Modelling and Characterization of a Distributed Feedback (DFB) Laser Diode

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Abstract—The aim of this work is the study of semiconductor lasers with distributed feedback (DFB), namely their usage as the light source in high bit rate optical communication systems (OCS). This is specially important in situations where direct modulation (DM) of the emitter is used.

There are two stages involved in this work. The first one is the study and modelling of the desired devices, through the coupled waves theory and the transfer matrix method (TMM). The second one is the optimization of new DFB structures, in order to achieve a performance far superior to the ones already reported in the literature.

The obtained results are systematically validated by comparing with results taken from literature and, to a certain extent, by the inclusion of some experimental measures.

In the end, optimized DFB structures are achieved, which are capable of maintaining side mode suppression ratio values as high as $50 \text{ dB}$, with very stable emitted wavelength, as well as good efficiency in the electro-optical conversion.

Index Terms—Laser, matricial methods, distributed feedback, semiconductor.

I. INTRODUCTION

The increasing demand for high-bandwidth communications has been responsible for the rapid development of Optical Communication Systems (OCS). These systems use fibre optics to transmit electromagnetic waves, and need to take into account the undesired effects of dispersion and attenuation on the bandwidth. Experimental data showed that the fibre has minimum attenuation for the wavelength $1.55 \mu m$ (see Fig. 1). In order to minimize dispersion, there is an increasing necessity of implementing a coherent light source.

In the OCS context, there are two types of modulation - external modulation and direct modulation. In the second one, which is the least expensive, the light source is an optoelectronic device which translates electrical current variations in optical power intensity. It needs to ensure a coherent and stable spectrum for different biasing currents.

A. The LED and the Laser

The Light Emitting Diode (LED) and the Light Amplification by Stimulated Emission of Radiation (LASER) are two alternative light sources. The LED is a cheap option but the spectral linewidth is too large because it mainly uses spontaneous emission. The laser uses mainly stimulated emission, thus it has a much more coherent spectrum.

B. DFB Laser

In the conventional Fabry-Perot (FP) laser, the principle of operation lies on the usage of mirrored facets at the extremities of the laser cavity. In a Distributed-Feedback (DFB) laser this reflection is performed throughout the entire laser cavity, using a grating which consists of a periodic variation of the refractive index (see Fig. 2).

Using a properly optimized DFB laser, a very coherent spectrum is achieved, with a narrow linewidth and a high Side-Mode-Suppression-Ratio (SMSR). This optimization is usually performed at threshold regime, where an high mode discrimination is desired as well as a flat intracavity electrical field profile. In this manner, in the above threshold regime, an high SMSR is achieved and the harmful effect known as Spatial Hole Burning (explained in I-D) is avoided.

Fig. 1 — Attenuation in fibre optics, depending on wavelength ([1, p. 3]).

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Fig. 2 — Diagram of laser cavities: (a) Fabry-Perot. (b) DFB.


C. Materials

In a DFB laser fabrication, there is an active zone made with a quaternary compound InGaAsP and a non-active zone made with the binary compound InP (see Fig. 3).

\[
\text{In}_{x}\text{Ga}_{1-x}\text{As}_{y}\text{P}_{1-y}/\text{InP heterojunction is used, where the values of } x \text{ and } y \text{ are defined by two factors:}
\]

- InGaAsP should emit the desired wavelength;
- The heterojunction should ensure a non-strained interface.

D. Spatial Hole Burning

When the distribution of the electrical field is non-uniform at threshold, there is a phenomenon occurring above threshold previously defined as SHB. Where the photon density is higher, the carrier density is lower and the refractive index is higher. These variations will undermine the laser behaviour, specially the emitted wavelength [2].

E. Figures of Merit

Throughout this document, the main figures of merit of a laser structure shall be:

- Selectivity (\(\mathcal{S}\)) - Difference between the normalized main mode gain and the normalized secondary mode gain;
- Flatness (\(\mathcal{F}\)) - Indicator of intracavity electrical field homogeneity;
- SMSR - Difference, in dB, between the main mode emitted power and the main secondary mode emitted power.

II. STATE OF THE ART

The conventional DFB laser corresponds to the simplest DFB structure, which is merely a periodic variation of the refractive index (see Fig. 4). The problem of such structure is the non-existence of a main mode in the spectrum, which impairs its usage in OCS. This issue is solved using some sort of phase adjustment in the grating.

A. PAR-DFB Laser

It is possible to use a Phase-Adjustment-Region (PAR) structure to introduce a distributed phase-shift [1, p. 89], [3]. In this structure, the width of the active zone is not constant (see Fig. 5).

B. MPS-DFB Laser

It is also possible to introduce localized phase-shifts (PS) on the grating (see Fig 6). The most popular structure is the Quarterly-Wavelength-Shift (QWS), which has a 90° PS in the center of the cavity [1, p. 95]. The QWS-DFB has a high \(\mathcal{S}\) but the electrical field is far from homogeneous.

