Investment decisions in generation assets: A portfolio theory approach

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Abstract — **Generation companies operating in a competitive market face price and volume risks that affect their return. Being able to identify and quantify these risks for each type of generation asset and how they interact is fundamental when making investment decisions regarding the portfolio of generation assets.**

This paper proposes a set of tools to support the decision of investment in a portfolio of generation assets, according to portfolio theory. The concept of efficient frontier of risk and return as set in that theory is used to find optimal generation portfolios.

Gas-, coal- and fuel- fired as well as hydroelectric and on-shore wind power plants are considered. Risk and return are estimated through Monte Carlo simulation of free cash flows. For that effect, exogenous variables such as fuel, electricity and emission prices, wind speeds and water inflows are modeled with stochastic processes.

Investment decisions are studied for two perspectives: fixed capital costs allocation and installed capacity allocation. Considering constraints for risk, return and capacity over the efficient Pareto frontier, decisions of buy and/or sell required to achieve optimal generation portfolios are pointed out.

Index Terms — generation assets, investment decisions, portfolio theory, stochastic processes

I. INTRODUCTION

A company owning a diversified set of assets – its portfolio of assets – faces a diversified set of uncertainties. A generation company's portfolio of assets is composed by the different types of generation plants with different sources of energy such as thermal, nuclear, hydro, wind and other renewables. Each type of asset has different drivers of uncertainty that are correlated with one another. These uncertainties and interactions must be taken into account when deciding over the composition of the portfolio of assets. Since generation companies have limited resources to invest, and given the multiplicity and the interaction of the different sources of uncertainty, as well as the diversity of assets and possible portfolios, it makes sense to use the concept of efficient Pareto frontier of risk and return from portfolio theory to support the investment decision of a generation company when managing its portfolio of assets.

The different uncertainties that the generation assets are exposed to relate with different types of risk such as price and volume risks. Volume risk usually denotes uncertain demand as a result of market conditions. In addition, renewable power plants such as hydro and wind face the volume risk of uncertain production. Price risk denotes the uncertainty of future fuel and emissions' prices as well as electricity price in the case of competitive market conditions.

Economics literature refers to risk meaning uncertainty quantified by its probability density function. More specifically, portfolio theory refers to risk as the dispersion of the return. Therefore, according to this theory, the probability density function of return needs to be determined.

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Considering return as the monetary outcome that the investment will provide, future free cash flows must be estimated. Monte Carlo simulation is used for that effect, as it is a commonly used technique that allows the probabilities of different outcomes of an investment project to be determined [1].

This paper aims to propose an approach for estimating risk and return of a portfolio of generation assets, in order to determine an efficient frontier. Moreover, it also aims to relate the set of efficient portfolios with strategic directions of generation companies by introducing constraints for risk, return and capacity on the efficient frontier.

The paper has the following structure: Section II shortly revises the basic concepts of portfolio theory; Section III describes the considered method to valuate generation assets and to estimate their risks and returns; Section IV considers 2 perspectives to study the investment decision: capital costs allocation and capacity allocation; this section also addresses the usage of constraints super imposed on the efficient frontier; Section V illustrates the overall approach to support the investment decision; finally, in section VI we draw some conclusions.

II. PORTFOLIO THEORY

Portfolio theory had its foundations established in the middle of the 20th century as a result of the PhD thesis of the Nobel prize winner Harry Markowitz. Since then it has evolved into a consolidated theory often used nowadays in allocation problems of financial assets under uncertainty, with a strong presence in most of the economic and financial management literatures.

This theory was revolutionary at the time of its birth by explicitly considering return, risk and the interactions of different assets in the composition of a portfolio. It launched a pioneer methodology for the optimization and the selection of portfolio of assets from a risk-return efficient frontier. It provided the evidence for diversification and for the different portfolio choices of different investors, based upon the trade-off between risk and return and different risk aversions.

A. Portfolio of assets

Considering a portfolio of N assets $(i = 1,...,N)$, its return r_P is the weighted average of the assets' return r_i, with w_i the weights of the investment in the i^{th} asset over the total investment.

$$r_P = \sum_{i=1}^{N} w_i r_i \tag{1}$$

Given the expected return μ_i and the standard deviation σ_i for each asset, as well as the correlation ρ_{ij} between them, one can infer the expected return μ_P and the risk σ_P of the portfolio of N assets according to:

$$E[r_p] \equiv \mu_P = \sum_{i=1}^{N} w_i \mu_i \quad , \quad \sum_{i=1}^{N} w_i = 1 \tag{2}$$

$$VAR[r_p] \equiv \sigma_P{}^2 = \sum_{i=1}^{N} w_i{}^2 \sigma_i{}^2 + \sum_{\substack{i,j=1 \\ i \neq j}}^{N} w_i w_j \sigma_{ij} \tag{3}$$

where σ_{ji} is the covariance between the assets i and j calculated according to:

$$COV[r_i, r_j] \equiv \sigma_{ij} = E[(X-\mu_X)(Y-\mu_Y)] = \rho_{ij}\sigma_i\sigma_j \tag{4}$$

A clarifying way of visualizing the domain of risk-return of a portfolio of assets it is a graphic risk-return, such as Fig. 1.

It must be noted that the resulting domain of risk-return of the portfolio depends on the characteristics of the assets that compose it. Fig. 1 illustrates the mentioned trade-off between risk and return.

B. *Portfolio selection*

From all possible portfolios of N assets, the selection must only consider the portfolios over the optimal Pareto frontier of the domain of options. Markowitz identified that frontier as the efficient frontier of risk and return [2].

The efficient frontier of risk and return contains all the portfolios that can not suffer an improvement in both risk and return among the alternative options. Therefore, the alternative selection of portfolios along the efficient frontier with higher returns implies, as well, higher risks (and vice-versa). These portfolios are considered optimal.

