Optimal model for warehouse location: a real case-study

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Abstract

Warehouse location has been studied since the beginning of the 20th century. In this paper, we approach again this problem with some not yet studied features and restrictions. An optimization model is developed to account for the optimal warehouse location of a real distribution system where different types of transport are considered, maritime and territorial. Inbound and outbound costs at the warehouse level as well as warehouse fixed location costs are accounted for. The final goal of the problem is the minimization of transportation and warehousing costs. The model is applied to the solution of a real case study of a pulp & paper mill distribution system.

Keywords: warehouse location, optimization, transportation

1. Introduction

Locating facilities is crucial for companies. These locations will determine what distance will be covered to arrive to the client. Consequently they will determine load transporting costs. In some cases when there are thousands of clients these costs can be very high, but if facilities are well located those costs can be reduced.

Arriving to an optimal structure is some times very difficult especially when the problems are very large. Adding up to this there is another great difficulty, the information in which the models are based is in constant mutation. So the problems have to be solved in a dynamic way in order to be more realistic.

Alfred Weber (1909) began the investigation about this theme, he developed the first model for locating a single factory. That model would just minimize the distance between the factory and some different clients in a chosen area.

In spite of Weber's model being such a basic one many years passed before researchers looked again seriously into the location problems. Some examples of studied problems are: the location of fire fighting vehicles (Valinsky 1955), the location of refuse collecting points (Wersan *et al.* 1962) and the location of factories (Burstal *et al.* 1962), among others.

The problem known as SPLP (Simple Plant Location Problem) was first formulated by Kuehn and Hamburger (1963). This model uses a heuristic that has as inputs the location of the clients and all the possible places for the plant location. The goal is to minimize the cost of transporting load between the factory and clients, minimizing at the same time the construction and working factory costs. The model proposes the construction of another factory for the case where the transport to a group of clients is so costly that it becomes cheaper building up of a new factory near those clients.

Chvatal (1979), formulated the so-called Greedy heuristic. This heuristic was some years later used by Guha and Kuller (1998) to solve the problem UFLP (Uncapacitated Facility Location Problem).

Sultan and Fazan (1989) also addressed the UFLP problem and used a meat-heuristic, "tabu search", to solve it.

Still related with location problem is the CFLP (Capacitated Facility Location Problem) problem. This is very close to the UFLP problem but with limited capacities. This problem is sometimes very large especially in real situations, so there are great difficulties to achieve an optimal solution. Some of the models developed for this problem were solved using Lagrangean Relaxation as in Holmberg *et al.* (1999) and some years after in Díaz and Fernandez (2001). For these models also developed decomposition algorithms. Primal and primal-dual were also used (e.g. Wentges (1996)). Chudak and Williamson (2004) developed a local search heuristic that by simplifying another algorithm released by, Plaxton, and Rajaraman (2000) reaches solutions close to the optimum in less time.

Another problem within the location problems is the Connected Facility Location Problem, where graph modulation has to be done. For the first time Swamy e Kumar (2004) use a primal-dual algorithm to solve this problem. Afterwards they used the same algorithm to solve other problems.

It is easy to understand that all location problems have much in common. In all cases the goal is to minimize costs of transport, construction, service, etc. So the research about warehouse location is strongly linked to the ones mentioned above.

As referred before, Alfred Weber (1909), published a paper called “Theory of the Location of Industries”, the first study about location problems. Some decades after Baumol and Wolfe (1957) developed a model to determine optimum locations for many warehouses. This model uses an exact algorithm arriving to an optimum solution.

Kuehn and Hamburguer (1963), bring forth a heuristic for spatial location of warehouses in large scale. Essentially based in linear programming and optimization this model brings great advantages to the ones develop until then.

Khumawala (1972) develops a mathematical model solved by an exact algorithm “branch-and-bound” to figure out the solution to the UWLP problem (Uncapacitated Warehouse (Facility) Location Problem).

Brandeau e Chiu (1989) presented a review were relevant articles about “Warehouse Location” are identified. They refer essentially the ones we referred above.

Later on with Perl and Daskin (1994) the problem is again discussed. A new methodology was proposed. This methodology joins an exact resolution with a heuristic approach. Dividing the problem in three parts this methodology solves the problem using the exact method and the heuristic sequentially.

A new method based on simulation to solve the problem UWLP appears with Hidaka and Okano (1997). This model that deals with the large scale problem (a simulation with more than 6800 clients is provided) arrives to a near optimal solution.

