Establishment of unit hydrographs

Nuno Miguel Silva Paço
Technical University of Lisbon (TULisbon). Instituto Superior Técnico (IST).Civil Engineering Lisbon,Portugal
Department (DECivil).(+351)936219293
nuno__paco@hotmail.com

Introduction and objectives

The hydraulic studies, including those needed by the implementation of hydraulic infrastructures, often require the flood analysis. One of the most common methods applied for that purpose is the unit hydrograph method (UH). When there are observed rainfall and runoff data, the UH can be derived by direct methods; otherwise synthetic hydrographs obtained by indirect methods can be applied.

The main goal of this thesis was to appreciate the applicability of the unit hydrograph model, more precisely of the synthetic unit hydrographs, to the flood analysis in Portuguese watersheds. For that purpose the time-area method, the Clark´s synthetic hydrograph and the Soil Conservation Service (SCS) synthetic hydrograph were applied. Also, two types of direct unit hydrographs were included in the analysis, namely the so-called average unit hydrograph and the triangular unit hydrograph. Two Portuguese watersheds were adopted as case studies, one located in the north of Portugal and the other in centre. Eleven floods events (characterized by the respective flood hydrographs and rainfall hyetographs) were used in each one of the previous watersheds. Five of those events were utilized to establish the direct unit hydrographs and to calibrate the storage coefficient of the Clark unit hydrograph (calibration stage) and the remaining six hydrographs to evaluate the performance of the different models (validation stage).

It should be pointed out that all the unit hydrographs considered in the study assumed an effective precipitation of 1 mm with duration of D=1h.

Basic concepts and models applied

Only the effective parcel of the total rainfall and the direct runoff component of the observed flood hydrograph are used to compute the unit hydrographs by direct methods. Also, the application of the unit hydrograph model requires availability of effective rainfalls. Therefore, it is necessary to calculate the rain losses and to split the observed hydrograph into its direct runoff and base flow components. For that purpose, several models have been developed.

The model applied in the research carried out to estimate the effective rainfall hyetograph considered the rainfall loss in each time step as a fixed percentage of the respective total rainfall [4]. To separate of direct and base flow components of each flood hydrograph a model based on a straight line connecting the two points that represent the beginning and the end the direct flow was applied. As after the end of the direct flow the shape of the hydrograph becomes approximately exponential, to identify the ending point of the direct flow a semi-logarithmic time-discharge plotting was used. According to [7] this procedure is a satisfactory and straightforward method that gives consistent results.

For each case study, several flood hydrographs and corresponding rainfall hyetographs were selected. For each flood event the effective rainfall hyetograph and the direct runoff hydrograph were estimated as well as the unit hydrograph obtained by applying a direct method to the previous data. The linear programming solution (PL) [1] was adopted. As the different flood events resulted in different unit hydrographs, to identify a UH some how characteristic of the watershed under consideration two techniques were applied: the so-called average unit hydrograph and the triangular unit hydrograph. Both techniques require the previous computation of the dimensionless S-hydrograph that corresponds to each UH according to the following equation:

$$S' = \frac{S}{I\,A} \qquad \textbf{(1)}$$

where S' is the dimensionless S-hydrograph; S the dimensional S-hydrograph computed using the UH provided by the LP solution; I the intensity of unit effective rainfall; and A the catchment area. In each instant, the average S-curve was given by the mean of the ordinates of the several dimensionless S-hydrographs. Based on that curve the corresponding unit-hydrograph (average unit hydrograph) was then computed. The triangular unit hydrograph was computed according to [3], based on three parameters: the base time, the time-to-peak and the peak discharge of the unit hydrograph.

As previously mentioned, different synthetic unit hydrographs were applied: the time-area method, the Clark's synthetic unit hydrograph and the Soil Conservation Service's (SCS) synthetic hydrograph. The Clark's UH was combined in two different models, as shortly described.

The SCS unit hydrograph is a dimensionless hydrograph, originally developed from observed data collected in small agricultural watersheds. Later, this dimensionless hydrograph was improved for more general applications. The general hydrograph is scaled by the lag time to produce the unit hydrograph, and considering the ratios of q/q_p (flow/peak flow) on the ordinate axis and t/t_p (time/time to peak) on the abscissa. This hydrograph considers that 37.5% of the runoff volume occurs before the peak flow and the base time of the hydrograph is five times the lag. The standard lag is defined as the time interval between the centroid of precipitation mass and the peak flow of the unit hydrograph. Often, the lag time is approximated by 60% of the time of concentration [2].

