INSTITUTO SUPERIOR TÉCNICO
Universidade Técnica de Lisboa

Dynamic behaviour of high speed railway bridges. Structural design specifications

Miguel Pereira*,¶

Department of Civil Engineering, Instituto Superior Técnico, Universidade Técnica de Lisboa, Portugal

Abstract

In this paper, the criteria for a proper conception of the bridges and viaducts of a high-speed railway system are developed based on current standards and theoretical investigations. The main purpose is to assist the elaboration of a set of recommendations for the Portuguese highspeed railway network. An extensive bibliographical survey of existing codes is taken for a full comprehension of the European experience and knowledge on the subject. More specifically, the dynamic phenomenon associated to high speed circulation is studied as an equivalent static action or through the equation of motion. The simpler approaches may suffer from inaccuracy and underestimation of the solution at resonance but help the designer in a rough evaluation of the variable action. The comparison of some of the methodologies is carried out and a new expression for specially built high-speed lines is proposed for EN 1991-2. Moreover, methodologies for solving the equation of motion for a simply supported beam are compared and some of its parameters are evaluated through sensitivity analysis and determination of design values. For a general framework, with multi degrees of freedom, the fundaments of dynamic analysis through the Finite Element Method are presented. A case study of an existing bridge undergoes to a number of limit states verifications with the aid of a computational program and the guidance of the EN 1991-2. The effect of ballast concludes the present study.

Keywords: Interoperability, railway codes, bridge dynamic behaviour, moving loads

1. Introduction

The Portuguese high-speed network is currently in a primordial stage. The network building will be left in charge of public and private partnerships, responsible for the financial and technical management of the infra structure. RAVE, as the regulating entity of the Portuguese highspeed network must give out some project guidelines in order to better evaluate and control its performance.

The current work is based on this specific task, which its sole objective is to clearly and concisely expose the main problems and design procedures related to the works of art, ones to be built or re-used in the high-speed network.

* E-mail: mop16900@gmail.com
¶ Master Student.

2. Existing documentation

The construction of a high-speed railway infrastructure is not new in Europe. Parallel to this experience, a set of norms and recommendations have already been written in order to alert the designer to the actions and dimensioning conditions. Among others, the TSI, CEN, CENELEC and UIC rules are also included.

TSI – Technical Specifications for Interoperability – these documents, of regulatory character, have the purpose of guiding the technical options of the competent authorities and to establish a common base. To every subsystem there is a corresponding article of the L245 publication of the European Communities Journal. About the bridges and viaducts, document L245/143 [1] demands that the evaluation of the infrastructural system must verify the safety of the vertical loads (static and dynamic effects), the transversal horizontal loads as well as the longitudinal loads throughout the project and the submission tests before putting into service.

CEN rules - European Committee for Standardization – these documents, of recommendatory status, surpass the civil engineering domain. In fact, among countless CEN norms, only some actually concern the matter of high-speed railway traffic. So the emphasizing rules would be the EN 199x specifically the EN 1991-2 [2], which defines the traffic actions on bridges.

UIC reports – created by the Union Internacionale de Chemins de Fer (UIC), they are a result of an investigation subsidized by the national railway entities, members of the UIC. Some of the reports were used to originate the aforementioned rules and, in particular, stresses the importance of the reports of the European

Rail Research Institute (ERRI) in the progress of the high-speed bridge behavioural study.

Other documents – similarly, some European countries have their own norms and documents, applicable only to their network. The ELTB (Germany), I/SC/PSOM/2298 (Italy), SAI 261 (Switzerland), BV-FS 2002 (Sweden) and the IAPF (Spain) [3] are some rules that must be italicized, although these latter ones are the ones that, due to geographical proximity and interoperability of the Lisboa-Madrid corridor, will serve most of the Portuguese needs.

3. Actions

A reading of the prior codes allows us to list the actions to take in account in the design of this kind of structure. The combination is made through the partial factor method, stated in EN 1991-2, and the ultimate and serviceability states of the structure are, thereby, verified.

The main action here is the one that serves the purpose of the creation of such structure: the railway traffic action. The design load models pointed out in the rules come from the envelope of static and dynamic effects of the existing rolling stock.

For the loading static diagram study, we recur to the LM71 model (Fig. 1) multiplied by a weight factor which takes in account the heavy traffic. According to the ATE [4], this factor assumes the value of 1.21. Even though new train models continuously pour out into the market, the weight of the LM71 has a sufficient gap between the heavier real trains: the weight ratio between the LM71 and current models are 4.10 (ICE2), 3.85 (Eurostar Class 373) or 2.85 (Virgin), to a model LM71 with the same train length as the ones above.

The same rules do not apply to the dynamic calculations: the load models HSLM (Fig. 2) are created surrounding the dynamic envelope of the rolling stock which dates back to decades ago. When choosing a new train, it is important to confirm that to every wavelength the dynamic signature of the train is inferior to the HSLM-A envelope. The author suggests a deeper investigation of the data of the most recent trains (e.g.: Alstom AGV, Siemens Velaro, Bombardier Zefiro) for load models so that, if necessary, more universal trains may be added to the HSLM set.

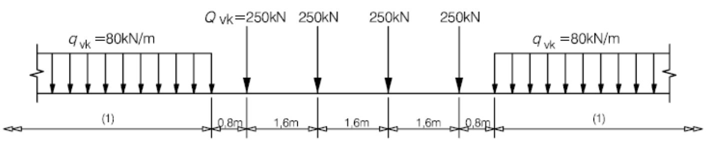


Fig. 1 – Load Model 71 with characteristic values

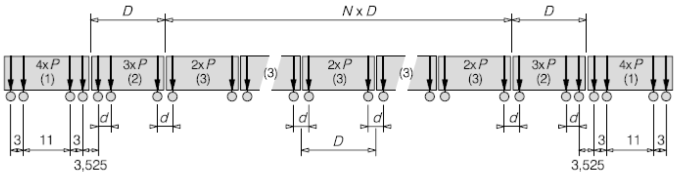


Fig. 2 – HSLM-A model

4. Methodologies for dynamic analysis of bridges and viaducts

4.1. State of the Art

One of the first more relevant studies in the subject dates all the way back to 1922, with the analytical solution for a single moving load on a simple supported beam, given by Timoshenko. As the knowledge progressed, new solutions were made up for more complex train and bridge models. What started off with the consideration of a moving load (Fig. 3), moved on to moving mass (Fig. 4), with the consideration of the inertial effect of the vehicle, finishing off onto a suspended mass model (Fig. 5), which refined, gave birth to the current models of vehicle-structure interaction. These last ones, with a higher number of degrees of freedom, allow the calculation of a variable that has been becoming predominant in this type of structural calculations: the acceleration of the vehicle. The limitation of response on this level allows to control the level of comfort experienced by the passenger, which with the increase of service speed, tends to condition the structural design.

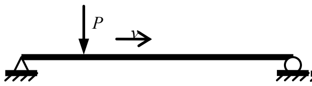


Fig. 3 – Moving load

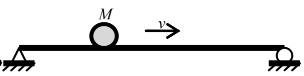


Fig. 4 – Moving Mass

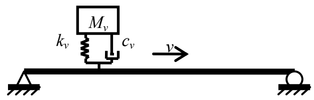


Fig. 5 – Moving Suspended Mass

New techniques of mitigation of the dynamic effect provoked by a high-speed vehicle on a bridge came up in the last couple of years. One way of succeeding is by augmenting the general suspension of the structure with energy dissipative devices, such as tuned mass dampers (TMD's) (Fig. 6) or fluid viscous dampers (FVD's) (Fig. 7).

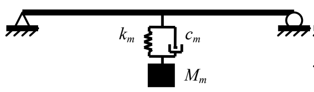


Fig. 6 – Structure with a Tuned Mass Damper

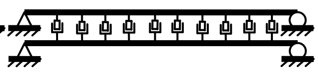


Fig. 7 – Structure with a set of Fluid Viscous Dampers

Calçada and Delgado [5] wrote the first Portuguese articles concerning the dynamic behaviour of high-speed railway traffic bridges, still based on moving loads model.

Later on, Delgado and dos Santos [6] presented iterative procedures for the solution of the matrix vehicle-structure system for each time step.

Following the protocol between Instituto Superior Técnico and Rede Ferroviária de Alta Velocidade, S.A. (RAVE), a campaign of studies taken on by Civil Engineering MSc students, focusing on the vehicles lateral dynamic behaviour (Dias, [7]), influence of the vertical support stiffness on the dynamic behaviour of high-speed railway bridges (Tavares, [8]) and vibration control of high-speed railway bridges through Tuned Mass Dampers (Henriques, [9]).