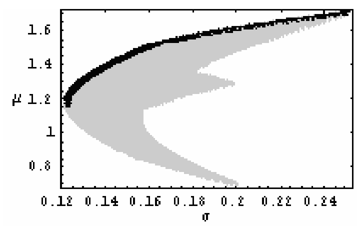

Fig. 1. Graphic risk-return of a portfolio of assets.

III. Valuation of Generation Assets

In this paper, the value of a set of generation assets is established through Monte Carlo simulation of each type of asset's operation under market conditions. Under this approach, a number of uncertain exogenous variables related to the plants operation need to be modeled (coal fired power plants depend on the price of coal; the same way as wind power production depends on the intensity of wind speed). Once the uncertainty is modeled, the free cash flows of each asset can be simulated and, as a consequence, their risk and return.

A. *Exogenous processes*

The uncertainty is modeled using stochastic processes. The following exogenous variables are considered: fuel, emission and electricity prices, wind speeds, and water inflows.

1) *Prices of fuel, electricity and CO_2 emissions*

It is assumed that fuel, electricity prices and CO_2 emission prices $s(t)$ follow mean reversion stochastic processes [3-5]. This process takes the form:

$$\frac{ds}{s} = \lambda[\phi - \ln(s)]dt + \sigma dz \tag{5}$$

where λ is the reversion strength, ϕ is the logarithm of the long-term value, σ is the volatility and z follows a Wiener process. It is assumed that the CO_2 emission prices also follow this process.

Considering the process $y = \ln(s)$, making use of Itô's lemma, a simpler process denoted Ornstein-Unlenbeck is obtained:

$$dy = \lambda(\Omega - y)dt + \sigma dz \tag{6}$$

where $\Omega = \phi - \frac{\sigma^2}{2\lambda}$.

Its properties are:

$$E[y_t] \equiv \mu_{Y_t} = y_{t_0} e^{-\lambda(t-t\ )} + \Omega\left[1 - e^{-\lambda(t-t_0)}\right] \tag{7}$$

$$VAR[y_t] \equiv \sigma^2_{Y_t} = \frac{\sigma^2}{2\lambda}\left[1 - e^{-2\lambda(t-t_0)}\right] \tag{8}$$

Using these properties along with the relation $s = e^y$, one can infer the properties of the original process of $s(t)$:

$$E[s_t] \equiv \mu_{S_t} = e^{\mu_{Y_t} + \frac{\sigma^2_{Y_t}}{2}} \tag{9}$$

$$VAR[s_t] \equiv \sigma^2_{S_t} = e^{2\mu_{Y_t} + \sigma^2_{Y_t}}\left(e^{\sigma^2_{Y_t}} - 1\right) \tag{10}$$

A suitable discrete-time autoregressive model of 1st order AR(1) often used for the description of the Ornstein-Uhlenbeck process (6) takes the form:

$$y_i = y_{i-1} e^{-\lambda\delta t} + \Omega\left[1 - e^{-\lambda\delta t}\right] + \varepsilon_i \tag{11}$$

where δt it is the time frame between y_i and y_{i-1}, and ε_i are random drawings from a normal distribution $N\left(0, \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda\delta t}\right)\right)$.

Once the parameters of process (6) are known, one can generate trajectories Y_n of the process using its discrete-version (11). Later, trajectories S_n for the original price process (5) are obtained calculating $S_n = e^{Y_n}$.

Correlation

Typically fossil fuel prices move in tandem, which means their processes are correlated. The way to impose the correlation is by correlating the random drawings ε_i of each process. The correlated Wiener processes ε_i^{ρ} are extracted from a matrix W^{ρ} such that:

$$W^{\rho} = A^{'}W \tag{12}$$

where W contains uncorrelated random drawings ε_i for each fuel process, and $A^{'}$ is obtained through the Cholesky decomposition of the covariance matrix C of the fuel processes, such that:

$$C = A^{'}A \tag{13}$$

The elements of the covariance matrix are obtained according to (4).

2) *Wind speed*

It is the random behavior of wind that determines how much power a wind turbine generates. Several studies show that wind speeds follow a Weibull distribution [6], according to:

$$f(x, k, \lambda) = \left(\frac{x}{\lambda}\right)^{k-1} \frac{k}{\lambda} e^{(-x/\lambda)^k} \quad (14)$$

where x is the wind speed, k the shape parameter and λ the scale parameter. These parameters are site specific. Particularly, the shape parameter is set such that the expected value of the Weibull distribution is the annual average of the wind speed at the n[th] location.

However, a simple draw of values from a Weibull distribution does not describe the autocorrelation between wind speeds of consecutive time instants, neither does describe the cross correlation among close locations. A process that deals with those questions is defined in [7].

The present work uses a process which is based in that process, but uses a uni-dimensional AR(1) filter to impose autocorrelation and does not consider cross correlation.

Considering $\{Z_t : t \geq 0\}$ a time series of wind speeds, $Z_t \sim Weibull(k, \lambda)$, the relation between these samples and normal distributions is:

$$Z_t = \left(X_t^2 + Y_t^2\right)^{(1/k)} \quad (15)$$

where $X_t, Y_t \sim N(0, \sigma^2)$ and $t(t = 1,...,T)$, such that:

$$\sigma^2 = \frac{\lambda^k}{2} \quad (16)$$

The properties of Z_t are known from stochastic theory:

$$E[Z_t] = \lambda \, \Gamma(1 + 1/\lambda) \quad (17)$$

$$VAR[Z_t] = \lambda^2 \left[\Gamma(1 + 2/\lambda) - \Gamma^2(1 + 1/\lambda)\right] \quad (18)$$

The second moment as a function of the correlation $\rho_{i,j}$ between the normal samples $X_i \; / \; X_j$ and $Y_i \; / \; Y_j$ is also known [8]:

$$E[Z_i Z_j] = \lambda \lambda \, \Gamma(1 + 1/\lambda)\Gamma(1 + 1/\lambda)\, {}_2F_1(-1/\lambda; -1/\lambda; 1; \rho_{i,j}^2) \quad (19)$$

where Γ is the gamma function and ${}_2F_1$ the hypergeometric function [(a)].

