Particle Propagation in a Medium

Medium induced modifications to parton evolution

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Abstract

The formation of a medium with a non-negligible spatial extension, composed of quarks and gluons, is one of the main characteristics of heavy-ion interactions. We pretend to study some of the modifications introduced by the presence of such a medium in the parton propagation and evolution. After a brief introduction, we describe the Deep Inelastic Scattering (DIS) experiments and the hadron-hadron interactions to recall some fundamental concepts of high energy physics (Parton Distribution Functions, the parton model, the DGLAP equations, ...). We then introduce a formalism developed to treat the propagation of particles inside a medium with which they can interact and use it to compute the medium induced gluon radiation. Finally, we apply this same formalism to the calculation of the process of emission of two gluons in the presence of a medium. We conclude that although the existing formalism allows us to obtain interesting results, further developments are requires since it cannot give answers to some fundamental questions yet.

1 Introduction

The theoretical description of heavy-ion (i.e. heavy nuclei) scattering experiments had to be modified during the twentieth century as experiments were performed at higher and higher energies. In the early nucleus-nucleus experiments performed by Rutherford in the 1910s the energy of the particles was of a few MeVs. At this energy, nuclei could still be considered point-like particles so the analysis of experiments was very simplified.

The exploration of the structure of heavy-ions and hadrons led to a modern approach to the study of experiments involving those particles. Although we do not see partons in the initial and final experimental states, at high enough energy we have to do calculations in terms of partons because they are the relevant degrees of freedom (this observation is correctly reproduced by the theory of QCD). Deep Inelastic Scattering (DIS) experiments were the first ones in which the connection between hadrons and partons was understood. Hence, a correct understanding of the DIS experiments and of the tools that were developed in the theoretical description of DIS is fundamental to approach the hadron-hadron and heavy-ion scattering cases. Since heavy-ions are composed of hadrons it is also important to understand the modern approach and the main features of hadron-hadron interactions before dealing with the more complex case of heavy-ions.

The main difference between hadron-hadron and heavy-ion interactions is the presence of a medium with a non-negligible spatial extension composed of quarks and gluons, with which the partons interact. These interactions are expected to modify the way partons evolve after the interaction. In this text, we study some of those modifications.

We will thus start by a quick review of DIS experiments to introduce some fundamental concepts. Then we focus on hadron-hadron interactions, to introduce the description of an hadronic final state. In section 4 we present a formalism to describe the propagation of particles inside a medium and use it to compute the medium induced gluon radiation. Finally, we try to apply this formalism to the calculation of the process of emission of two gluons in the presence of a medium. We finish with some conclusions and outlooks

2 Deep Inelastic Scattering and the Parton Model

2.1 The parton model

The DIS experiments consist in the scattering of a lepton on a hadron. Given that hadrons are not point-like particles, their role in the interaction is more difficult to describe than the one of the leptons. They are usually described by an hadronic tensor [1–3]:

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1\left(x, Q^2\right) + \left(p^\mu - q^\mu \frac{p\cdot q}{q^2}\right) \times \left(p^\nu - q^\nu \frac{p\cdot q}{q^2}\right) \frac{W_2\left(x,Q^2\right)}{M^2} \tag{1}$$

where q is the momentum exchanged in the interaction, $Q^2 = -q^2$, p is the initial hadron momentum, M is the hadron mass and $x = Q^2/(2p\cdot q)$. $W_1(x,Q^2)$ and $W_2(x,Q^2)$ are called structure functions and have to be extracted from experimental data. The advantage of DIS experiments is that we can learn about the hadronic tensor structure only by looking at the lepton initial and final momentum. The hadronic final state does not need to be known.

DIS experiments performed at SLAC showed $\nu W_2(x,Q^2)$ (where ν is the energy lost in the interaction by the lepton) was independent of Q^2 to a good approximation, and the structure functions were related by $2xMW_1 = \nu W_2$ [4]. The first result was called Bjorken scaling, and the second the Callan-Gross relation. By assuming the lepton interacted with almost free charged spin-$1/2$ point-like particles, the partons, the parton model was shown to reproduce correctly both results [5–7].

