Optimization of the Scheduling of Production of Paper Machines from a Portuguese Paper Plant

Pedro P. Mariano[1]

Abstract

Due to its importance to the production/transforming sectors companies and those that have to manage inventory, scheduling has been the target of several scientific contributions, since the eighties. This present thesis follows these contributions, studying the production of a Portuguese paper plant, and trying to optimize the scheduling of the orders to process in the three available paper machines. This thesis does so through the implementation of a model with continuous-time formulation and Resource-Task Network representation. It successfully achieves processing of all orders before their due dates, formation of blocks of production smaller than half of a day of production, and minimizes the makespan, by reducing the changeovers between different products in the same machine. The present study also makes the comparison between the achieved results and the performance of the plant, as well with a previous study of the same subject done using a discrete-time formulation.

It is verified that the model implemented achieves better schedules than both the actual production cycle of the plant, and the discrete-time model.

Keywords: Production, Scheduling, Optimization, Resource-Task Network

1. Introduction

Scheduling is concerned with allocation of resources over time so has to execute the processing tasks required to manufacture a given set of products[1,2]. Due to the critical role this type of operation plays in organizations working in production/transformation sectors and/or with considerable amounts of inventory, several scientific contributions in scheduling problems have been made in the last couple of decades. One sector which fits perfectly in the scenery cited above is the pulp and paper sector. This sector is the forth largest exporter in Portugal, only behind textiles, leather and wood. To this high ranking, the Group Portucel Soporcel contributes in large scale, being the biggest player in Portugal.

The Group Portucel Soporcel runs three paper pulp and/or paper plants in Portugal, more specifically, in Cacia, Figueira da Foz and Setúbal, being that the last one is where the study of this paper lies.

The Production Department of the Setúbal paper plant plans the budget for production and transforming (the second of two stages performed at the plant which includes cut, cut size and rewind) for a time horizon of one year. This plan is strongly influenced by the market behavior and the data acquired from the clients. Considering this plan and in the orders received, the Sales & Marketing Department conceives the scheduling for each month. In order to maximize the efficiency of the three available machines, each order has to have more than three tons. For the same reason, the scheduling cannot have a block of production (the production of a particular product) smaller than half a day of production. Figure 1 illustrates a generic month production plan.

The generic month plan consists in 25568 tons of paper produced in 90 days (30 per machine). It predicts 22 changeovers between different products distributed by the three machines (12 in MP1, 5 in MP2, and 5 in MP3).

This paper aims to optimize the scheduling for the three available machines, minimizing the total production time and the number of changeovers. This paper does so through the implementation of a continuous-time (CT) formulation RTN representation MILP model in GAMS/CPLEX. Another aim of this work is to compare the results achieved with the actual performance of the Group Portucel Soporcel, and with the performance of a discrete-time (DT) formulation applied to the same case study[4]. It comes to conclusion that the model implemented has more efficient results than the actual production cycle of the plant, and the results achieved with the DT formulation model.

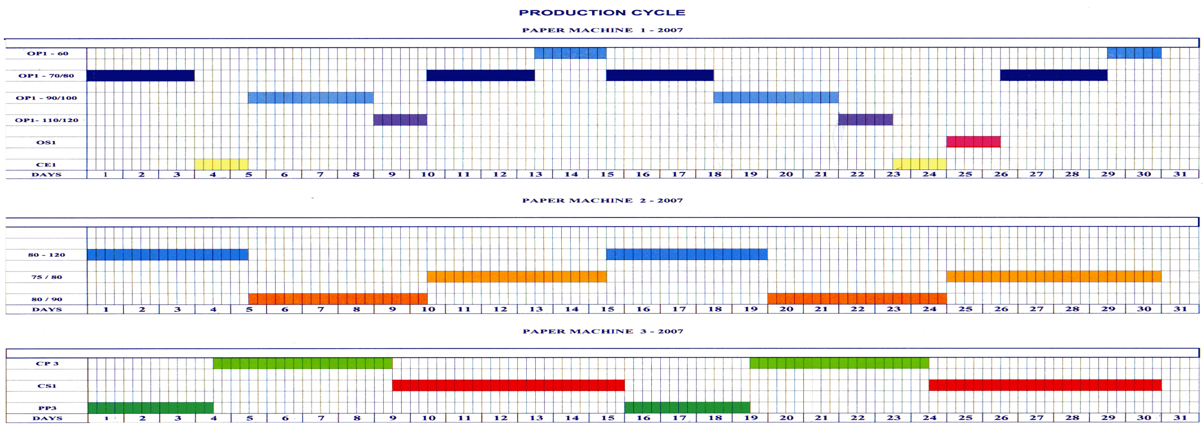

Figure 1 – Example of a monthly production cycle at the plant

The rest of the paper is structured as follows. Section 2 presents the problem definition. In section 3 is made a review of the state-of-art in the field of scheduling and its role on operations management. Section 4 shows the implementation and analysis of the CT formulation RTN representation model, including also a comparison with the performance at the plant. Section 5 compares the CT representation with the discrete one. Section 6 refers further efforts made in this paper, as the re-scheduling of the orders and the implementation of a control to the amount of stock in the warehouse. In section 7 the mathematical formulation of the implemented model is presented. Finally, the conclusions and some relevant work to do in the future are given.

