ABSTRACT

The electromagnetic propagation within photonic crystals, namely stratified periodic structures, behaves in the very same way as the localization of conduction electrons submitted to a potential caused by the interaction with neighbouring molecules and atoms. That constitutes a field of knowledge widely studied and deeply analyzed through decades of scientific investigation as it allowed the development and implementation of electronic circuits and devices based on semiconductors, which are modern life technology fundamental cells. The beginning of quantum mechanics in the past century, together with the arise of the paradoxical and non intuitive concept of matter with undulatory behaviour, described by the wave-particle duality, led researchers into suggesting the concept of artificially made lattices that would become the next step in terms of crystal structure organization. In fact, both semiconductors and photonic crystals are made of smaller periodically arranged units, in a diverse manner, as the latter are composed of common macroscopic media instead of atoms.

Therefore, the alluring idea of an analogy between these two apparently unfamiliar worlds has soon developed into practice. Notions like the Bloch’s theorem for periodic lattices, the Brillouin zone or the theory of bands have been tailored to fit the demands of this new kind of organized solid. The present thesis provides the tools for unveiling the properties, behaviour and phenomena exhibited by photonic crystals, specifically the one-dimensional alternate stratified case, allowing the characterization of the electromagnetic propagation throughout these structures. Great attention is devoted to the emergence of allowed and forbidden bands at a spectral level, according to the lattice’s dimensions and geometry as well as the angle of incident radiation, as it may be the basis for achieving attributes not available in existing natural materials.

With the purpose of further research, a recently discovered and intensely studied type of material, the DNG media, is introduced in that structure as a layer, providing intriguing innovative capabilities related with the confining of energy and reversal of the direction of phase velocity. Moreover, the numerical simulation analysis is carried out taking into account both dispersion and losses. Despite being considered negligible by some authors, these effects must be considered because DNG media operate in frequency bands close to the constitutive parameters resonances. In a near future, devices based on photonic crystals are expected to be capable of molding the flow of light, leading to compact all-optical microchips, which that shall revolutionize information and telecommunications.

Index Terms— Photonic Crystals, Periodic Unidimensional Stratified Structures, DNG Media, Dispersion Models, Losses, Theory of Bands, Bloch Waves, Bragg Condition.

1. INTRODUCTION

With the demand for technological resources, particularly with regard to information technology and communications, growing exponentially, science focuses in the design and development of optical devices, whose capacity, speed and bandwidth display a remarkably high and unexplored potential. However, this trend has found a barrier in the limitations of the optical properties of materials, responsible for an unwanted delay, given the inability to reproduce accurately the desired functions. The current situation in the optical field contrasts with the variety of solutions available within the electronics field, where the implementation of virtually any property or behaviour pattern is doable, through the parameterization and modelling of the available components. These are long and historically known and have been analyzed and tested under the most diverse range of conditions.

This phenomenon is related to the type of interaction established by the electrons with the networks of crystalline materials in the electrical circuit, which form the basis of their classification as metals, semiconductors or insulators [1]. Indeed, through occasional changes in the structure, it is possible to tune the properties provided by the media in order to face requirements. In this scenario, there is an obvious analogy between the optical and electrical worlds, that constitutes the basis for adapting the concept of crystalline electronic network to electromagnetic
propagation, resulting in the creation of the above mentioned photonic crystals.

The study of electromagnetic propagation in photonic crystals, name suggested by Yablonovitch in 1989 [2], is a scientific field in a process of considerable development, with the design, implementation and production of optical components being increasingly more common. With multiple layers, characterized by various thicknesses and electromagnetic characteristics, arranged in order to eliminate, alter or increase the refractive effects or change their properties of dispersion and polarization. This constitutes a strong pillar of motivation, indicating the proliferation of research in this area.

With the recent achievement of artificial structures characterized by a negative index of refraction, entailing the development of a whole new field of scientific analysis, based on the new concept of metamaterial or, more specifically, of DNG medium, rises the attractive challenge for testing and studying the influence that its inclusion in regular crystals can represent in terms of their inherent properties.