Nevertheless, more than one PS can be introduced, creating Multiple-Phase-Shift (MPS) structures, such as the 3PS-DFB proposed by H. Ghafouri-Shiraz [1, p. 96], where the \(\mathcal{S}\) is improved at the expense of a decrease in \(\mathcal{F}\). Other 3PS-DFB structures can attain both figures of merit as long as they are properly optimized [4].

C. DCC-DFB Laser

A higher \(\mathcal{S}\) can be achieved using Distributed-Coupling-Coefficient (DCC) structures, where the coupling coefficient has a modulated amplitude (see Fig. 7).

A DCC profile is usually combined with some structure with a phase-shift, enhancing their performance. Examples of such structures are QWS-DCC-DFB and 3PS-DCC-DFB [1], 1PS-DCC-DFB [5], or other more sophisticated structures [6].

D. CPM-DFB Laser

Structures that modulate the grating period in order to produce a distributed phase shift are called Corrugation-Pitch-Modulated (CPM) (see Fig. 8). In this manner, a high \(\mathcal{S}\) is achieved without jeopardizing a flat electrical field profile.

T. Fessant proposes two structures [7], CPM-DFB and CPM-DCC-DFB, where the second one is clearly advantageous. Other authors present detailed analysis of such CPM structures [8], [9].

E. GLTG-DFB Laser

Instead of using DCC profiles with discrete values on the coupling coefficient, continuous variations can be fabricated, creating structures called Gaussian-Like Tapered Grating (GLTG), explored by T. Fessant [10].

F. CG-DFB Laser

It is also possible to vary the grating period in a continuous manner, corresponding to the structures called Chirped-Grating (CG), producing interesting results [11].

G. Transversal Structure

It is possible to introduce discretizations in the laser structure. When a Bulk Laser (BL) is discretized in one dimension, a Quantum-Well (QW) structure is introduce. If two dimensions are discretized, a Quantum-Dash is created. If all the three dimensions are discretized, a Quantum-Dot (QD) structure is created. These approaches will greatly influence the threshold current of the DFB laser.
Fig. 4 – Simplified scheme of a conventional DFB.

Fig. 5 – Simplified scheme of a PAR-DFB.

Fig. 6 – Simplified scheme of a PS-DFB.

Fig. 7 – Simplified scheme of a DCC-DFB.

Fig. 8 – Simplified scheme of a CPM-DFB.
III. COUPLED-WAVE THEORY IN SEMICONDUCTOR LASERS

It is shown in the thesis that the coupled wave equations rule the behaviour of a DFB laser. Ultimately, after several mathematical developments of the coupled wave equations, for a uniform structure, these equations have a relatively simple solution:

\[
(\mathbf{T} \cdot L_{\text{cav}})^2 \cdot \mathcal{D} + \\
\left[ \begin{array}{cc}
K \cdot L_{\text{cav}} & \sinh^2 (\mathbf{T} \cdot L_{\text{cav}}) \cdot (1 - r_1^2) \cdot (1 - r_2^2) + \\
+j \cdot 2 \left( K \cdot L_{\text{cav}} \right) \cdot \left( r_1^2 \cdot r_2^2 \right) \cdot (1 - r_1^2) \cdot (1 - r_2^2) & \\
(\mathbf{T} \cdot L_{\text{cav}}) \cdot \sinh(\mathbf{T} \cdot L_{\text{cav}}) \cdot \cosh(\mathbf{T} \cdot L_{\text{cav}}) = 0
\end{array} \right]
\]

These variables are fully explained in the following section. This can be solve using numerical methods like the Newton method.

However, in order to analyse a sophisticated laser structure, it is required a very heavy analytical process in order to take into account all the boundary conditions involved. This problem leads to a more universal and systematic method, the Transfer Matrix Method (TMM) (see section IV).

IV. TMM AT THRESHOLD

As referred in III, a flexible and efficient method is required in order to simulate any sophisticated DFB structure. In the present section, the TMM is introduced, and explained in the near threshold regime. This method divides the laser cavity in several sub-sections, each one represented by a transfer (2x2) matrix.

A. Matrix of a Uniform Section

The fundamental matrix represents a uniform section (see Fig. 9) in the laser cavity, written as \( \mathbf{T}(z_{m+1}/z_m) \).

\[
\begin{array}{ccc}
\mathcal{E}_R(z_m) & \Lambda_m & \mathcal{E}_R(z_{m+1}) \\
\mathcal{E}_S(z_m) & & \mathcal{E}_S(z_{m+1})
\end{array}
\]

In Fig. 9, \( \Lambda_m \) is the grating period, \( \mathbf{K}_m \) is the coupling coefficient and \( \Omega_m \) is the phase residue, defined as:

\[
\Omega_m = \Omega_1 + 2 \cdot \sum_{k=1}^{m-1} \left( \frac{\pi}{\lambda_k} \cdot L_k \right) ; \quad 2 \leq m \leq M .
\]

