PUNCHING SHEAR STRENGTH OF ASYMMETRICALLY REINFORCED CONCRETE SLABS

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Dissertation to obtain the Master degree in Civil Engineering

Jury

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1 Introduction

Since last century, punching has been an important problem to researchers who tried to fully understand it. No matter how many experiments were made, some models and analysis relied on empiric results that did not characterise this phenomenon to its fullest. Test parameters covered either the geometry of the slabs, as well as the size of aggregates, the reinforcement properties and loading modes. Although all of these were executed and planned in an intelligent way, allowing a wide range of results, it is virtually impossible to cover all parameters.

This experimental campaign is going to provide some new cases.

Punching shear failure is characterised by a truncated-cone-shaped element that appears when concentrated or punctual forces (such as columns) are imposed in thin wide structures such as concrete slabs (common situation when using flat slabs systems). To better understand it, there is a schematic representation below.

![Figure 1 – Punching shear failure (Guandalini 2005)](image)

This phenomenon is of great danger since it is also characterised by a brittle failure that does not provide sufficient warning about the impending collapse.

Three experiments were made to better understand punching shear and find new evidences that can help to make better models for designing new structures and evaluate existing ones. This study investigated the effect of orthotropic reinforcement.

Aims

- Study the effect of the different quantities of flexural reinforcement in orthogonal directions in the punching shear strength of flat slabs;
- Comparison, between codes and test results, of the punching shear strength;
- Theoretical approach of prestressing.
2 Theoretical background

2.1 Mechanical behaviour

In flat slabs the load transfer between the slab and the column induces high stresses near to this last that incite to cracking and even failure. The punching shear failure is associated to the formation of a cone-shaped element. This shape is a result of the interaction between the shear effects and flexion in a region close to the column.

This is called a local mechanism associated to a brittle failure. Although the punching shear failure is a local phenomenon it can, sometimes, provoke a progressive failure extending the whole structure since one local failure increases the shear forces in the other columns.

In a flat slab under uniformly distributed loads, cracks will first appear near the columns. With the augmentation of the load, other cracks will appear parallel to the columns sides forming what know as tangential cracking. There will be radial cracks as well starting from the columns forming several radial parts.

When in failure, tangential cracks propagate in an inclined surface from the slab side in tension until the intersection between the column with the slab side in compression. This will then form the already mentioned cone-shaped element.

The punching shear strength in a flat slab depends on the slab geometry, loaded area, slab thickness, concrete strength and amount of reinforcement (either flexural or punching shear reinforcement). The transferred moments between the slab and the column, the slab particularities (e.g.: openings), and the position of the column (centre, edge or corner) influence the punching shear force as well.

2.2 Failure criterion (Muttoni 2003)

In 2003, Muttoni proposed a model for the punching shear strength of reinforced concrete slabs without shear reinforcement (Muttoni 2003). This rotation-based model has the rotation $\psi$ as dominant factor since it is observed that the deformations of the slab concentrate near the column edge.

The shear strength is negatively affected by the propagation of flexural cracks (Muttoni, Schwartz 1991) and therefore the punching shear strength is calculated as a function of the deformations in the critical region. According to the same publication the width of the critical crack is correlated with $\psi \cdot d$. The next equation shows how to calculate the shear strength as a function of $\psi \cdot d$:
With $\psi \cdot d \cdot k_{dg}$ in [mm], $V_R$ being the resistant punching shear force, $u$ the length of the control perimeter (Figure 2), $d$ the effective depth, $\tau_c$ the nominal shear strength with $\tau_c = 0.3\sqrt{f_c}$ and the concrete compressive strength $f_c$ in [MPa], $\psi$ the rotation, and $k_{dg} = \frac{48}{D_{\text{max}} + 16}$ representing the influence of the maximum aggregate size ($D_{\text{max}}$ in [mm]).

The control perimeter $u$ is situated 0.5$d$ from the column edge according to the Swiss code SIA 262:2003.

This equation can be compared with experimental results in the next image.

\[
\tau_R = \frac{V_R}{u \cdot d} = \frac{\tau_c}{0.4 + 0.125 \cdot \psi \cdot d \cdot k_{dg}}
\] (2.1)
A special remark goes to the lack of tests with large rotations. Such detail occurs because high flexural reinforcement ratios were generally used with the aim of avoiding the yielding of reinforcement in tension. (Guandalini 2005) preformed several punching shear tests to study the effect of yielding of flexural reinforcement in symmetrically reinforced slabs without shear reinforcement. He concluded that the punching criterion proposed by (Muttoni 2003) remains valid for punching shear failure after yielding of the flexural reinforcement.

### 2.3 Codes of practice

In all codes it is necessary to compare the load effects with the shear resistance (per unit of length):

$$V_d \leq V_{Rd} \quad 2.2$$

And to obtain the shear strength per unit of length it is necessary to divide the total shear strength $V_{Rd}$ for the control perimeter $u$:

$$V_{Rd} = \frac{V_{Rd}}{u} \quad 2.3$$

In fact, the parameter chosen to compare all codes to reality was the shear strength $V_{Rd}$ as it is the most logical parameter when dimensioning structures.

#### 2.3.1 SIA 262:2003

To calculate the shear strength, the SIA 262:2003 propose the equation 2.4):

$$V_{Rd} = k_r \cdot \tau_{cd} \cdot d \quad 2.4$$

Where: $k_r = \frac{1}{0.45 + 0.9 \cdot r_y}$ is a coefficient associated to the deformations attained next to the critical area and $r_y = 0.15 \cdot I \left( \frac{m_{bd}}{m_{Rd}} \right)^{\frac{3}{2}}$.

The control perimeter $u$ is the minimum possible at a distance not inferior to $0.5 \cdot d$ just like explained before in Muttoni 2003 since this one is based on the SIA 262:2003 (see Figure 2).
2.3.2 EC2, 2004

EC2, 2004 proposes a different expression for the shear strength estimation:

\[ \nu_{Rd,C} = C_{Rd,C} \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{\frac{1}{2}} \]  

2.5)

Where: \( C_{Rd,C} = \frac{0.18}{\gamma_c}, \quad k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \) (\( d \) in [mm]), \( \rho = \frac{A_d}{b_w \cdot d} \leq 0.02 \) and \( f_{ck} \) in MPa.