In this model, a lepton-hadron interaction cross-section σ_{lh} is written as the convolution of a lepton-parton cross-section σ_{li} and a function that describes the probability for the parton to have a given fraction of the hadron's momentum x, $f_i(x)$, the Parton Distribution Function (PDF):

$$\sigma_{lh}(Q^2) = \sum_i \int_0^1 dx f_i(x) \sigma_{li}(x, Q^2) \tag{2}$$

This is a first example of cross-section factorization: while σ_{li} depends on the high-energy process we are considering and is calculated in perturbation theory, the PDF is a property of the hadron that depends on non-perturbative (low energy) QCD, just as its mass, and can only be measured experimentally. The possibility to factorize the cross-section in such a way is thus a consequence of the very different behaviour of the strong coupling constant at different energy scales. The PDFs are related with the structure functions in the following way, [5–7]:

$$W_1(x) = \frac{1}{2M} \sum_i f_i^h(x) e_i^2 \, ; \, W_2(x) = \frac{1}{\nu} \sum_i f_i^h(x) e_i^2 x$$

so that Bjorken scaling is a consequence of the independence of $f_i(x)$ on the energy scale, which is explained by the fact that there were no parton-parton interactions, and the Callan-Gross relation is indeed verified.

However, while Bjorken scaling was only observed to 10% accuracy it is obtained in the parton model without approximations. This fact was later understood when partons were identified as being quarks, the fermions introduced by the theory of the strong interaction, Quantum Chromodynamics (QCD). Even though quarks interact through the exchange of gluons, their interactions are suppressed at high energy due to the logarithmic decrease of the value of the strong coupling constant $\alpha_S(Q^2)$ with the increase of the energy [8]. This explains why the parton model's predictions were correct to a good approximation: it is the zeroth order term in a perturbative QCD theory.

2.2 The DGLAP equations

Let us start by recalling that the PDFs are non perturbative quantities because they describe the hadron's structure, which is controlled by non-perturbative low-energy QCD processes. Perturbative QCD calculations can thus only predict how a PDF is modified by high-energy processes. To understand why Bjorken scaling is slightly violated at high energy, we have to go to first order in perturbation theory and consider the role of gluons.

Gluons can affect the quark's PDFs in two ways: a quark can emit a gluon and thus lose a fraction $(1-x)$ of its momentum, or a quark with a given momentum can be created in the fluctuation of a gluon in a $q\bar{q}$ pair. Each process is described by its splitting function, $P_{qq}(x)$ and $P_{gq}(x)$ respectively. Since gluons are also partons (although not directly visible in DIS experiments), they can be assigned a PDF on their own, that is affected by two processes: the loss of momentum through the emission of a gluon, or the creation of a gluon emitted by a quark. They are described by $P_{gg}(x)$ and $P_{qg}(x)$.

However, the splitting functions only describe the process of one splitting. Let us say we have measured the PDFs at an energy scale and want to see how it varies with the energy scale (t). We then have to sum all the contributions of all possible splittings. This is achieved with the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equations, that sum all leading logarithmic terms ($\propto \alpha_S(t)\ln(t)$) in the perturbative expansion, [9–12]:

$$t\frac{\partial}{\partial t}\begin{pmatrix} f_i(x,t) \\ f_g(x,t) \end{pmatrix} = \frac{\alpha_S(t)}{2\pi} \sum_{j=1}^{2n_f} \int_x^1 \frac{d\xi}{\xi} \times \tag{3}$$

$$\times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}, \alpha_S(t)\right) & P_{q_i g}\left(\frac{x}{\xi}, \alpha_S(t)\right) \\ P_{g q_j}\left(\frac{x}{\xi}, \alpha_S(t)\right) & P_{gg}\left(\frac{x}{\xi}, \alpha_S(t)\right) \end{pmatrix} \begin{pmatrix} f_j(\xi,t) \\ f_g(\xi,t) \end{pmatrix}$$

This equation is one of the main results of perturbative QCD and fundamental in the description of interactions involving hadrons because, along with the PDFs, it establishes the link between non perturbative QCD states (the hadrons we observe experimentally) and the partons we can treat perturbatively.