Krativa, Filipovie and Tosie (1998) introduced the so called genetic algorithms in location problem solving. With these algorithms it is possible to achieve a good result in a reasonable time.

Beyond genetic algorithms, the meta-heuristic tabu-search is applied to the UWLP problem by Michel e Hentenryck (2003). The results achieve a near optimum result in a short time.

More recently, Dupont (2006), analyses the location of warehouses, considering that costs of transport and warehousing follow a concave function that relates them with the quantity transported. Dupont (2006) develops a heuristic using some of the properties of exact methods.

Finally, Ghill and Bhatti (2007) bring forth a heuristic that divides the problem of warehouse location in two parts. Able to solve large problems this model builds a binary matrix that describes all allocation possibilities. And in a second step an algorithm finds the optimum solution within those possibilities.

In this paper a real case of a warehouse location is studied. An optimization model is developed based on the work of Ghill and Bhatti (2007). The problem goal is to minimize transportation and warehousing location costs. Two types of warehouses are considered, warehouses that can be located anywhere in the map and ports that obviously need to have access by sea (possibilities of warehouse/port location must be given to the model). Two stages of transportation are needed to reach the clients. The first stage is from factories to the warehouses or ports, called primary or inbound transportation. The secondary or outbound transportation is done between warehouse/ports and clients. The products might arrive to the ports by sea or by road, while to the other warehouses is just possible to arrive by road. From warehouse/ports to the clients there is just one mean of transportation available – the road type of transport. In terms of costs, warehousing cost is defined for each ton warehoused. Moreover, each warehouse/port has a fixed cost included that is paid every year. In conclusion the model picks up all the information about transporting and warehousing cost and builds up the optimal warehouse structure that minimizes the distribution system costs.

The rest of the paper is structured as follows. Following this introduction the mathematical formulation is explained. Then the characterization and solution of a case-study is presented where a real pulp & paper distribution system is optimized. Some conclusions are finally drawn.

2. Mathematical formulation

Having in mind the problem characteristics described above a warehousing location mathematical model is developed. This involves a number of sets, parameters, variables, restrictions and an objective function. These are the following:

Sets

e ={warehouses}

i ={factories}

j ={cities}

p ={harbours}

t ={time}

Beyond these sets there are two more auxiliary sets *e1* and *p1* that have the same domain as *e* and *p*, respectively.

Parameters

a0,a1, a2 – polynomial coefficients

a_i - factory *i* available capacity

$c1_{i,p}$- transportation cost between factory *i* and harbour *p*

$c2_{j,e}$- transportation cost per kilometre and per ton between warehouse *e* and city *j*

$c3_{j,p}$-c transportation cost per kilometre and per ton between harbour *p* and city *j*

$CExcp_{j,p}$- transportation cost (exceptional) per kilometre and per ton between harbour *p* and city *j*

$d1_{i,e}$- distance between factory *i* and warehouse *e*

$d2_{j,e}$- distance between city *j* and warehouse *e*

$d3_{j,p}$- distance between city *j* and harbour *p*

$dist2_{e,e1}$- binary table in which value 0 does not permit that warehouse *e* and warehouse *e1* are used simultaneously because they are too close

$dist3_{e,p}$- binary table in which value 0 does not permit that warehouse *e* and harbour *p* are used simultaneously because they are too close

$dist4_{p,p1}$- binary table in which value 0 does not permit that harbour *p* and harbour *p1* are used simultaneously because they are too close

$Excp_{j,p}$- transportation cost between harbour *p* and city *j* is an exception to the general formula

g- transportation cost between a factory and a warehouse per kilometre and per ton

h- fixed cost for each warehouse per year

h1- warehousing cost per ton

MH- minimum usage of a warehouse in tons

$p_{j,t}$- demand in city *j* at the time *t*

The values of parameters c2 and c3 are calculated by:

$$c2_{j,e} = a0 \times d2_{j,e}^2 + a1 \times d2_{j,e} + a2$$
$$c3_{j,p} = a0 \times d3_{j,p}^2 + a1 \times d3_{j,p} + a2$$

In the exceptional cases referred in table $Excp_{j,p}$, c3 has the value mentioned for each pair *(j,p)* in table $CExcp_{j,p}$. The exceptional cases are the ones in which ports or warehouses are closer than 100 km and therefore are not allowed to be chosen simultaneously.