Regarding the control perimeter \( u \), there is a great difference when comparing to the SIA262:2003. The minimal distance it is not 0.5 but 2. This is obviously well adjusted to the general expression where we can find also evidence of the influence of the reinforcement ratio \( \rho \).

2.3.3 ACI 318-05

As for ACI 318-05, the code explains that for a two-way action the slab shall be designed according to equation 2.6).

\[
\min \left[ V_c = \left( 1 + \frac{2}{\beta} \right) \cdot \sqrt{f_{c'} \cdot b_0 \cdot \frac{d}{6}} ; \quad V_c = \left( \frac{\alpha_s \cdot d}{b_0} + 2 \right) \cdot \sqrt{f_{c'} \cdot b_0 \cdot \frac{d}{12}} ; \quad V_c = \sqrt{f_{c'} \cdot b_0 \cdot \frac{d}{3}} \right]  
\]

2.6)

Again, like the SIA 262:2003, the control perimeter \( b_0 \) it is calculated at a 0.5 \( d \) distance from the column edge.

The advantage of this code is that it proposes three sub-equations which consider several parameters separately: the first and second consider the effect of the column section and the fact observed in some tests that \( V_c \) decreases as the ratio \( \frac{b_0}{d} \) increases and the last sub-equation is the minimum shear stress value considering only the concrete compressive strength.
3 Summary of experimental results

This chapter describes the main results acquired from the experimental campaign carried out in the framework of this thesis. The experiments consisted of three tests performed on 3x3m slabs without punching shear reinforcement. The specimens were subjected to eight concentrated loads applied near the free edges. All slabs formed a shear strength typical truncated-cone element. Several factors influenced the type of behaviour such as the reinforcement ratio. Results show which models are the best and which factors influence each of these most.

3.1 Slabs

All slabs were made the 30th of March 2007 by the Proz Matériaux company at Riddes (VS, Switzerland). Each one measured 3x3x0.25 m and required nearly 2.5 m³ of concrete.

3.1.1 Geometry

As alleged, all slabs are 3x3x0.25 m. The column was simulated by a central steel plate with a 0.26x0.26 m area. Eight symmetrical holes were created during casting using small metallic tubes. These had the purpose of allowing the loading of the slabs using four hydraulic devices. The distance between the centre of the holes and the edge of the slab is 12 cm. This assured that there were no problems with the crushing of concrete. In the next page we can see an example that depicts the basic geometry of a slab (the slabs only differ in diameter of reinforcement and spacing).
Figure 4 – Geometrical characteristics of slabs. Dimensions in [mm]
3.1.2 Reinforcement

The difference between slabs consists in the spacing and reinforcement diameters, which results in different reinforcement ratios. The following images enlighten the particularities of each slab.

Slab 01

In the next image it is represented the reinforcement disposition including the cover and the absence of punching reinforcement. Each hole mentioned above it is reinforced by two bars of Ø18 mm with 60 cm of anchorage.

Figure 5 – Reinforcements of slab 01, plan
Slab 02

The second slab is identical to the first concerning the absence of punching reinforcement and the characteristics of the mentioned holes.
Slab 03

The third slab is identical to the first and second regarding the already referenced characteristics.
For a general view of the differences between slabs, in the next images and tables it is possible to identify the exact characteristics of each one of them, namely the reinforcement spacing, diameter and ratio. The concrete cover is of 20 mm for all slabs.
The next two tables show the main properties of reinforcement steel.

<table>
<thead>
<tr>
<th>Øx (mm)</th>
<th>s x (cm)</th>
<th>Øy (mm)</th>
<th>s y (cm)</th>
<th>d x (m)</th>
<th>d y (m)</th>
<th>dx (m)</th>
<th>r x</th>
<th>r y</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab01</td>
<td>16</td>
<td>0.125</td>
<td>20</td>
<td>0.1</td>
<td>0.222</td>
<td>0.204</td>
<td>0.213</td>
<td>0.72%</td>
<td>1.54%</td>
</tr>
<tr>
<td>Slab02</td>
<td>16</td>
<td>0.125</td>
<td>16</td>
<td>0.125</td>
<td>0.222</td>
<td>0.206</td>
<td>0.214</td>
<td>0.72%</td>
<td>0.78%</td>
</tr>
<tr>
<td>Slab03</td>
<td>10</td>
<td>0.115</td>
<td>16</td>
<td>0.125</td>
<td>0.225</td>
<td>0.212</td>
<td>0.219</td>
<td>0.30%</td>
<td>0.76%</td>
</tr>
</tbody>
</table>

Table 1 – Slab Characteristics (top reinforcement)

<table>
<thead>
<tr>
<th>Øx (mm)</th>
<th>s x (cm)</th>
<th>Øy (mm)</th>
<th>s y (cm)</th>
<th>d x (m)</th>
<th>d y (m)</th>
<th>dx (m)</th>
<th>r' x</th>
<th>r' y</th>
<th>r'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab01</td>
<td>10</td>
<td>0.125</td>
<td>10</td>
<td>0.1</td>
<td>0.225</td>
<td>0.215</td>
<td>0.22</td>
<td>0.28%</td>
<td>0.37%</td>
</tr>
<tr>
<td>Slab02</td>
<td>10</td>
<td>0.125</td>
<td>10</td>
<td>0.125</td>
<td>0.225</td>
<td>0.215</td>
<td>0.22</td>
<td>0.28%</td>
<td>0.29%</td>
</tr>
<tr>
<td>Slab03</td>
<td>10</td>
<td>0.115</td>
<td>10</td>
<td>0.125</td>
<td>0.225</td>
<td>0.215</td>
<td>0.22</td>
<td>0.30%</td>
<td>0.29%</td>
</tr>
</tbody>
</table>

Table 2 – Slab Characteristics (bottom reinforcement)

It was considered $\rho$ and $\rho'$ as geometric averages, \(\sqrt{\rho_x \cdot \rho_y}\) and \(\sqrt{\rho'_x \cdot \rho'_y}\) respectively.