3 Hadron-hadron interactions

3.1 The cross-section

We now extend what we said about DIS experiments to hadron-hadron interactions. The property of factorization, which we used for writing Eq.(2) and derives from the different behaviour of the strong coupling constant at different energy scales, is once again fundamental. In a parton model approach to these interactions, we want to write the cross-section with the help of a more fundamental parton-parton interaction. Since we have two hadrons, we must have two PDFs, one for the parton that comes from each hadron. In this type of interactions we must of course also describe the hadronic final state. This requires the introduction of a new object: the Fragmentation Functions (FF) $D_{i\to h_f}(z)$ of a parton of type

i into a hadron h_f carrying a fraction z of the parton's momentum [2, 13]. They are for the final state what the PDFs are for the initial state: they connect the partons produced in the hard interactions with the hadrons in the final state.

The hard interaction cross-section is the convolution of all these terms:

$$\sigma_{h_1 h_2 \to h_f}(P_1, P_2) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^{h_1}(x_1, \mu^2) \times$$
$$\times f_j^{h_2}(x_2, \mu^2) \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, Q^2/\mu^2) D_{k \to h_f}(z, \mu^2) \quad (4)$$

Just as the PDFs, they are non perturbative quantities and one can only hope to compute their evolution due to high energy processes. In fact, the processes that modify the FFs are exactly the same that modify the PDFs so that by summing all the possible splittings, we once again recover the DGLAP equations, this time for the FF evolution:

$$t \frac{\partial D_i(x,t)}{\partial t} = \frac{\alpha_S(t)}{2\pi} \sum_j \int_x^1 \frac{dz}{z} P_{ji}(z) D_j\left(\frac{x}{z}, t\right) \quad (5)$$

3.2 Jets

Jets are the final state signature of hadrons who originated in the fragmentation of partons produced in a hard-interaction. The hard interaction can be a parton-parton interaction, like in hadron-hadron collisions, or a lepton-lepton annihilation, like in a e^+e^- collision [13].

The high energy partons produced in the hard interaction, called the leading particles, will suffer multiple splittings with a probability controlled by the splitting functions. The emitted partons will form a parton shower around the leading particle, that will then hadronize into hadrons. Several theoretical arguments, like angular ordering, guaranty that the hadrons will be contained inside a well defined cone around the leading particle [3, 13].

Jets constitute one of the fundamental experimental observables when there is a hadronic final state. Their correct definition and identification has been the subject of a lot of efforts in recent years. Particularly in hadron-hadron interactions, the background composed of non-interacting or soft-interacting partons greatly increases the difficulty of the task of jet identification, [14].

4 Particle Propagation in a Medium

4.1 Heavy-ion collisions

The main difference between hadron-hadron interaction and heavy-ion interactions is the number of partons present during the scattering process. For a lead-lead collision, for instance, there are more than 200 nucleons present in the scattering. All this nuclear strong interacting matter is expected to have some effects in the cross-section. For instance, the splitting of the partons produced in the hard interaction is expected to occur inside this medium, since due to the large number of partons present in the collision, the spatial extension of the medium is not negligible.

The usual approach to these interactions is to keep the general expression for the cross-section, Eq.(4), and redefine the terms that have to be modified. For instance, the PDFs will not be hadronic PDFs, but nuclear parton distribution functions because the initial state parton evolution is also affected by the presence of the other hadrons. In this work, we will focus on the modification of the final state evolution. In particular, the high energy particles produced in the hard interaction are expected to lose energy in interactions with the medium. This effect is called "final state energy loss" or "jet quenching". A good understanding of jet quenching is fundamental not only for the correct analysis of experimental data, but also because it constitutes an indirect probe of the medium created in heavy-ion interactions, which has yet to be fully understood.

We base ourselves in the work presented in [15] and also in [16–18]. After obtaining expression for the propagation of high energy partons inside the nuclear medium, we compute the medium induced corrections to the gluon radiation spectrum. The medium is modeled by a semi-classical field distribution $A_\mu(x)$ that is not affected by the interactions with the high energy particle. The only effect is to induce colour rotation, without modifying the helicity or polarization. We neglect the recoil during the interaction.

