Abstract

In this paper, we analyze some of the key processes associated to the optical communication systems operation: emission, transmission, dispersion management and amplification.

Following the route of the optical signal, from the emitter to the detector, in a first step we characterize the light emitter: a semiconductor laser. In this topic we study the power coupling to the system, expressed in terms of a gain, as well as its direct modulation through the injection current.

In a second step, at the transmission level, we analyze the problem of the group velocity dispersion and its perturbation of the signal, which is supposed to keep its characteristics along the fiber. Within this environment, we have developed a numerical simulator for the calculation of the pulse propagation in the linear regime.

In the field of the dispersion management, we address the fiber optics non-linearity. We analyze the behaviour of a soliton type shaped pulse along the fiber. Then, we present an iterative analytical model suitable to calculate the output signal produced at the end of the fiber, based on its initial conditions. At this level, we study the fundamental soliton, and 2\textsuperscript{nd} and 3\textsuperscript{rd} order, as well as the interference generated by the interaction between solitons.

In order to transmit optical pulses that reach their final destination with a given required quality, it is necessary to amplify them so that these can cover long distances efficiently. We address the amplification issue by introducing the EDFA’s. To amplify a certain WDM signal, we need to design the EDFA system by taking into account the balance between the needed gain and the optimal length of the fiber.

Keywords: Fiber Optics, Semiconductor Lasers, Solitons, Time Dispersion, Nonlinear Regime, EDFA.

1. Introduction

In 1870, John Tyndall, gave the first step in the research of the guided propagation of the light. Tyndall demonstrated that the light could be guided, for internal reflection, through a water spurt proceeding from a bucket with a source of light and an orifice, for one another container [1].

The technology of fiber optics would only come back to suffer significant developments in the second half of the 20\textsuperscript{th} century.

The fiber optic was defined as a transmission environment (in general a cable with some pairs of plastic or fiber glass) where the information is carried under the form of light impulses.
In 1957, studies of Physics allies to the Optics had allowed the discovery of a new form of using light - the Laser (Light Amplification by the Stimulated Emission of Radiation). Gordon Gould described the device as an intense source of light that produces monochromatic electromagnetic radiation (narrow wavelength spectrum) and coherent (well defined relations of phase) that it spreads under a low-divergence beam.

1.1 Five fiber generations
The communications systems of fiber optics had been developing throughout generations. Since 1980 four generations of fibers have been studied with the main goals of: increase the bit rate of communications, minimize the system attenuation and increase the repeater distances to reduce the cost of long distance fiber systems. The fourth generation has as main characteristics the fact of working in the optical domain with the application of WDM - Wavelength Division Multiplexing - that made it possible to increase the speed of transmission. In these years we expect a fifth generation. If on the one hand we can supplant losses with fibers amplifiers, on other hand we still have the problem of dispersion.

With this scenario, the evolution of the communications will certainly count with the use of fiber optic and the photonics will be the main support of the highways of the information [3, 10].

2. Semiconductors Lasers
Lasers semiconductors use the electric chain to produce light. The pure spectrum that they produce, allied to the low volume, reduced costs and high durability make lasers ideal sources for communications applications. The laser (whose acronym means *Light Amplification by Stimulated Emission of Radiation*) has the capacity to produce electromagnetic radiation with very special characteristics: a narrow wavelength spectrum (monochromatic light), is coherent and it spreads throughout a beam) [4, 6].

Nowadays, the computational development permits to process complexes numerical methods of calculation, makes it possible to develop mathematical models that reproduce with high precision the behavior of lasers semiconductors.

2.1. Laser Model
The produced models are based on Maxwell’s laws that can be reduced to the dominated by rate equations, which in stationary regime,

$$\frac{dS}{dt} = \frac{dN}{dt} = 0$$  \hspace{1cm} (2.1)

has the following appearance

$$\begin{cases}
G_0S_0 + \beta_{sp}N_0 \left( \frac{S_0}{\tau_p} \right) = \frac{S_0}{\tau_p} \\
G_0S_0 + \frac{N_0}{\tau_c} = \frac{I_0}{q}
\end{cases} \hspace{1cm} (2.2)$$

Where the gain, or liquid elementary rate of stimulated radiation, is given by

$$G_0 = \frac{G_N(N_0 - N_l)}{1 + \varepsilon S_0}$$  \hspace{1cm} (2.4)

And $N_0$ e $S_0$ represent the number of electrons and photons respectively. Then, with this information and the main characteristics of laser, we can resume the threshold current
For a simulation of a laser, consider an injection current like the figure 2.