2. Problem Definition

In this paper, the short-term scheduling problem of a singlestage, multiproduct continuous flow in a paper plant is considered. Given are a set of 420 orders of 12 different products (P1-P12) which have to be processed in one of the three available machines belonging to the set $m \in M$ in a time horizon of one month. Given are also the duration of processing tasks, $p_{i,m}$, the cleaning tasks, $c_{li,i',m}$, the due dates, d_i and the amounts to process, qtd_i.

The original number of orders is well above the current capabilities of state-of-the-art scheduling models, so a two-stage solution method was applied to reduce the complexity. In the first case, orders belonging to the same product and sharing the density (g/m^2), size and due date were placed in the same group. This process reduced the number of orders to 191, still too high to start to solve the current problem. In the latter case, groups of orders with duration as close as possible to a day of production were formed, establishing a trade-off between flexibility (more orders to process) and computer effort (less orders). The result was the reduction of the number of orders to 73, an acceptable one, considering the state-of-the-art scheduling models capabilities.

The allocation order-machine is already stated by the plant's Production and Sales & Marketing Departments. In this way, orders belonging to P1-P6 are processed in MP1 (the first of the available machines); orders belonging to P7-P9 in MP2; and those belonging to P10-P12 in MP3.

The objective will be to find the optimum allocation for all of the three available machines *m* reducing the current number of changeovers (which can last between 10 and 30 minutes each) through the minimization of the makespan (in order to maximize productivity) or total earliness (in order to reduce inventory costs) and still accomplishing the due dates of all orders *i* and the maximum availability for storage in the plant's warehouse. The problem representation is shown in figure 2.

3. State-of-the-art of Optimization Works for Short-term Scheduling Problems

As referred in section 1, planning and scheduling (and in particular the short-term scheduling) of production have received considerable attention by the scientific community in the last twenty years,[6-12] in large part due to the vital role they play in operations management.

The DT formulation (firstly in STN representation and then followed by RTN representation) models evolved sooner than the continuous-time ones, however in the last decade several studies with this formulation had been published[13-18]. More recently, two works were published[19], where multiple-time grids are presented and compared with uniform-time grids. In all of these cases, the performance of the multiple-time grids achieved more success. This paper adopts the multiple-time grids presented in these two works.

Although all this publications are concerned to the enhancement of the practices taking place at a specific plant or to the resolution of some general problems, more than one methodology can be applied. One of the main methodologies applied has been the use of mathematical formulations, typically of the mixed-integer linear programming type (MILP). This kind of mathematical formulation is able to optimize key performance indicators such as stock levels, idle times, operating costs, profit, makespan, etc.

Nowadays, it is clear that there is not a single best approach for all problem types and researchers are still drawing the map that will tell which model to use as a function of the characteristics of the real problem.

4. Continuous Formulations: Implementation and Analysis

4.1 Implementation

To achieve the objectives of the present work (stated in section 1) imply the implementation of a continuous-time