The metamaterials can be simply defined [3] as an artificial structure of materials combined to obtain favourable and rare properties. Using an analogy, one can assume that they are composed of parts in the same way as the material consists of atoms, and that they represent the next level of organizational structure. Indeed, the prefix 'meta' attaches several connotations like 'after', 'beyond' or even 'superior type'.

Presently, among the most studied and analyzed classes of metamaterials, one should highlight the artificial dielectric and magnetic media, the chiral, anisotropic and bianisotropic materials, and above all, the double negative media, or DNG, or Veselago media [4]. This designation stems from the fact that they have been suggested for the first time in 1968 by the Russian physicist, who studied the electromagnetic propagation in media characterized by electric permittivity and magnetic permeability both negative [5]. He concluded that the direction of the Poynting vector of a monochromatic plane wave propagating along the media was opposed to the phase velocity and the wave vector direction, suggesting the notion of BW, or Backward Wave, and the associated concept of negative index of refraction. These materials are also known as LHM, or Left Handed Media because, unlike the common Double Positive media, or DPS, where the trihedral composed of the electric and magnetic fields and the wave vector is right, in a DNG medium the electromagnetic energy flows through a direction diametrically opposite to the phase of radiation, and the trihedral is left.

2. THE THEORY OF BANDS IN CRYSTALS

The unveiling of the parallel, simultaneous and indivisible undulatory and corpuscular nature of the electromagnetic propagation is one of the most remarkable revolutions offered to the world of physics by the scientific community of the 20th century. Following the studies of the black body radiation of Max Planck, the theory of the photoeffect of Albert Einstein and the Compton effect, it has become undeniably evident that the energy is transmitted, transported and mutated always in the form of a multiple of its fundamental indivisible quantity, or quantum, commonly known as the photon, calculated using the innovative and challenging relationship $\mathcal{E} = h\omega$ [6], where $h$ is the reduced Planck constant and $\omega$ the angular frequency, suitable for any particle. Starting from this premise, it immediately follows that a particle with energy $\mathcal{E}$ presents an angular frequency $\omega$, in a seemingly paradoxical mixture of matter and wave characteristics. The constant of proportionality between these two variables was determined on a trial basis. In the context of these discoveries, rises a clear necessity to reformulate the Newtonian or classic mechanics, driven by purely corpuscular principles, and even the electromagnetic relations proposed by J. Maxwell, built on the basis of continuous energy and taking a purely undulatory concept of radiation. As part of a whole new outlook regarding the guiding principles of physics, Einstein's theory of special relativity, a particular case of general relativity in which the gravitational effects are neglected, is essential in the unravel of this issue.

The fundamental principle of wave mechanics and more specifically, of the wave-particle duality, is the evidence that the electromagnetic propagation is characterized by a seemingly contradictory mix of properties associated with waves and matter, illustrated by these two fundamental equations

$$\begin{align*}
\mathcal{E} &= h\omega \\
p &= \hbar \omega.
\end{align*}$$

In his thesis, Louis de Broglie generalizes these assumptions to the whole matter, and considers that photons constitute a special case of the wave-corpuscle duality [7]. He adds that any particle should be characterized by its own energy and momentum and that a wave vector can be associated to any element of matter. Thus, the simultaneous corpuscular and wave-like nature is an inescapable concept of the new universe governed by undulatory mechanics and one can even admit that the mass of the photon is not null. This concept is not intuitively perceivable at a macroscopic basis, as the residual value of Planck's constant makes particularly noticeable the separation between the two concepts.

Nevertheless, even at a microscopic level, the wave-particle duality is insufficient from an axiomatic point of view, since the assumption that the matter has wave-like properties lacks rationality. Thus, the contributions of physicists like K. Heisenberg and E. Schrödinger, have proven to be decisive, as they were involved in the achievement of wave mechanics and later the quantum theory, widely accepted by the scientific community. In this context, it has been developed the perception [8] that it is not possible to know accurately and simultaneously the
position and the momentum of a certain particle. The Heisenberg’s Uncertainty Principle implies a rejection of the traditional notion of trajectory. Moreover, the particles are characterized by a wave function $\Psi$, in the form of a beam of fundamental plane waves, which is a function of the time and of their positions. Within wave mechanics, this concept provides no physical interpretation, and the adoption of a stochastic approach is essential. In fact, the square of the wave function module attached to a certain particle reflects a probability distribution function [9] that tailors its location. The unidimensional Schrödinger’s equation,

$$\frac{i\hbar}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi(x, t),$$  \hspace{1cm} (2)

where $m$ is the mass and $t$ is the time dimension, allows the description of the dynamics of a quantum system if there is a unique spatial dimension $x$; which is the case of interest in the study of the energy bands. In classical mechanics, the state of a particle is entirely defined by specifying the position and speed. In the context of quantum theory, this state is characterized by its wave function.