4.2. Impact or dynamic factor

A simple way to consider the dynamic amplification of the response consists in multiplying a static load by a impact factor, with the following equation:

$$I = \frac{R_d - R_s}{R_s} \qquad \text{Eq. (1)}$$

where

R_d Maximum dynamic response

R_s Maximum static response

The assumption of these factors for generic situations allows the running of quasi static analysis, which requires fewer resources. As such, recent codes suggest some expressions for this impact factor, of which the following stand out:

EN 1991-2

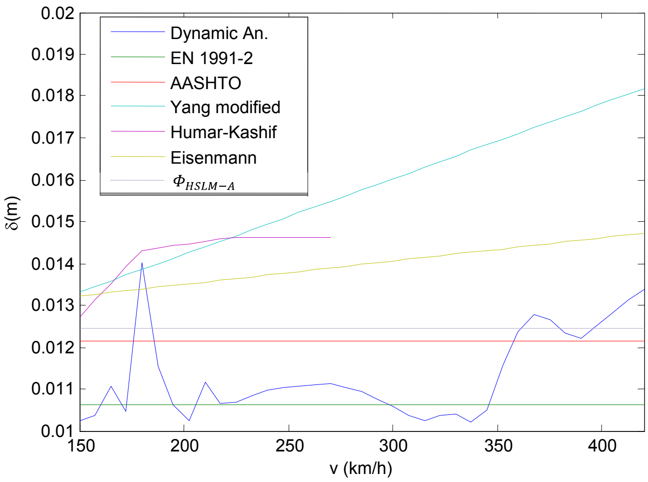
$$\Phi_2 = \frac{1.44}{\sqrt{L_\Phi} - 0.2} - 0.18 \text{ , for well maintained track} \qquad \text{Eq. (2)}$$

$$\Phi_3 = \frac{2.16}{\sqrt{L_\Phi} - 0.2} - 0.27 \text{ , for track with standard maintenance} \qquad \text{Eq. (3)}$$

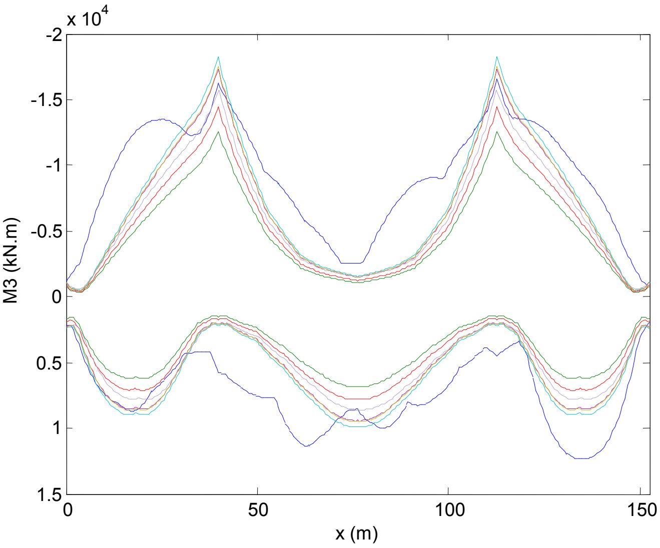
AASHTO [10]

$$I = \frac{15.24}{L + 38.1} \leq 0.3 \qquad \text{Eq. (4)}$$

Some changing proposals for the above expressions are suggested by other researchers, such as, Yang [11], Humar and Kashif [12] and Eisenmann [13].



(a)



(b)

Fig. 8 – (a) Maximum mid-span displacements for the proposed quasi-static analysis. The adopted values were: $L_\Phi = 66$m (all methods), $\omega_1 = 11.94$ rad/s, $\alpha = 0.004v/3.6$ (Yang and Humar-Kashif), $m_{veic} = 771.4$ ton, $m_{ponte} = 6758.7$ ton , $\kappa = 0.10$, $S = 27$ m, $S/L_\Phi = 0.20$ (Humar-Kashif), $t = 1$ and $s = 0.20$ (Eisenmann). See forward to $\boldsymbol{\Phi_{HSLM-A}}$

(b) Bending moment M3 envelope obtained for the proposed quasi-static analysis. The adopted values are the same as the aforementioned.

So, a comparison between the various presented methodologies was made, based on the passage of the HSLM-A10 on the bridge from the case (Fig. 8).

The first conclusion reached is neither of these methodologies consider sudden variation of the response at resonance – they underestimate the response at resonance and overestimate the response off resonance.

Other conclusions are:

- The American and European codes show an obvious limitation by not having a dynamic factor according to speed. For a bridge with this span length, the American regulation is more conservative and, for this particular case with better results, not only in terms of displacements but also in terms of forces. The intersection in the dynamic factors are found between the 15 and 20 meters span length, in which for smaller span lengths, the EN 1991-2 equations fall better in place.

In relation to displacements, the codes suggest a constant value that corresponds to the approximate average of obtained displacement in off resonance speeds.

On forces, as they are calculated for the resonance situation, both codes underestimate their value in every way.

-Yang's modified method shows better results than previous codes. The main advantage comes from considering speed in the dynamic coefficient calculation. Yet, a slight underestimation of the maximum displacement is noticed. Since the answer is directly proportional to speed, the displacements are highly overestimated if the method is applied to the highest

traffic speed, 420km/h. It is believed that this phenomenon will increase with the shortening of the span lengths. Consequently, the application of this method is especially effective if the resonance speeds are known right from the start.
This way, it is understandable that the obtained forces may be overestimated in relation to the static analysis of the problem, except for bending moments at mid-span.
The author suggests an early stabilization of the impact factor to a lower speed parameter.

- The Humar-Kashif method was the one that came closest to the dynamic analysis results. The explanation comes from the most refined train model. Even though the dynamic analysis uses the moving loads model, a train representation with two bogies with the distancing of S allows the consideration of the repetition of loads effect in a smaller scale. Unfortunately, this model was only studied up to 280km/h, where it can be of general interest the derivation of new design curves for speeds up to 420km/h. in terms of force, this method, similar to Eisenmann's, came up with results that were very close to the ones obtained by the dynamic analysis for the shear force and bending moment, with the exceptions of the values registered at mid-span.
- The Eisenmann method, in its modified version, was the only one in railway engineering considered here, due to not knowing the characteristics of the suspended and non-suspended weights of the HSLM-A model. This method does not consider at all the particularities of the bridge, noting that its adaptation skills may not always be the best suited. It just so happens that in this case in particular, a long span bridge with low dynamic amplification, the results are satisfactory, whether them being displacements or forces. The method, as proved in a later analysis, deteriorates itself in short span bridges.

The estimated equations by EN 1991-2 are applied to the HSLM-A10 model with unsatisfactory results. Either way, these were originally written to magnify the effect of the LM71 model, in accordance to the Eq. (4.2).

Therefore, it will be pertinent to work out, through the statistical principals given by EN 1991-2, a new proposal for the dynamic coefficient in bridges specially built for high-speed traffic.

The new method should apply to the same domain as the EN 1991-2 dynamic factor Φ for speeds above 200 km/h: simply supported bridges, with any span length and first vertical bending mode of vibration as the fundamental and main mode for the structure response. The LIR method, which returns the dynamic displacement, is appropriate for bridges with these characteristics and, hence, used by the author to create an envelope for mid-span displacements due to the passage of the ten HSLM-A train sets as a function of the span length. The analyses are limited to HSLM-A models thus the author does not recommend a quasi static analysis for bridges with a span length under 7 meters.

Along with the span length, it is also crucial to know the mass and bending stiffness of the structure to determine the dynamic response through the LIR method. These are the solutions found to measure the previous parameters as a function of the span length:

- Based on the ERRI bridges data, the author performed a linear regression between the mass per length unit and span length, with the final expression given by:

$$m = 0.66L + 4.47 \qquad \text{Eq. (5)}$$

with $r^2 = 0.97$ (coefficient of regression).

- Consider the interval of frequencies defined by Fig. 6.10 of EN 1991-2. This consists on a confidence interval between 95% and 98% for the exponential regression, undertaken by Fryba [18], which correlates the fundamental frequency of vibration with the span length.