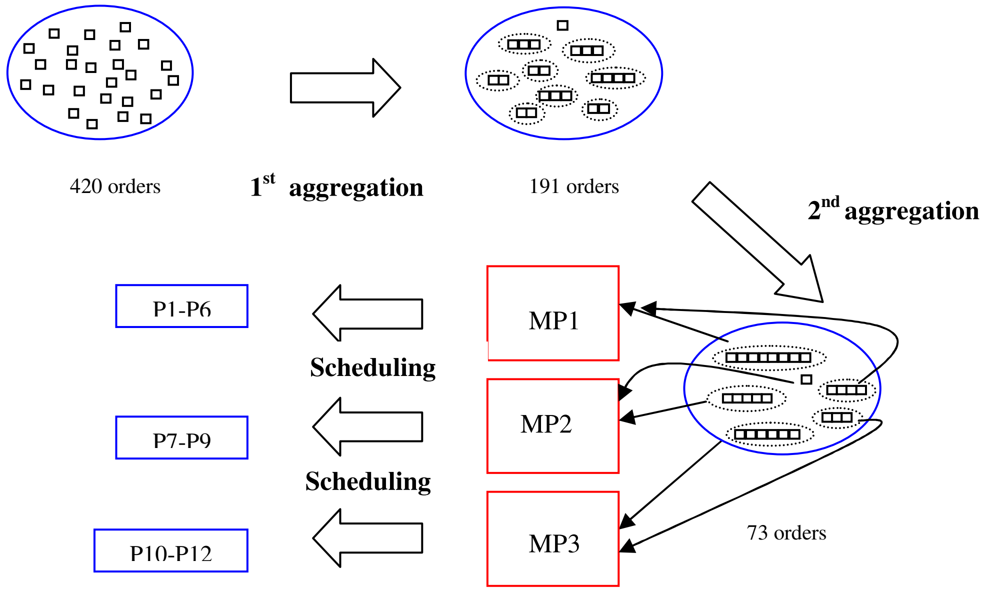

Figure 2 – Problem representation

formulation model. The first step in this effort was to decide how to divide the time horizon. When handling time in a continuous-time way, there are two alternatives to consider: a uniform time grid for each machine or a multiple-time grid[3] for all of machines. Both keep track of all events taking place, but in different ways. In the present case, there are not common resources and all the available machines are independent, so the choice lied in the application of a multiple-time grid. All |M| time grids are totally independent, meaning that no relation is assumed between time points of different grids. Nevertheless, all of them have the same lower and upper bounds on the first and last time points. As a result of the number of time points to use in each machine being known (equal to the number of pre-allocated orders plus: 34 in MP1, 16 in MP2 and 26 in MP3) and therefore not having the extra problem of choosing how many time points to use on a particular machine, the three time-grids have different dimensions.

Two continuous-time formulations in RTN representation models were implemented: the CT4I and the CT3I taken from Castro et al, 2006[1]. This type of formulations does not consider a flow of materials hence the matter of keeping the amount of stock below the capacity of the warehouse was postponed to the end of this study. The formulation of RTN models has direct impact in the respective behavior of the model. Thus both formulations were applied to the features of the present problem, but firstly to a half-month time horizon set of orders (which includes 36 orders and more or less 8500 tons of paper), and only optimizing each machine at time, by minimizing the makespan (Equation 1), in order to retrieve faster conclusions about which of both is the one more suitable for the present case study.

$$MS \geq \mathrm{Te}_{t,m} + \sum_{t' \in \mathrm{Te}, t' \geq t, t' \neq |\mathrm{Te}|} \sum_{i \in \mathrm{Im}} [\mathrm{N}_{i,m,t} \, (p_{i,m} + cl^{\min}{}_{i,m} \big|_{|\mathrm{Te}|-11})] \quad \forall m \in \mathrm{M}, t \in \mathrm{T} \quad (1)$$

4.2 Analysis

As one can see in Table 1, the CT4I formulation achieved better results for all machines (in CPU seconds) than CT3I. The latter behaves in worst way than CT4I due to substantial higher number of nodes produced and its resultant increase in complexity, even if the number of variables and constraints is significantly smaller. Consequently it was the CT4I formulation which was applied to the month time horizon set of orders with its 73 orders and 16752 tons of paper to process. The performance of CT4I (shown in as well in Table 1) decreases considerable, since it was not possible to achieve any solution when optimizing all machines at one time. When doing it one machine at time, the optimum solution for MP1 and MP3 was not found and the computer effort for this two machines increases considerably (in terms of makespan and CPU seconds). Despite the fact of CT3I achieves worst results to a half-month time horizon problem than CT4I, it was applied to the month time horizon problem in order to find if it is feasible solving all machines at time, and also in order to find if it was possible to achieve a better performance when optimizing each machine at a time. The results show that CT3I can solve the problem in both ways: each machine at time and all at the same time. Regardless of producing more nodes than CT4I, these nodes are solved much faster and allied to the smaller number of variables and constraints created by CT4I; make CT3I the best approach to solve this case study. The scheduling created by CT3I for the three equipments can be seen in figure 3.

4.3 Comparison with the plant performance

To establish a comparison between the performances in the plant and the CT3I continuous formulation, the 73 orders to process were allocated in the blocks of production presented in figure 2, ordered by its due date. Once the blocks were allocated with one order, they were produced in their totality to maintain the current politics of the company. The blocks of production to which no orders were allocated, ceased to exist, as is the case of a block in P1, P7, P8 and P12. Such represent a reduction in the quantity of paper to produce from about 25568 tons to 22800 and an almost proportional (not exactly the proportional because different products suffered different variations in production) reduction in production time.