4.2 Propagation of a high energy particle in matter

We here present the formalism introduced in [15]. All calculations are made in the eikonal approximation: the initial momentum of the particle is equal to the final momentum ($p = p'$), and the particle is not allowed to propagate in the transverse space ($\boldsymbol{p}_\perp, \boldsymbol{p'}_\perp \simeq 0$). We use light-cone variables [20], and the particle propagates in the positive x_3 directions, so that $p_+ \gg p_-$.

With these assumptions, the S-matrix for one quark-medium interaction is given by:

$$S_1(p, p') = \int d^4x \frac{1}{2} \sum_{\lambda\lambda'} e^{i(p'-p)\cdot x} \bar{u}(p', \lambda') \times$$
$$\times ig A_\mu^a(x) T^a \gamma^\mu u(p, \lambda) \delta_{\lambda\lambda'} \quad (6)$$

which can be simplified to get:

$$S_1(p, p') = 2\pi\delta\left(p'_+ - p_+\right) 2p_+ \int d^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{p}'-\boldsymbol{p})_\perp \cdot \boldsymbol{x}_\perp} \times$$
$$\times \left[ig \int dx_+ A_-(x_+, \boldsymbol{x}_\perp) \right] \quad (7)$$

where $A_-(x_+, \boldsymbol{x}_\perp) \equiv A_-^a(x_+, \boldsymbol{x}_\perp) T^a$ (because of Lorentz contraction, the field does not depend on x_-).

Computing the S-matrix for two quark medium interactions, we get:

$$S_2(p,p') = 2\pi\delta(p'_+ - p_+)2p_+ \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{x}_\perp\cdot(\boldsymbol{p}-\boldsymbol{p}')_\perp} \times \\ \times \frac{1}{2}\left[ig\mathcal{P}\int dx_+ A_-(x_+, \boldsymbol{x}_\perp)\right]^2 \tag{8}$$

where $\mathcal{P}$ means we take the fields in the correct path ordered way. The generalization to n scatterings is straightforward:

$$S_n(p,p') = 2\pi\delta(p'_+ - p_+)2p_+ \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{x}_\perp\cdot(\boldsymbol{p}-\boldsymbol{p}')_\perp} \times \\ \frac{1}{n!}\mathcal{P}\left[ig\int dx_+ A_-(x_+, \boldsymbol{x}_\perp)\right]^n \tag{9}$$

so that the total S-matrix (the sum of the contributions with all possible values of n) is just:

$$S_q(p,p') = 2\pi\delta(p'_+ - p_+)2p_+ \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{x}_\perp\cdot(\boldsymbol{p}-\boldsymbol{p}')_\perp} W(\boldsymbol{x}_\perp) \tag{10}$$

where $W(\boldsymbol{x}_\perp)$ is the fundamental Wilson line, defined as:

$$W(\boldsymbol{x}_\perp) \equiv \mathcal{P}\exp\left[ig\int dx_+ A_-(x_+, \boldsymbol{x}_\perp)\right] \tag{11}$$

For the propagation of a gluon in the eikonal approximation, the only change is that everything is in the adjoint representation:

$$S_g(p,p') = 2\pi\delta(p'_+ - p_+)2p_+ \times \\ \times \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{x}_\perp\cdot(\boldsymbol{k}-\boldsymbol{k}')_\perp} W^A(\boldsymbol{x}_\perp) \tag{12}$$

where:

$$W^A(x_\perp) = \mathcal{P}\exp\left[ig\int dx_+ A^A_-(x_+, \boldsymbol{x}_\perp)\right] \tag{13}$$

If we want to allow the particle to propagate in the transverse plane, the calculations are technically more complicated, but follow exactly the same steps [15, 16]. For a gluon (this is the only case we will need), we have:

$$S_g(p,p') = 2\pi\delta(p'_+ - p_+)2p_+ \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{x}_\perp\cdot(\boldsymbol{p}'_\perp-\boldsymbol{p}_\perp)} \times \\ \times G^A(\boldsymbol{x}_{0\perp}, x_{0+}; \boldsymbol{x}_{f\perp}, x_{f+}|p_+) \tag{14}$$

where:

$$G^A(\boldsymbol{x}_{0\perp}, x_{0+}; \boldsymbol{x}_{f\perp}, x_{f+}|p_+) = \\ = \int \mathcal{D}\boldsymbol{r}(\xi)\exp\left[i\frac{p_+}{2}\int d\xi\left(\frac{d\boldsymbol{r}}{d\xi}\right)^2\right] W^A(\boldsymbol{r}(\xi)) \tag{15}$$

with:

$$W^A(\boldsymbol{r}(\xi)) = \quad \mathcal{P}\exp\left[i\int d\xi A^A_-(\boldsymbol{r}(\xi),\xi)\right] \tag{16}$$

The Wilson line is thus replaced by the product of a Wilson line and a Feynman free propagator for the movement in the transverse space [21].