Using Eq.(5) to determine the mass, i.e., independent of the stiffness of the structure, the lower bound for the interval of frequencies corresponds to the minor of the stiffnesses of the system and, therefore, to the highest displacements. So, in a conservative way, the final expression for the frequency is given by:

$$f_0 = 80L^{-1} \quad \text{to } 4 \leq L \leq 20\ m \qquad \text{Eq. (6)}$$

$$f_0 = 25.58L^{-0.592} \quad \text{to } 20 \leq L \leq 100\ m \qquad \text{Eq. (7)}$$

Table 1

Proposal for a dynamic factor to apply to any HSLM-A model to perform a quasi static analysis in a bridge specially built for high-speed traffic

Steel bridges	
$\boldsymbol{\Phi_{HSLM-A}} = \begin{cases} \mathbf{-0.051(L-6)^2 + 1.401(L-6) + 2.459} & 7 < L < 20 \quad (\mathbf{r^2 = 0.877}) \\ \mathbf{12.55 \times e^{-0.09(L-20)}} & \mathbf{L} > 20 \quad (\mathbf{r^2 = 0.950}) \end{cases}$	Eq. (8)
Reinforced concrete bridges	
$\boldsymbol{\Phi_{HSLM-A}} = \begin{cases} \mathbf{-0.055(L-6)^2 + 1.234(L-6) + 3.087} & 7 < L < 20 \quad (\mathbf{r^2 = 0.790}) \\ \mathbf{10 \times e^{-0.09(L-20)}} & \mathbf{L} > 20 \quad (\mathbf{r^2 = 0.884}) \end{cases}$	Eq. (9)
Prestressed concrete bridges	
$\boldsymbol{\Phi_{HSLM-A}} = \begin{cases} \mathbf{-0.047(L-6)^2 + 1.054(L-6) + 1.949} & 7 < L < 20 \quad (\mathbf{r^2 = 0.810}) \\ \mathbf{8 \times e^{-0.1(L-20)}} & \mathbf{L} > 20 \quad (\mathbf{r^2 = 0.670}) \end{cases}$	Eq. (10)

The effects of track irregularities (φ''/2) was also considered.

The dynamic factor $\boldsymbol{\Phi_{HSLM-A}}$ was derived for the displacements, since the dynamic amplification of displacements is higher compared to forces, according to some of the mentioned proposals.

The three equations presented (Table 1), based on type of structure and respective damping, were consequently tested in the case study bridge, with very good results (see Fig. 8).

4.3. Dynamic structural behaviour for simply supported beams

The studies were followed with a new comparison, this time between the different existing methodologies for the solution of the equation of motion on a simply supported beam. Of the presented analytical solutions, the single moving load solution, the general solution of Fryba [14] to a series of moving loads and the particular solution of Yang [11] for the equidistant front and rear axles were studied.

Additionally, the DER and LIR methods, proposed in the ERRI D214/RP9 [15], are included in the analysis.

The results of this comparison are in Table 2.

- The DER and LIR methods give back close results, where there is no particular preference for either one, even though in all four cases the LIR method showed a lower maximum for the resonance speed. About the DER method, it is proved that for off resonance and close to cancellation speeds, it tends to give back null results, something that can be suppressed by using its modified version. The same can be said about LIR, which clearly underestimated the off-resonance accelerations.

- The Yang method has the obvious limitation of neglecting the damping, which is not wise to admit in a common steel or reinforced concrete bridge behaviour. The ratio between maximum accelerations for a damping coefficient of 0.1% and 4% is eight for a short span bridge and four for a long span bridge. This number is too high to be used as a safety coefficient. Another simplification dissuades one from using Yang's method for the response calculation: the equidistance between axles and the omission of the power cars produces acceleration twice as superior and the one obtained by the simplified methods, with the same damping coefficient, in a short span bridge. The only field advantage one can point out in this method consists in the simple and intuitive analytical exposure of the resonance phenomenon in a solvable mathematical expression with low computational resources.

- Fryba's method solved in MATLAB shows slightly different results compared to the indicated values in ERRI D214/RP9. For case b) (Table 1) the DER, LIR and Fryba solutions all present the same shape. The first resonance speed in the domain is the one presented by ERRI. However, response peaks are observed for higher speeds, which lead us to believe that speeds above the first one may have not been considered by ERRI. In any case, the solution given by Fryba is superior to the ones from DER and LIR. The explanation for this comes from the 25 modes of vibration in the Fryba solution, built in opposition to only one fundamental mode of vibration to the DER and LIR methods. That way, one may not be on the safe side by utilizing the simplified methods towards the structure responses, so a safety gap to the maximum acceleration permitted is advised. It is interesting to understand that, in accordance with this statement, it is for higher resonance speeds (with the excitation of higher vibration modes) that the deviation between the methods augments.

4.4. Resonance phenomena

The resonance effect of a vehicle-structure system can be divided in the following categories according with the mechanisms that generate them:

- Bridge resonance induced by periodically loading of moving load series. For the resonance analysis on the bridge, the series of loads must consist not only in the vertical forces on the axles from the weight of the train, but also the lateral forces transmitted in the contact points due to the centrifugal acceleration or wind pressure acting on the vehicle. To this type of resonance there is a corresponding critical speed, given to us by:

$$v_{cr} = \frac{d \times \omega_n}{2\pi i}, \quad n = 1,2,3,\ldots \text{ e } i = 1,2,3,\ldots \qquad \text{Eq. (11)}$$

Table 2

Solution comparison for the Eurostar train passage in: (a) Bridge 1 from ERRI (L=5 m, EI = 453919 kN.m^2, n_0=16 Hz, m=7 ton/m), ξ=0.1% (b) Bridge 1 from ERRI, ξ=4%, (c) Bridge 2 from ERRI (L=20 m, EI = 20750590 kN.m^2, n_0=4 Hz, m=20 ton/m), ξ=0.1%, (d) Bridge 2 from ERRI, ξ=4%

	DER Method		LIR Method		Yang Method		Fryba Method		Fryba - ERRI	
	$a_{máx}$ (m/s^2)	v (km/h)	$a_{máx}$ (m/s^2)	v (km/h)	$a_{máx}$ (m/s^2)	v (km/h)	$a_{máx}$ (m/s^2)	v (km/h)	$a_{máx}$ (m/s^2)	v (km/h)
a)	132	272	132	272	260	360	167	272	-	
b)	16.3	420	15.7	420	-		23.2	360	18	269
c)	33.5	278	33.4	278	33.5	270	34.7	280	-	
d)	7.05	278	6.85	278	-		7.57	276	6.3	269

- Bridge resonance induced by loading rate of moving load series. To this type of resonance there is a corresponding critical speed, given to us by:

$$v_{cr} = \frac{\omega_n L}{n\pi}, \ n = 1,2,3,\ldots \quad \text{Eq. (1)}$$

with values between 600 and 800km/h for simply supported beams with fundamental natural frequency of vibration given by 100/L.

- Bridge resonance owing to the sway forces of train vehicles (with the same critical speed as the first type of resonance with the single replacement of d with L_v, the characteristic wavelength of the track irregularities distribution).

– Vehicle resonance by the periodic action of continuous spans of regular length and its deformations. To this type of resonance a corresponding critical speed is given to us by:

$$v_{cr} = \frac{\omega_{vehicle} L}{2\pi} \quad \text{Eq. (2)}$$

with values between 57 and 216 km/h for coaches with a fundamental natural frequency of vibration between 0.8 and 1.5 Hz and span lengths between 20 and 40 meters. As such, it is not recommended to repeat identical spans in a continuous high speed railway bridge.

4.5. Dynamic Analysis of a Multi Degree of Freedom Systems (MDOF)

Based on the Zienkiewicz *et al.* [16], the resolution of the dynamic problems of a multi degree of freedom system is presented through the finite elements method (FEM).

Starting from FEM's fundamental equation for the static problem, the final form of the matrix equation of motion is achieved.

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{f} \quad \text{Eq. (3)}$$

The solution of this equation can be achieved through two types of methods:

- Methods of direct integration
- Methods of mode superposition

Using the first one usually requires larger computation resources since the step-by-step integration of the previous equation is needed. Among the explicit methods (excluding the inversion of the stiffness matrix) the author emphasizes the method of central difference and within the implicit ones (with inversion of the stiffness matrix) Newmark and Hilber-Hughes-Taylor's methods stand out as the ones with better results.

The superposition methods consist of combining the answer of the structure's various modes of vibration towards the final response. Limiting the analysis to 30Hz, the number of considered modes of vibration ends up minimizing the necessary resources, and consequently leading this method to be the most adequate to the dynamic analysis performed by the author in the case study. These analyses were conducted in SAP2000 combined with a pre-processor developed by Henriques [9] with the following default settings:

- The model is made out of finite elements mesh with a maximum dimension of 1 meter (reference value also used in previous works in the field of structural dynamic and seismic engineering). The convergence of the solution is assured for a mesh with these dimensions
- The adopted integration time step is T/10 = 0.032s, where T is the lowest period in the system (Bento *et al.* [17])
- The speed step is 7.5km/h based on the sensitivity of the results from the DER and LIR methods and line limitations from the Microsoft EXCEL® spreadsheet.
- The modal damping is constant and with the value of 1% (recommended value in Fig. c) for pre-stressed concrete structures).
- The performed dynamic analysis assumes a physical and geometrical linearity of the system.