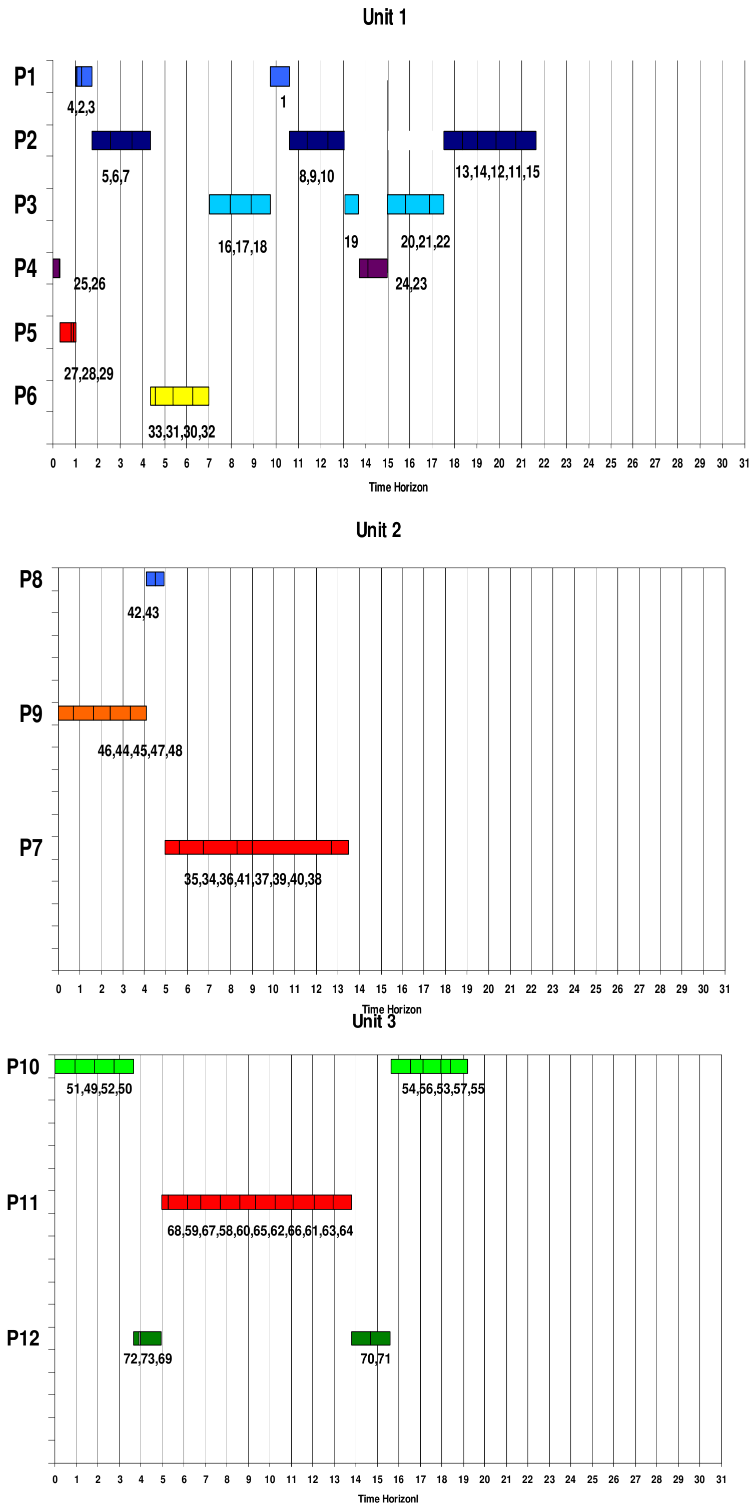

Figure 3 – Scheduling created by the continuous-time formulation

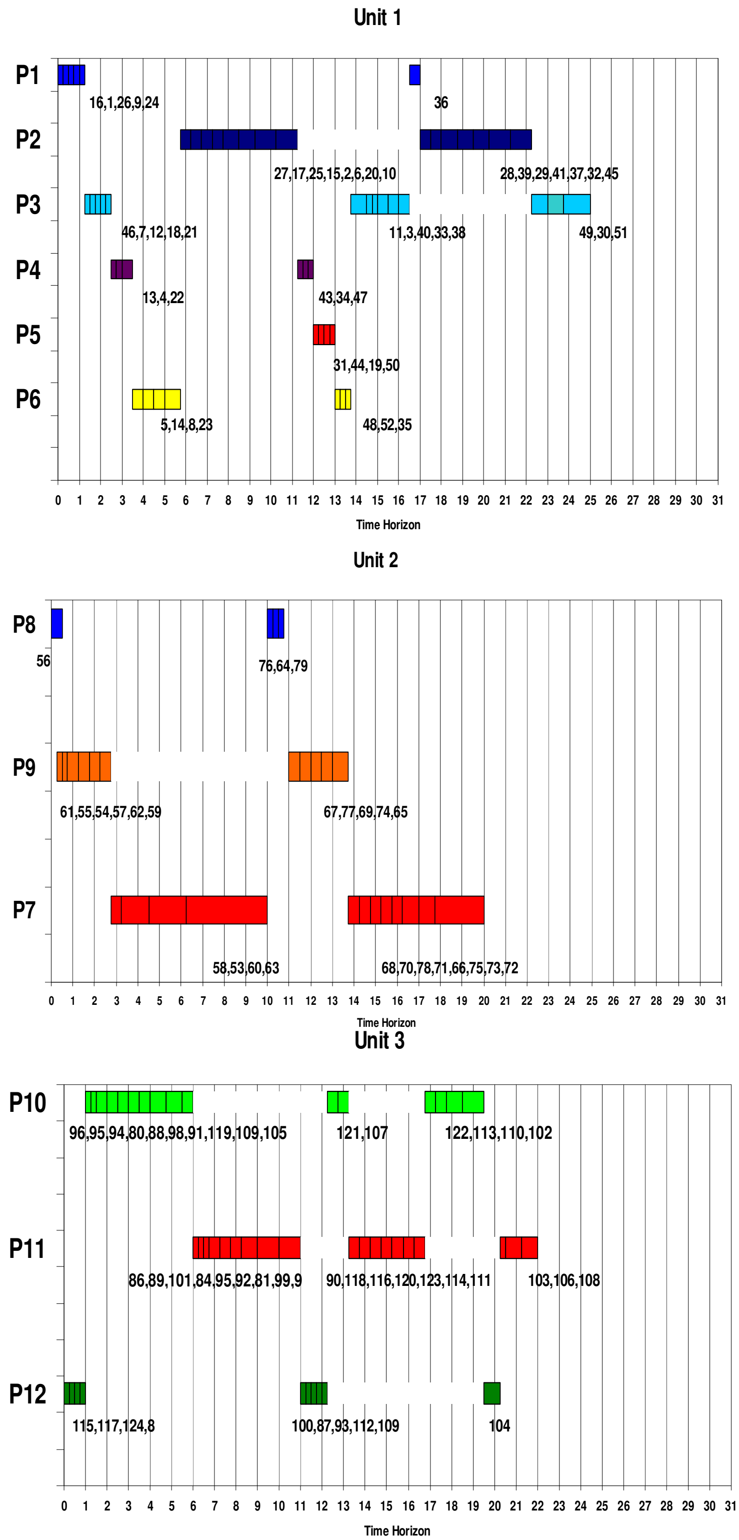

Figure 4 – Scheduling created by the discrete-time formulation

However, this quantity is by far greater than the amount produced by the CT formulation (16752 tons). Comparing the results reached with the continuous formulation CT3I against the performance achieved in the plant (briefly stated in section 1), it is come to conclusion that the former reduces the changeovers in all three machines: 12 to 11 in MP1, 5 to 2 in MP2 and 5 to 4 in MP3, which brings more efficiency to the process. The elimination of 5 changeovers and 4 blocks of production result in decrease of the total makespan from 90 to 57.69 days. More than the reduction of the total makespan, the implemented model proves that is possible to have a more flexible solution, confirming that this type of models bring flexibility to the planning and scheduling. One negative feature of the CT lies in the rise of stock occupation in the warehouse from (5.96 days/per order in MP1, 5.02 in MP2 and 6.70 in MP3 to 9.00 in MP1, 6.65 in MP2 and 8.34 in MP3). Nevertheless, the low occupation of the stock in the real case it is partially due to the allocation of orders by their due date and such is not always verified. In this way, it is possible that the real occupation of stock is nearer the occupation caused by the CT formulation than the values achieved.

The minimization of the makespan was not the only objective function studied, having been applied also the minimization of the total earliness (equation 2). As expected, the stock in the warehouse decreases significantly (to 0.50 days/per day in MP1, 0.20 in MP2 and 0.38 in MP3). Nevertheless, the number of changeovers increases to 22 in MP1, 8 in MP2 and 17 MP3. Worst than the increase in the number of changeovers is the fact that this objective function creates very small blocks of production (very few longer than 2 days) and several times no product is being processed. This is not tolerable in a continuous flow like a paper plant, where as the name implies always some kind of paper is always being produced (excluding periods of maintenance). For these reasons, the minimization of makespan was the objective function adopted to solve the present case study.

$$earl = \sum_{\mathrm{i}} \mathrm{due_i} - \sum_{\mathrm{t,t \neq |T|}} (Te_{t,m} + \sum_{i} N_{i,m,t} * p_{i,m}) \quad \forall \mathrm{m} \in \mathrm{M} \qquad (2)$$

5. Continuous Formulation versus Discrete Formulation

5.1 Discrete Formulation

As mentioned in section 1, this study includes a comparison between the performances of a discrete formulation[4] applied to the present case study and the continuous CT3I.