4.3 Medium induced gluon radiation

We now want to compute the spectrum of the medium induced gluon radiation. In the presence of a medium, two amplitudes contribute to this process: the emission of a gluon by a quark outside the medium, and the emission of a gluon by a quark inside the medium. We define y_+ as the position of the gluon emission vertex, x_{0+} as the beginning of the medium, and L_+ as the end of the medium. All calculations are made in the leading order of $1/z$, where z is the fraction of momentum taken by the emitted gluon.

The amplitude for the case where the gluon is emitted outside the medium is easy to obtain. The emission of a gluon from a quark in the vacuum is given by:

$$M_{rad} = -2gT^a\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{\epsilon}^a_\perp}{\boldsymbol{k}^2_\perp} \tag{17}$$

Since the quark has to cross all the medium before emitting the gluon, we must multiply this result by a Wilson line for the quark propagation. The final expression for the amplitude is thus:

$$M^q_{rad} = -2gT^a\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{\epsilon}^a_\perp}{\boldsymbol{k}^2_\perp}W(\boldsymbol{0}, x_{0+}, L+) \tag{18}$$

For the gluon emission inside the medium, the calculations are a little more difficult. We will thus give the final result and motivate the meaning of each term. After some calculations:

$$M^g_{rad} = 2g\frac{1}{k_+}\int_{x_{0+}}^{L_+} dy_+ \int d^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp} \times \\ W(\boldsymbol{0}; x_{0+}, y_+)T^b\boldsymbol{\epsilon}_\perp\cdot\frac{\partial}{\partial\boldsymbol{y}_\perp}\times \\ \times G^b(\boldsymbol{y}_\perp = \boldsymbol{0}, y_+; \boldsymbol{z}_\perp, z_+ = L_+)W(\boldsymbol{0}; y_+, L_+) \tag{19}$$

We can recognize a Wilson line for the propagation of the quark from the beginning of the medium (x_{0+}) to the gluon emission vertex (y_+), the gluon emission vertex itself (the product of the T-matrix and the partial derivative), the G function that describes the propagation of the gluon from the emission vertex to the end of the medium when it is allowed some freedom in the transverse plane, and finally a Wilson line for the eikonal propagation of the quark from y_+ to L_+.

To obtain the spectrum, we have to compute the squared amplitude. In doing that, we have to be careful with the relative position of the gluon emission vertex in the amplitude (y_+) and the conjugate amplitude ($\bar{y}_+$) because we can have $y_+ < \bar{y}_+$ or $y_+ > \bar{y}_+$. In fact, it can be proven that we take both cases into account if we fix $y_+ < \bar{y}_+$ and write:

$$M^2_{rad} = 2\,\mathfrak{Re}\left\{M^g_{rad}\left(M^g_{rad}\right)^\dagger + M^g_{rad}\left(M^q_{rad}\right)^\dagger\right\} + \\ + M^q_{rad}\left(M^q_{rad}\right)^\dagger \tag{20}$$

The spectrum is obtained from the squared amplitude by:

$$k_+\frac{dI}{dk_+d^2\boldsymbol{k}\perp} = \frac{\langle M^2_{rad}\rangle}{(2\pi)^3} \tag{21}$$