Variables

Positive continue variables

$y_{i,e,t}$- quantity transported between factory *I* and warehouse *e* at the time *t*

$x_{e,j,t}$- quantity transported between warehouse *e* and city *j* at the time *t*

$w_{i,p,t}$- quantity transported between factory *i* and harbour *p* at the time *t*
$k_{p,j,t}$- quantity transported between harbour *p* and city *j* at the time *t*

Binary variables

n_e -is 1 if the warehouse *e* is used and 0 if not
$n1_p$- is 1 if the harbour *p* is used and 0 if not

Auxiliary variables

z1- transportation cost to warehouses and harbours
z2- warehousing cost
z3- transportation cost from warehouses and ports to cities
z4- fixed cost of using a warehouse or a port
z- total cost

Using the above data and problem characteristics the following restrictions describe the model:

$$\sum_{j,t} x_{e,j,t} \geq MH \times n_e \qquad \nabla e \tag{1}$$

$$\sum_{j,t} k_{p,j,t} \geq MH \times n1_p \qquad \nabla p \tag{2}$$

$$\sum_{j,t} x_{e,j,t} - n_e \times M \leq 0 \qquad \nabla e \tag{3}$$

$$\sum_{j,t} k_{p,j,t} - n1_p \times M \leq 0 \qquad \nabla p \tag{4}$$

$$\sum_{e} y_{i,e,t} + \sum_{p} w_{i,p,t} = a_i \qquad \nabla i \tag{5}$$

$$\sum_{e} x_{e,j,t} + \sum_{p} k_{p,j,t} = p_{j,t} \qquad \nabla j,t \tag{6}$$

$$\sum_{j} x_{e,j,t} - \sum_{i} y_{i,e,t} = 0 \qquad \nabla e,t \tag{7}$$

$$\sum_{j} k_{p,j,t} - \sum_{i} w_{i,p,t} = 0 \qquad \nabla p,t \tag{8}$$

$$\sum_{p} n1_p \geq 1 \tag{9}$$

$$n_e + n_{e1} \leq dist2_{e,e1} + 1 \qquad \nabla e, e1 \tag{10}$$

$$n_e + n1_p \leq dist3_{e,p} + 1 \qquad \nabla e, p \tag{11}$$

$$n1_p + n1_{p1} \leq dist4_{p,p1} + 1 \; \nabla p, p1 \tag{12}$$

Restrictions 1 and 2 define the minimum usage for each warehouse or harbour. Restrictions 3 and 4 use Big M to increment to 1, $n1_p$ and n_e, if warehouse *e* or harbour *p* are used (remember that n_e and $n1_p$ are binary). Restriction 5 assures that deliveries are limited by the capacity of production a_i. Restriction 6 compels the model to satisfy all the demands while in restrictions 7 and 8 it becomes compulsory that all the load that enters in a warehouse or port is used while satisfying demand. Restriction 9 compels the model to choose at least one port to be used. Finally, restrictions 10, 11, 12 limit the proximity between warehouses and ports, the binary tables dist reveal which warehouses/ports can not be used at the same time

Having defined the model restrictions the model objective function is defined which consists on the minimization of the distribution system cost. This is given by:

$$z1 = \sum_{i,e,t} g \times d1_{i,e} \times y_{i,e,t} + \sum_{i,p,t} c1_{i,p} \times w_{i,p,t} \qquad 13$$

$$z2 = h1 \times (\sum_{e,j,t} x_{e,j,t} + \sum_{p,j,t} k_{p,j,t}) \qquad 14$$

$$z3 = \sum_{e,j,t} c2_{j,e} \times d2_{j,e} \times x_{e,j,t} + \sum_{p,j,t} c3_{j,p} \times d3_{j,p} \times k_{p,j,t} \qquad 15$$

$$z4 = h \times (\sum_{e} n_e + \sum_{p} n1_p) \qquad 16$$

$$z = z1 + z2 + z3 + z4 \qquad 17$$

In equation 13 the calculation of the primary transport (factories – warehouses/harbours) cost is performed while equation 14 gives the cost of warehousing. The calculation of secondary transport (warehouses/harbours – cities) cost is done in equation 15 and equation 16 calculates the total location cost. The latter considers only fixed costs but this can be easily extended to include variable costs related to the designed warehouse capacity. Finally, equation 17 calculates the total cost of the distribution system.