4.6. Vehicle-bridge interaction

The last breakthroughs in the field of dynamic analysis have been on the vehicle-bridge interaction models. These models lead to detailed presentation of the train, in order to recreate the existing train and bridge systems interaction, each one with its mass matrixes, damping and stiffness. Delgado and dos Santos [6] define the equation of motion for the interaction between both systems:

$$\begin{bmatrix} M_b & 0 \\ 0 & M_v \end{bmatrix} \begin{Bmatrix} \ddot{u}_b(t) \\ \ddot{u}_v(t) \end{Bmatrix} + \begin{bmatrix} C_b & 0 \\ 0 & C_v \end{bmatrix} \begin{Bmatrix} \dot{u}_b(t) \\ \dot{u}_v(t) \end{Bmatrix} + \begin{bmatrix} K_b & 0 \\ 0 & K_v \end{bmatrix} \begin{Bmatrix} u_b(t) \\ u_v(t) \end{Bmatrix} = \begin{Bmatrix} F_b(t) \\ F_v(t) \end{Bmatrix} \quad \text{Eq. (4)}$$

where M, C and K represent the mass matrixes, damping and stiffness, F is the external force vector, u is the nodal displacement vector, and the b and c indexes concern to the bridge and vehicle, respectively.

The common point of both equations are the obtained displacements: the moving load action F_b causes a u_b displacement on the bridge which, in the same instant, turns into a u_v base displacement for the vehicle.

According to EN 1991-2, the accounting of the dynamic effect of vehicle-structure interaction tends to decrease the resonance response for bridges with a span shorter than 30 meters. In order to considerate this effect, the aforementioned document suggests two alternative procedures:

- Perform an interactive vehicle-structure dynamic analysis
- Increase the assumed damping coefficient for the structure: $\Delta\zeta = \frac{0.0187L - 0.00064L^2}{1 - 0.0441L - 0.0044L^2 + 0.000255L^3}$

5. Dynamic Behaviour according to EN 1991-2

A reading of EN 1991-2 leads one to the listing of the main factors that influence the dynamic behaviour of a structure. In this chapter, these same factors are studied.

5.1. Speed of traffic

The safety verifications must be done in the speed interval between 0 and 1.2 V km/h, where V is the maximum design speed. The safety coefficient of 1.2 comes into play with the speed variation and so, with the possible resonance speeds with a value nominally superior to the predicted maximum.

Even though in the proposed flow chart in EN 1991-2 one admits that the resonance effects do not exceed the predicted dynamic increments for speeds up to 200km/h, it is proved by the author that such assumption is nothing but empirical and that by resorting to the DER method for the HSLM-B train effect analysis (d=2.5m e N=10) crossing bridge 1 from ERRI, the opposite is stressed out (Fig. 10).

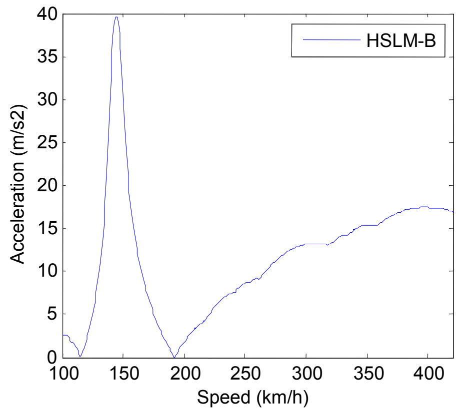


Fig. 10 – Maximum mid-span Acceleration in bridge 1 from ERRI (see Table 1, ξ=1%) due to the passage of the universal HSLM-B train (d=2.5m e N=10), according to DER method. Notice that the first and main resonance speed is 150km/h, approximately. Agreeing with the flow chart from EN 1991-2 the dynamic analysis would not be required.

A precaution measure led to consider a minimum speed of 150 km/h in all upcoming tests.

5.2. Span length

Contrary to response static analysis, where the cube of the span length is proportional to mid-span displacement, the structure's response in a dynamic analysis stops registering this proportionality. In fact, in terms of accelerations, the beam's response tends to diminish with the span increase, being the maximum value to do a dynamic analysis is 43 meters, is accordance to Fig. 11 (this value is close to the allowed minimum from EN 1991-2 for an equivalent static analysis).

In the same way, the relation L/d is determining for the structure's dynamic response. It is not a direct induction since the resonance condition depends on other factors, such as traffic speed and natural frequency of the structure.

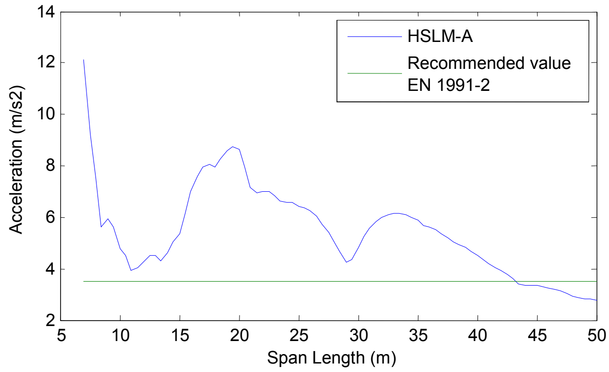


Fig. 11 – Maximum mid-span accelerations registered for HSLM-A envelope in simply supported beams, according to DER method. The damping values (steel bridge), natural frequency and mass per length are the ones defined in EN 1991-2, $n_0= 133.006L^{-0.911}$ and $m=0.66L+4.47$, $r^2 = 0.97$ *(linear regression for ERRI's bridges),* respectively.

5.3. Train Signature

In both DER and LIR, the $G(\lambda)$ term represents the loading spectrum of a train that, when multiplied by the $A(L/\lambda)$ term, returns the aggressiveness in agreement with EN 1991-2 E.2(6). If in the $G(\lambda)$ term one is to assume a null damping, it is possible to obtain an independent expression which returns the dynamic signature of a given train.

$$S_0(\lambda) = \max_{i=1\,a\,M} \sqrt{\left[\sum_{k=1}^{i} P_k \cos\left(\frac{2\pi x_k}{\lambda}\right)\right]^2 + \left[\sum_{k=1}^{i} P_k \sin\left(\frac{2\pi x_k}{\lambda}\right)\right]^2} \qquad \text{Eq. (5)}$$

This measurement allows evaluating in a simple way the dynamic effect of each real train. Granting for all wave lengths the dynamic signature of a given train is inferior to the envelope of HSLM trains signatures, the designed structure's safety is granted for these universal trains.

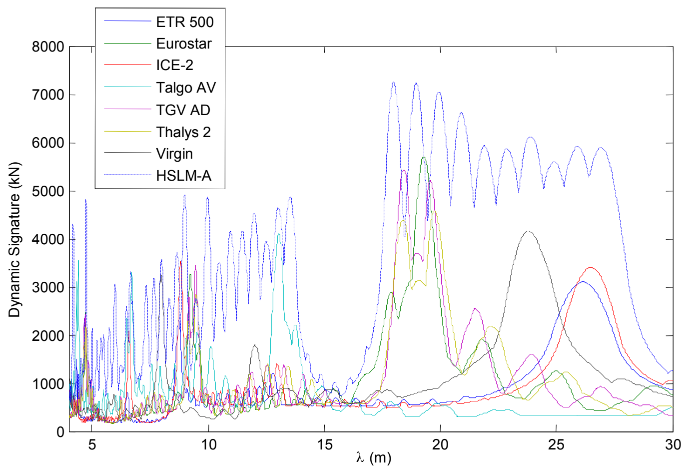


Fig. 12 – Dynamic signature of some of the existing trains in the European network and the envelope of the dynamic signature of HSLM-A models.

5.4. Damping

The damping in structures occurs due to a loss of energy during each oscillation cycle.