2.2. Laser “on”

\[ I_0 = 1.1 I_{th} \text{ e } I_p = I_{th} \]

We analyze the evolution of electrons in a laser when influenced by the injection current (Figure 2). We verify that when the injection current is bigger than threshold current, the system works without interruptions. We have to enhance the oscillatory spectrum that we can see on the Figure 4. When the pulse ended the variation tends to stabilize for a constant value.

2.3. Laser “on” in a short time

\[ I_0 = 0.8 I_{th} \text{ e } I_p = I_{th} \]

In the second analysis, the laser only has the injection current upper from the threshold in a short time, more precisely at \( t = 0 \) when \( I_0 + I_p > I_{th} \). This situation provokes a delay from the laser and in consequence an increase of photons comparatively with the previous one. In conclusion, this condition makes with that the number of occurred photons is insignificant and insufficient for the emission occurrence.
After the pulse, the device enters in a transitory regime that will take quickly the number of photons tendency to zero.

3. Time Dispersion

The propagation of the light through fiber optics suffers some difficulties that make with that the optical systems of communications cannot be considered ideal. This fact has a relationship with time dispersion concept which imposes some limitations to the implementation of the optical communications [2, 7, 8, 9, 10].

3.1. Linear Regime

For analyze the phenomenon of dispersion in fiber optics occurs at pulse propagation and starting from a pulse \( A(0, t) \),

\[
A(0, t) = H \left( t + \frac{T}{2} \right) \left[ 1 - \exp \left( - \frac{t + T/2 \lambda_{0}}{\tau_0} \right) \right] - H \left( t - \frac{T}{2} \right) \left[ 1 - \exp \left( - \frac{t - T/2 \lambda_{0}}{\tau_0} \right) \right]
\]  

we deduce the equation that translates the development of the pulses throughout a monomodal optical fiber in linear regime.

\[
\frac{\partial A}{\partial z} + \sum_{m=1}^{\infty} \frac{i^{m-1}}{m!} \beta_m \frac{\partial^m A}{\partial t^m} + \frac{\alpha}{2} A = 0
\]  

where \( \beta_m \) is obtained from (3.3)

\[
\beta_m = \frac{\partial^m \beta_1}{\partial \omega^m} \bigg|_{\omega = \omega_0}
\]  

Developing the summation above, we get

\[
\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} + \frac{\alpha}{2} A = 0
\]  

And the coefficients \( \beta_m \) are obtained from (3.3)

\[
\beta_1 = \frac{1}{v_g(\omega_0)}
\]

\[
\beta_2 = - \frac{1}{v_g^2(\omega_0)} \frac{\partial v_g}{\partial \omega} \bigg|_{\omega = \omega_0}
\]  

where \( v_g \) represents group velocity and the \( \beta_2 \) coefficient is group velocity dispersion (GVD).
3.2. Numerical Simulation

For a numerical simulation of pulse propagation in linear regime, we disdained the losses, $\alpha = 0$, and other dispersion effects, $\beta_2 = 0$, and then we obtained (3.2) equation in a more simple form:

$$\frac{\partial A}{\partial Z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} = 0$$  \hspace{1cm} (3.7)

With some algebrical manipulation and in a normalized form, we obtain the equation (3.8)

$$\hat{A}(\zeta, \xi) = \hat{A}(0, \xi) \exp \left( i \frac{1}{2} \text{sgn}(\beta_2) \xi^2 \right)$$  \hspace{1cm} (3.8)

Now, we can enunciate three stages of numerical resolution that lead the calculation of the spectral value of the impulse in any position $\zeta$ from the initial impulse:

(i) Calculation of FFT

$$\hat{A}(0, \xi) = \text{FFT}[A(0, \tau)]$$  \hspace{1cm} (3.9)

(ii) Calculation of $\hat{A}(\zeta, \xi)$

$$\hat{A}(\zeta, \xi) = \hat{A}(0, \xi) \exp \left( i \frac{1}{2} \text{sgn}(\beta_2) \xi^2 \right)$$

(iii) Calculation of IFFT

$$A(\zeta, \xi) = \text{IFFT} \left[ \hat{A}(\zeta, \xi) \right]$$  \hspace{1cm} (3.10)

3.3. Simulation

3.3.1. Gaussian Pulse

Consider a pulse shown in equation (3.1), where $H(t)$ is function Heaviside, with $T = 5 \tau_0$ and a distance of 1250km.

In fact, face to group velocity dispersion (GDV) existing, we can verify a gradual widening of the impulse throughout the optical fiber. All frequencies that compose the pulse will go to spread with different speeds, and this reality will provoke that we will have components with delay relatively to others.

$$A(0, t) = \exp \left[ - \frac{1 + i C}{2} \left( \frac{t}{\tau_0} \right)^6 \right]$$  \hspace{1cm} (3.11)

In Figure 10 we can see that this pulse is much sensitive to dispersion than the previous one. This is relative to the inverse proportionality between time and frequency that will record a bigger number of components in interference with their neighbors.
pulse. However, the chirp can only be introduced at the entrance of the fiber what makes this effect limited, finishing when is reached the maximum nip of the pulse. From there onward the dispersive effect comes back to dominate the structure of the impulse making with that the pulse would gradually be widely.