Clark, 1945, developed an instantaneous unit hydrograph, IUH, based on the assumption that the unit hydrograph can be derived from the direct runoff hydrograph caused by an instantaneous unity excess precipitation, which is then routed through a linear reservoir to account for the storage effect of the basin. According to Clark, the translation component can be described by the time-area diagram (TAD) which represents the fraction of watershed area that contributes to the discharge at the watershed outlet as a function of time since the start of the effective precipitation. The time-area diagram is bounded in time by the watershed time of concentration, t_c, and can be obtained by drawing the equal flow-time lines or isochrones. The flow attenuation can be represented by means of linear reservoir.

Based on the previous assumptions and on the continuity principle, the Clark instantaneous unit hydrograph is given by following equations:

$$Q_2 = C\overline{I_2} + (1-C)Q_1 \quad \textbf{(2)}$$

$$C = \frac{2\Delta t}{2K + \Delta t} \quad \textbf{(3)}$$

where I_2 is the inflow rate at the end of each time step, Δt, and Q_1 and Q_2 are the outflow rates at the beginning and at the end of that step. It should be stressed that, in order to prevent negative ordinates, the previous equations implicitly lead to a minimum value of the Clark´s storage coefficient, that minimum being equal to half of the time step.

To obtain the unit hydrograph with the duration D, UHD, from the Clark's IHU the following equation can be used [6], where Q denotes discharges and the sub scripts identify the type of discharge under consideration:

$$HUD_t \cong \frac{1}{2}\left(HUI_t + HUI_{(t-D)}\right) \quad \textbf{(4)}$$

The time-area method transforms an effective storm hyetograph into a runoff hydrograph. The time-area method is based, like Clark´s IUH, in a time-area diagram, but only accounting for the translation component, thus neglecting the storage effect. Therefore, the hydrographs calculated by the time-area method show less diffusion, resulting in higher peak discharges than those that would have been obtained if storage had been taken into account, like in Clark´s unit hydrograph [5].

In this study an unit hydrograph based in the time-area method was used. This particular UH was derived from the Clark´s unit hydrograph but considering a watershed storage parameter equal to zero (K=0h).

Both the Clark's and time-area methods were combined with two different time-area diagrams. One of these diagrams it will be further denoted by HEC TAD as it uses the time-area diagram developed by Hydrologic Engineering Center US Army Corps of Engineers given by the following definition (5):

$$\frac{A}{A_t} = \begin{cases} 1.414\left(\dfrac{t}{t_c}\right)^{1.5} & t \le \dfrac{t_c}{2} \\ 1-1.414\left(1-\dfrac{t}{t_c}\right)^{1.5} & t > \dfrac{t_c}{2} \end{cases} \quad \textbf{(5)}$$

where A denotes the watershed area that contributes to the discharge at the outlet section at instant t, A_t is the watershed area and t_c the time of concentration.

The second time-area diagram was established based on the isochrones – proposed TAD. To obtain the isochrones a specific technique was developed supported by the available topographic information – maps at the 1:25000 scale. According to that technique, the flow-time travel required to draw the isochrones was calculated by the difference between the times of concentration of successive sub-basins. For this purpose a huge number of sub-basins were considered and the Temez's equation for the time of concentration was applied.

To derivate the two direct unit hydrographs and to calibrate the storage parameter of Clark's unit hydrograph five storm hydrographs were used in each one of the case study. To calibrate the storage coefficient, an objective function was developed by combining five goodness-of-fit or performance indicators. The same performance indicators were used to compare the observed hydrographs and hydrographs simulated by the different models (validation stage).

The performance indicators and their units were the following ones:

SQD – sum of squared residuals (m^3/s);

CORREL – correlation coefficient (-);

FSF – shape synchrony indicator (m^3/s);

Δt_p – difference between observed and simulated time-to-peak (h);

ΔQ_p – difference between observed and simulated peak flow (%).