In the thesis, it is shown that

\[
\mathbf{T}(z_{m+1}/z_m) = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}
\]

where \( t_{11}, t_{12}, t_{21} \) and \( t_{22} \) are given, respectively, by

\[
t_{11} = \frac{\xi_m - \gamma_m^{-1}}{(1 - \rho_m) \cdot \xi_m} ; \quad t_{12} = -\frac{\rho_m (\xi_m - \xi_m^{-1}) e^{-j\Omega_m}}{(1 - \rho_m) \cdot \xi_m} ;
\]

\[
t_{21} = \frac{\gamma_m \cdot (\xi_m - \xi_m^{-1}) e^{j\Omega_m}}{(1 - \rho_m) \cdot \xi_m} ; \quad t_{22} = \frac{\rho_m^2 \cdot \xi_m - \xi_m^{-1}}{(1 - \rho_m^2) \cdot \xi_m^{-1}}
\]

with \( \xi_m = e^{\pi z_m} \) and \( \zeta_m = e^{j\beta_m \cdot z_m} \). The propagation constant, \( \beta_m \), and the complex propagation constant, \( \tau_m \), are given, respectively, by

\[
\beta_m = \frac{\pi}{\Lambda_m} ; \quad \tau_m = \sqrt{(\alpha - j\delta_m)^2 + \rho_m^2} ,
\]

where \( \rho_m \) and \( \delta_m \) are given by

\[
\rho_m = \frac{-j \tau_m}{\alpha - j \delta_m + \tau_m} ; \quad \delta_m = \delta + \pi \left( \frac{1}{\Lambda_1} - \frac{1}{\Lambda_m} \right),
\]

with \( \alpha \) and \( \delta \) being, respectively, the gain and detuning for the propagation modes taking the left section as a reference.

B. Matrix of a Phase Shift

There is also the need to model a PS on the laser structure (see Fig. 10).

\[
\begin{array}{ccc}
\mathcal{E}_R(z_m^-) & \theta & \mathcal{E}_R(z_m^+) \\
\mathcal{E}_S(z_m^-) & & \mathcal{E}_S(z_m^+)
\end{array}
\]

The matrix that represents this PS is given by:

\[
\mathbf{T}(z_m^+/z_m^-) = \begin{bmatrix} \exp (j \cdot \theta) & 0 \\ 0 & \exp (-j \cdot \theta) \end{bmatrix}
\]

C. Matrix of a Facet

There is an uncertainty on the length of the corrugation, relatively to the number of periods. Thus, a phase is implicitly created at the facets when they are reflective facets (see Fig. 11).

\[
\begin{array}{ccc}
\mathcal{E}_R(0^-) & \mathcal{F}_1 & \mathcal{E}_R(0^+) \\
\mathcal{E}_S(0^-) & & \mathcal{E}_S(0^+)
\end{array}
\]
The reflection coefficients are defined by the complex numbers $\hat{r}_1$ and $\hat{r}_2$, where:

$$\hat{r}_1 = r_1 \cdot \exp \left( j \cdot \varphi_1 \right)$$

$$\hat{r}_2 = r_2 \cdot \exp \left( j \cdot \varphi_2 \right)$$

The matrices that represent the left and right reflectivities are given by:

$$M_1 = \frac{1}{t_1} \begin{bmatrix} \exp \left( j \cdot \frac{\varphi_1}{2} \right) & r_1 \cdot \exp \left( j \cdot \varphi_1 \right) \\ r_1 \cdot \exp \left( -j \cdot \frac{\varphi_1}{2} \right) & \exp \left( -j \cdot \varphi_1 \right) \end{bmatrix}$$

$$M_2 = \frac{1}{t_2} \begin{bmatrix} \exp \left( j \cdot \frac{\varphi_2}{2} \right) & -r_2 \cdot \exp \left( -j \cdot \frac{\varphi_2}{2} \right) \\ -r_2 \cdot \exp \left( j \cdot \varphi_2 \right) & \exp \left( j \cdot \varphi_2 \right) \end{bmatrix}$$

where $t_1$ and $t_2$ are given by:

$$t_1 = 1 + r_1$$

$$t_2 = 1 - r_2$$

### D. Oscillation Condition

The fields at both ends of the cavity are connected by the elementary matrix product

$$\begin{bmatrix} \overline{E}_R(L_{cav}) \\ \overline{E}_S(L_{cav}) \end{bmatrix} = \mathbf{T}_{\text{Total}} \cdot \begin{bmatrix} \overline{E}_R(0) \\ \overline{E}_S(0) \end{bmatrix}$$

where

$$\mathbf{T}_{\text{Total}} = \prod_{m=M}^{1} \mathbf{T}(z_{m+1}/z_m)$$

The oscillation condition corresponds to the vanishing of the incoming waves and it is determined by the following requirement

$$t_{22}^{\text{Total}} (\alpha, \delta) = 0$$

where $t_{22}^{\text{Total}}$ is the fourth element of the matrix $\mathbf{T}_{\text{Total}}$, given by (12). The solutions are the mode gain, $\alpha$, and the detuning, $\delta$, and are related to the modes that are allowed to propagate inside the cavity. For the main mode their values are, respectively, the threshold gain, $\alpha_{th}$, and the threshold detuning, $\delta_{th}$.