When a particle, such as an electron, is in a process of interaction with a crystalline network of atoms and molecules represented, for instance, by a potential $V(x)$ described by the Kronig-Penney model [10], i.e., a Dirac comb, two factors must be considered. On the one hand, the frontier conditions of the system, which are periodically repeated, and on the other hand the Bloch’s theorem [11],

$$\Psi(x) = e^{i\mathbf{q} \cdot \mathbf{r}} u(\mathbf{r}),$$ \hspace{1cm} (3)

which states that the wave function of a particle submitted to a periodic potential, known as Bloch wave, can be split in the form of a product of a function of magnitude $u(x)$ and a periodic function with a period equivalent to the period of the crystal. Combining (2), (3) and the Kronig-Penney model, an eigenvalues system emerges. When solved, originates the fundamental equation of the energy bands,

$$\cos(\zeta) = \cos(\xi) + \frac{\lambda \sin(\xi)}{\zeta},$$ \hspace{1cm} (4)

where $\xi$ and $\zeta$ establish normalizations for the wave number in free-space $k$ and in the periodic medium $q$, respectively, and $\lambda$ models the magnitude of the potential Dirac comb. It defines a relation between the propagation modes in the periodic crystal, through the Bloch wave number, and the energy of the system, through the free-space wave number. Since the relations of the wave-particle duality (1) postulate that the energy is equivalent to the angular frequency, apart from the Planck constant, it follows that (4) sets a dispersion equation of the wave function.

In this context, the concept of energy band emerges. The values of the free-space wave number, and consequently the energy, that lead to values of the first member of (4) within the $[-1, +1]$ range, creating real Bloch wave numbers, constitute the so-called allowed bands of energy. In contrast, the remaining values of the free-space wave number conduct to complex Bloch wave numbers, meaning no solution for the wave function. These intervals, shown as shadowed areas in Figure 1, are usually known as hiatus or forbidden bands. The solutions for $q$ are evanescent, that is to say, there are certain energy levels at which particles are not allowed to propagate along the structure.

The corresponding dispersion equation, relating the Bloch wave number $q$ to the energy of the particle and, consequently, its angular frequency, is depicted in Figure 2. The width of the forbidden bands of energy is variable and depends on the intensity of the potential $\lambda$. However, their locations in the diagram are fixed in the values of the normalized Bloch wave number multiples of $\pi$.

3. UNIDIMENSIONAL PHOTONIC CRYSTALS

Similarly to the interaction of electrons in a crystal, the electromagnetic waves that propagate along periodic structures are organized in photonic bands, separated by gaps corresponding to evanescent modes. That is to say, groups of frequencies for which a destructive interference develops between the incident and the reflected waves in successive layers. This phenomenon is known as Bragg reflection or Bragg condition. It is within this paradigm of analogies that arises the conceptualization of photonic forbidden bands of frequency or PBG, Photonic BandGaps, and the consequent transposition of concepts, such as the reciprocal networks, the Brillouin zones, the dispersion equations or the wave functions from electronic crystalline...
networks to the context of electromagnetic waves. In both cases, a periodically spaced potential causes the creation of forbidden bands of energy within the dispersion relation, respectively for the wave functions of photons and electrons.

3.1. Spectral Decomposition

Once the analysis is restricted to the behavior of non-magnetic photonic crystals and since the periodicity of the medium stems from a spatially homogeneous variation of the electric permittivity tensor, the general unidimensional layered photonic crystal sketched in Figure 3, where \( n_i \) stands for the index of refraction of the layer \( i = [1,2] \), can be characterized by a Fourier series expansion,

\[
\varepsilon(r) = \sum \varepsilon_i \exp(-iG \cdot r) ,
\]

where \( r \) represents the spatial dimension, \( x \) if unidimensional, \( \Lambda \) is the period of the structure, and \( G \) runs all the vectors of the reciprocal network \( \mathbf{G} \).

\[
g = \frac{2\pi}{\Lambda} \Rightarrow G = lg = \frac{2\pi}{\Lambda} x ,
\]

incorporating the multiple spatial variation of the periodic structure.