Since the components of the correlation matrix S are given by:

$$s_{i,j} = \frac{E[Z_i Z_j] - E[Z_i]E[Z_j]}{\sqrt{VAR[Z_i] \; VAR[Z_j]}} \quad (20)$$

Substituting (17), (18) and (19) into (20), one can obtain the relation of the auto correlation $s_{i,j}$ between the wind speed samples Z_i and Z_j with the auto correlation $\rho_{i,j}$ of the normal samples $X_i \; / \; X_j$ and $Y_i \; / \; Y_j$:

$$s_{i,j} = \frac{\Gamma(1 + 1/\lambda)\Gamma(1 + 1/\lambda)\left[{}_2F_1(1/\lambda, 1/\lambda; 1; \rho_{i,j}^2) - 1\right]}{\sqrt{\left[\Gamma(1 + 2/\lambda) - \Gamma^2(1 + 1/\lambda)\right]\left[\Gamma(1 + 2/\lambda) - \Gamma^2(1 + 1/\lambda)\right]}} \quad (21)$$

Autocorrelation

In order to impose the autocorrelation, the autoregressive filter AR(1) is considered to generate normal samples V_t:

$$V_t = \phi V_{t-1} + \varepsilon_t \quad (22)$$

where ε_t are random drawings of a normal distribution $N(0, \sigma^2)$ and ϕ is the autocorrelation of the normal samples X_t and Y_t and is determined through (21), once the autocorrelation $s_{i,j}$ of the wind speed samples is known.

It is proven from stochastic theory [9] that in case of a stationary process of the form (22) $(|\phi| < 1)$, its properties are such:

$$E[V_t] = 0 \quad (23)$$

$$VAR[V_t] = \frac{\sigma^2}{1 - \phi^2} \quad (24)$$

Thus, considering $\sigma^2 = 1 - \phi^2$, one has auto correlated normal samples $V_t \sim N(0,1)$.

Knowing the autocorrelations of the wind speed series, the scale parameter and the annual average wind speed, a number of steps should be taken in order to generate the wind speed time series. First, one must determine the scale parameter through (17), given the annual average wind speed. Second, the auto correlations (ϕ) to use regarding the normal samples are determined through the implicit function (21). Given ϕ, the next step is to generate 2 autocorrelated normal samples through (22). Afterwards, the normal samples are multiplied by σ^2 (obtained through (16)), to assure the aimed properties. Finally, one obtains the wind speed samples using the pair of normal samples according to (15).

3) Water inflows

Monthly natural water inflows typically show seasonality. Seasonality is present not only in monthly mean values and standard deviation, as well as in the autocorrelation structure. A widely used process that deals with periodic seasonality of the inflows it is the multivariate periodic autoregressive model of 1[st] order PAR(1) [10 - 11].

Considering $\{Q_t : t \geq 0\}$ a seasonal time series of inflows with period s, the time index may be regarded as a function of the year $T(T = 1,...,N)$, the season $m(m = 1,...,s)$ such that

(a)

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-x} dt \quad , \quad x \in \Re \quad ; \quad {}_2F_1(a,b;c;x) = \sum_{k=0}^{\infty} \frac{a_k b_k}{c_k} \frac{x^k}{k!}$$

$t=(T-1)s+m$. The process $\{X_t : t \geq 0\}$ PAR(1) considered takes the form:

$$X_t = \phi_m X_{t-1} + \varepsilon_{t,m} \quad (25)$$

where $\varepsilon_{t,m}$ are random drawings of a normal distribution, ϕ_m is the autocorrelation between season m and season $m-1$, thus assuring the periodic correlational structure of the inflows and X_t is a standardization of the logarithm of the inflows that assures stationarity and normality of the process (25):

$$X_t = \frac{L_t - \mu_m}{\sigma_m} \quad (26)$$

$$L_t = \ln(Q_t) \quad (27)$$

where μ_m is the mean and σ_m the standard deviation of the logarithm of the inflows L_t, for each season m.

After the standardization with (26), the variance of X_t is unitary, hence, according to (24):

$$\sigma_m{}^2 = (1-\phi_m{}^2) \quad (28)$$

Considering (26) and (28) into (25), one obtains the process to generate the logarithm of the inflows:

$$L_t = \mu_m + \phi_m \sigma_m \left(\frac{L_{t-1} - \mu_{m-1}}{\sigma_{m-1}} \right) + \left(1-\phi_m{}^2\right)^{1/2} \sigma_m \varepsilon_{t,m} \quad (29)$$

Once the parameters of (29) have been estimated, using a historic time series of inflows, one can generate through (29) a series of the logarithm of the inflows L with a Monte Carlo simulation. Afterwards, the series of inflows Q is obtained through the inverse function of (27), $Q = e^L$

B. Power

Wind and hydroelectric power are generated through the respective wind speed and water inflows time series. For thermal power, we will consider an annual average of equivalent hours working at maximum capacity.

1) Wind power

Wind power production finds its source in the kinetic energy of the wind. Knowing the wind speed that flows through the wind turbine, the maximum generated power is determined as follows:

$$P_E = \frac{1}{2} \rho_{air} v^3 \pi (D/2)^2 \quad (30)$$

where P_E is the maximum generated power, ρ_{air} is the density of the air, v the wind speed and D the diameter of the turbine.