As it is shown, there is not a great variation in the lower reinforcement ratios. On the contrary, it is on the top layers that we have a logical variation of this factor as it is here that the reinforcement is going to be of some use. Therefore it is possible to observe that Slab 01 has a much higher reinforcement ratio than Slab 03, either in each direction as in the average between both directions.

Slab 02 works as a reference to these two, because it represents the most common solution used in real structures (symmetrical).
3.2 Materials

Concrete

The concrete has a superplasticizer (GLENIUM 21) that helps the filling of normal concrete. Due to this admixture the Water / Cement ratio is of 0.32 and other mechanical characteristics are greatly enhanced namely the resistance, the permeability, the aesthetics, workability. This last is also achieved due to the maximal diameter of the aggregates being $D_{\text{max}} = 16$ mm. This aggregate it is adequate for the bar spacing and concrete cover. A better understanding can be seen in the following resuming table:

<table>
<thead>
<tr>
<th>Composition</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel 8/16</td>
<td>390 kg/m$^3$</td>
</tr>
<tr>
<td>Gravel 4/8</td>
<td>525 kg/m$^3$</td>
</tr>
<tr>
<td>Sand 0/4</td>
<td>885 kg/m$^3$</td>
</tr>
<tr>
<td>Cement</td>
<td>147 kg/m$^3$</td>
</tr>
<tr>
<td>Water</td>
<td>47 lt/m$^3$</td>
</tr>
<tr>
<td>Addition: Glénium 21 fluidifier</td>
<td>1.13 kg/m$^3$</td>
</tr>
</tbody>
</table>

Table 3 – Concrete Properties

To measure the concrete properties, there were made two specific tests at the time of concreting: the Slump test and the Flow Table test. Both of them provide information about the consistence of the concrete in function of the slump measured and the spread attained, respectively. The specimens were concreted at the same time, thus, the results are the same for all slabs and can be seen in the following figures and table.

Figure 12 and 13 – Slump (left) and Table Flow (right) tests
Later on another tests were performed, namely resistance tests. Four cylinders for each slab were made during concreting. These were tested approximately 2 weeks after (to evaluate the evolution of the concrete compression strength) and the day of each experiment (to have a more precise value of the real resistance on that specific day). In the next figures and table it is possible to see the 16cm-diameter and 32cm-high cylinders manufacture and results.

<table>
<thead>
<tr>
<th>Test</th>
<th>Values</th>
<th>Concrete consistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slump test</td>
<td>4.3 cm</td>
<td>plastic</td>
</tr>
<tr>
<td>Table Flow test</td>
<td>50 cm</td>
<td>soft</td>
</tr>
</tbody>
</table>

Table 4 – Test results

<table>
<thead>
<tr>
<th>Slab</th>
<th>Concreting date</th>
<th>Test date</th>
<th>Age [days]</th>
<th>$f_{cm, test}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>30.03.2007</td>
<td>11.05.2007</td>
<td>42</td>
<td>67.5</td>
</tr>
<tr>
<td>02</td>
<td>30.03.2007</td>
<td>04.05.2007</td>
<td>35</td>
<td>67.0</td>
</tr>
<tr>
<td>03</td>
<td>30.03.2007</td>
<td>22.05.2007</td>
<td>53</td>
<td>66.0</td>
</tr>
</tbody>
</table>

Table 5 – Test results

Figure 14 and 15 – Manufacture of the concrete cylinders

Figure 16 – Concrete resistance evolution
Steel

Three different diameters of steel bars were used in the experiments: $\varnothing = 10$ mm, $\varnothing = 16$ mm and $\varnothing = 20$ mm. The smaller diameters were submitted to three tension tests and the largest only to two.

The steel type was not the same for all diameters. It was used a TOPAR 500R (cold-worked) bar type for the $\varnothing = 10$ mm and TOPAR 500S (hot-rolled) for all the others.

The strains were measured using a 100 mm sensor in the central part of each bar. The length of each tested bar was 900 mm.

In the next figures and table it possible to identify the stress-strain relationships of all bars.

![Stress-strain graphs for different diameters](image)

**Figure 17 – Steel $\sigma - \varepsilon$ behaviour a) $\varnothing = 10$ mm; b) $\varnothing = 16$ mm; c) $\varnothing = 20$ mm**

<table>
<thead>
<tr>
<th>$\varnothing$ [mm]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [MPa]</th>
<th>$\varepsilon_{su}$ [%]</th>
<th>$E_s$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>568</td>
<td>647</td>
<td>7.8</td>
<td>203</td>
</tr>
<tr>
<td>16</td>
<td>552</td>
<td>645</td>
<td>8.7</td>
<td>201</td>
</tr>
<tr>
<td>20</td>
<td>597</td>
<td>681</td>
<td>8.0</td>
<td>205</td>
</tr>
</tbody>
</table>

**Table 6 – Test results** ($f_y$ - yield strength; $f_u$ - ultimate strength $\varepsilon_{su}$ - deformation under maximal load; $E_s$ - Young’s modules)
3.3 Test set-up

The load was applied symmetrically near the free edges. Each two application points of the same side were connected by a RHS tube and were separated by 1200 mm. To apply the load it was used one Dywidag Ø36 bar for each application point and another from the RHS to the jacks below the test set-up. This last one had a load cell on the top, above the RHS tube and below, above the jacks. The type of jacks used was the BIERI kind, with 1000 KN as maximal strength. Supporting the slab there was the pseudo-column composed by steel plates of 26x26x5 cm. Between this and the specimen it was made a thin layer of plaster with 1 to 2 mm thickness to level the surface of the plate to avoid concentrated irregularities. Below to the steel plates three load cells were placed (2000 KN each). A schematic representation of the test set-up can be seen in the following images.

Figure 18 – Test set up, plan [mm]
B-B Cut

Figure 19 – Test set up, B-B cut [mm]
3.4 Sensors

Firstly, all plans were executed bearing in mind that the most important and interesting axis was the weakest (regarding only the top reinforcement ratio). Therefore, the majority of sensors were placed along this axis.

Secondly, there were chosen four different sensors (LVDTs, inclinometers, omega-shaped extensometers with a base length of 50 and 100 mm and load cells) to assure all the required measures.

Thirdly and finally, these were all brought together and analysed to make sure that all measurements were performed.