Furthermore, the electric field that flows along the medium can also be described by a Fourier integral, decomposing it in coefficients of the propagation constant \( k \),

\[
E = \int A(k) e^{-i(k \cdot r) d^2k} .
\]

The above mentioned Bloch’s theorem can be applied to a photonic structure,

\[
E = e^{-i(k \cdot r)} E_k(r) ,
\]

with a phase factor controlled by the Bloch wave vector \( K \) and a periodic magnitude component \( E_k(r) \). Knowing that the electromagnetic propagation obeys the wave equation [12], which can be written as

\[
\nabla \times (\nabla \times E) - \omega^2 \mu \varepsilon E = 0 .
\]

Considering (5), (7), (8) and (9), under the Bragg condition,

\[
\begin{align*}
|k - g| &= K \\
\left( k^2 - \omega^2 \mu \varepsilon \right) &= \frac{1}{2} \theta - \frac{\pi}{\Lambda} .
\end{align*}
\]

the normalized Bloch wave vector module is a multiple of \( \pi \), like in the electronic crystalline network case, meaning a destructive interference between the incident and reflected waves, and a system of two equations is attained [13],

\[
\begin{pmatrix}
K^2 - \omega^2 \mu \varepsilon \epsilon_0 \\
-\omega^2 \mu \varepsilon \epsilon_{-1}
\end{pmatrix}
\begin{pmatrix}
A(K) \\
A(K - g)
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\]

When unraveled, it originates the dispersion equation that describes the spectral behavior of a general photonic crystal,

\[
(K^2 - \omega^2 \mu \varepsilon \epsilon_0)(K - g)^2 - \omega^2 \mu \varepsilon = 0 .
\]

Applying the Bragg condition (10) to (12), one can obtain the frequency limits of the first forbidden band as a function of the electromagnetic characteristics of the material,

\[
\omega_1 = \left( \frac{1}{2} \theta \right)^2 \frac{1}{\mu(\varepsilon_0 - \varepsilon_{1})} \\
\omega_2 = \left( \frac{1}{2} \theta \right)^2 \frac{1}{\mu(\varepsilon_0 + \varepsilon_{1})}
\]

as in Figure 4, for a structure of two layers of identical thickness \( a = b = 0.5 \Lambda \) and an arbitrary period of \( \Lambda = 0.185 \mu m \), with the dashed line representing the complex component of the Bloch wave number in the first forbidden band.

3.2. Matrix Analysis

The case of interest within the photonic crystals consists of a periodic structure of alternating layers of materials characterized by different indexes of refraction and a non-orthogonal angle of incidence, which creates a dependence of the dispersion equation on the polarization. In Figure 5, it is assumed that the electromagnetic energy propagates along the \( xx \) plane, that \( x \) is the spatial dimension associated with
the periodicity of the structure, the $K_y$ component of the Bloch wave vector is a constant and the $K_z$ component is null. Moreover, it is to be considered that the electric field in each layer of the structure can be written in the form of a column vector of incident and reflected waves.

The distribution of the electric field in the structure is described by the expression

$$E(x, z) = \left[ a_n^{(I)} e^{-ikx(x-z)} + b_n^{(R)} e^{-ikx(x-z)} \right] e^{-ikz} ,$$  \hspace{1cm} (14)

with the longitudinal wave number given by

$$K_z^2 = k_{nx}^2 + K_z^2 + K_x^2 \Rightarrow k_{nx}^2 = \left[ \frac{n_n}{\epsilon} \right]^2 - K_z^2 ,$$  \hspace{1cm} (15)

and the transversal component of the Bloch wave number,

$$K_x = KS \sin(\theta) .$$  \hspace{1cm} (16)