However, not all the kinetic power of the wind is transformed into electric power. One must take into account the efficiency of the power generator as well as the wind speed range in which the turbine generates power, between the *cut in* and the *cut out*. This relation is expressed in the power curve of each turbine model. Knowing the time series of wind speeds, simulated with the process mentioned in sub subsection III.A.2), one obtains the time series of power through the power curve.

2) Hydropower

Hydroelectric plants find their energetic source in the potential energy of the natural water inflows. Knowing the difference of the height of the upstream and downstream water level that the natural inflows suffer, taking into account the efficiency of the power generator as well as the losses during the fall of the inflows, one can determine the maximum electric power generated:

$$P_H = q \rho_{água} g \Delta h (1-\alpha) \eta \quad (31)$$

where P_H is the maximum generated power, q is the water inflow, ρ_{H_2O} is the water density, g is the acceleration of gravity, Δh is the difference of the up and downstream water levels, α accounts for the losses during the fall, and η is the efficiency of the generator.

Knowing the time series of inflows simulated with the process mentioned in sub subsection III.A.3) and the hydro power plant specifics regarding efficiency, one obtains the time series of power through (7).

C. Annual free cash flows

The estimation of annual free cash flows is fundamental for the assessment of an investment. It represents the cash that a company is able to generate after laying out the money required to maintain or expand its asset base. It is the balance between the operating cash flows and the capital expenditures.

The operating cash flows of a power plant depend on the value of its production p_e, the electricity, the fuel costs p_{fuel}, the CO_2 emissions em and its costs p_{CO_2}, the costs of operation and maintenance c_{OM}, and the efficiency η. The free cash flows fcf, apart from the operating cash flows, depend as well on the tax rate tx and the value of depreciation of the plant dep:

$$fcf_t \big|_{Thermal} = \left[\left(p_{el\,t}.e_t - \frac{(p_{fuel\,t} + p_{CO_2}.em)e_t}{\eta} - c_{OM} \right) - dep \right] (1-tx) + dep \quad (32)$$

In case of a renewable power plant, there are no fuel costs, neither costs of emission:

$$fcf_t \big|_{Renewables} = \left[\left(p_{el\,t}.e_t - c_{OM} \right) - dep \right] (1-tx) + dep \quad (33)$$

In case of thermal plants, trajectories of free cash flows can be generated using trajectories of electricity, fuel and CO_2 prices generated with the processes mentioned in subsection III.A. In the case of renewable power plants, the free cash flows trajectories use trajectories of wind power or hydropower generated as described in subsection III.B.

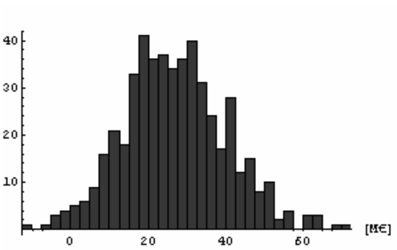

Fig. 2. Histogram of absolute frequencies of illustrative annual free cash flows of a thermal power plant, out of a sample of 1000 trajectories, considering 4500 equivalent hours working at nominal power of 400MW.

D. Metrics for risk and return

In order to compare investments in assets with different life times, according with the portfolio theory's criteria of risk-return, the correlation between the return of different investments must be estimated. Therefore, the returns should be referenced to a common time frame. For that effect, let us consider as a measure of return r_t the ratio between the annual free cash flows fcf_t and the annualized fixed capital cost cc, subtracted by a unit, to have the portion of investment that refers to added value:

$$r_t = \frac{fcf_t}{cc_t} - 1 \tag{34}$$

Assuming that the investment is concentrated in the beginning of the project and considering that it can be equally distributed along the lifetime of the plant, the annualized fixed capital cost cc may be regarded as an annuity, determined as follows:

$$cc = \frac{i(1+i)^{lt}}{(1+i)^{lt} - 1} I \tag{35}$$

where i is the discount rate expected by the investors, lt is the life time of the plant, and I the investment made in time 0.

Knowing the probability density function of the annual free cash flows, constructed as depicted in subsection III.C, and the annual fixed capital cost of a plant, determined through (35), one can infer the probability density function of the annual return and determine its expected value μ_{r_t} and its variance $\sigma_{r_t}{}^2$ according to:

$$E[r_t] \equiv \mu_{r_t} = \frac{E[fcf_t]}{cc} - 1 \tag{36}$$

$$VAR[r_t] \equiv \sigma_{r_t}{}^2 = \frac{VAR[fcf_t]}{cc^2} \tag{37}$$

It must be pointed that since the measurement of risk adopted is through the dispersion of the annual return, the adequate discount rate to use in (35) is the riskless one, assuring no previous considerations on risk [12].

Once the probability density function of the return of each type of asset is known for different years, one can define proxies of the metrics of risk and return needed for portfolio theory's analysis on section II – the expected value and standard deviation of return. Considering N risk-return sets estimated for N years:

$$\hat{E}[r_i] \equiv \mu_{r_i} = \frac{\sum_{t=1}^{N} E[fcf_{t,i}]}{cc_i . N} - 1 \tag{38}$$

$$\sqrt{\hat{VAR}[r_i]} \equiv \sigma_{r_i} = \frac{\sum_{t=1}^{N} \sqrt{VAR[fcf_{t,i}]}}{cc_i . N} \tag{39}$$

$$\hat{E}[r_i, r_j] \equiv \sigma_{ij} = \frac{\sum_{k=1}^{N} \left[(\mu_{r_{i,k}} - \mu_{r_i})(\mu_{r_{j,k}} - \mu_{r_j})\right]}{N} \quad , \quad i \neq j \tag{40}$$

where i identifies each type of asset, μ_{r_i} its expected value, σ_{r_i} its standard deviation, and σ_{ij} the covariance between assets i and j.