For a grating with a first-order Bragg diffraction, the mode gain and the detuning can be expressed, respectively, as [1, p. 151]

$$\alpha(z) = \frac{\Gamma g(z) - \alpha_{th}}{2}$$

and

$$\delta(z) = \frac{2 \pi}{\lambda} n(z) - \frac{2 \pi n_g}{\lambda \lambda_A} \left( \lambda - \lambda_A \right) - \frac{\pi}{\Lambda(z)}$$

where $\Gamma$ is the optical confinement factor, $\alpha_{th}$ is the total loss, $n$ is the effective index, $\lambda_A$ is the Bragg wavelength, $\lambda$ is the lasing mode wavelength, $n_g$ is the group effective index and $g$ is the material gain, given by [1, p. 151]

$$g(z) = A_0 \cdot (N(z) - N_0) - A_1 \cdot \left( \lambda - \left( \lambda_0 - A_2 (N(z) - N_0) \right) \right)^2$$

In (16), $N$ is the carrier concentration, $A_0$ is the differential gain, $N_0$ is the carrier concentration at transparency ($g = 0$), $\lambda_0$ is the peak wavelength at transparency and $A_1$ and $A_2$ are parameters used in the parabolic model assumed for the material gain. Using the first-order approximation for the effective index $n$, one obtains [1, p. 151]

$$n(z) = n_0 + \Gamma \frac{\partial n}{\partial N} N(z)$$

where $n_0$ is the effective index at zero carrier injection and $\partial n/\partial N$ is the differential index. The photon concentration ($S$) and $N$ are coupled together through the steady-state carrier rate equation [1, p. 152]

$$\frac{I}{q V_{act}} = A N(z) + B N^2(z) + C N^3(z) + \frac{v_g g(z) S(z)}{1 + \varepsilon S(z)}$$

where $I$ is the injection current, $q$ is the modulus of the electron charge, $V_{act}$ is the volume of the active layer, $A$ is the spontaneous emission rate, $B$ is the radiative spontaneous emission coefficient, $C$ is the Auger recombination coefficient, $\varepsilon$ is a non-linear coefficient to take into account saturation effects and $v_g = c/n_g$ is the group velocity, with $c$ being the free space velocity.

In an purely index-coupled DFB laser cavity, which happens to be the case considered along this paper, the mutual interaction between the coupled waves $\overline{E}_R(z)$ and $\overline{E}_S(z)$ can be neglected in the rate of total power change [1, p. 59], [12]. Therefore, the local photon density inside the cavity can be expressed as [1, p. 152]

$$S(z) \approx \frac{2 \varepsilon_0 n(z) n_g \lambda}{h c} \cdot \frac{\varepsilon_0}{c_0^2} \left[ |\overline{E}_R(z)|^2 + |\overline{E}_S(z)|^2 \right]$$

where $\varepsilon_0$ is the free space permittivity, $h$ is the Planck’s constant and $c_0$ a dimensionless coefficient that allows the determination of the total electric field at the above threshold regime, taking into account that the normalization

$$|\overline{E}_R(0)|^2 + |\overline{E}_S(0)|^2 = 1$$

has been imposed. The equation (20) and the boundary conditions imposed at the left facet allow the calculation of the two counter running waves, $\overline{E}_R(z)$ and $\overline{E}_S(z)$, at $z = 0$. The use of the TMM allows the calculation of the longitudinal electric field profile. The output power at the right facet can be determined as [1, p. 152]

$$P = \frac{d w}{T} \cdot v_g \cdot \frac{h c}{\lambda} \cdot S(L)$$

where $d$ and $w$ are the thickness and width of the active layer, respectively.

From the solutions of the oscillation condition (13), $\alpha_{th}$ and $\delta_{th}$ are determined. Using equations (14)–(17), the carrier concentration at threshold ($N_{th}$), the effective index at threshold ($n_{th}$), the threshold wavelength ($\lambda_{th}$) and $\lambda_0$ are successively evaluated. Threshold current ($I_{th}$) is obtained from (18), assuming that $S$ is negligible at threshold. Within this assumption, the $z$ dependence described in eqs. (14), (16)
and (17) is also neglected. The same assumption is valid for eq. (15), except for CPM structures where a $z$ dependence is included in $\Lambda(z)$.

V. TMM RESULTS AT THRESHOLD

A. Known Structures

In this sub-section the simulator which was implemented in Matlab is validated. Some results from the literature are replicated and match the original ones.

1) H. Ghafouri-Shiraz: The results related to the electrical field distribution from Ref. [1, pp. 132, 146] are replicated in Fig. 12, where an exact match is visible. Many other results are replicated in the thesis.