By applying the frontier conditions regarding the continuity of the components of the electric field as well as its derivatives, one can obtain a transfer matrix $M$ that relates the magnitude of the incident and reflected waves in two adjacent cells of the structure,

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = M \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} ,$$  \hspace{1cm} (17)

with the coefficients

$$\begin{align*}
A &= e^{ik_{nx}z} \cos(k_{nx}b) + \frac{1}{2} \left( \frac{k_{nx}^2 - k_{nx}^2}{k_{nx}^2 + k_{nx}^2} \right) \sin(k_{nx}b) \\
B &= e^{-ik_{nx}z} \left[ \frac{1}{2} \left( \frac{k_{nx}^2 - k_{nx}^2}{k_{nx}^2 + k_{nx}^2} \right) \sin(k_{nx}b) \right] \\
C &= e^{ik_{nx}z} \left[ -\frac{1}{2} \left( \frac{k_{nx}^2 - k_{nx}^2}{k_{nx}^2 + k_{nx}^2} \right) \sin(k_{nx}b) \right] \\
D &= e^{-ik_{nx}z} \cos(k_{nx}b) - \frac{1}{2} \left( \frac{k_{nx}^2 + k_{nx}^2}{k_{nx}^2 + k_{nx}^2} \right) \sin(k_{nx}b) \end{align*}$$  \hspace{1cm} (18)

The above described Bloch’s theorem can be adopted again in the context of the matrix characterization of the photonic crystal,

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = E(x + A) \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} + e^{-ik_{nx}z} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} ,$$  \hspace{1cm} (19)

which, in association with (17), conducts to an eigenvalues system,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{ik_{nx}z} \begin{pmatrix} a_n \\ b_n \end{pmatrix} ,$$  \hspace{1cm} (20)

that leads to the general expression of the dispersion equation of an unidimensional photonic crystal with two layers

$$K = \frac{1}{\Lambda} \cos^{-1} \left( \frac{A + D}{2} \right) ,$$  \hspace{1cm} (21)

as a function of both the angular frequency and the angle of incidence of the electromagnetic energy, as shown in Figures 6 and 7, since the coefficients of the matrix $M$ (18) depend on these variables, as demonstrated by (15) and (16).
dispersion characteristic assumes a sinusoidal shape, suggesting the classical equation of the theory of bands (4).

When the direction of the electromagnetic energy flow is orthogonal to the structure, the dispersion relation (21) comes simply

$$\cos(\kappa \Lambda) = \cos^2 \left( \frac{\omega t}{2} \right) - \frac{1}{2} \frac{n_2}{n_1} \sin \left( \frac{\omega t}{2} \right) \sin \left( \frac{\omega t}{2} \right).$$

The quarter-wave stack is a particular case of the unidimensional layered photonic crystal in which the thickness of each layer is a function of its index of refraction,

$$n_1 a = n_2 b = \frac{\pi}{2},$$

resulting in a simplified version of the general dispersion equation (22).

$$\cos(\kappa \Lambda) = \cos^2 \left( \frac{\Omega}{2} \right) - \frac{1}{2} \frac{n_2}{n_1} \sin \left( \frac{\Omega}{2} \right) \sin \left( \frac{\Omega}{2} \right).$$

with the normalized angular frequency $\Omega = \omega/c$. The dispersion characteristic of this structure, depicted in Figure 8, is characterized by a remarkable symmetry and regularity of its forbidden and allowed bands, bringing obvious advantages in what concerns the study of the refractive properties of photonic crystals. This is illustrated by Figure 9, where the transmissivity of the quarter-wave stack is sketched, as proposed by Tretyakov [14].

The transmissivity is given by

$$T = 1 - R = \frac{1}{1 + |\tau|^2} \frac{\sin(N\kappa\Lambda)}{\sin(K\Lambda)},$$

where the reflectivity $R$ is the square of the reflection coefficient $r_\phi$, with $N$ standing for the number of cells in the structure, which in turn is calculated through the Chebyshev identity [15] and the matrix $M$ (17),

$$r_\phi \equiv \left( \begin{array}{c} b_h \end{array} \right)_{h=0} = \frac{\sin(N\kappa\Lambda)}{N \sin((N-1)\kappa\Lambda)}.$$

4. DISPERSIVE DNG MEDIA

Concerning the influence over the electromagnetic propagation, the main effect that most materials cause is the tailoring of the distribution of fields, in a well determined way, which can be described by the electric permittivity and the magnetic permeability. With the advent of the metamaterials, these characteristics became more complex and widely moldable, by artificial means. This underpins the pursuit of properties and attributes desired for the operation of certain optical devices.