The second stage of the aggregation procedure is made in different way than the approach taken for the CT formulation. More specifically, in the latter case the duration is made as close as possible to the day, establishing a trade-off between flexibility (more orders to process) and computer effort (less orders). In the former case, the selection is done so that the duration of the group of orders is as close as possible to a multiple of the interval length, a quarter of a day, in order to minimize idle times (see section 2). This leads to the production of extra paper until a multiple of a quarter of a day is reached. So, the DT formulation created does not use the same amount of paper to produce as the month time horizon set of orders stated in section 4 (16572 tons), but a bit more (19428 tons).

In this problem, 73 group of orders resulted for CT and 124 for DT, which brings a natural increase in the complexity of resolution of the model to implement in the latter case.

All the orders were produced before their due date and it was proved that is possible to reduce the processing time (to 67 days of production as one can see in figure 4) at the cost of reducing a proportional quantity of paper produced. This work also managed to keep the amount of paper in inventory below the maximum capacity of the warehouse (6500 tons), despite the model did not take in account the flow of materials. Another positive measure in the DT formulation lied in the reduction of the number of changeovers in MP1 (12 to 11). The other two machines did not behave so well as MP1, for the reason that the number of changeover remained in MP2 (5) and increased in MP3 (5 to 8). Figure 4 represent the scheduling achieved by the DT formulation.

5.2 Comparison between the two formulations

Both formulations are able to process all orders before the respective due dates and in both cases the maximum capacity of the warehouse is not exceeded (despite the two formulations do not consider flow of materials). Any similarities end here, since the performance in terms of makespan, CPU's, production and stock occupation of the two formulations is quite dissimilar.

The DT formulation produces more changeovers than the continuous one in MP2 and MP3 (3 more in the former and 4 in the latter) and the same number of changeovers in MP1. Knowing that changeovers imply idle times (between 10 and 30 minutes each), the DT formulation has more time without any product being processed (0.43 days versus 0.25). In the same way, the efficiency (time of production on makespan) of the CT formulation is higher than the discrete one (99.56% versus 99.36%). The solution of the CT is closer to the optimum than the DT, with a relative gap of 0.55% for all three machines (whereas the DT has a relative gap of 4.84% for MP1 and 1.50% for MP3).

Another advantage of the CT formulation lies in the computer effort. The DT formulation spends 14516 CPU seconds to process all the orders of the three equipments against 3434 CPU seconds of the CT. Great fraction of the time spent by the DT is caused by the utilization of the time horizon technique, necessary to reduce the relative gap in MP1 and MP3. The stock produced by the continuous formulation is also lower than the discrete one (8.52 days/per order versus 10.33 in MP1, 6.44 versus 10.92 in MP2 and 8.14 versus 9.30 in MP3). None of the formulations consider the allocation of orders on basis of its due date; hence the continuous formulation occupying less stock has to be considered a fluke. This and other results can be consulted in table 2.

The differences stated in the paragraph above can be explained by the different input (regarded to the number of orders to process) each formulation receives caused by the different second different stage of aggregation. More orders to process and more quantity of paper to produce (therefore more time to process those orders) due to the obligation of production to a multiple of the chosen slot has been reached,

Table 1 – CT3I versus CT4I

*solved with CPLEX10.2.0		CT3I	CT4I
Half-monthly time horizon*	Makespan [days]	28.86	28.86
	CPU [s]	5042	3599
	Discrete Variables	1551	19212
	Single Variables	1595	20805
	Constraints	1664	3085
	Nodes	4.02E06	5.49E05
Monthly time horizon*	Makespan [days]	57.60	57.77
	CPU [s]	15577	25175
	Discrete Variables	6443	167331
	Single Variables	6524	173179
	Constraints	6117	6219
	Nodes	1.41E06	2.07E05

leads to an increase in the complexity of the current problem. Another reason to explain such disparity between the two formulations is could lie in the application of a different objective function. The CT tried to minimize the total makespan of the three equipments through the equation 1, whereas the DT applies the equation 3, which has the purpose to minimize the sum of the finishing time of the orders to process. This objective function considers also a second term which stands for the penalties when changeovers occur.

$$Z=\sum_{i,p,m,t} N_{i,p,m,t} \times (t+\tau_i-1)\times\delta + \sum_{p,p',m,t} 2Mu_{p,p',m,t} \quad \forall i\in I, p\in P, m\in M, t\in T \quad (3)$$

6. Further studies

6.1 Re-scheduling

In order to not limit this paper to the actual allocation order/equipment decided by the Departments of Production and Sales & Marketing, we decided to release this constraint to a certain number of products in the CT. The choice depended on: a) the number of orders (many would lead to a great increase in the computer effort); b) some products with the maximum duration of changeover (30 minutes) with the other products in the same machine. Hereupon, the constraint order/equipment was released for P5 and P6, which represent 7 of 73 orders to process. It was considered that if these products were processed in MP2 or MP3, the rate of processing would be the slower of the respective equipment in the way of respecting eventual non-existing economies of experience.