![Fig. 12 – Electrical field profiles for the known DFB structures under analysis.](image)

2) T. Fessant: This author [7] also analyzes CPM structures, not analyzed by H. Ghafouri-Shiraz [1]. Some results related to these laser structures are presented in Ref. [13], and successfully match the original results.

B. Optimized Structures

1) 1PS-DCC-DFB: This structure has a grating with two coupling coefficients, $\overline{K}_s$ and $\overline{K}_c$, separated by $K_{p_1}$ and $K_{p_2}$. It has a period $\Lambda$ and a PS with value $\theta$ and position $\theta_p$. The optimization results on the following values: $\overline{K}_{av} L_{cav} = 1.7$, $K_{p_1} = 0.24$, $K_{p_2} = 0.797$, $\Lambda_{ratio} = 7$ and $\theta = 129^\circ$.

Such structure is studied in Refs. [5] and [14] and its figures of merit are listed in Table I.

2) 3PS-DFB: This structure has a constant grating period $\Lambda$ and a constant coupling coefficient $K$, but has three PS with values $\theta_1$, $\theta_2$, and $\theta_3$, located in the positions $\theta_{p_1}$, $\theta_{p_2}$ = 0.5 and $\theta_{p_3}$. The optimization results on the following values: $\overline{K}_{av} L_{cav} = 1.7$, $\theta_2 = 60^\circ$, $\theta_{p_1} = 0.127$, $\Lambda = 110.7^\circ$, $\theta_{p_3} = 0.64$ and $\theta_3 = 100^\circ$.

Such structure is studied in Refs. [15] and [16] and its figures of merit are listed in Table I.

3) Asymmetric CPM-2DCC-DFB: This structure has two coupling coefficients, $\overline{K}_s$ and $\overline{K}_c$, separated by $K_{p_1}$ and $K_{p_2}$. It has two grating periods $\Lambda_s$ and $\Lambda_c$, separated by $\Lambda_{p_1}$ and $\Lambda_{p_2}$. It has no PS. The optimization results on the following values: $\overline{K}_{ratio} = 8.5$, $\overline{K}_{av} L_{cav} = 1.7$, $K_{p_1} = 0.2190$, $K_{p_2} = 0.8235$, $\Delta \Lambda = 9.5625 \times 10^{-4}$, $\Lambda_{p_1} = 0.4225$ and $\Lambda_{p_2} = 0.6075$.

Such structure is studied in Refs. [17] and [18] and its figures of merit are listed in Table I.

4) Symmetric CPM-3DCC-DFB: This structure has three coupling coefficients, $\overline{K}_s$, $\overline{K}_c$, and $\overline{K}_e$, separated by $K_{p_1}$, $K_{p_3}$, $K_{p_2} = 1 - K_{p_3}$ and $K_{p_2} = 1 - K_{p_3}$. It has two grating periods $\Lambda_s$ and $\Lambda_c$, separated by $\Lambda_{p_1}$ and $\Lambda_{p_2}$. It has no PS. The optimization results on the following values: $\overline{K}_{ratio} = 10.0$, $\overline{K}_{av} L_{cav} = 1.7$, $K_{p_1} = 0.1578$, $K_{p_2} = 0.2362$, $\Lambda_{p_1} = 0.4014$ and $\Delta \Lambda = 9.8571 \times 10^{-4}$.

Such structure is studied in Refs. [19] and [20] and its figures of merit are listed in Table I.

5) HR-AR-DCC-DFB: This structure has a DCC profile with 2 coupling coefficients, $\overline{K}_s$, $\overline{K}_c$, separated by $K_{p_1}$ and $K_{p_2} = 1 - K_{p_1}$, it has one grating period $\Lambda$, a high-reflective (HR) facet and an anti-reflective (AR) facet. The optimization results on the following values: $K_{p_1} = 0.28$, $\overline{K}_{ratio} = 10.0$, $\overline{K}_{av} L_{cav} = 0.645$ and $\varphi_2 = \pi/2$.

Such structure is studied in Ref. [21] and its figures of merit are listed in Table I.

C. Performance Summary

The table I summarizes the threshold performance of several structures, the known ones and the optimized ones.