IV. Selection of Portfolios of Generation Assets

A. Investment allocation

Taking proxies (38), (39) and (40) into definitions (2) and (3), for a portfolio of N assets:

$$E[r_P] \equiv \mu_{r_p} = \sum_{i=1}^{N} w_i \, \mu_{r_i}$$

$$VAR[r_p] \equiv \sigma_{r_p}{}^2 = \sum_{i=1}^{N} w_i{}^2 \sigma_{r_i}{}^2 + \sum_{\substack{i,j=1 \\ i \neq j}}^{N} w_i w_j \, \sigma_{ij}$$

$$w_i = \frac{cc_i}{\sum_{i=1}^{N} cc_i} \tag{41}$$

satisfying $w_i \geq 0$ and $\sum_{i=1}^{N} w_i = 1$.

Using these equations, the sets of risk-return of possible portfolios with the N considered assets can be mapped. The mapping is done iteratively, changing the different weights w_i in each iteration.

B. Capacity allocation

The mapping of the risk-return sets of possible portfolios mentioned above is done considering the relative weight of each type of asset in the total annual fixed capital cost. However, this relative weight is not the most adequate reference to assess investments on generation portfolios. On the one hand one must take into account the total amount of investment required; on the other hand it should be possible to equate the portfolio of assets with a strategic orientation of the generation company such as, for example, installed capacity.

For that effect, let us redefine the weights defined in (41) as a function of the installed capacity of each of the N assets:

$$w_i = \frac{\lambda_i p_i}{\sum_{i=1}^{n} \lambda_i p_i} \tag{42}$$

where λ_i is specific annual fixed capital cost of the each asset (€/MW.year) and p_i its installed capacity. Considering a desired total installed capacity, P_T, and the relative weight of each asset w_i in annual fixed capital costs, one obtains the installed capacity of each asset solving the linear system $Ap = S$, where:

$$A = [a_{ij}]_{n,n} : a_{ij} = \begin{cases} \lambda_i (w_i - 1) & , \quad i = j \wedge i < n \\ w_i \lambda_j & , \quad i \neq j \wedge i < n \\ 1 & , \quad i = n \end{cases} \tag{43}$$

$$S = [s_i]_n^T : s_i = \begin{cases} 0 & , \quad i < n \\ P_T & , \quad i = 1 \end{cases} \tag{44}$$

$$p = [p_i]_n^T$$

Each asset installed capacity p_i is obtained solving:

$$p = A^{-1} S \tag{45}$$

Having the weights of annual fixed capital cost of each asset related with its installed capacity, it is possible to relate the assessment of the investment in a portfolio of generation asset with the strategic goals of attaining a given installed capacity P_T as well as establishing a minimum or maximum capacity of a given type of asset. It is also possible to identify actions of buy and/ or sell generation assets towards optimal portfolio and determine the correspondent required investment.

Constraints for installed capacity, risk and return

In order to take into account considerations of risk and return of the investor, one can superimpose constraints for risk and return on the efficient frontier, once this is mapped. Assuming that the current weights of each type of asset w_i^* reflect adequate levels of risk and return, it is natural to constraint the selection of portfolios over the efficient frontier to portfolios that represent a Pareto improvement from the current portfolio. The selection is therefore limited to portfolios with higher return and lower risk than the current one: $\mu_P \geq \mu_P^* \wedge \sigma_P \leq \sigma_P^*$.

In addition, with the new definition of the relative weights in (42), solving (45), one can introduce criteria for minimum percentage of, for example, renewable power plants. This allows the investor to take into account a number of new factors (e.g., environmental concerns, EU guidelines and regulations, state subsidies, etc).

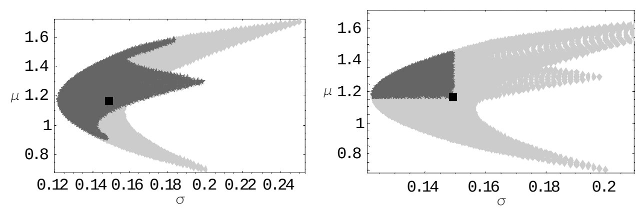

Fig. 3. At the left, we show portfolios of the domain of risk and return constrained for installed capacity. At the right, we show portfolios of the domain of possible portfolios constrained for risk and return. The constraint for risk and return only considers Pareto improvements from the current portfolio.

[*] Combined cycle gas

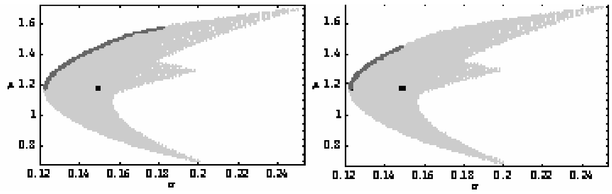

Fig. 4. At the left, we show portfolios of the efficient frontier constrained for installed capacity. At the right, we show portfolios of the efficient frontier constrained for risk and return.

C. Decisions to buy and/ or sell towards an optimal portfolio

Given an optimal portfolio of the efficient frontier, considering constraints for risk, return and installed capacity, one can identify decisions of buy and/ or sell generation assets according to:

$$\Delta p_i = p_{i,2} - p_{i,1} \tag{46}$$

$$\begin{cases} \Delta p_i > 0, & buy\ \Delta p\ of\ the\ i^{th}\ asset \\ \Delta p_i < 0, & sell\ \Delta p\ of\ the\ i^{th}\ asset \end{cases}$$

where $p_{i,1}$ and $p_{i,2}$ are the current and the optimal installed capacity obtained as described in subsection IV.B.

V. Illustration

As an illustration, let us consider 5 types of generation assets as possible investments, with specific investment τ_i, specific annual capital cost λ_i and lifetime lt as depicted in Table I.