Load cells

Eleven load cells were used with ranges from 1000 to 2000 KN. Eight (1000 KN) were placed above the jacks and below the slab and the remaining three (2000 KN) were placed under the pseudo-column as depicted in the following figure. This allowed for redundancy in the force measurements.
LVDTs

The LVDTs were used to read the deflection (along the weak axis and at the tip of each side of the slab) and the thickness change of the slab (near the column). Two aluminium tubes supported by a metallic structure were set in place to hold the LVDTs.

Figure 21, 22 and 23 – Structure used to organise sensors and details
A total of 31 LVDTs were placed along the weak axis (15 on the top side and 16 on the bottom one, 2 of which were under the column), 2 in the East-West extremities, 2 others below the column (on the strong axis) to measure any deformations this could have, and a last one horizontally against the column to measure possible displacements of the same. For the thickness change there were placed 2 sensors together with small shanks close to the column where were expected some alterations.

The following figures depict the standard scheme adopted for all inductive sensors in all slabs.
Figure 25 – Inductive sensors plan (top side)

Figure 26 – Inductive sensors plan (lower side)
Omega-shaped extensometers

Similarly to the LVDTs, the omega-shaped extensometers were placed along the weak axis in order to obtain the measures of the largest deformations. These were used to measure radial and tangential deformations which served as an indicator of the failure load. With these sensors on the top and bottom side of the slab it was possible to identify the actual behaviour of the slab before failure.

![Figure 27 – Omega-shaped extensometers (top side)](image)

Figure 27 – Omega-shaped extensometers (top side)

![Figure 28 – Omega-shaped extensometers (bottom side, radial on the left, tangential on the right)](image)

Figure 28 – Omega-shaped extensometers (bottom side, radial on the left, tangential on the right)
Figure 29 – Omega-shaped extensometers plan (top side)

Figure 30 – Omega-shaped extensometers plan (lower side)
Inclinometers

Five of these sensors were placed above the slab, one in each cardinal direction and a fifth inclinometer in a North-East orientation as shown in the following figure. Because of this even distribution it was possible to eliminate the rigid body rotation of the slab.

Figure 31 – Inclinometers plan

Finally the type and range of all sensors is shown in the next table:

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Measure</th>
<th>Type</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td>Strength</td>
<td>Losinger</td>
<td>0 ÷ 2000 kN (under the column)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 ÷ 1000 kN (all others)</td>
</tr>
<tr>
<td>Inclinometer</td>
<td>Rotation</td>
<td>Wyler</td>
<td>± 1°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>± 10°</td>
</tr>
<tr>
<td>Inductive</td>
<td>Deflection</td>
<td>HBM W5</td>
<td>± 5 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HBM W10</td>
<td>± 10 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HBM W20</td>
<td>± 20 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HBM W50</td>
<td>± 50 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HBM W100</td>
<td>± 100 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td></td>
<td>As for deflection</td>
<td>As for deformation</td>
</tr>
<tr>
<td>Jauge Omega</td>
<td>Deformation</td>
<td>TML PI-2-50</td>
<td>± 2 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TML PI-2-100</td>
<td>± 2 mm</td>
</tr>
</tbody>
</table>

Table 7 – Sensors range
3.5 Results

<table>
<thead>
<tr>
<th></th>
<th>$\psi_{x,max}$ [%]</th>
<th>$\psi_{y,max}$ [%]</th>
<th>$V_{Rd,C}$ (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab01</td>
<td>11.2</td>
<td>9.9</td>
<td>975</td>
</tr>
<tr>
<td>Slab02</td>
<td>14.5</td>
<td>16.8</td>
<td>983</td>
</tr>
<tr>
<td>Slab03</td>
<td>45</td>
<td>9.3</td>
<td>596</td>
</tr>
</tbody>
</table>

Table 8 – Maximal rotations and failure load

It is possible to state that the rotation was always larger in the weakest direction. One of the most important analyses is the $V-\psi$ (shear force-rotation) relationship.

Figure 32 and 33 – $V-\psi$ behaviour for slab 01 and 02
Next is also possible to observe the V-δ (shear force-deflection) relationship. These results were possible using the most distant from column LVDTs below the slab on each side (300 mm from the edge)
Figure 36 – $V$-$\delta$ behaviour for slab 02

Figure 37 – $V$-$\delta$ behaviour for slab 03
(Note: both $V$ and $\delta$ axis have a different scale than the figures above)
Observing these plots it is possible to conclude that there is an important rotation before punching shear failure. The first two cases are quite comparable since both have a classic behaviour before failure with rotation around 10-15 % (visible to the naked eye). It is also possible to observe that on these two first cases the rotation in one direction is not very different from the other, even for the case were the reinforcement ratio was nearly the double in one direction (Slab 01).

An analogy can be done when observing the deflection plots. Slabs 01 and 02 have deflections quite similar. The deflection in one direction is also reasonably close to the one in the other direction for each of these first two cases.

The third case is the most peculiar and interesting to analyse. It appears to be mostly a type of failure associated to one-way shear than with punching shear (see Figure 40). The substantial rotation and deflection in the weak axis is due to its weak reinforcement.

Three widely open flexural cracks (> 10 mm) formed along the strong axis, above the column, leading to yielding and a clearly defined yield-line mechanism.

The failure pattern did not fully developed around the column (as indicated in Figure 40), but only perpendicular to the weak direction.

It was only after cutting the slab that a truncated-cone pattern was visible.

The following images depict the crack configuration of every slab.

![Figure 38 – Cracks after testing on slab 01 (top surface)](image-url)
Figure 39 and 40 – Cracks after testing on slab 02 and 03 (top surface)
As mentioned before, cutting the slabs allows to better observe the punching shear cone. The next images and table show the effect of the reinforcement on the cone-shaped element result of the punching shear failure.

A special remark goes once again to the slab 03 (see Figure 41). In this case it is possible to observe the significant difference between the two directions, while the first two slabs present inclinations more or less similar. In any case a relation between the reinforcement ratio and the angles is observed. For high reinforcement ratios the angles are larger.

