconducting phase of heavy fermions materials. We used the lattice Anderson model with $U = \infty$ which can be done using slave-boson formalism. The result for the thermal current and conductivity was obtained using the same as for the previous problem since the Green's function can be written very similarly to the d-wave superconductor propagators except they are in the momentum space. However, since the hamiltonian has more terms, the complexity in this thermal conductivity has also increased.

The expressions obtained in this thesis will be applied shortly to the full numerical solution of the vortex positional disorder plus random impurities. The thermal conductivity for the clean case will also be compared to the results from the thermal conductivity of Vafek $et.al$ [16] which will also be computed. As a future work, the optical conductivities, for finite frequencies, are also expected to be obtained.

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