As illustrated in Figure 10, a DNG medium, or Double Negative, can be defined as a material with negative real components of the electromagnetic parameters,

$$\epsilon = \text{Re}(\epsilon) + i \text{Im}(\epsilon) = \epsilon' + i \epsilon''.$$  

$$\mu = \text{Re}(\mu) + i \text{Im}(\mu) = \mu' + i \mu''.$$  

4.1. Negative Index of Refraction

To obtain an expression for the index of refraction of a DNG medium, it proves necessary to carry out an analysis of the behavior of the electromagnetic parameters in the context of the complex plane. In fact, it is possible to decompose $n$ in its electric permittivity and magnetic permeability components

$$n = n_\epsilon n_\mu.$$

According to the scheme depicted in Figure 11,

$$n_\epsilon = n'_\epsilon + i n''_\epsilon = p_{\text{A}} \sin \left( \frac{\theta_\epsilon}{2} \right),$$

$$n_\mu = n'_\mu + i n''_\mu = p_{\text{B}} \cos \left( \frac{\theta_\mu}{2} \right).$$

Figure 8 – Quarter-wave stack dispersion characteristic.

Figure 9 – Quarter-wave stack transmissivity characteristic.
one can obtain expressions for the variables

\[
\begin{aligned}
\rho_\epsilon &= \sqrt{\left(\epsilon'\right)^2 + \left(\epsilon''\right)^2}, \\
\cos(\theta_\epsilon) &= \frac{\rho_\epsilon}{\epsilon'}, \\
\sin(\theta_\epsilon) &= \frac{\rho_\epsilon}{\epsilon''}.
\end{aligned}
\] (30)

and the real and imaginary components of \( n \),

\[
\begin{aligned}
n_r' &= \left(\frac{\epsilon'}{2}\right)^\frac{1}{2} \left[1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2\right] \frac{1}{2} \left[1 + \sgn(\epsilon')\right] \left[1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2\right]^{\frac{1}{2}}, \\
n_r'' &= \left(\frac{\epsilon'}{2}\right)^\frac{1}{2} \left[1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2\right] \frac{1}{2} \left[1 - \sgn(\epsilon')\right] \left[1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2\right]^{\frac{1}{2}}.
\end{aligned}
\] (30)

The analysis of the magnetic permeability component of the index of refraction leads to analogous results. From (30), it is clear that \( n_r' > n_r'' \) and \( n \) can be written as

\[
n = n_r n_\mu = -\left(n_r' n_r'' - n_r' n_r''\right) + \left(n_r'' n_r'' + n_r' n_r''\right).
\] (31)

This proves that a DNG medium is characterized by a negative real component of the index of refraction and explains why it is called a NIR, or Negative Index of Refraction. In reality, the DNG spectral region is a subset of the NIR spectral region [16], as it is illustrated in Figure 12, with a Lorentz model for the electromagnetic parameters and arbitrary values for its variables.

### 4.2. DPS-DNG Photonic Crystals

Once defined the conditions that serve as basis for the definition of DNG media, it is appropriate to insert a material of this kind in a photonic crystal, as sketched in Figure 13, creating a structure that allows a more flexible control of the flow of the electromagnetic radiation.

This structure is characterized by a particular case of the dispersion equation (22)

\[
\cos(kA) = \cos(k_1 a) \cos(k_2 b) + \frac{1}{2} \left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right) \sin(k_1 a) \sin(k_2 b)
\] (32)

or, specifically for the quarter-wave stack,

\[
\cos(kA) = \cos^2 \left(\frac{\Omega \pi}{2}\right) + \frac{1}{2} \left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right) \sin^2 \left(\frac{\Omega \pi}{2}\right).
\] (33)

As it is clear in (31), this structure provides discrete allowed bands, meaning the forbidden bands expand through the entire spectrum, except for periodically spaced points. This results in peaks of transmissivity, as depicted in Figure 14, that are the narrower the larger the number of cells of the photonic crystal.