<table>
<thead>
<tr>
<th>Laser structure</th>
<th>$\Theta$</th>
<th>$\overline{\Lambda}$</th>
<th>$\alpha_{th} \cdot L_{cav}$</th>
<th>$\delta_{th} \cdot L_{cav}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QWS-DFB from [1]</td>
<td>0.73</td>
<td>0.301</td>
<td>0.70</td>
<td>0</td>
</tr>
<tr>
<td>QWS-DCC-DFB from [1]</td>
<td>1.09</td>
<td>0.168</td>
<td>0.93</td>
<td>0</td>
</tr>
<tr>
<td>3PS-DFB from [1]</td>
<td>0.33</td>
<td>0.012</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>3PS-DCC-DFB from [1]</td>
<td>0.49</td>
<td>0.016</td>
<td>1.54</td>
<td>0.35</td>
</tr>
<tr>
<td>CPM-DCC-DFB from [7]</td>
<td>0.99</td>
<td>0.019</td>
<td>1.28</td>
<td>0.84</td>
</tr>
<tr>
<td>Asym. 1PS-DCC-DFB</td>
<td>1.85</td>
<td>0.019</td>
<td>1.50</td>
<td>2.24</td>
</tr>
<tr>
<td>Asym. 3PS-DFB</td>
<td>0.77</td>
<td>0.008</td>
<td>1.19</td>
<td>3.71</td>
</tr>
<tr>
<td>Asym. CPM-2DCC-DFB</td>
<td>2.18</td>
<td>0.017</td>
<td>1.42</td>
<td>1.70</td>
</tr>
<tr>
<td>Sym. CPM-3DCC-DFB</td>
<td>2.54</td>
<td>0.019</td>
<td>1.48</td>
<td>1.53</td>
</tr>
<tr>
<td>Opt. HR-AR-DCC-DFB</td>
<td>1.17</td>
<td>0.014</td>
<td>0.59</td>
<td>0</td>
</tr>
</tbody>
</table>

| TABLE I -- $\Theta$, $\overline{\Lambda}$, $\alpha_{th} \cdot L_{cav}$, and $\delta_{th} \cdot L_{cav}$ for several laser structures. |

From all these structures, the 3PS-DFB has the best $\overline{\Lambda}$ and the CPM-3DCC-DFB has the best $\Theta$. The HR-AR-DCC-DFB structure does not present optima results but it is expected to present a good opto-electrical conversion efficiency above threshold due to the HR-AR profile.
VI. TMM ABOVE THRESHOLD

In the above-threshold regime, \( S(z) \) is high enough to induce important non-uniformities in \( N(z) \) and \( n(z) \). Despite the SHB effect can be minimised by an adequate design of the DFB structure, the interdependence of \( S(z) \), \( N(z) \) and \( n(z) \) induces strong longitudinal inhomogeneities that forces the division of each section into several sub-sections, in order to ensure a correct evaluation of the above-threshold characteristics. According to Ref. [1, p. 153], for a cavity with \( L = 500 \mu \text{m} \), at least a total of \( M = 5000 \) sub-sections are needed.

In this paper, the numerical procedure for the above-threshold calculations follows closely the method developed in Refs. [1, p. 149], [7]. However, in order to ensure a quick convergence in the evaluations of the laser characteristics, an adequate strategy is now proposed, which is fully described below.

1) Lasing mode analysis: For each bias current \( I \), the numerical above-threshold calculations concerning the lasing mode are summarised as follows:

(i) Successive \( (G \times G) \) grids are created in the \( (c_{0i}, \lambda) \) plane. The \( i \)-th grid is centred at \( (c_{0_{\text{th},i}}, \lambda_{\text{th}}) \) and is enclosed in the region defined by the limits \( c_{0_{\text{min},i}}, c_{0_{\text{max},i}}, \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \). For the initial grid \( (i = 1) \)

\[
\lambda_{\text{th}} = \frac{h c (I - I_{\text{th}})}{2 q V_{\text{act}} V_{g} g_{th} n_{th} n_{g} \lambda_{\text{th}}} \tag{22}
\]

\[
c_{0_{1}} = \sqrt{\frac{h c (I - I_{\text{th}})}{2 q V_{\text{act}} V_{g} g_{th} n_{th} n_{g} \lambda_{\text{th}}}}, \tag{23}
\]

\[
c_{0_{\text{min},i}} = c_{0_{1}} - \Delta c_{0_{1}} ; \quad c_{0_{\text{max},i}} = c_{0_{1}} + \Delta c_{0_{1}} \tag{24}
\]

\[
\lambda_{\text{min}} = \lambda_{c} - \Delta \lambda ; \quad \lambda_{\text{max}} = \lambda_{c} + \Delta \lambda \tag{25}
\]

where \( \Delta c_{0_{1}} \leq 0.1 \) nm and \( \Delta \lambda \) are adequate in order to prevent an eventual convergence towards a local minimum. This is a critical aspect of the proposed analysis, since an inadequate choice would prevent the numerical convergence;

(ii) For each one of the \( G \) \( G \) pairs of the \( i \)-th grid \( (c_{0_{i,l}}, \lambda_{i,l}) \) with \( k; l = 1 \ldots G \), the equations (16)\((19)\) are self-consistently solved, in order to determine the material gain, carrier density, photon density and effective index for each one of the \( j \)-th sub-section, respectively, \( g_{i,l}, N_{i}, S_{i} \) and \( n_{i} \), with \( 1 \leq j \leq M \);

(iii) The equations (14) and (15) are solved in order to determine the lasing mode gain and detuning for the \( j \)-th sub-section, respectively, \( \alpha_{j} \) and \( \delta_{j} \). The transfer matrix of the \( j \)-th sub-section, \( T(z_{j+1}/z_{j}) \), is then calculated;