TABLE I
GENERATION ASSET'S CHARACTERISTICS

i	Description	τ_i [M€/MW]	λ_i [m€/year.MW]	lt [years]
1	Coal	0.475	48.38	20
2	CCGT*	0.475	48.38	20
3	Fuel-oil	0.475	48,38	20
4	Hydro	0.897	73.36	50
5	Wind	1	101.85	20

Let us now consider the current portfolio of a generation company as shown in Table II.

TABLE II
CURRENT PORTFOLIO

i	Nr. plants	Installed capacity[M€/MW]
1	1	400
2	1	400
3	1	400
4	1	390
5	1	24
		1614

The expected return and risk of the considered type of assets as well as of the current portfolio are registered in Table III. They were estimated according to (38) and (39), through a Monte Carlo simulation of the respective annual free cash flows, as depicted in subsection III.C, using the stochastic processes mentioned in subsection III.A. The parameters of those processes, the correlations of the returns, the plants parameters and other assumptions for estimating the free cash flows are shown in Tables 7-14.

In the case of thermal power plants it is assumed an annual fixed demand equivalent to 4500 hours working at nominal power. It is also assumed that its entire production is sold through pool and that the generation company must buy CO_2 emission certificates for all of its thermal production.

In the case of hydro and wind power plants, it is assumed that the entire production is consumed at a fixed tariff applicable throughout its lifetime, respectively 60 and 70 €/MWh.

In both cases – thermal and renewable, the operation and maintenance costs c_{OM} are considered negligible.

TABLE III
EXPECTED RISK AND RETURN OF EACH TYPE OF ASSET AND CURRENT PORTFOLIO

i	Description	Scenario 1		Scenario 2	
		μ_{r_i} %	σ_{r_i} %	μ_{r_i} %	σ_{r_i} %
1	Coal	35.31	69.70	54.98	74.93
2	CCGT*	85.49	61.77	106.85	67.86
3	Fuel-oil	8.01	76.91	28.61	81.15
4	Hydro	30.93	47.00	30.93	47.00
5	Wind	8.28	2.90	8.28	2.90
Current portfolio		38.10	38.57	51.51	39.06

Let us look at 2 scenarios, considering a difference of 4€/MWh in the long-term pool price of electricity, keeping the rest of the parameters constant.

Under both scenarios, the following constraints for risk, return and installed capacity are superimposed on the efficient Pareto frontier of risk-return of the domain of portfolios for each scenario:

$$\sigma_{r_p} \le \sigma_{r_p}{}^* \wedge \mu_{r_p} \ge \mu_{r_p}{}^* \wedge \sum_j p_j \ge 0.3$$

where $\left(\sigma_{r_p}{}^*, \mu_{r_p}{}^*\right)$ is the risk-return set of the current portfolio and p_j is the installed capacity of the asset j, a renewable asset. The resulting optimal alternative portfolios are depicted in Fig. 5.

From this point on, the choice of one portfolio out of the set of optimal portfolios resulting from the imposed constraints over the efficient frontier of risk-return depends upon the risk aversion of the decision makers, as well as other constraints such as the availability of licenses to build certain type of assets, the opportunities of buying and/ or selling other types of assets, constrains of budget, etc.

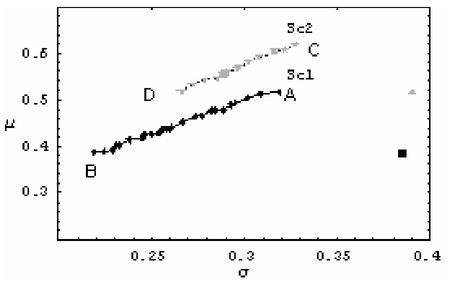

Fig. 5. The lines illustrate the set of optimal portfolios of the efficient frontier under scenario 1 and 2, respecting the constraints for risk, return and capacity. B/D and A/C indicate the lowest and highest sets of risk-return for scenario 1 and scenario 2. The isolated points ate the right represent the current portfolio under each scenario.

As an illustration of the alternative optimal portfolios, let us look with more detail into the portfolios with lowest B/D and highest A/C sets of risk-return of each scenario.

TABLE IV
WEIGHTS OF INSTALLED CAPACITY OF EACH TYPE OF ASSET OF PORTFOLIOS WITH HIGHEST AND LOWEST SET OF RISK-RETURN, UNDER SCENARIO 1 AND 2

i	Scenario 1		Scenario 2	
	A %	B %	C %	D %
1	6.33	13.85	12.49	13.37
2	63.17	41.56	56.21	46.78
3	0	0	0	0
4	12.50	18.27	16.47	17.63
5	18.00	26.32	14.83	22.22
μ_{r_P}	51.63	38.68	61.84	51.98
σ_{r_P}	31.88	21.95	32.83	26.64

It can be seen that the trade-off between risk and return among A/C to B/D relates to a higher amount of CCGT power plants in the former as a contrast to a higher amount of wind power plants in the latter. This result is coherent with the idea that riskier portfolios are composed of riskier assets.

Considering a goal of total installed capacity 20% higher than the current one, which results in a new target P_T = 1936MW, solving (45) and then (46) and (47), one determines the actions of buy and/ or sell, as well as the investment required to achieve each of the optimal portfolios depicted in Fig. 5 – A, B, C and D, which are registered in Table V.

TABLE V
DECISIONS OF BUY AND SELL ASSETS [MW].

i	Scenario 1		Scenario 2	
	A	B	C	D
1	-278	-132	-158	-141
2	824	404	689	506
3	-400	-400	-400	-400
4	-148	-36	-71	-49
5	324.7	486	263	406

Considering data from Table I, for the mentioned total installed capacity, the expected free cash flows and their standard deviation estimated for each of those portfolios are in Table VI.