![Figure 41](image)

**Figure 41 – Cone-shaped element [strong a) and weak b) axis of slab 01, slab 02 c) and strong d) and weak e) axis of slab 03]**

<table>
<thead>
<tr>
<th>Slab</th>
<th>$\rho$</th>
<th>Left 2</th>
<th>Left 1</th>
<th>Column</th>
<th>Right 1</th>
<th>Right 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab 01</td>
<td>1.54%</td>
<td>14°</td>
<td>38°</td>
<td>32°</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.72%</td>
<td>-</td>
<td>20°</td>
<td>17°</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Slab 02</td>
<td>0.72%</td>
<td>35°</td>
<td>10°</td>
<td>30°</td>
<td>13°</td>
<td></td>
</tr>
<tr>
<td>Slab 03</td>
<td>0.76%</td>
<td>37°</td>
<td>49°</td>
<td>35°</td>
<td>20°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.33%</td>
<td>-</td>
<td>40°</td>
<td>12°</td>
<td>35°</td>
<td></td>
</tr>
</tbody>
</table>

**Table 9 – Crack inclinations (to respect to Figure 41)**
4 Analysis

As known, there are various possible models to approach reality in which every engineer falls upon when dimensioning a structure. In this chapter, based on the Swiss (SIA 262:2003), European (EC 2, 2004) and American code (ACI 318-05), the test values are compared with the predictions of various expressions provided by these codes or other models.

The experiment focused essentially on the reinforcement aspect. Although neither of the slabs possessed punching shear reinforcement, the flexural reinforcement was very different between slabs.

4.1 Comparison with codes of practice

Neither the SIA 262:2003, nor the EC 2, 2004, nor the ACI 318-05 was able to fully predict the ultimate load of all slabs accurately.

On one hand the results produced by the usual codes were quite good regarding normal and highly reinforced slabs (01 and 02), but on the other hand the last slab, which had a low reinforcement, lead into an undervalue by those same codes. Another important remark lies on the parameters which each code uses. In SIA 262:2003 it is possible to observe the good accordance with the reinforcement ratio although it still produced undervalued results for the last slab. It is important to note the big difference regarding the other codes that were unable to provide close results to the same specimen.

EC 2, 2004 counts both the effect of effective depth and the reinforcement ratio. The ACI 318-05 is based only in effective depth and as a result the values acquired were quite similar among themselves.

In the EC 2, 2004 equation it was intentionally used $\gamma_c = 1.0$ to obtain actual values and $f_{ck}$ was changed to $f_{cm}$ as to approach in a quite simple way the reality.

Table 10 and Figure 42 show the results of the different calculations and test values.

<table>
<thead>
<tr>
<th>Slab</th>
<th>EC2</th>
<th>Test/EC2</th>
<th>SIA</th>
<th>Test/SIA</th>
<th>ACI</th>
<th>Test/ACI</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>1163.3</td>
<td>0.84</td>
<td>906.8</td>
<td>1.08</td>
<td>996.7</td>
<td>0.98</td>
<td>975</td>
</tr>
<tr>
<td>02</td>
<td>1043.7</td>
<td>0.94</td>
<td>928.9</td>
<td>1.06</td>
<td>1000.0</td>
<td>0.98</td>
<td>983</td>
</tr>
<tr>
<td>03</td>
<td>922.0</td>
<td>0.65</td>
<td>679.0</td>
<td>0.88</td>
<td>1021.8</td>
<td>0.58</td>
<td>596</td>
</tr>
</tbody>
</table>

| Average | 0.81 | 1.00 | 0.85 |
| Standard Deviation | 0.15 | 0.11 | 0.23 |
| Coef. Variation | 0.19 | 0.11 | 0.27 |

Table 10 – Codes shear force results [KN]
In the next figure it is shown the $V_R - \rho$ relationship for EC2, 2004 and all slabs. It is possible to see that this code has a minimum of shear strength even for low reinforcement ratios. Slab 03 approaches this zone due to its reinforcement ratio and it presents a shear strength already under the EC2, 2004 minimum.
4.2 Failure criterion (Muttoni 2003)

Muttoni 2003 proposed this theory based on the parameter rotation. Such parameter can be estimated using a finite element method. A software package (ANSYS®) was used to model the slabs, using 4 node isoparametric shell element (see Figure 44).

For the linear elastic uncracked model, the Young’s modulus of concrete was used in both directions (uncracked stiffness).

For the linear elastic cracked model, the bending stiffness was reduced to consider the effect of cracking in both directions, accordingly to equation 4.2) and 4.3). For this case the in-plane shear stiffness is reduced to consider the effect of cracking, as indicated in the equation 4.1).

\[ G = \frac{1}{8} \cdot \frac{E}{2(1+\nu)} \] \hspace{1cm} \text{with } \nu = 0.2 \hspace{1cm} (4.1)

To assist this programme there were used two expressions to easily calculate the depth of compression zone \(x\) and the cracked stiffness corresponding to direction \(x\) and \(y\).

\[ x = d \cdot (\rho + \rho') \cdot n \left( 1 + 2 \frac{\rho + \rho' \frac{d'}{d}}{n(\rho + \rho')^2} - 1 \right) \] \hspace{1cm} (4.2)

and

\[ EI_{cr} = b \cdot d^3 \cdot E \left[ \frac{1}{3n} \left( \frac{x}{d} \right)^3 + \rho \left( 1 - \frac{x}{d} \right)^2 + \rho' \left( \frac{d'}{d} - \frac{x}{d} \right)^2 \right] \] \hspace{1cm} (4.3)

![Figure 44 – Square-shaped element used in ANSYS® mesh (shell 43)](image)

Table 11 and Figure 45 show the comparison between the model using the failure criterion (Muttoni 2003) and the test results.
<table>
<thead>
<tr>
<th>Models</th>
<th>Test/ME</th>
<th>Mod Crack</th>
<th>Test/MC</th>
<th>Mod Plastic</th>
<th>Test/MP</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab01</td>
<td>1545.6</td>
<td>1074.1</td>
<td>0.91</td>
<td>1232.8</td>
<td>0.79</td>
<td>975</td>
</tr>
<tr>
<td>Slab02</td>
<td>1507.5</td>
<td>986.8</td>
<td>1.00</td>
<td>1107.7</td>
<td>0.89</td>
<td>983</td>
</tr>
<tr>
<td>Slab03</td>
<td>1478.1</td>
<td>843.8</td>
<td>0.71</td>
<td>545.3</td>
<td>1.09</td>
<td>596</td>
</tr>
</tbody>
</table>

Average 0.56 0.67 0.92
Standard Deviation 0.14 0.15 0.15
Coef. Variation 0.24 0.17 0.17

Table 11 – Models shear force results [KN]

Figure 45 – Models shear force results

It is possible to conclude that the failure criterion (Muttoni 2003) with a linear elastic (cracked) behaviour is the best method since, generally, it approaches the actual values the most.