The above performance indicators are calculated by the following equations:

$$SQD = \frac{\sum (Q_o - Q_s)^2}{\sum Q} \quad \textbf{(6)}$$

where $\sum Q$ represents indifferently the sum of the observed (Q_o) and of the simulated Q_s discharges which, according to the mass equation, must be equal;

$$CORREL \Leftrightarrow \rho_{Q_o,Q_s} = \frac{COV(Q_o,Q_s)}{\sigma_{Q_o}\sigma_{Q_s}} \quad \textbf{(7)}$$

where COV is the covariance and σ the standard deviation.

$$FSF = \frac{Min(\sum (Q_o - Q_s)_i^2)}{\sum Q} \quad \textbf{(8)}$$

The FSF indicator is similar to the SQD, but it evaluates the best shape fit between observed and simulated hydrographs by testing sequential translations (in axis) of the simulated hydrograph comparatively to the observed one;

$$\Delta t_p = (t_{ps} - t_{po}) \quad \textbf{(9)}$$

$$\Delta Q_P = \frac{(Q_s(\text{time peak}) - Q_o(\text{time peak}))}{Q_o(\text{time peak})} \times 100 \quad \textbf{(10)}$$

The objective function applied to calibrate the storage coefficient of Clark´s UH and to judge the fitness between observed and estimated hydrographs is given by:

$$K = F(Min(SQD), Max(CORREL), Min(FSF), Min(\Delta t_p), Min(\Delta Q_p)) \quad \textbf{(11)}$$

In the calibration stages, the objective function indicates the value of the storage coefficient that leads to the best results for three global performance indicators (SQD,CORREL and FSF) and two specific performance indicators (Δt_p and ΔQ_p).

The notation adopted to identify the unit hydrographs applied in the research carried out was the following one:

- proposed TAD – application of the time-area diagram obtained by drawing the isochrones;

- HEC TAD – application of the time-area diagram developed by HEC;

- Clark (proposed TAD) – Clark´s model combined with the proposed TAD;

- Clark (HEC TAD) – Clark´s model combined with the HEC TAD;

- SCS – Soil Conservation Service unit hydrograph;

- PL – average unit hydrograph resulting from the linear programming;

- triangular – triangular unit hydrograph model.

Case studies

As case studies, two Portuguese watersheds were considered. The first is the Vez River at the stream gauging station of Pontilhão Celeiros (Figure 1), located in the north of Portugal, with an basin area of 170 km^2. The second basin is the Alenquer River at the stream gauging station of Ponte Barnabé (Figure 1), located in the centre of Portugal with an area of 113 km^2.

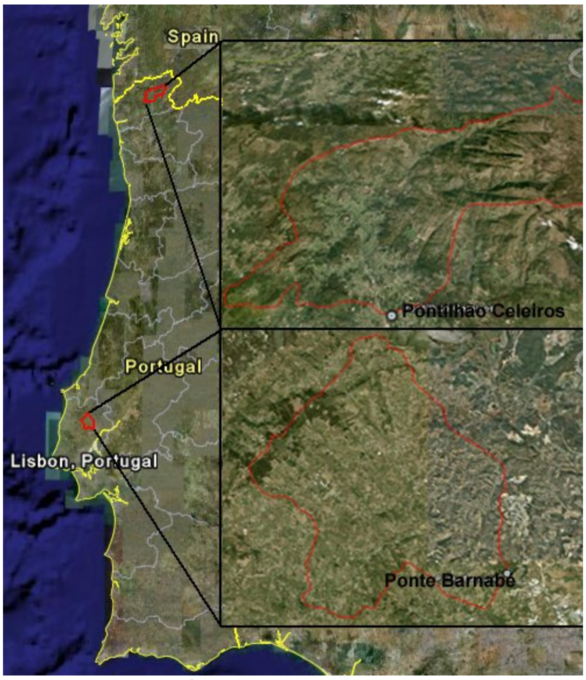

Figure 1 – Case studies general location. Vez River basin at the stream gauging station of Pontilhão Celeiros and Alenquer River basin at the stream gauging station of Ponte Barnabé.

Results

The Figure 2 contains, for both case studies, the time-area diagrams obtained based on the isochrones drawing – proposed TAD – and developed by the Hydrologic Engineering Center – HEC TAD.

To exemplify the results from the calibration of the Clark´s storage coefficient (K) one flood event was chosen in each case study as represented in Figure 3.

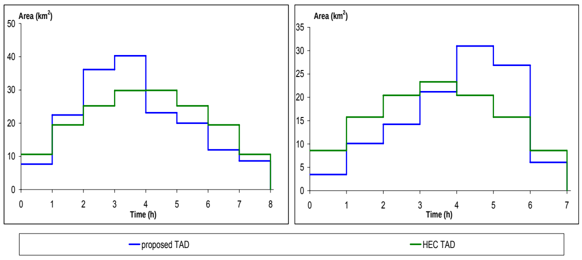

Figure 2 – Proposed TAD and HEC TAD for the Vez River basin at the stream gauging station of Pontilhão Celeiros (left side) and for the Alenquer River basin at the stream gauging station of Ponte Barnabé (right side).