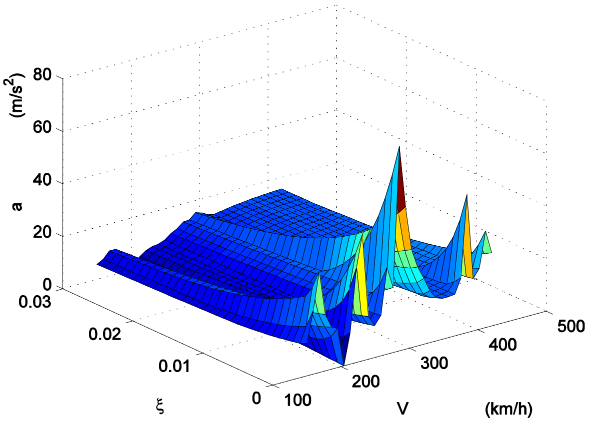


Fig. 13 – Sensitivity analysis for Damping. Accelerations obtained through LIR method for the problem of a Eurostar train crossing the bridge 1 from ERRI.

Concordant to Fig. 13, one can conclude that the lower the damping, the higher the system response. This is trivial knowledge in the dynamic field. Still, in this case, one must define the lower limits to ensure a good estimate of the dynamic effect in resonance.

It is expected that the damping will increase over the years, especially due to friction phenomena: structure deterioration, loosening of the joints and contamination of the ballast. Among the recommended service tests by TSI, a measurement of the bridge's damping coefficient must be included.

5.5. Mass of the structure

Another important parameter in the definition of the structure is its own mass. In fact, for an oscillator with multi degrees of freedom, the resolution of the equation of motion imposes the calculation of the mass matrix (nxn) associated to the multiple degrees of freedom. Observing Fig. 14, one can conclude the larger the mass, the higher the system response. This is because the smaller the mass, the higher the frequency (note that the expression for the natural frequency of an oscillator of 1 degree of freedom), and consequently the higher the system's natural frequency, the higher the response. So a lower bound must be defined for the analysis.

However, one must stress that the (resonance) critical speed is given to an equality between d (axle distance) and $v_{cr} \times T$ (critical speed x period), to equally spaced load systems. For minor frequencies, there are less critical speeds. Indeed, a superior limit for calculation purposes should be defined.

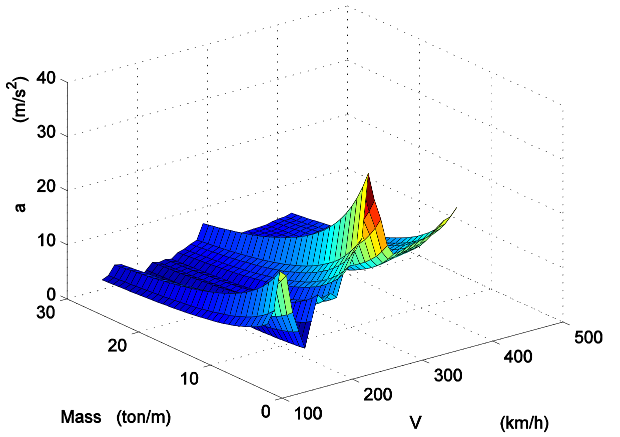


Fig. 14 – Sensitivity analysis for Mass. Accelerations obtained with same presets as Fig. 14 $\zeta = 1.0\%$

As written in 6.4.6.3.2 (2) of EN 1991-2, the following hypothesizes are considered:

- To determine the tray's maximum acceleration, the ballast is admitted with its dry specific weight and minimum thickness.

- To determine the lower resonance speeds, the ballast is admitted with the saturated specific weight and, if predicted, the rails' weight for a new track.

The recommended values for material density are indicated in Annex A of the EN- 1991-1-1.

5.6. Stiffness of the structure

To solve the differential equation of motion, one must determine the stiffness matrix, K.

Through the DER method's expression which expresses the acceleration, one understands the frequency's quadratic term cancels with K*, making the acceleration of the system independent of its stiffness, as shown in Fig. 15 a). It may be a little unsafe subscribing this affirmation since, by the DER method's simplification, the structure's first mode of vibration is admitted as a bending mode, which happens in most cases. Only then can it be said that K* is equal to the simply supported beams bending stiffness.

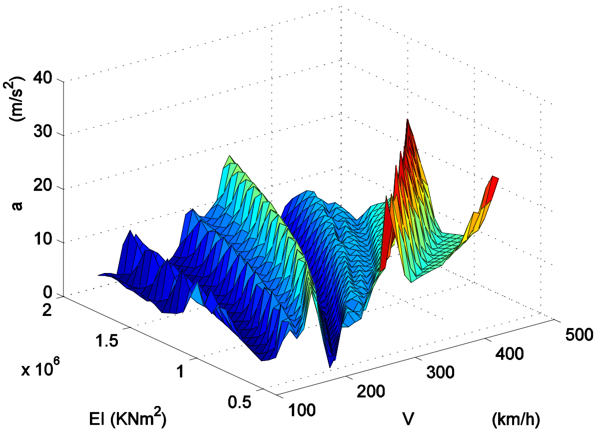
On the other hand, for a stiffer structure, higher natural frequency, higher critical speed.

Like Fig. 15 b) indicates, for more stiffness, lower vertical displacements. This happens because the speed parameter lowers with the increase of frequency. The static displacement itself lowers with higher stiffness: $\{q\} = [K]^{-1} \{F\}$.

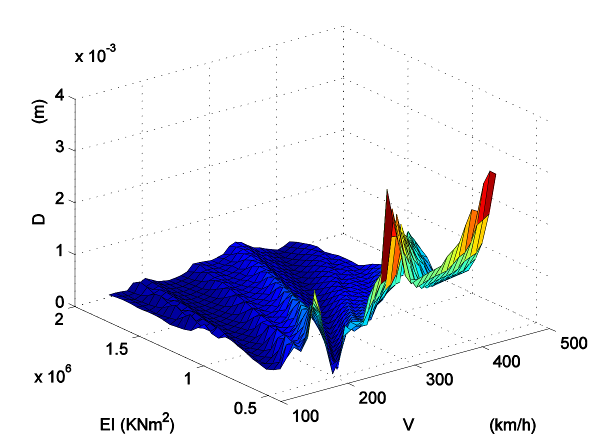
Hence, it is important to determine a lower limit for analysis purposes.

The Young's modulus of steel is 210 GPa, for either static or dynamic behaviour. The distortion module is 80GPa.

For concrete, regard the ERRI D214/RP9 conclusions based on Ammann and Nussbaumer's works: for dynamic actions, the Young's modulus increases when compared with the static value (an increase of about 15% is admitted., valid for concrete with f_{cm} up to 60MPa (C50/60)). On another perspective, REBAP's article 17.2 predicted a 25% increase of the concrete's Young's modulus for fast actions, such as moving loads. The recommendation falls into the Ecm (chart 3.1 do EN 1992-1-1) as the recommended value for the stiffness' inferior limit, just like the distortion modulus that results from a Poisson coefficient of 0.2 for uncracked concrete or 0 for cracked concrete.



(a)



(b)

Fig. 15 – Sensitivity analysis for Stiffness. Accelerations (a) and displacements (b) obtained with same presets as Fig. 14 $\zeta = 1.0\%$

5.7. Other parameters

Other parameters were also studied, such as the structure's natural vibration frequencies effect and track and vehicle irregularities.

The resolution of the dynamic problem with mode superposition techniques involves the determination of the structure's modes of vibration and their respective frequencies. The simple methods calculate the structure's response based on the vertical vibration's fundamental frequency.

Concerning the track irregularities (modifications of the track original geometry), they are a source of a bridge excitation during a vehicle passage. The dynamic amplification due to track irregularities increases with speed and decreases for long span bridges and is mainly caused by the vehicle's unsuspended mass which contacts directly to the rail profile.

Mathematically speaking, the formulation of these irregularities is, in a way, abstract. Fryba [18] categorizes the irregularities on two types: periodic and random. In order to avoid such track complex longitudinal profile presentations, UIC has defined factor φ'' which represents the additional dynamic effect due to track irregularities.

Amidst the most common vehicle irregularities, outstands the wheel corrugation which clearly has a harmful effect in the dynamic amplification (Nielsen, [19]), augmenting the same way as speed increases.

6. Dynamic analysis of a railway bridge

In this final chapter, the author verified the safety of an existing conventional line for high speed train traffic. The structure was designed for the load model LM71 with an α coefficient of 1.21 (to predict heavy traffic) and dynamic factor Φ of 1.1 and should now obey the following conditions (as agreed on EN 1991-2 6.4.6.5):

- Maximum deck acceleration of 0.35g in case of ballast tracks or 0.5g for slab tracks. This verification must be checked with the HSLM load models and, if known, for the real train load model that will circulate on those tracks.