The linear elastic (uncracked) behaviour is not representative of reality. There are no perfectly elastic materials and cracks (loss of resistance) need to be considered since it makes the elements weaker and therefore overvalued results appeared.

All details were carefully reproduced in ANSYS®. In the next figures it is possible to observe some of the outputs acquired.
Figure 46 – Total rotation [mrad] for a charge of 670 KN in ANSYS® ($E_x = E_y$)

Figure 47 – Total rotation [mrad] of slab 01 for a charge of 670 KN in ANSYS® (cracked)
Figure 48 – Total rotation [mrad] of slab 02 for a charge of 670 KN in ANSYS® (cracked)

Figure 49 – Total rotation [mrad] of slab 03 for a charge of 670 KN in ANSYS® (cracked)
Figure 50 – 3D image of slab 02 rotation [mrad] for a charge of 670 KN in ANSYS® (cracked)

In all figures the rotation is represented. Figure 46 is the output from the linear elastic (uncracked) behaviour (symmetric between axis) and it has a lower rotation when comparing with Figure 47, Figure 48 and Figure 49 (asymmetrical) which are outputs from the linear elastic (cracked) behaviour.

When using the failure criterion (Muttoni 2003) there were used the maximal rotations obtained from ANSYS® at the slabs edges considering $\theta = \sqrt{\theta_x^2 + \theta_y^2}$.

Using the values obtained in ANSYS® it was possible to recalculate the ultimate shear force according to Muttoni 2003. The results demonstrate a better approximation for the shear force in all cases.
Figure 51 – Comparison between Muttoni 2003 cracked, non-cracked and plastic models (Slab 01)

Figure 52 – Comparison between Muttoni 2003 cracked, non-cracked and plastic models (Slab 02)

Figure 53 – Comparison between Muttoni 2003 cracked, non-cracked and plastic models (Slab 03)
It is possible to observe that in the case of the third test the plastic model is better when approximating shear strength resistance and the rotation of the slab, even though this last one is only half of the one observed.
4.3 Yield-line analysis

Finally, mainly due to the behaviour of slab 03, it was performed yield-line analysis to attain a different study and to have another mean of comparison with all mentioned methods.

It was chosen the following failure case since it was the closest to the one observed during the test (for slab 03) but also because it was the one who, respecting the physical interaction between column and slab, gave lower results and thus better ones (for slab 01 and slab 02):

![Yield-line model](image)

Figure 54 – Yield-line model [mm]
It was followed the next procedure (the present results are for slab 03):

\[ M_y = (0.9 \cdot d) \cdot \frac{P}{100} \cdot (d \cdot 1) \cdot f_y \]

\[ W_e = \sum P_i \cdot \delta_i \]

\[ W_e = \frac{Q}{4} \cdot \frac{\delta}{1.37} \cdot 1.25 + \frac{Q}{4} \cdot \frac{\delta}{1.37} \cdot 0.47 + (25 \cdot 0.25) \cdot \frac{1.37 \cdot 3}{2} \cdot \delta = 0.314 \cdot Q \cdot \delta + 12.84 \]

\[ W_i = \int m_p \cdot \theta \cdot dx = 100 \cdot \frac{\delta}{1.37} \cdot 3 = 2.19 \cdot m_p \cdot \delta \]

\[ W_e = W_i \]

\[ 0.628 \cdot Q \cdot \delta + 12.84 = 4.38 \cdot m_p \cdot \delta \]

\[ Q_{flex} = \left( \frac{4.38 \cdot m_p - 12.84}{0.628} \right) = 565.8 \text{ KN} \]

Since it is difficult to know exactly the state of the yielded reinforcement it was considered the average between \( f_y \) and \( f_u \) when calculating the \( m_p \) of slab 03.

This result although inferior to expected it is rather close to reality and thus produces a good estimation of the actual load.

According to Muttoni 2007, it is possible to approximate the slab behaviour through the following equation: \( \psi = 1.5 \cdot \frac{r_s}{d} \cdot \frac{f_y}{E_s} \cdot \left( \frac{V}{V_{flex}} \right)^{\frac{1}{2}} \), here designated as \( V_{flex} \)-based behaviour.

In the next figures it is possible to compare this theoretical expression with Muttoni 2003 and the real results from the experimental campaign.
Figure 55 – Comparison between Muttoni 2003 and V\textsubscript{flex}-based behaviour

Figure 56 – Comparison V\textsubscript{flex}-based behaviour and Slab 01
Figure 57 – Comparison $V_{\text{flex}}$-based behaviour and Slab 02

Figure 58 – Comparison $V_{\text{flex}}$-based behaviour and Slab 03
5 Theoretical approach of prestressing

This last chapter was put together to study the influence of prestressing in the tested slabs. It was thought, as a starting point, to submit the slabs to a $\sigma_n = 3.0\, MPa$ and then proceeded to realistic case to obtain the gain of resistance in each code.

5.1 Introduction

Prestressing any structure is a decision taken to improve its performance, to allow thinner elements and to produce a better optimisation of materials. When applying it to a flat slab it is possible to produce two favourable effects regarding the punching phenomenon: firstly, the inclination of the cable produces forces that, oriented to the column, produce a shear force that opposes the one in the unstressed structure, thus reducing $V_d$ where it matters most, near the column; secondly, due to the cable position and compression strength, the deformations and cracks are greatly diminished what results in a larger shear strength resistance.

5.2 Design

There are several aspects to be considered to provide a good durability of the structure such as the protection of the steel and concrete, the grouting, the concrete cover and quality of it.