3. Case Study

The previous model is applied to the solution of a real portuguese pulp & paper distribution system. The company wants to define the optimal warehouse location that minimizes both the inbound and outbound transports to/from the warehouses as well as the costs of warehouse location. The region under study is within central Europe and comprises Belgium, Holland, Luxemburg and Germany. The production is performed in Portugal. Although there are several factories in Portugal, due to the country size, a single production geographical centre was considered as origin of the products to be distributed in central Europe

This case study involves two phases of development:

Phase 1 – Analysis of the present distribution system: the company wants to compare the actual system implemented and the optimal.

Phase 2 – Taking into account that there will be in 2008 an increment on the paper production, due to the build up of a new paper machine, is important to analyse the possibility of restructuring the present system.

Phase 3 - Increment of the market demand for 2010.

It is important to note that the location of new warehouses considered is a simple contract established by the company and the distribution partners, therefore a possible restructuring , in the distribution system, exists.

The model was implemented in GAMS and solved through CPLEX (ILog®) for a margin of optimality of 0,5%.

3.1 Transportation and warehousing cost

Before analysing the different scenarios we have just to make a small explanation on the different types of cost and how they are calculated.

Warehousing cost – This cost is always 12 m.u. (monetary units)/ton. Obviously not every ton of load is the same amount of time in the warehouse. But as this model does not use an inventory schedule we have estimated the average time spent in the warehouse and assumed that each ton pays this value.

Primary transportation cost – As explained there are two types of primary transport maritime and territorial.

For the maritime transport, the different costs of transporting one ton to all the available ports are in this table:

Table 1 – Cost of the maritime primary transport

Port	Cost (m.u./ton)
Hamburg	110
Antwerpen	100
Moerdijk	100
Roterdam	100
Bremen	115

Considering now the primary road transportation we had access to the cost of transporting a ton to three different cities (where now a day warehouses are located). In the next table the cost per ton per km is calculated for these routes.

Table 2 - Cost of primary road transportation

Port/Warehouse	Cost (m.u./ton)	Distances to Portugal	Cost (m.u./ton.km)
Moerdijk	105	2012	0.052
Frankfurt	118	2309	0.051
Nuremberg	130	2315	0.056

Because the costs per ton per km are not much different we decided to use the average cost which is 0.053 m.u./ton.km.

Secondary transportation cost – All the secondary transportation is done by road. We have to extrapolate the costs from some routes given to us initially. So the model can compare the costs in every possible location.

Table 3 - Cost of secondary transport in the present routes

Port/Warehouse	Country of destination	Medium distance	Cost (u.m./ton.km)
Moerdijk	Holland	147	0.1
Moerdijk	Belgium	125	0.09
Moerdijk	Germany	237	0.16
Frankfurt	Germany	206	0.13
Nuremberg	Germany	147	0.1

To extrapolate from these values we had to do an equation relating distance and cost per ton per km. This equation will be used to find the cost of every possible route between all the clients and the entire possible warehouse and port locations.

The equation found is a quadratic function as it is shown next:

$$\text{Cost} = a0 \times \text{distance}^2 + a1 \times \text{distance} + a2$$

The coefficients are :

a0 = 3.44573641E-06
a1 = 6.41193383E-04
a2 = 1.17596970E-01

The graph illustrates the equation:

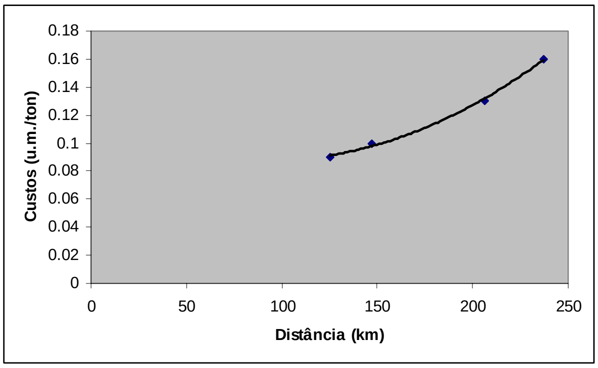

Fig. 1 – Cost curve of secondary transport is function of distance

To figure out the transportation cost by road, both the primary and secondary, we have to know the distance between the two places a specific route links.