- The vertical load effects given by the most unfavourable of both:

$$\left(1+\varphi'_{din}+\frac{\varphi''}{2}\right)\times(HSLM\ ou\ RT) \qquad \text{Eq. (11)}$$

$$(1+\Phi)\times(LM71) \qquad \text{Eq. (12)}$$

- The additional security to fatigue by the least favourable of both cases mentioned above

The fatigue effect for high-speed was not studied in this dissertation. For a better comprehension of the subject, refer to Annex D of EN 1991-2

As there is no knowledge on the actual trains used on Portuguese tracks, only the ten HSLM-A models are picked for the design indicated for a continuous structure.

6.1. Bridge description

The railway bridge has the following attributes:

- The bridge cross-section has two tracks and a total width of 12.3 metres (Fig 16).

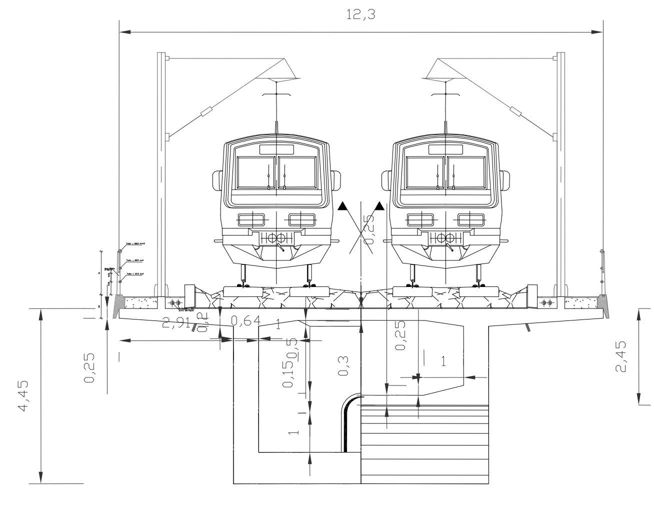


Fig. 16 – Bridge cross-section

- The longitudinal layout of the bridge has three spans of 40 + 72.5 + 40m (Fig 17).

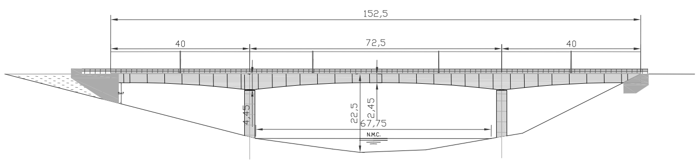


Fig. 17 – Balanced cantilever bridge longitudinal layout. The vertical lines define a new constructional segment

- The concrete is class C35/45, the reinforcing steel is A500NR and the prestressing steel is A1670/1860.
- The superstructure-infrastructure connection is made through Neoprene bearings with free rotation and longitudinal displacement (except for one of the abutments).
- In the intermediate and end support sections diaphragms a metre thick were concreted.

The dynamic factor Φ used in the quasi static analysis of the conventional line was obtained through the following formula:

$$\Phi = \frac{2.16}{\sqrt{L_\Phi} - 0.2} - 0.27 = 0.002 < 0.1 \Rightarrow \Phi = 0.1 \qquad \text{Eq. (12)}$$

where L_Φ is the characteristic length, case 5.2 of the 6.2 of EN 1991-2.

In the same way, one of the high-speed model alterations consists on the tracks' axle distance of 4.6m (opposing the conventional 4.1m).

For modal analysis it was considered not only the bridge self weight but also the weight of non structural members (e.g.: ballast, rails).

6.2. Bridge FE modelling

For the practical case, two models were chosen for the analysis: The Beam FE Model and the Shell FE model (ordered by refinement). The beam model can impose certain exaggerations of simplification: for example the deck slab behaviour, with cylindrical bending when the load is applied far from the web, is not accounted for in this model, the deck behaves as the top flange. It is easy to understand that the more detailed the structure representation, with its local phenomena, the more accurate the results are. The innovative usage of the beam model, that supposedly has an unknown results deviation, constitutes the common practice in design offices. The difference with the shell model will be analyzed.

After the design of the bridge object as a frame object (beam model), it was necessary to additionally consider a polar moment of inertia in each section, so that one may consider the mass parted from the centroid and so be able to include the torsional modes of vibration. A moment of inertia in the direction of 1-1 was assigned to the midpoint of each bridge segment with the value of:

$$I_{rx} = L_{segment} \times \frac{J_i + J_{i+1}}{2} \times \frac{\gamma}{g}\ (kN.m.s^2) \qquad \text{Eq. (12)}$$

where,

$L_{segment}$ Segment length (m)

J_i Torsional constant for the start section of the segment

J_{i+1} Torsional constant for the end section of the segment

γ Segment specific weight (=25 kN/m^3)

g Gravitational constant (=9.8 m/s^2)

6.3. Beam and shell models manipulation

As the shell model is more refined, the beam model is forced to give back the same results over the careful and justified manipulation of the structure's stiffness in certain sections since this model overestimates the bending and, mainly, torsion stiffnesses.

It was attempted, through SAP2000, to modify the section properties in order to decrease deviations between the frequencies of the original beam model and shell model and the deviations between the midspan displacements of each model. After a number of iterations, optimum multiplier values were reached: 0.9 and 0.32 for the moment of inertia about 3 axis and torsional constant, respectively.

About the static analysis, an improvement in the results is obvious– see Table 4. The decrease of vertical bending stiffness through the adjustment of the moment of inertia about 3 axis makes the deviation between the vertical displacement u_z lesser. In the same way, decreasing the torsion stiffness, in a more drastic way, resulted in a balancing of the deviations of the torsion rotations Θ_x for the extreme (negative) and central (positive) half span nodes.

As to the modal analysis, the result improvement was also notorious – see Table 3. Nevertheless, it is impossible to reduce the modes of vibration deviations to zero, by general manipulation (non-local) of the structure. One of the limitations in the inertia reduction about 3 axis was on the 3rd mode of vertical vibration which mass participation factor about z is the highest. This way, a new correction for the multipliers, based on *in situ* values, is recommended.

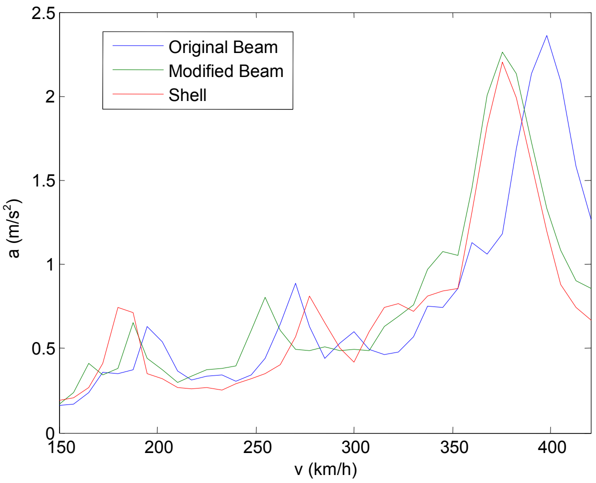
Table 3

Modal Analysis results for the three models

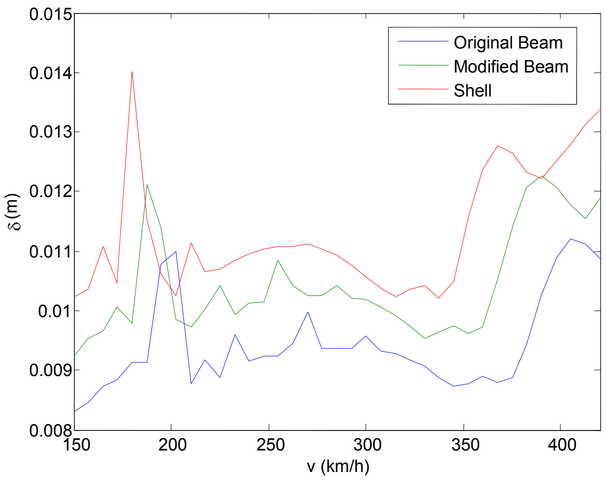
Original beam model			Modified beam model			Shell model	
f (Hz)	Mode type	$\varepsilon_{beam-shell}$ (%)	f (Hz)	Mode type	$\varepsilon_{beam-shell}$ (%)	f (Hz)	Mode type
2.10	1ºV	10.53%	2.10	1ºV	10.53%	2.10	1ºV
4.06	2ºV	5.73%	4.06	2ºV	5.73%	4.06	2ºV
4.53	1ºH	-3.41%	4.53	1ºH	-3.41%	4.53	1ºH
5.44	3ºV	-3.89%	5.44	3ºV	-3.89%	5.44	3ºV
7.45	4ºV	14.26%	7.45	4ºV	14.26%	7.45	4ºV
9.49	1ºM	4.86%	9.49	1ºM	4.86%	9.49	1ºM
…			…			…	
20.24	2ºM	44.16%	13.48	2ºM	28.26%	20.24	2ºM
…			…				
22.69	1ºT	28.26%	20.51	1ºT	44.16%	22.69	1ºT

A set of conclusions can be deducted from the previous analysis:

- The Beam model modification results in a better proximity to the Shell model, in terms of dynamic moving load analysis (see Fig. 18 a) and b))



(a)



(b)

Fig. 18 – Comparison of the maximum acceleration (a) and displacement (b) for the HSLM-A10 traffic in the Beam and Shell models

- When the Shell model is resonant, there is a difference between the beam models, especially in relation to the original Beam model. For off resonance speeds, the difference between models tends to diminish, logically.