The complexity of the mathematical problem increases significantly (from 3434 to 20411 CPU seconds), because now the model has to decide to which equipment these products will be processed, but the total production time decreased 3.29%, from 57.69 to 55.73 days – see table 2). This happened because P5 and P6 (which were produced in MP1) were transferred to MP3, the faster of the available machines.

6.2 Control of stock

Despite all of the reached solution keep their levels of stock under the maximum capacity of the warehouse, in neither of the CT or DT the flow of materials was considered. Therefore, and because the control of the stock produced is a vital matter in most companies who have to manage their inventory, to the CT (without the modifications of the section 6.1) implemented were added the necessary constraints in order to accomplish such goal.

It were added six constraints (equations 10 to 15) and three new sets of variables ($Y_{t,dd}$, $Si_{t,i}$ e S_t) which inflict a vast increase in the computer effort (from 3434 to 16322 CPU seconds). The capacity of the warehouse (6500 tons) was fractioned by the three machines, according to the quantities each had to process. Thus, the maximum capacity for each unit was 2000 tons to MP1, 500 to MP2 and 400 to MP3. Another motive which can explain such increase in the computer effort is due to the complication in comparing inventories of different time grids, since they have different time points. Some negative features of this solutions lie on the great number of changeovers (27 in MP1, 4 in MP2 and 7 in MP3) which cause more than half a day of idle time (0.57 days) and in the occurrence of times where there is no production or changeovers.

7. Continuous Formulation: Constraints

The CT formulation is composed by the following constraints:

Equation 4 establish that the equipment m can only be available in the time point t, if no orders start to be processed in the same time point. Equation 5 takes place in order to reduce solution degeneracy and to improve the performance of the model. If the equipment m will be available, $R_{m,t}$ is equal to 1, otherwise, is equal to 0.

$$R_{m,t} = 1 - \sum_{t\neq|Te|} Ni,m,t \quad \forall m\in M, t\in T, tm_{m,t} \quad (4)$$

$$R_{m,t} \leq R_{m,t-1} \quad \forall m\in M, t\in T, t\neq 1, t\neq |T|, tm_{m,t} \quad (5)$$

Equation 6 ensures that the difference between two time points is greater or equal than the duration of the task plus any eventual changeover.

$$Te_{t+1,m} - Te_{t,m} \geq \sum_{i} N_{i',m,t} * p_{i'm} + (N_{i,m,t} * clmax_{i,m})\Big|_{t\neq|T|-1} - \sum_{i',t\neq|T|-1} N_{i',m,t+1} * cldelta_{i,il,m}$$
$$\forall m\in M, i\in I, t\in T, t\neq 1, t\neq |T|, tm_{m,t} \quad (6)$$

Table 2 – Results achieved

*solved with CPLEX11.0.1			DT	CT	CT – Earliness	CT-Rescheduling	CT – Control of Stock	Group Portucel Soporcel
Monthly time horizon*	Makespan [days]		67.00	57.63	58.27	55.74	57.92	90.00
	Relative gap	MP1	11.9%	0.64%			4.84%	
		MP2	0	0		0.39%	0.37%	
		MP3	2.88%	0.43%			1.50%	
	CPU [s]		14516	3434	962	20411	163222	
	Production [ton]		19428	16752	16752	16752	16752	25567
	Stock [days/per order]	MP1	10.33	8.52	0.50	9.67	5.68	5.96
		MP2	10.92	6.44	0.20	7.23	6.37	5.02
		MP3	9.30	8.14	0.38	7.69	7.62	6.70

Equation 7 and 8 guarantee that the absolute time of point *t* in equipment *m* is greater or equal than its minimum value, and lower or equal than its maximum value.

$$T_{t,m} \geq \sum_{i} N_{i,m,t} * lb_{i,m} \quad \forall m \in M, t \in T, t \neq |T|, tm_{m,t} \quad (7)$$

$$T_{t,m} \leq \sum N_{i,m,t} * hb_{i,m} + H*(1-\sum_{i} N_{i,m,t}) \quad \forall m \in M, t \in T, t \neq |T|, tm_{m,t} \quad (8)$$

Equation 9 assures that all orders are processed once.

$$\sum_{t,t \neq |T|} \sum_{m} N_{i,m,t} = 1 \quad \forall i \in I \quad (9)$$

Equations 10 to 12 are destined to control the stock. The first defines the balance of materials for each time point, where this is equal to the stock of previous time point, plus what is produced in the previous time point, minus what is taken from the warehouse in that time point. The second claims that the total value of stock is equal to the sum of the stocks of all the orders. The latter stipulate that the total value of stock in the warehouse has to be lower than the maximum capacities of each unit.