After some calculations, [15–18], we obtain:

$$k_+\frac{dI}{dk_+d^2\boldsymbol{k}\perp} = \frac{C_F\alpha_S}{\pi^2}2\,\mathfrak{Re}\left\{\frac{1}{k_+^2}\int_{x_{0+}}^{L_+}dy_+\int d^2\boldsymbol{z}_\perp\int_{y_+}^{L_+}d\bar{y}_+\int d^2\bar{\boldsymbol{z}}_\perp\int d^2\boldsymbol{w}_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{z}-\bar{\boldsymbol{z}})_\perp}\times\right. \\ \times\frac{\partial}{\partial\boldsymbol{y}_\perp}\frac{\partial}{\partial\bar{\boldsymbol{y}}_\perp}\frac{1}{N^2-1}\langle tr\left[W^A(\boldsymbol{0};y_+,\bar{y}_+)G(\boldsymbol{y}_\perp=\boldsymbol{0},y_+;\boldsymbol{w}_\perp,\bar{y}_+)\right]\rangle\times \\ \frac{1}{N^2-1}\langle tr\left[G(\boldsymbol{w}_\perp,\bar{y}_+;\boldsymbol{z}_\perp,L_+)G(\bar{\boldsymbol{y}}_\perp,\bar{y}_+;\bar{\boldsymbol{z}}_\perp,L_+)\right]\rangle - \frac{\boldsymbol{k}_\perp}{k_+\boldsymbol{k}_\perp^2}\int_{x_{0+}}^{L_+}dy_+\int d^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp}\times \\ \left.\times\frac{\partial}{\partial\boldsymbol{y}_\perp}\frac{1}{N^2-1}\langle tr\left[W^A(\boldsymbol{0};y_+,L_+)G(\boldsymbol{y}_\perp=\boldsymbol{0},y_+;\boldsymbol{z}_\perp,z_+=L_+)\right]\rangle\right\} + \frac{C_F\alpha_S}{\pi^2}\frac{1}{\boldsymbol{k}_\perp^2} \tag{22}$$

In this expression, all spin and color sums and averages have been performed, and we have written it as a product of terms that can be computed doing medium averages [15–19]. Although it is an important issue to extract medium properties from experimental results, it is outside the scope of what we pretend to do in this text. More information can be found in the references.

Finally, we have to say that the above expression can be used to redefine the P_{qg} and P_{qq} splitting functions to include medium effects [22–24]. Its use in Monte Carlo simulations seems to correctly reproduce energy loss of the leading particle [25].

5 Two Gluon Emission in the Presence of a Medium

There is a subtlety we did not mention when introducing the DGLAP equations. For them to be valid, it has to be possible to factorize subsequent emissions. For instance, it has to be possible to write the emission of two gluons by a quark as the emission of one gluon times the emission of the other gluon. Although in the vacuum this has been proven to be true [26, 27], we do not know if it is still true in a medium, or at least if it is true to some approximation. One of the fundamental tools we need to solve this problem is the amplitude for the process of two gluon emission in the presence of a medium.

We now apply the formalism of the previous section to this new calculation. Our goal is to understand what we can already compute, which new tools need to be developed and if the presented formalism is suitable for this calculation.

5.1 The amplitudes

The diagrams that contribute to this process in the vacuum case are the ones given in Fig. 1, see [3]. The distinction between Fig. 1(a) and Fig. 1(b) is made so that when we compute the squared amplitude we do not forget diagrams where the gluons cross (non-planar diagrams).

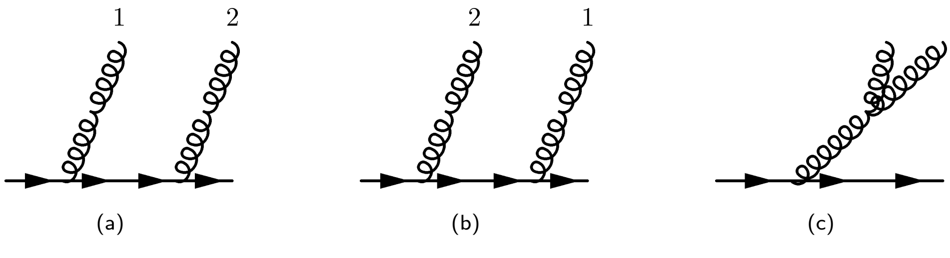

Figure 1: Amplitudes contributing to the two gluon emission by a quark process

If a medium is present, each one of the diagrams in Fig. 1 gives three contribution: for Fig. 1(a) and Fig. 1(b), we must distinguish de cases in which all emissions occur outside the medium, the case in which one gluon is emitted outside and one inside the medium, and the case in which both gluons are emitted inside the medium. For Fig. 1(c), we distinguish the cases in which everything happens outside the medium, the quark emits the gluon inside the medium but the gluon splits outside the medium, and all the emissions occur inside the medium.