TABLE VI
EXPECTED FREE CASH FLOWS AND STANDARD DEVIATION [M€]

i	Scenario 1		Scenario 2	
	A	B	C	D
E[FCF]	278.6	239.6	283.5	236.9
$VAR[FCF]^{1/2}$	58.6	37.9	57.5	46.3

A clear visualization of the difference between the highest and the lowest set risk-return (A and B/ C and D) is through the probability density function of the free cash flows. As an illustration for scenario 1, those cases are depicted in figure 6.

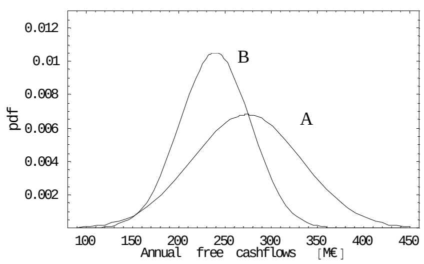

Fig. 6. Probability distribution of the free cash flows of the portfolios with lower and higher risk-return sets.

VI. Conclusion

The investment decision in a portfolio of generation assets has been addressed in this paper. An approach to display optimal combinations for the decision maker to take into consideration when deciding where to invest is proposed.

Each different selection among those optimal portfolios always represents a trade-off between risk and return, as in the efficient frontier of the portfolio theory.

In addition to adding constraints for risk and return over the efficient frontier, this paper also describes a way to introduce constraints for capacity over the domain, making it possible to relate the investment decision with strategic guidelines of the generation company.

The return and risk of the portfolios are computed as defined in portfolio theory. Monte Carlo simulation is used to estimate the return and risk of each type of asset, as well as their interaction, through the estimation of free cash flows.

The approach presented is general and other models of valuation as well as other stochastic processes may be considered for estimating the return and risk of the generation assets in a more sophisticated and detailed manner.

Summing up, this approach, when properly framed, can bring clarity when the time comes to decide what generation assets to buy and to sell regarding the implication those actions may have on levels of risk and return.

References

[1] R. A. Brealy, S. C. Myers, *Principles of Corporate Finance*, 7th ed., McGraw-Hill/Irwin Series in Finance, Insurance, and Real Estate

[2] H. Markowitz, *Portfolio Selection*, The Journal of Finance, Vol. VII, No.1, Março 1952

[3] Vincent Kaminski, *Energy Modeling: Advances in the Management of Uncertainty*, Risk Books; 2a Ed, 2005.

[4] J. C. Hull, *Options, Futures, and Other Derivatives*, 5th ed., Finance Series. Upper Saddle River, NJ: Prentice Hall

[5] P. F. Correia, P. M. S. Carvalho, L. A. F. M. Ferreira, J. Guedes, J. Sousa, *Power Plant Multi-Stage Investment Under Market Uncertainty*, 2007

[6] [online] www.windpower.org

[7] P. F. Correia, P. M. S. Carvalho, L. A. F. M. Ferreira, J. Guedes, J. Sousa, *On the Returns from a Portfolio of Wind Turbines*, working paper, 2007

[8] W. J. Szajnowski, *The Generation of Correlated Weibull Clutter for Signal Detection Problems*", IEEE transactions on aerospace en electronic systems, VOL. AES-13,No.5, September 1997

[9] Bento J. F.Murteira, Daniel A.Muller, K. Feridun Turkman, "Análise de Sucessões Cronológicas", McGraw Hill, Janeiro de 2000

[10] D.L.D.D. Jardim; M.E.P. Maceira; D.M. Falcão; J.M. Damázio, *Stochastic Streamflow Model for Hydroelectric Systems using Clustering Techniques*, VIII SEPOPE (Symposium of specialists in electric operational and expansion planning), Maio 2002

[11] J. D. Salas, J. A. Ramirez, P. Burlando, Sr. R. A. Pielke, *Stochastic simulation of precipitation and streamflow processes*, Chapter 33 in *Handbook of Weather, Climate, and Water*, T.D. Potter and B.R. Colman, Eds., Wiley-Interscience, John Wiley and Sons Inc., New York, NY, 607-640

[12] J. O. Soares, A. V. Fernandes, A. A. Março, J. P. P. Marques, *Avaliação de Projectos de Investimento na Óptica Empresarial*, 1ª edição, 1999, Edições Sílabo

[13] [Online] National Information System for Hydrological Resources www..snirh.pt

Other bibliography

[13] J. P. S. Paiva, *Redes de Energia Eléctrica – uma Análise Sistémica*, Colecção Ensino da Ciência e da Tecnologia, IST Press

[14] [online] www.mathworld.com

[15] [online] www.inag.pt

[16] A. Damodaran, *Investment Valuation – Tools and techniques for determining the value of any asset*, Wiley Frontiers in Finance, John Wiley & Sons, Inc

[17] F. Hoss, *Asset Management: Using Efficient Frontier Analysis to Select Projects With Achievable Returns While Operating Within Acceptable Risk*, GE Energy

[18] H. M. Markowitz, *Foundations of Portfolio Theory*, Nobel Lecture, December 1990

[19] M. Liu, F. F. Wu, *Portfolio Optimization in Electricity Markets*, Electric Power Systems Research, Elsevier

[20] [online] W. Sharpe, *Macro Investments* Analysis, Stanford University http://www.stanford.edu/~wfsharpe/mia/mia.htm

[21] H. M. Markowitz, *Foundations of Portfolio Theory*, Nobel Lecture, Dezembro 1990

[22] A. K. Dixit, R. S. Pindyck, *Investment under Uncertainty*, Princeton, NJ: Princeton University Press, 1994

[23] Anne Ku, *Risk and Flexibility in Electricity: Introduction to the Fundamentals and Techniques*, Risk Books, 2003