- In what concerns accelerations, the modified Beam and Shell models show peak results with an average deviation of 10% with some dispersion but, for the same resonance speeds, in general. By comparing to the original Beam model, as the natural vibration frequencies are not perfectly aligned and the excitation frequency is the same, the resonance is given at different speed (average speed deviation of 3%) which implies variation in the maximums. The speed deviation has a positive value, for it expectable that the resonance speeds to be higher in this last model (according to Eq. (5))

- As to displacements, the modified Beam model shows inferior maximums records, (8% in average), as one would expect due to the models higher stiffness, but still better than the original Beam model.

Table 4

Static Analysis results for the passage of LM71 in the three models

	Original beam model		$\varepsilon_{beam-shell}$ (%)		Modified beam model		$\varepsilon_{beam-shell}$ (%)		Shell model	
	Node 72	Node 352	72 - 862	352-366	Node 72	Node 352	72 - 862	352-366	Node 862	Node 366
u_z (Abs) (cm)	0.76	2.78	-15.73%	-14.57%	0.83	3.04	-8.09%	-6.56%	0.90	3.25
Θ_x (Max) (mrad)	0.14	0.32	-74.86%	-64.03%	0.43	1.10	-21.31%	21.83%	0.55	0.88

NOTE: Node 72 (BM) and Node 862 (SM) : (x,y,z) =(17.5,0,0) – Peak response section in the lateral span. Due to the bridge variable depth, the peak dislocates from 5/8 span (supported-encastrated beam) to mid-span span.

Node 352 (BM) and Node 366 (SM) : (x,y,z) =(76.25,0,0) – Peak response section in the central span. Mid-span.

Once completed, the previous models pass through structure's security verification according to EN 1991-2 and EN 1990 – A2.2.4 [20] criteria. The only considered criteria are the ones that demand additional verification for HSLM universal trains.

6.4. Ultimate Limit State

Firstly, the produced forces in the structure were studied, through the least favourable case:

$$\left(1+\varphi'_{din}+\frac{\varphi''}{2}\right)\times HSLM = HSLM \qquad \text{Eq. (12)}$$

$$(1+\Phi)\times(LM71\times\alpha) = (1+1.1)\times(1.21\times LM71) = 1.331\times LM71 \qquad \text{Eq. (13)}$$

where,

$$\varphi'' = 1\times\left[0.56e^{-\left(\frac{66.08}{10}\right)^2}+0.50\left(\frac{2.0\times 66.08}{80}-1\right)e^{-\left(\frac{66.08}{20}\right)^2}\right]\approx 0$$

Through the comparison of the LM71 and HSLM-A2 (heaviest HSLM load model) weights, one can conclude that the first is two and a half times heavier than the second which, in similarity to Fig. 19, allows concluding that, even with dynamic amplification, the produced forces by the LM71 will be more penalizing to the structure, especially in long span bridges, as is in this case.

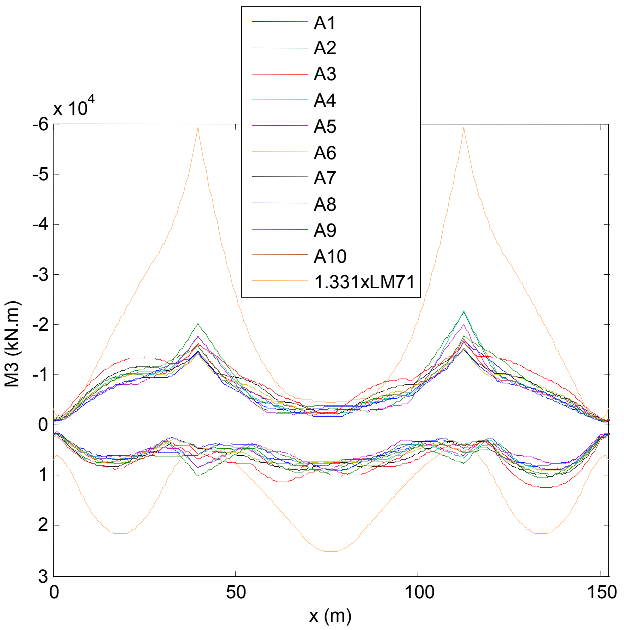


Fig. 19 – Envelope of the bending moment M3 for the train action only: $Q_{sd}=\gamma_{Q,1}Q_{k,1}=1.5Q_{k,1}$ The analysis are undertaken in the modified Beam model.

Therefore, once designed for conventional railway traffic, the bridge confirms the safety for the ultimate limit states in case of high-speed railway traffic.

6.5. Serviceability Limit State

The first safety standard is to avoid the ballast instability and the reduction of the contact between the rail and the wheel. For this particular case, ballasted track, the allowed maximum vertical acceleration is 3.5m/s^2. And so, the studies proceeded with the determination of the vertical accelerations produced by the ten HSLM-A models and assurance that they were in accordance to the recommendations.

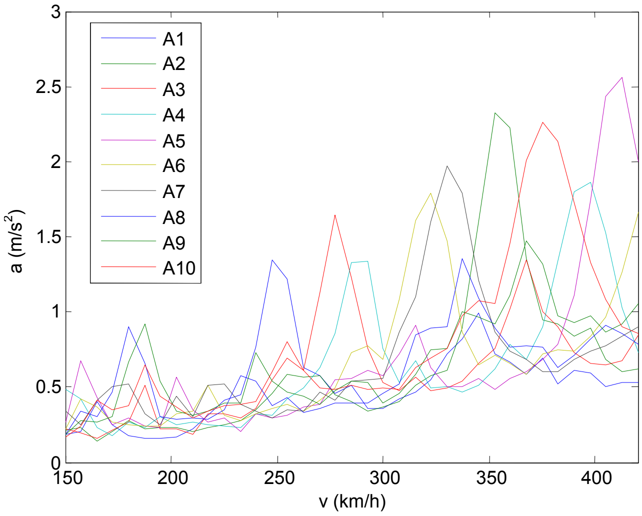


Fig. 19 – Maximum vertical accelerations for the ten HSLM-A trains in the modified Beam Model

For the case study, the bridge confirms the safety for maximum deck vertical accelerations: 27% below the limit value for the modified Beam model and 37% for the Shell model.

The criteria for displacement limitation are to avoid the modification of the track's vertical geometry and to ensure the geometric linear analysis for robust structures (lines a) and b)) in general. On the other hand, for a system without vehicle-bridge interaction, like the case study, the same criteria can be used to assure the passenger comfort (line c)).

a) According to A2.4.4.2.3 of the EN 1990 code, one must ensure that, for all possible load configurations with its respective characteristic values, the maximum vertical displacement along the track due to railway traffic will not surpass $L_\Phi/600$ (=5.5cm for the LM71).

b) According to the IAPF 4.2.1.1.3, one must ensure that, with the same conditions of the previous line, the maximum deck rotation in its supports is, for a ballasted track, equal to $\theta = 6.5\times10^{-3}$ rad (end support) or (θ_1 + θ2=10×10−3 rad (intermediate support).

c) According to A2.4.4.3 of the EN 1990 code, one must ensure that, in case of non consideration of the vehicle-structure interaction, the maximum vertical displacement along the track is given by Fig. 6.10 and it is function of the span length, train speed, number of spans and bridge configuration (=2.8cm for the HSLM-A).