### 3.4. Result analysis

The widening of the pulses is directly related with inter-symbolic interference (ISI) existing at pulse propagation. This fact affects the binary debit of digital transmission.

In order to be able to compare the performance of each one of the gaussian pulses in analysis, we introduce the definition of the moment:

$$\langle t^q \rangle = \frac{\int_{-\infty}^{\infty} t^q |A(\zeta, \tau)|^2 d\tau}{\int_{-\infty}^{\infty} |A(\zeta, \tau)|^2 d\tau}$$  \hspace{1cm} (3.12)

And define pulse effective with as

$$\sigma(\zeta) = \sqrt{\langle t^2 \rangle - \langle t \rangle^2}$$  \hspace{1cm} (3.13)

The straighten of the pulse is expected to occur when it verifies the condition $\beta_2 C < 0$. Is we analyze Figure 13 in detail we see that the pulse that have negative chirp is the worst one.
The negative chirp provokes even bigger width distance of the pulse. With positive chirp we see that in a first phase we see the reduce of spectral width. After a determined distance the effect of chirp is over, and the better solution is the one without chirp’s influence.

4. Non-Linear Regime

The development of the studies made during the beginning of the decade of 90 made with that the solitons gave a important step of modulation in the systems of optical communications. The possibility to keep solitons in the fiber optics was the reached commitment between Group Dispersion Velocity (GDV) and Self-Phase Modulation (SPM).

4.1. Soliton Propagation

If we restrict the analysis to anomalous dispersion zone, we can define equation 4.1 that conducts the pulse propagation at fiber in non-linear regime.

\[ i \frac{\partial u}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} - i \cdot \kappa \frac{\partial^3 u}{\partial \tau^3} + |u|^2 u = -i \Gamma \cdot \frac{1}{2} u \]  

(4.1)

Split-Step Fourier Method (SSFM) is a used numerical analysis method of resolution of non-linear equations of pulse propagation in fiber optics. The name of the method has origin in its two stages of resolution.

\[ u(\zeta, \tau) = u_0(\tau) = u(\zeta = 0, \tau) \]  

(4.2)

\[ v(\zeta, \tau) = u(\zeta, \tau) \cdot \exp \left( -\frac{h}{2} \Gamma \right) \cdot \exp(i \cdot h \cdot |u|^2) \]  

(4.3)

\[ \tilde{v}(\zeta, \xi) = TF[v(\zeta, \tau)] \]  

(4.4)

\[ \tilde{u}(\zeta + h, \xi) = \tilde{v}(\zeta, \xi) \cdot \exp \left( -i \cdot \frac{h}{2} \cdot \xi^2 \right) \cdot \exp(i \cdot h \cdot \kappa \cdot \xi^3) \]  

(4.5)

\[ u(\zeta + h, \tau) = TF^{-1}[\tilde{u}(\zeta + h, \xi)] \]  

(4.6)

From equation 4.2 to 4.6 we define the four steps that resume the SSFM.

4.2 Initial pulse at fiber’s entrance  
4.3 Non linear effect analyze  
4.4 FFT application  
4.5 Linear effect analyze  
4.6 IFFT application

This iterative method comes back to equation 4.3 with \( u(\zeta, \tau) = u(\zeta + h, \tau) \) - so many times as we can detail the intended results.

4.1.1. Fundamental Soliton

The fundamental soliton preserves his main attributes throughout fiber propagation.

4.1.2. Second order soliton

Contrarily to the previous one, we verify that the second order soliton has some variations in its characteristics of shape and amplitude throughout the fiber propagation. In this situation, the relationship between the AMF and the DVG are changeable in constant intervals of time. The energy conservation is
present in this system and when we have pulse
nip, we verify a peak at the amplitude (Figure
15).

![Figure 15: 2nd order soliton propagation](image)

4.1.3. Third order soliton

In this situation, the previous phenomenon
occurs at half of normalized distance.

![Figure 16: 3rd order soliton propagation](image)

4.1.4. Result analysis
If we consider the three previous solitons in
telecommunications applications, we conclude
that the fundamental one is the unique ideal for
use, however its observed behavior don’t
include losses.

4.2. Soliton interaction
The coexistence between solitons is a real
situation in a telecommunications system. In
communications we are interested to send a
group of bits instead of an isolated pulse.
Thus, we study the influence between pulses
when they are in presence of other solitons.
For that we analyzed a set of situations varying
amplitude and phase shift.