[24] R. M. Vogel, J. R. Stedinger *The Value of Stochastic Streamflow Models in Overyear Reservois Design Applications*, Water Resources Research, Vol.24, Num.9, 1483-1490, Setembro 1998

[25] D. Koutsoyiannis, *Stochastic Simulation of Hydrosystems*, The Encyclopedia of Water

[26] *Engineering and Design - Hydrologic Frequency Analysis*, Engineer Manual 1110-2-1415, Department of the Army - U.S. Army Corps of Engineers, Washington, DC 20314-1000, Março 1993, Capítulo 12 *Stochastic Hydrology*

[27] D. Harte, M. Pickup, *Stochastic Models for Hydro Catchment Inflows; an Exploratory Analysis*, A report undertaken for the New Zealand Electricity Commission

[28] W. T. Song, L. Hsiao, *Generation of Autocorrelated Random Variables with a Specified Marginal Distribution*, Proceedings of the 1993 Winter Simulation Conference

[29] M. Denton; A. Palmer; R. Nasiello; P. Skantze, *Managing Market Risk in Energy*, IEEE transactions on power systems, Vol. 18, No. 2, May 2003

[30] S. N. Siddiqi, *Project Valuation and Power Portfolio Management in a Competitive Market*, IEEE transactions on power systems, Vol. 15, No. 1, February 2002

[31] E. Sun; E. Liu, *Generation Asset Valuation Under Market Uncertainties*, IEEE, 2007

[32] E. Caruso; M. Dicorato; A. Minoia, M. Trovato, *The Valuation of New Generation Assets in a Competitive Electricity Market*, IEEE MELECON 2004, May, Dubrovnik, Croatia

[33] H. Y. Yamin; J. D. Finney; W. Rutz; H. Holly, *Gencos Portfolio Management using "Strategist" in Deregulated Power Markets*, IEEE, 2001

X. Guan; J. Wu; F. Gao; G. Sun, *Optimal Generation Portfolio Management for Futures and Spot Market*, IEEE, 2006

Appendix

TABLE VII
PLANTS PARAMETERS

i	Efficiency η	CO_2 emmissions [ton/GJ]
1	36%	0.0767
2	57%	0.056
3	32%	0.074
4	87%	-
5	Fig.7	-

TABLE VIII
PRICES' PROCESS PARAMETERS

	Prices				
	Electricity [€/MWh]	CO_2 [€/ton]	Coal [€/GJ]	Gas [€/GJ]	Oil [€/GJ]
S(0)	58	17	1.48	3.3	3.3
λ	1.93	0.05	0.1	0.1	0.1
ϕ	Log(40) /Log(44)	Log(30.6)	Log(0.9)	Log(1.5)	Log(1.6)
σ	0.6	0.15	0.25	0.4	0.35

TABLE IX
FOSSIL FUELS' MATRIX OF CORRELATION

	Coal	Gas	Oil
Coal	1	0.2	0.2
Gas	0.2	1	0.2
Oil	0.2	0.2	1

TABLE X
HYDROELECTRIC POWER PARAMETERS

Δh	58 m
α	5%

Monthly averages and standard deviations of water inflows, as well as monthly autocorrelations were computed using the data available for a hydroelectric power plant located in Douro river [13].

TABLE XI
WIND SPEED' PROCESS PARAMETERS

Autocorrelation	0.4
Weibull shape parameter	2.2
Annual mean wind speed	5.5 m/s

TABLE XII
ESTIMATED CORRELATIONS BETWEEN THE RETURNS UNDER SCENARIO 1

i	1	2	3	4	5
1	1	0	0.6	0	0
2	0	1	0.6	0	0
3	0.6	0.6	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1

TABLE XIII
ESTIMATED CORRELATIONS BETWEEN THE RETURNS UNDER SCENARIO 2

i	1	2	3	4	5
1	1	0	0.7	0	0
2	0	1	0.2	0	0
3	0.7	0.2	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1

TABLE XIV
ASSUMPTIONS FOR ESTIMATING FREE CASH FLOWS

Riskless discount rate	8%
Tax rate	35%

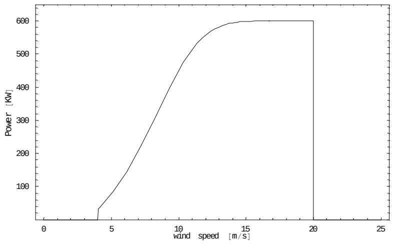

Fig. 7. Power curve of a Vesta V44 600/44, with nominal capacity of 600kW.

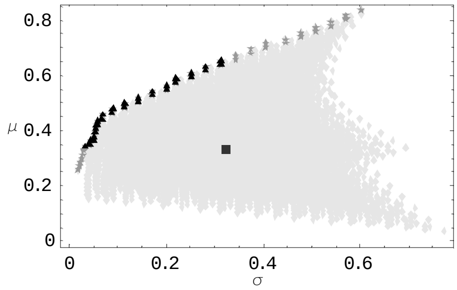

Fig. 8. For scenario 1, the domain of risk-return is represented in a lighter grey; the efficient frontier in a darker grey; and the portfolios constrained for risk, return and capacity are represented in black.

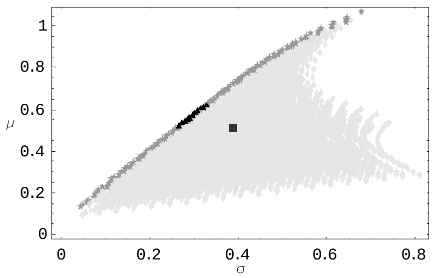

Fig. 12. For scenario 1, the domain of risk-return is represented in a lighter grey; the efficient frontier in a darker grey; and the portfolios constrained for risk, return and capacity are represented in black.