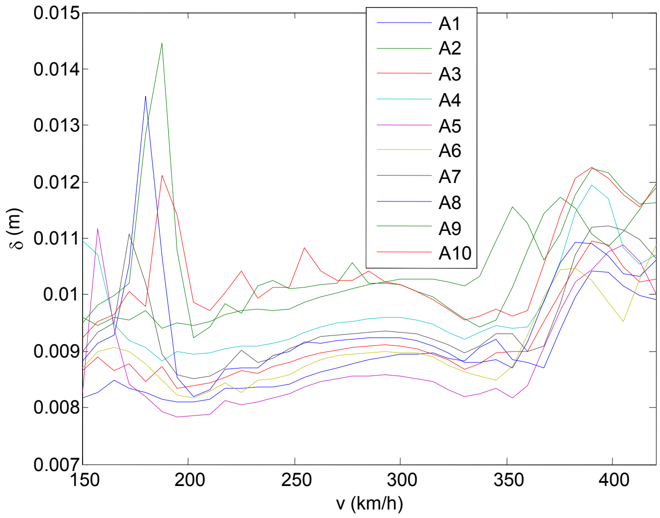


Fig. 20 – Maximum vertical displacements for the ten HSLM-A trains in the modified Beam Model

In relation to support rotations, it was measured for the maximum train displacement, HSLM-A9 at the speed of 187.5km/h (Beam model) and 180km/h (Shell model).

For the case study and, in accordance with Fig 6.11, Chart 6.3 and Chart 6.5, the bridge confirms the safety for a maximum deck vertical displacement:

LM71 – 49% below the limit value for the modified Beam model and 45% for the Shell model,

HSLM-A – 48% below the limit value for the modified Beam model and 46% for the Shell model

In what concerns maximum deck rotation:

End support - 86% below the limit value for the modified Beam model and 85% for the Shell model

Intermediate support - 92% below the limit value for the modified Beam model and 90% for the Shell model

6.6. Ballast modelling

A more detailed model of the bridge was created to represent the ballast, concrete sleepers and rails.

The ballast has an important role in the dynamic behaviour of the structure, since its integrity serves as a dynamic designing criterion.

The configuration of the ballast follows the Spanish practice, regulated by the Ministerio do Fomento [21]. The same document defines the layers (from the deck to the surface) and mechanical parameters of the materials:

-30cm of tout-venant, ($E = 19.6\text{MPa}$, $\nu = 0.30$, $\gamma = 35°$, $\rho = 16.5\text{KN/m}^3$)

-30cm of sub-ballast, ($E = 117.6\text{MPa}$, $\nu = 0.30$, $\gamma = 35°$, $\rho = 16.5\text{KN/m}^3$)

-30cm of normal ballast, ($E = 127.4\text{MPa}$, $\nu = 0.20$, $\gamma = 45.5°$, $\rho = 20\text{KN/m}^3$)

The ballast occupies an area of 152.5 x 7.75m centred in the bridge y axis, according to the original cross section.

The remaining considered elements in the refined analysis were:

- 2 rails UIC-60 with a 1.435 meters offset
- Equivalent B70 sleepers half a meter wide and each parted by 1 meter.

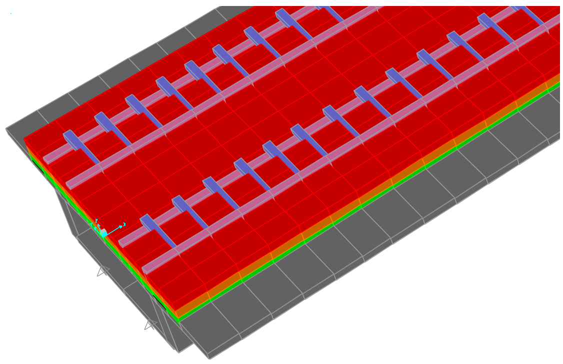


Fig. 21 – Bridge object with ballast: Green – Tout-venant, Orange – Sub-ballast, Red – ballast, Purple – Sleepers, Pink - Rails

For the ballast definition in the FE program (Fig. 21), three new materials with the mechanical properties above were created and a liner elastic behaviour was admitted.

As each layer has a mesh of finite elements, and these have similar dimensions in all three directions, the author selected Serendipian solid elements with 8 nodes to define the ballast mesh.

Concerning the load model, it was decided to divide the train car weight by both rails. This decision allows performing the last of the verifications of the serviceability limit state: the deck twist.

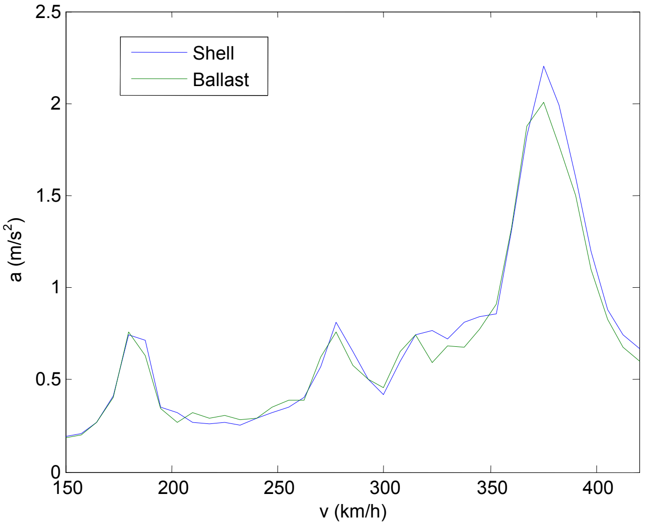
As in Beam models, static and modal analysis are carried out to compare the Shell model and the Ballast model.

About the static analysis, an average deviation of -50% on the vertical displacement caused by LM71 is consequence of the distribution of loads within the ballast depth. This is a clear advantage of a ballast track.

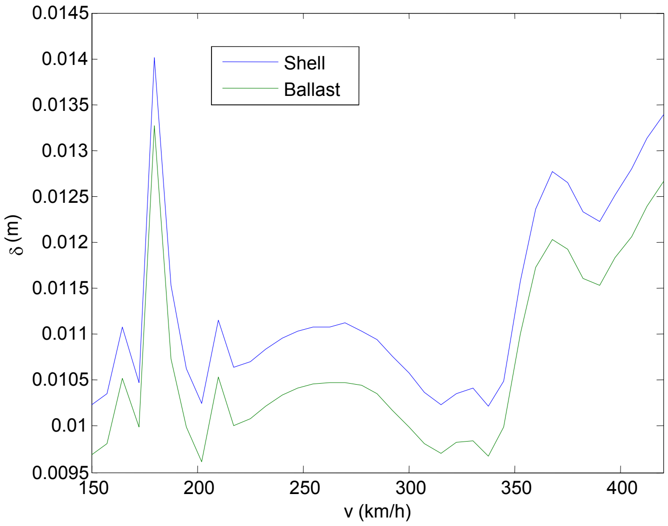
As on the modal analysis, the frequencies of vibration are practically the same (as expected by the low Young's module of the ballast components), except for lateral modes: this occurs due to the lateral constraint of the ballast given by the beam alongside the ballast borders.

At last, the dynamic analysis of the Ballast model is undertaken for the ten HSLM-A models (Fig. 22).

The slight difference of maximum accelerations between the two models has an explanation: the ballast works at a certain level as an energy dissipative device. This difference increases for more flexible ballasts. Regardless of a harsher permitted deck vertical acceleration, the option for a ballast track, besides anticipated noise reduction, leads to lower deck accelerations and, in a dual point of view, lower vehicle accelerations (Yau, [22]).



(a)



(b)

Fig. 22 – Comparison between maximum vertical accelerations (a) and displacements (b) for the passage of HSLM-A10 trainset on Shell and Ballast models

This chapter ends with the deck twist criterion verification.

The twist was measured in the section with the highest vertical displacement and to the most unfavourable train: intermediate midspan and HSLM-A9. The nodes 4648, (x,y)=(76.25,-1.6), and 9407, (x,y)=(79.25,-3,0115), were chosen.

The author's procedure was to determine the instant in the time history plot where the deviation between the displacements of the nodes 4648 and 9407 was the highest: at 9.024 seconds, the value for deck twist is 0.42 mm/3m, 66% below the limit value (1.5 mm/3m) recommended in EN 1991-2.

7. Conclusions

The purpose of this work was to present the advances in the analysis of high speed railway bridges and viaducts through a survey of the most recent studies or investigations conducted by the author. The general and comprehensive nature of the studies was always maintained along the dissertation to converge into the designer's perspective.

Acknowledgments

The author wishes to express his gratitude to Professor Jorge Proença (Auxiliar Professor, Department of Civil Engineering, Instituto Superior Técnico) for his help and encouragement during the development of this investigation and, also, to my colleague João Henriques for kindly dispose his pre-processor to SAP2000 dynamic analyses.

Finally, a special word to all my colleagues who helped me with their saviour or friendship during my MSc in Técnico.

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