The geographical distance is calculated through the following formula that uses the GPS coordinates of clients and the production centre or the warehouses.

$$Distance = 1.2 \times \frac{\sqrt{((Latitude1 - Latitude2) \times 1852)^2 + ((Longitude.c1 - Longitude.c2) \times 1852)^2}}{1000}$$

The primarily terrestrial transportation cost is found by multiplying the distance between the centre of production and the clients by the cost per km. Secondary transportation cost is calculated also by multiplying the distance and the cost per km. The distance is calculated by the same formula as given above. When finding the distance we multiplied per 1.2 because the geographical distance is not the road distance. So we admitted that road distance would be more 20% than the real one. In the primary terrestrial transportation we did not just used this formula directly to figure out the distance between production and warehouse because there is the sea in the middle. We had to make a two step calculation with an auxiliary point in the border between Spain and France.

More information about the cost calculation and data treatment is in Figueiredo (2007) master thesis.

3.2 – Case-Study Results

Phase 1 – Analysis of the present scenario

Presently the demand in every client city of the studied area is represented in the next map. Easily we can see that the greatest concentration is on the west, in spite of the fact that Germany is the biggest product buyer.

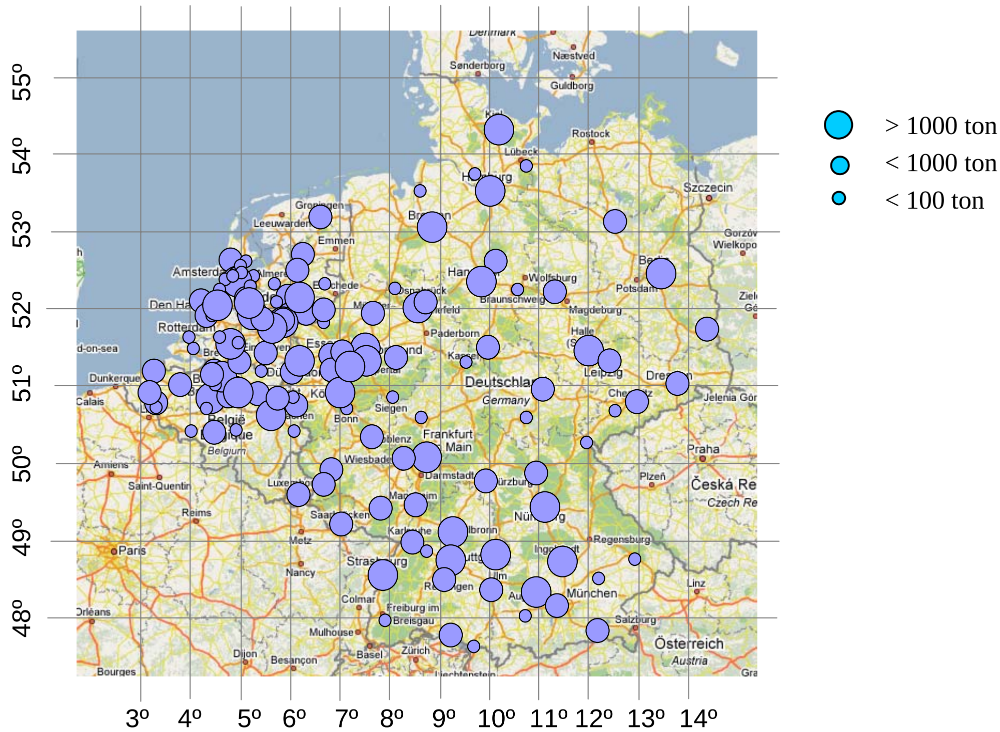

Fig. 2 – Demand density map

Now a day the company uses the port of Moerdijk, and two warehouses one in Frankfurt and the other in Nuremberg. The following map shows the warehouses and port and the main routes of distribution.

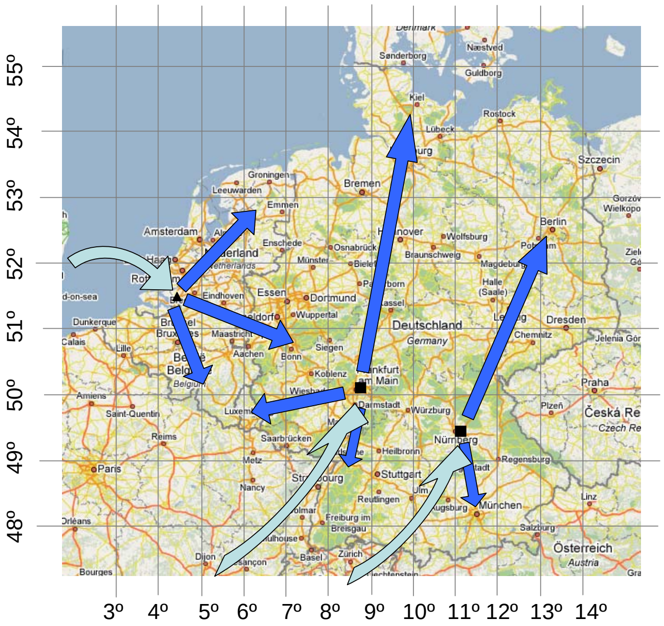

Fig. 3 – Current port and warehouse locations and main distribution routes

It is not difficult to understand that this is not probably the best distribution system since there are two warehouses in the south and none in the north.