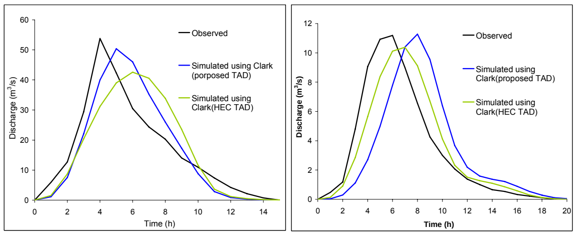

Figure 3 – Calibration of the Clark´s storage coefficient for Vez River basin at the stream gauging station of Pontilhão Celeiros (left side) and for Alenquer River basin at the stream gauging station of Ponte Barnabé (right side).

The Clark´s storage coefficient that resulted from the calibration stage was K=30 min for both case studies.

To appreciate the applicability of the different unit hydrographs six storm events were considered in each case study. The results thus achieved for each watershed are exemplified based on one storm event (considered as most representative): Table 1 and Figure 4, for Vez River, and Table 2 and Figure 5, for Alenquer River.

Table 1 – Vez River at the stream gauging station of Pontilhão Celeiros. Validation stage. Performance indicators. Best models resulting from each indicator.

Performance indicators	Storm hydrograph 11 - 03 - 2004							
	Observed	Simulated						
		proposed TAD	HEC TAD	Clark(proposed TAD)	Clark(HEC TAD)	SCS	PL	Triangular
SQD (m^3/s)		2.58	4.83	3.77	6.55	9.63	5.65	5.58
Qp (m^3/s)	74.07	65.01	59.86	63.75	58.66	48.08	61.91	63.58
ΔQp (%)		-12.23	-19.20	-13.94	-20.81	-35.09	-16.43	-14.16
tp (h)	4.00	5.00	6.00	6.00	6.00	7.00	5.00	6.00
Δtp (h)		1.00	2.00	2.00	2.00	3.00	1.00	2.00
FSF (m^3/s)		2.24	4.15	3.28	3.65	3.42	2.87	2.93
CORREL (-)		0.960	0.922	0.940	0.893	0.834	0.909	0.911
Best model for each performance indicator								
Direct and synthetic unit hydrographs	Min(SQD)	proposed TAD	Direct unit hydrographs	Min(SQD)	Triangular	Synthetic unit hydrographs	Min(SQD)	proposed TAD
	Max(CORREL)	proposed TAD		Max(CORREL)	Triangular		Max(CORREL)	proposed TAD
	Min(FSF)	proposed TAD		Min(FSF)	PL		Min(FSF)	proposed TAD
	Min(Δtp)	proposed TAD, PL		Min(Δtp)	PL		Min(Δtp)	proposed TAD
	Min(ΔQp)	proposed TAD		Min(ΔQp)	Triangular		Min(ΔQp)	proposed TAD

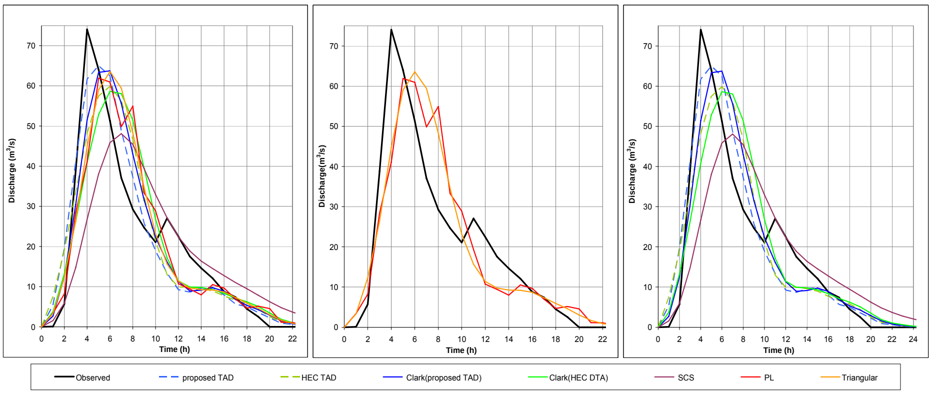

Figure 4 - Vez River at the stream gauging station of Pontilhão Celeiros. Validation stage. Comparison between observed and simulated hydrographs.

Table 2 – Alenquer River at the stream gauging station of Ponte Barnabé. Validation stage. Performance indicators. Best models resulting from each indicator.