At this stage we can already mention one limitation of the formalism presented above: we have no tools to describe a tri-gluon vertex inside a medium. All the other eight amplitudes can be computed, so in this first approach to this calculation we will focus on them.

The non-planar terms are known to be suppressed by a factor of $1/N$ in the vacuum. In a medium, this will have to be proven. However, the difficulties in the calculation of non-planar diagrams are the same as with planar diagrams, so we will leave the non-planar diagrams as future work.

The diagrams that can be straightforwardly computed with the formalism presented in the last section are the ones of the type of Fig. 1(a).

5.2 The infinite medium case

If we focus on the amplitudes of the type of Fig. 1(a), we can compute nine terms for the squared amplitude. Of those, however, the case that presents the more subtleties in the calculations is the one where the two gluons are emitted inside the medium in both the amplitude and the conjugate amplitude, which we write as $M^{gg}_{med}(M^{gg}_{med})^\dagger$. It is the only term present in an infinite medium. All the others can be trivially obtained once this calculation as been understood.

We start by defining y_{1+} ($\bar{y}_{1+}$) as the first gluon emission vertex in the amplitude (conjugate amplitude) and y_{2+} ($\bar{y}_{2+}$) as the second gluon emission vertex. Just as when we computed the medium induced gluon radiation spectrum, we will have to be careful when computing $M^{gg}_{med}(M^{gg}_{med})^\dagger$ to not forget any contribution. The only constrain we have is that $y_{1+} < y_{2+}$ and $\bar{y}_{1+} < \bar{y}_{2+}$, because it was implicitly assumed in the definition of the amplitudes. We are thus left with six different ordering possibilities. It can be shown that all contributions are taken into account if we fix $y_{1+} < \bar{y}_{1+}$ and then take two times the real part of the three remaining terms.

Applying directly Eq.(19) and making all the Wilson line products simplification, we get:

$$
\begin{aligned}
M^{gg}_{med}(M^{gg}_{med})^\dagger = \frac{16g^4}{k^2_{1+}k^2_{2+}}\frac{1}{2N}2\,\mathfrak{Re}\Bigg\{ & \\
&\left(\int_{x_{0+}}^{L_+} dy_{1+}\int_{y_{1+}}^{L_+} dy_{2+}\int_{y_{2+}}^{L_+} d\bar{y}_{1+}\int_{\bar{y}_{1+}}^{L_+} d\bar{y}_{2+}\int d^2\boldsymbol{z_1}_\perp\int d^2\bar{\boldsymbol{z}}_{1\perp}\int d^2\boldsymbol{z_2}_\perp\int d^2\bar{\boldsymbol{z}}_{2\perp}\right.\\
&\times tr\left[W^\dagger(\mathbf{0};y_{1+},\bar{y}_{1+})T^{c_1}W_{a_2a_3}(\mathbf{0};y_{1+},y_{2+})T^{d_1}W(\mathbf{0};y_{2+},\bar{y}_{2+})T^{\bar{d}_1}W^\dagger(\mathbf{0};\bar{y}_{1+},\bar{y}_{2+})T^{\bar{c}_1}\right]+\\
&+\int_{x_{0+}}^{L_+} dy_{1+}\int_{y_{1+}}^{L_+} dy_{2+}\int_{y_{1+}}^{y_{2+}} d\bar{y}_{1+}\int_{y_{2+}}^{L_+} d\bar{y}_{2+}\int d^2\boldsymbol{z_1}_\perp\int d^2\bar{\boldsymbol{z}}_{1\perp}\int d^2\boldsymbol{z_2}_\perp\int d^2\bar{\boldsymbol{z}}_{2\perp}\\
&\times tr\left[W^\dagger(\mathbf{0};y_{1+},\bar{y}_{1+})T^{c_1}W(\mathbf{0};y_{1+},\bar{y}_{1+})T^{\bar{c}_1}\right]tr\left[W(\mathbf{0};y_{2+},\bar{y}_{2+})T^{\bar{d}_1}W^\dagger(\mathbf{0};y_{2+},\bar{y}_{2+})T^{d_1}\right]+\\
&+\int_{x_{0+}}^{L_+} dy_{1+}\int_{y_{1+}}^{L_+} dy_{2+}\int_{y_{1+}}^{y_{2+}} d\bar{y}_{1+}\int_{\bar{y}_{1+}}^{L_+} d\bar{y}_{2+}\int d^2\boldsymbol{z_1}_\perp\int d^2\bar{\boldsymbol{z}}_{1\perp}\int d^2\boldsymbol{z_2}_\perp\int d^2\bar{\boldsymbol{z}}_{2\perp}\\
&\left.\times tr\left[W^\dagger(\mathbf{0};y_{1+},\bar{y}_{1+})T^{c_1}W(\mathbf{0};y_{1+},\bar{y}_{1+})T^{\bar{c}_1}\right]tr\left[W^\dagger(\mathbf{0};y_{2+},\bar{y}_{2+})T^{\bar{d}_1}W(\mathbf{0};y_{2+},\bar{y}_{2+})T^{d_1}\right]\right)\\
&\times\exp\left[-i\boldsymbol{k}_{1\perp}\cdot(\boldsymbol{z}_{1\perp}-\bar{\boldsymbol{z}}_{1\perp})\right]\exp\left[-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{z}_{2\perp}-\bar{\boldsymbol{z}}_{2\perp})\right]\frac{\partial}{\partial\boldsymbol{y}_{1\perp}}\frac{\partial}{\partial\boldsymbol{y}_{2\perp}}\frac{\partial}{\partial\bar{\boldsymbol{y}}_{2\perp}}\frac{\partial}{\partial\bar{\boldsymbol{y}}_{1\perp}}\times\\
&\times G_{c_1c}(\boldsymbol{y}_{1\perp}=\mathbf{0},y_{1+};\boldsymbol{z}_{1\perp},z_{1+}=L_+)G_{d_1d}(\boldsymbol{y}_{2\perp}=\mathbf{0},y_{2+};\boldsymbol{z}_{2\perp},z_{2+}=L_+)\times\\
&\times G_{\bar{d}_1d}(\bar{\boldsymbol{y}}_{2\perp}=\mathbf{0},\bar{y}_{2+};\bar{\boldsymbol{z}}_{2\perp},z_{2+}=L_+)G_{\bar{c}_1c}(\bar{\boldsymbol{y}}_{1\perp}=\mathbf{0},\bar{y}_{1+};\bar{\boldsymbol{z}}_{1\perp},z_{1+}=L_+)\Bigg\}
\end{aligned}
\tag{23}
$$

where all spin and color averages have been performed.

With the developed formalism we can thus get an expression in which we understand where each term comes from. The only problem that still as to be solved in this expression is the simplification of the trace of the product of four Wilson lines and four T-matrices (the other traces can be simplified, [15, 16]).

Although we are still far from having a final expression for the squared amplitude of the two gluon emission in the presence of a medium process, this formalism seems to be appropriate for the calculations, because it is easy to use and produce clear results.

6 Conclusions and Outlooks

The formalism presented in section 4 has the great advantage of being very easily used. The expressions we get with it, particularly Eq.(22), have been used to modify the DGLAP equations to include medium induced effects (such as jet quenching) with good results, [22–25]. However, a formal proof that this can be done has yet to be given. In order to do that, one of the fundamental process that has to be computed is the process of two gluon emission in the presence of a medium, to understand if subsequent splittings can be factorized. Even though the formalism presented in [15] allows us to compute some terms that contribute to the squared amplitude of this process, we showed that other diagrams required the development of new tools.

Hence, although it is to early to understand if it is possible to factorize subsequent splittings that occur inside a medium, or at least in which approximation it can be made, the work that remains to be done is clear: develop tools to treat all the diagrams contributing to the two gluon emission process and to be able to further simplify the obtained expressions. Once this is done, it will hopefully be possible to have an expression for the squared amplitude of that processes that allows to verify or refute the factorization hypothesis.

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