In the specific case of a flat slab it important to check the serviceability limit state of deflection and the ultimate limit state of punching shear.

The prestress can be considered whether as external action or as part of the resistance. The first case is the most used since the balancing load technique is a good method used in design and because it can be used for SLS (Serviceability Limit States) and ULS (Ultimate Limit States) verifications. The second case it can be only used in ULS verification and it is also necessary to consider the hyperstatic effects as an action.

Some methods of analysis pass through linear elastic non-cracked and cracked models, plastic models and finite elements models, all of them used in this thesis. It was also used the yield-line analysis which is commonly used when the failure mechanism is well known.

The study carried out it did not go through all this procedure since the objective was to estimate the gain of resistance when submitting the slabs to currently used value of prestressing.
5.3 Codes Approach

Each code has a particular way of considering the benefits of prestressing. The SIA 262:2003 uses a very similar expression than the one used without prestress,

\[ \nu_{Rd} = k_r \cdot \tau_{cd} \cdot d \]

with the only difference in \( r_y = 0.15 \cdot I \cdot \left( \frac{m_{bd} - m_{pd}}{m_{rd}} \right)^{3/2} \) where \( m_{pd} \) is the moment produced by the prestress cables and \( m_{rd} \) has already the prestress influence.

The EC 2, 2004 adopts the simplest approach of the three analysed codes considering simply a percentage of the prestress, \( k_1 \) which final expression is:

\[ \nu_{Rd,C} = C_{Rd,C} \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{1/2} + k_1 \cdot \sigma_{cp} \cdot \]

When analysing the ACI 318-05 expression, \( V_c = \left( \beta_p \cdot \sqrt{f_c} + 0.3 \cdot f_{pc} \cdot b_0 \cdot d + V_p \right) \) it is possible to acknowledge some particularities. Taking the normally determinant expression \( V_c = \sqrt{f_c} \cdot b_0 \cdot d / 3 \) from the non-stressed case, the final expression consists in simply adding the prestress effect in it. Since the ACI 318-05 is strongly based on empirical results, \( f_c' \) as an upper limit of 35 MPa due to the lack of tests with higher values and \( f_{pc} \) is also limited, although it does not present a problem in this analysis.

Another interesting remark goes to the final variable \( V_p \) that represents the vertical component of the prestress in the critical section, which is zero when using straight prestressed cables.

5.4 Adopted Model

It was decided to use 30mm-high ducts founded in Freyssinet catalogue to obtain all the measurements required to determinate the load capacity gains.

The schematic model can be seen in the following figure.

![Figure 59 – Code-applied model](image-url)
In Table 12 it is possible to state the average gains of resistance in each code.

<table>
<thead>
<tr>
<th>Slab01</th>
<th>Slab02</th>
<th>Slab03</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC2</td>
<td>EC2(PS)</td>
<td>EC2(PS)/EC2</td>
</tr>
<tr>
<td>1163.3</td>
<td>1400.8</td>
<td>1.20</td>
</tr>
<tr>
<td>906.8</td>
<td>1789.1</td>
<td>1.97</td>
</tr>
<tr>
<td>996.7</td>
<td>1021.5</td>
<td>1.02</td>
</tr>
<tr>
<td>SIA</td>
<td>SIA(PS)</td>
<td>SIA(PS)/SIA</td>
</tr>
<tr>
<td>928.9</td>
<td>1775.8</td>
<td>1.91</td>
</tr>
<tr>
<td>1000.0</td>
<td>1027.7</td>
<td>1.03</td>
</tr>
<tr>
<td>ACI</td>
<td>ACI(PS)</td>
<td>ACI(PS)/ACI</td>
</tr>
<tr>
<td>679.0</td>
<td>1820.3</td>
<td>2.68</td>
</tr>
<tr>
<td>1021.8</td>
<td>1064.7</td>
<td>1.04</td>
</tr>
<tr>
<td>Average</td>
<td>1.44</td>
<td>2.19</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.21</td>
<td>0.43</td>
</tr>
<tr>
<td>Coef. Variation</td>
<td>0.14</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 12 – Gain of resistance due to prestressing

Due to all the particularities already referred it is possible to understand why the ACI 318-05 produced such minor gains (mostly due to the limited $f'_c$, which reduced the normal resistance to values around 600KN). The EC 2, 2004 presented a good percentage of gains normally attained through prestress, nevertheless these were merely deducted from taking a percentage of the prestress into account. The SIA 262:2003 it was the one who produced larger gains. They were probably the most realistic too since this code is the only one that considers all the previous parameters (unlike EC 2, 2004) and has applicability without restraints (unlike ACI 318-05).
6 Conclusions

There are several factors that could be matter of discussion.

One of the most important in the code analysis was the hypothesis considered in all codes: the expressions extracted from them were created for design and not to replicate reality (they are only supposed to produce safety margins), and when changing them from design values to real or average ones they become deranged. The $V_r - \rho$ relationship for EC2, 2004 is an example of that were slab 03 does not attain the minimal shear strength defined by the code.

Regarding the codes it is accurate to say than none of the codes can produce a reliable solution for slab 03 although SIA 262:2003 approached this value quite better than the other two codes.

The yield-line analysis method can estimate the failure load for slab 03 (with steel yielding).

Muttoni 2003 produced good estimations for the rotation of slab 03 (at failure) and thus providing good approximations (it was used the maximal rotation criteria $\theta = \sqrt{\theta_x^2 + \theta_y^2}$ in the model).

In general all methods predicted the punching shear strength of slab 01 and 02 considerably well with special remark for Muttoni 2003 together with elastic-cracked analysis and ACI 318-05 that even with much simpler equations had good approximations.

Only the EC 2, 2004 and the SIA 262:2003 adjusted to the variation of reinforcement through the slab. This is logical since only these two take into account the reinforcement ratio, although the last one was closer to reality.

The first two slabs failed by punching shear even though with different deformations and ductility.