### 4.3. The Influence of Dispersion and Losses

Although a quarter-wave stack structure provides very interesting properties concerning the dispersion and transmissivity characteristics, it is not doable, as the thicknesses of the layers depend on the respective index of refraction, which in turn is a function of the frequency, because the phenomenon of dispersion cannot be ignored. Admitting, firstly, that the losses are negligible and the dispersion is tailored by a simple linear model,
one reaches a total distortion transmissivity characteristic, as shown in Figure 16. In Figure 17, the slope of the linear dispersion model is increased, enlarging the magnitude of this effect. Figure 15 depicts the legend, which is valid for all transmissivity graphs of Figures 16, 17, 19 and 20.

Nevertheless, the causality principle impels the consideration of the losses of the material together with the dispersive effect. This assumption is supported by the Hilbert, or Kramers-Kronig transform [17],

\[
\begin{align*}
   n'(\omega) &= 1 + 2\omega \left( \frac{x n''(x)}{x + \omega} \right) \\
   n''(\omega) &= -2\omega n''(\omega)
\end{align*}
\]

(35)

that attest the interdependence of the real and imaginary components of the index of refraction in a linear time invariant system. Once one of them is set, the other is immediately determined by (35).

Then, it proves essential to include losses in the dispersive model in order to attain a correct vision of the way the phenomena involved influence the refractive characteristics of the structures. The Lorentz model,

\[
\begin{align*}
   \varepsilon(\omega) &= 1 + \frac{\Omega_P^2}{\Omega_P^2 - i\Omega_P\omega - \Omega^2} \\
   \mu(\omega) &= 1 + \frac{\Omega_P^2}{\Omega_P^2 - i\Omega_P\omega - \Omega^2}
\end{align*}
\]

(36)

is widely used as a proxy for the electromagnetic behavior of this type of materials, because of its resemblance to reality. The model is sketched in Figure 18 for arbitrary values of the variables in (36) and it is possible to observe once again the DNG region as a subset of the NIR region.

Figure 19 shows the transmissivity plot of a photonic crystal with its DNG layers modeled by the Lorentz model, but without the consideration of losses, whereas Figure 20 illustrates the same graph for a non null losses coefficient.

The propagation constant becomes complex in the entire spectrum and the differentiation between allowed and forbidden bands lacks sustainability, forcing the definition of a ratio between the imaginary and the real components of \( K \) [18], as shown in Figure 21. In fact, propagation emerges as evanescent for all the frequencies, leading to an overall decrease in the module of the transmissivity characteristic.
This brings out that it is unavoidable to consider a Lorentz model or an equivalent when tailoring and analyzing the refractive properties of unidimensional layered photonic crystals, as the dispersion distorts the shape of the transmissivity and the material losses substantially reduce the percentage of electromagnetic energy that crosses the structure, therefore limiting its extension, and namely the number of cells.

5. CONCLUSION

The photonic crystals are artificially made structures in which the regular flow of radiation and electromagnetic energy can be shaped and controlled, similarly to when the bands of energy are manipulated in semiconductor electronic crystals. Developments in this area involve the discovery and creation of networks of crystal with dimensions, electromagnetic properties and geometrical arrangements in such a way that properties considered useful are attained, like the emergence of photonic forbidden bands, PBG.

Future applications include the control of spontaneous emission, the generalization of the obtained results to 2D and 3D periodic structures, which implies the consideration of a numerical simulation approach, and the PCF or Photonic Crystal Fibers, optical fibers in which the core is simply air or vacuum instead of glass, covered by a 3D photonic crystal with a PBG operating at a desired frequency, what is likely to limit material losses.

Finally, in the field of photonic crystals, one of the most attractive future applications regards the dream of full integration. Indeed, the creation of an integrated photonic circuit, or PIC, capable of processing a purely optical signal, gained a remarkable boost with the advent of photonic crystals. This concept is based on the photonic properties of artificial structures, designed to remove, retain or amplify the flow of light over optical circuits, in line with the definition of logic functions and, ultimately, the creation of photonic transistors in forthcoming years.

6. REFERENCES