(iv) Using the TMM, the two counter-running waves at the output of the \( j \)-th sub-section, \( E_{\text{out}} \) and \( E_{\text{side}} \), are obtained. For the \( M \)-th sub-section, the discrepancy found between those values and the laser right facet boundary condition is represented by \( \varepsilon_{k} \). This value is evaluated and stored for each pair \( (c_{0_{i,k}}, \lambda_{i,k}) \) of the \( i \)-th grid. The error associated to the \( i \)-th grid is given by \( \varepsilon_{i} = \min (\varepsilon_{k}) \):

\[
\alpha_{\text{av}}(I) = \frac{1}{M} \sum_{j=1}^{M} \alpha_{j}(I) ; \quad \delta_{\text{av}}(I) = \frac{1}{M} \sum_{j=1}^{M} \delta_{j}(I) . \tag{26}
\]

Notice that the sequential analysis (i)-(v) assumes a one-mode propagation laser behaviour. This approach is itself a good assumption, since the present analysis focuses on DFB structures that must ensure SLM operation. Otherwise, different strategies must be adopted.

When studying the \( I \) influence on the laser characteristics, a considerable CPU time reduction can be achieved if, for each subsequent current, instead of using (22), \( \lambda_{c} \) is taken as the solution found in the previous bias current;

2) Side mode analysis: \( S(z) \), \( N(z) \) and \( n(z) \) profiles are settled for each \( I \) by the lasing mode profiles obtained in sub-section VI-1. At threshold, these distributions are nearly uniform along the cavity, assuming average values, respectively, \( 0, N_{\text{th}} \) and \( n_{\text{th}} \). The gain mode and detuning associated with the side-mode, at threshold, respectively, \( \alpha_{\text{side}} \) and \( \delta_{\text{side}} \), are settled. In the one-mode approximation the use of the eq. (15) leads to

\[
\lambda_{\text{R}}(\delta_{\text{side}}) = \frac{2 \pi \lambda_{\Lambda} (n_{\text{th}} + n_{g})}{\delta_{\text{side}} \lambda_{\Lambda} + 2 \pi n_{g} + \frac{\pi \lambda_{\Lambda}}{\alpha_{\text{av}}}}, \tag{27}
\]

\(^{1}\) Notice that, according to (20), \( c_{0_{k}} \) is numerically equal to \( \sqrt{\frac{h c (I - I_{\text{th}})}{2 q V_{\text{act}} V_{g} g_{th} n_{th} n_{g} \lambda_{th}}} \).
where \( \lambda_{av} \) is the average grating period given by

\[
\lambda_{av} = \frac{\sum_{m=1}^{M} L_m \cdot \lambda_m}{L_{cav}}.
\]

This assumption means that \( \lambda_{th} (\delta_{side}) \) would be the threshold wavelength if \( \delta_{side} \) would correspond to the lasing mode. On the other hand, regarding the side-mode gain, eq. (14) imposes that

\[
2 \alpha_{side} = \Gamma g_{side} - \alpha_{loss},
\]

where \( g_{side} \) is obtained from (16), making \( N(z) = N_{th} \) and \( \lambda = \lambda_j \) \( (\alpha_{side}) \). This would be the wavelength in the one-mode approach if \( \alpha_{side} \) would correspond to the threshold gain. It will be designated by the side-mode effective wavelength. Similarly, for the lasing mode, it is obtained

\[
2 \alpha_{th} = \Gamma g_{th} - \alpha_{loss},
\]

where \( g_{th} = A_0 \left( N_{th} - N_0 \right) \). Then, from equations (29) and (30), it can be shown that

\[
\lambda_I(\alpha_{side}) = \lambda_{th} + j \lambda_T(\alpha_{side}),
\]

where

\[
\lambda_T(\alpha_{side}) = \sqrt{\frac{2 \left( \alpha_{side} - \alpha_{th} \right)}{A_1 \Gamma}}.
\]

A \((G \times G)\) grid is created in the plane \((\lambda_T, \lambda_R)\) in a similar way as done for the plane \((c_0, \lambda)\), in sub-section VI-1. The initial grid is centered in \((\lambda_T^{(1)}, \lambda_R^{(1)})\), where \(\lambda_T^{(1)}\) and \(\lambda_R^{(1)}\) are given, respectively, by (32) and (27). The limits of the initial grid are defined by \(\lambda_T^{(1)} \pm \Delta \lambda_T^{(1)}\) and \(\lambda_R^{(1)} \pm \Delta \lambda_R^{(1)}\). \(G = 10, \Delta \lambda_T^{(1)} \approx 0.01 \text{ nm} \) and \(\Delta \lambda_R^{(1)} \approx 0.1 \text{ nm}\) seem reasonable for most of the DFB structures but, as before, a readjustment may once in a while be necessary to avoid the mode hopping. Usually \(\Delta \lambda_T^{(1)}\) is one order of magnitude lower than \(\Delta \lambda_R^{(1)}\) because the difference between the gains associated with different modes is about one order of magnitude lower than the difference between their detunings. Successive \((G \times G)\) grids are defined in the wavelength plane, centering the \(i\)-th grid in \((\lambda_T^{(i)}, \lambda_R^{(i)})\) and enclosing it in the region defined by the limits \(\lambda_T^{(i)} \pm \Delta \lambda_T^{(i)}\) and \(\lambda_R^{(i)} \pm \Delta \lambda_R^{(i)}\).