Slab 03 had a peculiar failure behaviour which could be understood as a punching shear or as bending one. A flexural failure exhibits a smooth decrease of the load carrying capacity contrary to a punching failure that a sudden decrease of the same, and there is a formation of a yield-line mechanism on the first and inclined punching crack on the second (Menétry 1995). The decrease of load carrying capacity in slab 03 was neither sudden nor smooth and there was the formation of both yield-line mechanism and inclined punching crack. Both SIA 262:2003 and the yield-line method produce close approximations to the load capacity and the $V_{\text{flex}}$-based with Muttoni 2003 analysis results lead into an unclear failure mechanism.
Slab 01 and 02 had a brittle punching shear failure without (or limited) yielding of the flexural reinforcement.

A final conclusion goes to the control perimeter which is influenced by different reinforcement ratios and consequently the distribution of shear forces.
Notation

Roman capital letters

\( A_{sl} \) longitudinal steel area
\( C \) constant; factor
\( D_{max} \) maximal diameter; dimension
\( E \) modulus of elasticity
\( I \) inertia
\( P \) force
\( V \) shear force
\( W \) work

Roman small letters

\( b \) width
\( b_0 \) length of control perimeter (ACI)
\( d \) effective depth (traction)
\( d' \) effective depth (compression)
\( f_c \) concrete strength
\( f_{ck} \) concrete characteristic strength
\( f_s \) steel strength
\( k \) constant; factor
\( m_{bd} \) reference moment
\( m_p \) plastic moment
\( n \) \( \frac{E_s}{E_c} \) ratio
\( r \) radius
\( r_y \) radius of plasticized area
\( u \) length of control perimeter
\( x \) depth of compression zone; coordinate axis
\( y \) coordinate axis

Greek letters

\( \psi \) rotation
\( \tau \) shear strength
\( \nu \) shear strength per unit of length
\( \gamma_c \) concrete safety factor
\( \rho \) reinforcement ratio (traction)
\( \rho' \) reinforcement ratio (compression)
\( \varepsilon \) strain
\( \theta \) angle
\( \delta \) deflection

Special characters

\( \text{Ø} \) diameter
Bibliography

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Menétrey Ph, *Flexural and punching failure experiments in reinforced concrete slabs*, EPFL, Lausanne, Switzerland, Mar., 1995


*Menétrey Ph, Flexural and punching failure experiments in reinforced concrete slabs*, EPFL, Lausanne, Switzerland, Mar., 1995


## Appendix

Legend:

<table>
<thead>
<tr>
<th>Data</th>
<th>Intermediary equation</th>
<th>Final Result</th>
<th>Previous result</th>
</tr>
</thead>
</table>

### EC2

\[ V_{s} = \sqrt{f_{c}'(100 - \alpha_{p}^2)\gamma_{v}} \]

<table>
<thead>
<tr>
<th>Slab01</th>
<th>Slab02</th>
<th>Slab03</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>0.125</td>
<td>0.125</td>
<td>0.115</td>
</tr>
<tr>
<td>20</td>
<td>16</td>
<td>16</td>
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<tr>
<td>0.1</td>
<td>0.125</td>
<td>0.125</td>
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<tr>
<td>0.222</td>
<td>0.222</td>
<td>0.225</td>
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<tr>
<td>0.213</td>
<td>0.214</td>
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<td>0.0106</td>
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<tr>
<td>1.969</td>
<td>1.967</td>
<td>1.957</td>
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<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
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<tr>
<td>1.469</td>
<td>1.308</td>
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</tr>
<tr>
<td>1163.3</td>
<td>1043.7</td>
<td>922.0</td>
</tr>
</tbody>
</table>

### SIA 262

\[ V_{s} = K_{r} \cdot \gamma_{v} \cdot d \]

<table>
<thead>
<tr>
<th>Slab01</th>
<th>Slab02</th>
<th>Slab03</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.58</td>
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</tr>
<tr>
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<td>1.03</td>
<td>0.74</td>
</tr>
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<td>2.46</td>
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</tr>
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<tr>
<td>1.709</td>
<td>1.712</td>
<td>1.279</td>
</tr>
<tr>
<td>996.8</td>
<td>928.9</td>
<td>679.0</td>
</tr>
</tbody>
</table>

### ACI 318

\[ \min\left(\sqrt{f_{c}'}(1+2/b)\cdot d; V_{s} = v_{R_{d,c}} = (a_{s} \cdot d/b) \cdot \sqrt{f_{c}'} \cdot b_{0} \cdot d/12; v_{R_{d,c}} = \sqrt{f_{c}'} \cdot b_{0} \cdot d/3 \right) \]

<table>
<thead>
<tr>
<th>Slab01</th>
<th>Slab02</th>
<th>Slab03</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.213</td>
<td>0.214</td>
<td>0.2185</td>
</tr>
<tr>
<td>1.712</td>
<td>1.712</td>
<td>1.726</td>
</tr>
<tr>
<td>986.7</td>
<td>1009.0</td>
<td>1021.8</td>
</tr>
</tbody>
</table>

### Table 12 – Codes

<table>
<thead>
<tr>
<th>( f )</th>
<th>( t )</th>
<th>( \alpha_{p} )</th>
<th>( \gamma_{v} )</th>
<th>( \gamma_{v}' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab01</td>
<td>67.5</td>
<td>40</td>
<td>0.213</td>
<td>1.712</td>
</tr>
<tr>
<td>Slab02</td>
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<td>40</td>
<td>0.214</td>
<td>1.712</td>
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<tr>
<td>Slab03</td>
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<td>40</td>
<td>0.2185</td>
<td>1.726</td>
</tr>
</tbody>
</table>

### Table 13 – Failure criterion (Muttoni 2003) + Linear elastic (uncracked) behaviour

<table>
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</tr>
</thead>
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<td>3.4</td>
<td>3.4</td>
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<tr>
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<td>0.214</td>
<td>0.2185</td>
</tr>
<tr>
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<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
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<td>1.11435</td>
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<td>1.7264</td>
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<tr>
<td>1.443321</td>
<td>1.57636</td>
<td>1.784102</td>
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<tr>
<td>1545.57</td>
<td>1507.49</td>
<td>1476.07</td>
</tr>
</tbody>
</table>
\[
\text{Cracked} \\
\text{w} = \left(1 + \frac{h}{a} + \frac{a_s}{a} \right)^{1/2} \left(1 + \frac{a_s}{a} \right)^{1/2} \left(1 + \frac{a_s}{a} \right)^{1/2} \\
\]