Performance indicators	Storm hydrograph 23 - 03 - 2006							
	Observed	Simulated						
		proposed TAD	HEC TAD	Clark(proposed TAD)	Clark(HEC TAD)	SCS	PL	Triangular
SQD (m^3/s)		2.29	0.79	4.04	1.73	4.02	0.30	0.31
Qp (m^3/s)	22.14	21.01	18.59	19.91	18.56	14.80	19.88	20.00
ΔQp (%)		-5.11	-16.06	-10.09	-16.18	-33.16	-10.20	-9.66
tp (h)	6.00	7.00	6.00	8.00	7.00	8.00	5.00	6.00
Δtp (h)		1.00	0.00	2.00	1.00	2.00	-1.00	0.00
FSF (m^3/s)		0.28	0.45	0.33	0.50	1.33	0.17	0.23
CORREL (-)		0.906	0.968	0.832	0.928	0.825	0.988	0.987
Best model for each performance indicator								
Direct and synthetic unit hydrographs	Min(SQD)	PL	Direct unit hydrographs	Min(SQD)	PL	Synthetic unit hydrographs	Min(SQD)	HEC TAD
	Max(CORREL)	PL		Max(CORREL)	PL		Max(CORREL)	HEC TAD
	Min(FSF)	PL		Min(FSF)	PL		Min(FSF)	proposed TAD
	Min(Δtp)	HEC TAD		Min(Δtp)	Triangular		Min(Δtp)	HEC TAD
	Min(ΔQp)	proposed TAD		Min(ΔQp)	Triangular		Min(ΔQp)	proposed TAD

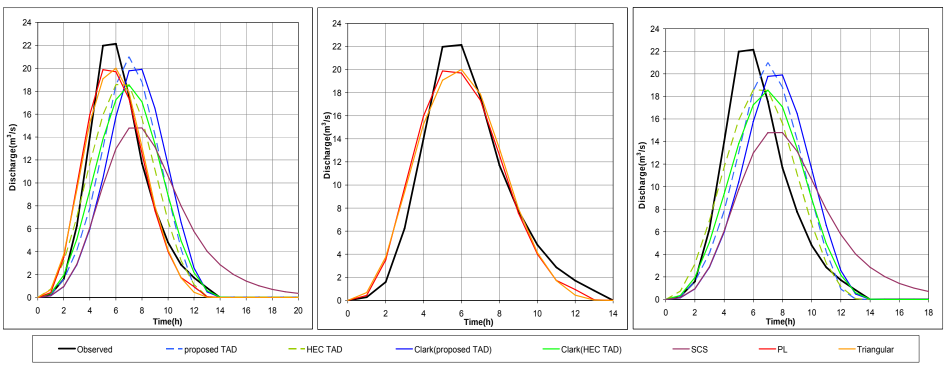

Figure 5 - Alenquer River at the stream gauging station of Ponte Barnabé. Validation stage. Comparison between observed and simulated hydrographs.

Conclusions

First of all, it should be stressed that the results achieved are closely related with the models applied to describe the rainfall losses and to identify the direct flow. If models different from the ones adopted were utilized the results would be more likely different from those achieved.

In general terms, the following aspects may be highlighted:

1- The unit hydrographs based on direct methods showed good performance. However the interest and consequently the applicability of such models are very limited as more often there is not the data required by their establishment. Also, such type of models is out of the scope of the research carried out which was mainly focused on synthetic unit hydrographs. Accordingly the unit hydrographs based on direct methods will not be further mentioned.

2- More frequently, the hydraulic studies intend to estimate peak flood discharges instead of other flood characteristics, such as the shapes of the hydrograph or the times to the peak. For that purpose the time-area diagram computed based on the isochrones – proposed DTA – proved to be the more suitable model.

3- In what concerns the time-to-peak, the smaller differences between observed and simulated times were obtained using the proposed DTA, for the Vez River at the stream gauging station of Pontilhão Celeiros, and the HEC DTA, for the Alenquer River at the stream gauging station of Ponte Barnabé.

4- The better adjustment between the shapes of the observed and of the simulated hydrographs was achieved by applying the proposed DTA, either in association or not with the Clark's unit hydrograph.

5- For any of the case studies, the SCS model showed always the worst results. Particularly, this model underestimates the peak flood discharges (average "gap" between observed and estimated discharge of 30 and 33% in the Vez River and in the Alenquer River, respectively.

Therefore, it seems valid to conclude that the model that ensures the best global performance was the time-area model obtained by drawing the isochrones – proposed DTA. The Clark´s model associated to the proposed time-area – Clark (proposed TAD) – also seems to be a promising model, though more case studies need to be considered.

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