Modified Gravity Theories with non-minimal coupling between curvature and matter - Energy Conditions and Dolgov-Kawasaki Criterion

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I. SUMMARY

Recently, there has been a revival of interest in gravity models where the linear scalar curvature term of the Einstein-Hilbert action is replaced by a function, $f(R)$, of the scalar curvature. This interest is due to the possibility of accounting for the late time accelerated expansion of the universe without the need of explicit additional degrees of freedom such as dark energy and also to possibly replace dark matter at galactic level and beyond.

In this thesis, one examines models where, besides the extension in the geometric part of the action, there is also a direct coupling of a new function of the Ricci scalar to the matter Lagrangian density

$$S = \int \left[ \frac{1}{2} f_1(R) + f_2(R) \mathcal{L}_m \right] \sqrt{-g} d^4x, \quad (1)$$

where $f_i(R)$ (with $i = 1, 2$) are arbitrary functions of the curvature scalar $R$ and $\mathcal{L}_m$ is the Lagrangian density corresponding to matter. This theory exhibits interesting features. The most intriguing is the fact that due to the interaction between geometry and matter an extra force shows up and the motion of a point-like test particle turns non-geodesic.

However, as any modified gravity theories, there may exist anomalies which may eventually turn the theory physically meaningless. In this context, one undertakes a study in what respects the energy conditions, fundamental to restrict energy-momentum tensors, and the Dolgov-Kawasaki criterion, fundamental to ensure the stability of the theory, for these new modified models of gravity with non-minimal coupling between curvature and matter.

As one shall see, the energy conditions allow one to establish under which conditions gravity remains attractive, along with the demands that the energy density is positive and cannot flow faster than light, while the Dolgov-Kawasaki stability criterion helps one ruling out some classes of modified models.

This thesis is organized as follows. In Chapter 1, one presents a briefly introduction on
the issues concerning Einstein’s general relativity and the paths being pursued to obtain a theory capable of describing the observed universe. In Chapter 2, one introduces the standard $f(R)$ theories and their equivalence to scalar-tensor theories. In Chapter 3, one introduces the $f(R)$ theories with the non-minimal curvature-matter coupling. One begins by deriving the field equations in the metric formalism and writing them in a way that resembles the Einstein’s field equations with an effective energy-momentum tensor. By comparing the effective energy-momentum tensor with the one describing the perfect fluid one defines an effective energy density, $\hat{\rho}$, and an effective pressure, $\hat{p}$. An effective coupling constant has also been defined and has been used to obtain an additional condition for attractive gravity. In the same Chapter, one presents the key features of the theory induced by the non-minimal coupling such as the fact that the energy-momentum tensor might not be covariantly conserved, leading to non-geodesic motion, and the broken degeneracy of the matter Lagrangian density. The study of the equivalence with a scalar-tensor theory has also been considered and shown that the equivalence with a standard $f(R)$ theory is not possible due to the non-minimal coupling.

One then discusses the central topic of this thesis – the Energy Conditions and the Generalised Dolgov-Kawasaki instability criterion for this class of gravity models. In what concerns the energy conditions, one derives the strong energy condition and the null energy condition directly from geometric principles, through the use of the Raychaudhuri equation, along with the requirement that gravity is attractive. It is shown that both conditions could be derived directly from their equivalent in general relativity through the transformations $\rho \rightarrow \rho + \hat{\rho}$ and $p \rightarrow p + \hat{p}$. Applying these transformations to the dominant energy condition and the weak energy condition of general relativity, one obtains the generalized conditions for the gravity models under study. One then applies the resulting conditions to a broad class of $f_i(R)$ functions. As expected, all the generalized conditions for a suitable energy-momentum tensor are dependent on the geometry, on the coupling constants of the model and on the matter Lagrangian density.

With respect to Dolgov-Kawasaki instability criterion, one reviews the derivation of this criterion in the context of the $f(R)$ theories with a non-minimal curvature-matter coupling and study the viability of two distinct models. As in the study of the energy conditions, the Dolgov-Kawasaki criterion will depend on the coupling constants of the model, on the matter Lagrangian density and also on the space-time under consideration.
Through the application of the encountered conditions to a specific kind of models one notices that all the energy conditions together with the positiveness of the effective gravitational coupling and the Dolgov-Kawasaki criterion can be unified in one single inequality with specific parameters.

Chapter 4 contains our conclusions.

The original part of the work presented in this thesis follows the one developed in Ref. [1].