Then, for each pair \((k, l)\) of the \(i\)-th grid, i.e. \((\lambda_T^{(i)}, \lambda_R^{(i)})\), the mode gain and detuning for each one of the \(j\) \((j = 1, \ldots, M)\) sub-sections of the cavity are obtained for a given current \(I\) as, respectively,

\[
\alpha_{side_{k,l}}(I) = \alpha_j(I) + \left( \lambda_T^{(i)} \right)^2 A_1 I \frac{\Gamma}{2},
\]

\[
\delta_{side_{k,l}}(I) = \frac{2 \pi}{\lambda_R^{(i)}} n_g(j) - \frac{2 \pi n_g}{\lambda_R^{(i)}} \left( \lambda_T^{(i)} - \lambda_A \right) - \frac{\pi}{\lambda_j}.
\]

\(\alpha_j(I)\) and \(n_j(I)\) are, respectively, the lasing mode gain and the refractive index associated with the \(j\)-th sub-section for a biasing current \(I\), achieved in subsection VI-1. Besides, \(\lambda_j\) is the corrugation period of the \(j\)-th sub-section.

Similarly as in subsection VI-1, steps (iii)–(v) are then sequentially followed. The side-mode analysis is quicker than the lasing mode analysis since the step (ii) described in subsection VI-1 is not necessary.

Table II summarizes all the laser parameters considered during the implementation of the simulator. It is possible to perform an extended set of experimental measures in order to retrieve updated values for these inputs.

### VII. TMM Results Above Threshold

#### A. Known Structures

In order to study the relation between photon density and carrier density, the results from Fig. 6.12 and Fig. 6.13 from Ref. [1, pp. 164, 165] are replicated in Fig. 13(a) and Fig. 13(b), respectively. These results validate the simulator in the context of the SHB effect.

According to Fig. 13, and as it was expected, there is an inverse relation between the photon density and the carrier density. When the current increases, the number of photons increase but not in a homogeneous profile, leading to an inhomogeneous distribution of carriers.

The spectrum of a QWS-DFB laser presented in Fig. 7.13 from Ref. [1, p. 183] is replicated in Fig. 14. This figure demonstrates that an increase in the biasing current leads (due to the SHB effect) to a distortion of the emitted spectrum. More specifically, there is a blue-shift in the wavelength as well as a decrease of the SMSR because the secondary modes reach a higher output power.
B. Optimized Structures

The SMSR ($I/I_{th}$) characteristic of the optimized structures is presented in Fig. 15(a). The structures with an asymmetric DCC profile have strong discontinuities on the SMSR, namely the 1PS-DCC-DFB structure. Generally, the structure with the highest and most stable SMSR is the CPM-3DCC-DFB.

The $P(I)$ characteristic of the optimized structures is presented in Fig. 15(b). The laser with the highest electro-optical conversion efficiency is the HR-AR-DCC-DFB, as it was expected. It outperforms the remaining lasers.

![Figure 13](image13.png)

Fig. 13 – Distributions of (a) carriers and (b) photons for the QWS-DCC-DFB structure.

![Figure 14](image14.png)

Fig. 14 – Above threshold performance of the QWS-DFB laser: The emitted spectrum.

![Figure 15](image15.png)

Fig. 15 – Above threshold results for the optimized structures: (a) SMSR ($I/I_{th}$) and (b) $P(I)$. The caption of (b) also applies to (a).

VIII. EXPERIMENTAL RESULTS

Aiming to validate the simulator under development, some experiments were made at Air Force Academy, in Sintra, using the electro-optics laboratory. Although some of the equipment was not functioning properly, it was possible to retrieve the $P(I)$ characteristic of a DFB laser.

The laser characteristic is represented in Fig. 16. It is possible to see the dependence of such characteristic with the operating temperature.
coupled-wave theory as well as the transfer matrix method. Also be used for the study of vertical-cavity-surface-emitting-structure. The knowledge gained throughout this work could be used for predicting the static behaviour of any DFB laser.

A detailed analysis of DFB lasers was carried out, using the coupled-wave theory as well as the transfer matrix method. A large amount of already known results was replicated in order to validate the simulator that was implemented. Besides, some new structures were optimized and presented, in order to overcome the performance of the already known structures from literature.

This work produced an enhanced simulator, capable of predicting the static behaviour of any DFB laser. As future work, it would be convenient to explore the dynamic behaviour of the laser as well as its transversal structure. The knowledge gained throughout this work could also be used for the study of vertical-cavity-surface-emitting-lasers (VCSEL).

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