![Figure 17: Distance between solitons in throughout
propagation distance](image)

By observation of figure 17, we can affirm that
the black graphic is the one that presents the
best results. However the oscillation, the shift
is limited and can be controlled.

In summary, the solitons allow to transmit data
by a channel to a rhythm much more raised,
since they do not suffer dispersion effects.
However, the multichannel transmission
implementation has some limitations.

5. Amplification

For signals arrive at its destination with one
determined quality demanded, it is necessary
that the same ones are amplified.
In 1985 occurred a great discovery in
propagation technology. The possibility to use
chunks of fiber optics where the signals are
amplified, without the necessity to convert the
original signal to electrical mode, has
revolutionized the study of the amplification.
Years later, the doped amplifying fiber
introduction with erbium (EDFA - Erbium Doped to Fiber Amplifiers) showed a great innovation in systems of optic communication. The EDFA is a doped fiber optic with ions of erbium, which makes its functioning in the third window around 1550nm. The input power is maximum at the moment of the injection, gradually decrease throughout the time distributing itself for the channels introducing power in the fiber through the amplification. This share is carried out by a laser that excites ions for superior levels of energy [10, 11, 19, 20, 21].

5.1. Amplification gain

The amplification’s gain is given by gain coefficient that can be calculated with following expression:

\[ g_k(\lambda_k) = (\alpha_k + \gamma_k) \frac{N_2}{\rho} - \alpha_k \]  

(5.1)

Where \( \alpha_k \) is the absorption coefficient represented by

\[ \alpha_k = \Gamma_k \sigma_{ak} \rho \]  

(5.2)

\( \Gamma_k \) is the optical confinement factor, \( \sigma_{ak} \) is efficient section of emission and \( \rho \) is the effective ray of erbium ions concentration.

In parallel, the parameter \( \gamma_k \) is the emission coefficient given by

\[ \gamma_k = \eta_k \alpha_k \]  

(5.3)

Where \( \eta_k \) is the coefficient that relates the efficient sections of emission and absorption.

5.2. Amplification of a WDM signal

The amplification of a WDM signal can be translated through the following pair of equations:

\[ \frac{dp_k}{dz} = g_k p_k \]  

(5.4)

\[ g_k = \frac{\alpha_k}{1 + \sum_j p_j} \left( -1 + \sum_{j \neq k} \eta_k \eta_j \frac{p_j}{1 + \eta_j p_j} \right) \]  

(5.5)

5.3. Optimal length

The optimal length of the EDFA, \( L_{opt} \), is the value for which is guaranteed the maximum gain in the end of the amplification for one determined input power, that is, it verifies the expression:

\[ \frac{dp}{dz}_{z=L_{opt}} = 0 \]  

(5.6)

The optimal length can be obtained by

\[ L_{opt} = \frac{1}{\alpha_p U_s} \left( U_p q_0 - 1 \right) - \frac{1}{\alpha_s} \left[ \ln G_s + (G_s - 1) p_0 \right] \]  

(5.7)

In Figures 18 we can see a WDM signal with four channels represented at different colors. Although identical input power, we can see different gains and maximums for each channel centered at different wavelength.

Figure 18: Evolution of the power introduced in a WDM signal throughout the length of the amplification

Finishing, the EDFA is very sensible to its length and the wavelength of transmission.
Depending on the type of signal to amplify, it is necessary to choose a specific optimal length to optimize the behavior of the EDFA.

6. Conclusions

In this article we analyze many aspects of pulse propagation in fiber optics, namely:
Chapter 2 we made a first introduction to the semiconductors lasers as emission devices. We conclude that when the laser is in emission, the population of electrons and photons has an oscillatory spectrum throughout the duration of the impulse. When this finishes, these numbers tends to stabilize.
In chapter 3 we talk about pulse propagation in linear regime. The main results verify that the propagation is influenced by Inter-symbolic interference and that directly depends on the widening of the pulses and the dispersion.
The chapter 4 was dedicated to the non-linear regime. We started for introducing SSFM (Split-Step Fourier Method) used in the resolution of non-linear equations of pulse propagation in fiber optics. Additionally we verify that the fundamental soliton is the only one that is indicated for pulse propagation and when we talk about interaction, we reached best results when we use solitons in phase but with different amplitudes.

In chapter 5 we introduce the subject of the amplification. The amplification is realized with EDFA’s that, between other things, has the main characteristic to amplify signal in all-optical domain. We verify that the gain is no uniform and for obtain the maximum gain we have to equilibrate all gains produced by EDFA in each channel of signal.

References