$$Si_{t,i} = Si_{t-1,i}|_{t \neq 1} + \sum_{m} N_{i,m,t-1} * qtd_{i} - \sum_{dd|td(dd),m} eep_{i,m,dd} * Y_{t,dd} \quad \forall i \in I, t \in T, t \neq 1 \quad (10)$$

$$S_{t} = \sum_{i} Si_{t,i} \quad \forall i \in I, t \in T, t \neq 1 \quad \forall m \in M, t \in T, t \neq 1, maqact_{m}, tm_{m,t} \quad (11)$$

$$S_{t} \leq caparm1|_{m=1} + caparm2|_{m=2} + caparm3|_{m=3} \ \forall m \in M, t \in T, maqact_{m}, tm_{m,t} \quad (12)$$

Equations 13 to 15 are referred to due dates and the respective withdraw of the order *i* from the warehouse in that same date. The first ensures that to each due date exists only one time point *t*. The second and third assure that the order *i* is not removed from the warehouse before its due date. Only

Equation 15 has a big-M constant, since the same would be relaxed in equation 14.

$$\sum_{t,t \neq 1} Y_{t,dd|td(dd,m)} = 1 \quad \forall m \in M, dd \in DD, td|_{dd,m} \quad (13)$$

$$Te_{t,m} \geq \sum_{dd|td(dd,m)} dd * Y_{t,dd} \quad \forall m \in M, t \in T, t \neq |T|, tm_{m,t} \quad (14)$$

$$Te_{t,m} \leq \sum_{dd|td(dd,m)} dd * Y_{t,dd} + H*(1-\sum_{dd|td(dd,m)} dd * Y_{t,dd}) \quad \forall m \in M, t \in T, t \neq |T|, tm_{m,t} \quad (15)$$

8. Conclusions and future work

This paper implemented and compared a continuous-time formulation of a short-term scheduling problem with a discrete-time formulation and the current performance at the plant where lies the problem. The results (figures 3 and 4 and tables 2) proves that the CT formulation obtains better results than its discrete counterpart, as in the form of the makespan, the number of changeovers, efficiency, computer effort, relative gap and stock produced. Such is explained by a different second stage in the process of aggregation of the orders to process; by the features of discrete-time models, which divide the time horizon in slots; and by a different objective function, perhaps less adequate to short term scheduling problems. Comparing to the current performance of the plant, the stock increased but the changeovers and therefore the inefficiency of the process decreased. It was proved that P5 and P6 should be processed in MP3 and not in MP1, and a control for the stock was implemented: a novelty when comparing to discrete-time formulation.

In future works in the same case study the aggregation process could be enhanced, in particular the aggregation of orders belonging to the same product (P1-P12) but with different rates of processing. Another topic to enhance lie on the set of orders utilized (the same of the DT). To achieve a more precise comparison between the CT and the performance at the plant, it would be necessary that the same set of orders would be applied (25567 tons and not 16752 tons as this paper applied).

To explore in the same case study are the incorporation of a fourth unit (installed in August of 2008) and of the second stage in the production of paper, the transforming one, which includes the cut, the roll and the cut size of the paper produced.

Nomenclature

Sets/Indices

DD/dd = possible due dates
I/i = process orders
T/t = points of the time grid
M/m = process equipment units
$tm_{m,t}$ = points of the time grid belonging to equipment *m*
$t_{dd,m}$ = existing due dates in equipment *m*

Parameters

cl_{max} = maximum changeover time from order *i* in equipment *m*
$cldelta_{i,il,m}$= difference between maximum and actual changeover from order *i* to *i'* in equipment m
$clmin_{i,m}$ =minimum changeover time from order *i* in equipment m
due_i = due date of order *i*
$eep_{i,m,dd}$ = quantity to remove of order *i* in equipment m in due date *dd*
$hb_{i,m}$ = highest time at which order *i* can start to be processed in equipment m
$lb_{i,m}$ = lowest time at which order *i* can star to be processed in equipment m
maq_i = equipment m where order *i* is processed
$p_{i,m}$ = processing time of order *i* in equipment m
qtd_i = quantity to process of order *i*

Variables

MS = makespan
$N_{i,m,t}$ = binary variable that assigns the start of order *i* (followed by *i'*) in equipment m at time point *t*
$R_{m,t}$ = excess amount of machine m at time point *t*
$S_{it,I}$ = Stock of order i at time point *t*
S_t = total stock at time point *t*
$T_{et,m}$ = absolute time of event point t in unit *m*
$Y_{t,dd}$ = binary variable that assigns if the quantity *qtdi* is removed from stock at time point t in the due date *dd*

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