The model was then solved for the present market demand. The main results are depicted in Figure 4 where one can see that some differences exist when compared to the current scenario (Figure 3). The location of one warehouse in the south (between Nuremberg and Frankfurt) is observed and two ports (Moerdijk and Hamburg) as well as the main routes of distribution.

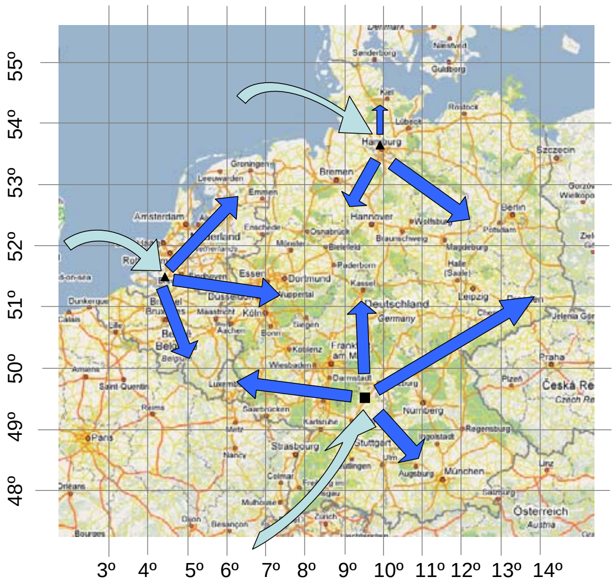

Fig. 4– Location of warehouses (square), ports (triangles) and major routes of distribution.

Phase 2 – 2008 scenario

For 2008, the predicted demand increase is shown is Table 4:

Table 4 – Increase in the demand for the different countries

Country	%
Belgium	27
Holland	9
Germany	28
Luxemburg	28

When this situation is optimized within the developed model the results are the same as the ones observed in phase 1 and depicted in Figure 4. Therefore, the costs of changing locations that eventually would exist are avoided.

Phase 3 – 2010 scenario

To complete this case study we studied the expected scenario in 2010. The demand for this year is expected to grow in the different countries as is shown in Table 5:

Table 5 – Increase in the demand for the diferent countries

Country	%
Belgium	54
Holland	18
Germany	56
Luxemburg	56

Using this new data the model is again run and the result is shown in figure 5.

Fig. 5 – Location of warehouses, ports and major routes of distribution

The locations observed (Figure 5) for the previous two cases in the cases of the ports are kept namely in Moerdijk and Hamburg. In terms of warehouses and in order to decrease the secondary transportation cost a new warehouse is located near the German city of Plauen. The warehouse located in the south is moved in the southern direction to near the city of Reutlingen.

In order to quantify the results obtained previously is important to show the values involved in the objective function. Table 6 presents this values. For each year two rows are shown, one has the costs if the present locations (the ones in operation actually) are accepted and the other has the costs if the model proposal will be implemented (model).

Table 6 - Table summing all the costs and savings

Costs (u.m.)	Present 2006	Model 2006	Present 2008	Model 2008	Present 2010	Model 2010
Primary transportation cost	1.42E+07	1.38E+07	1.88E+07	1.81E+07	2.39E+07	2.32E+07
Warehousing cost	1.57E+06	1.57E+06	1.92E+06	1.92E+06	2.25E+06	2.25E+06
Secondary transportation cost	5.05E+06	2.18E+06	7.13E+06	3.01E+06	9.66E+06	3.07E+06
Total cost	2.09E+07	1.76E+07	2.78E+07	2.30E+07	3.59E+07	2.85E+07
Total savings	3.28E+06		4.82E+06		7.31E+06	

Every year the costs obtained by the model are lower than if the present situation is kept. Therefore the savings have the tendency to grow in each year. This is an important result and has been received very positively by the company that is going to implement the choices optimized by the model.

The following table sums up the main computer statistics. The computer used in this work is a Compaq Deskpro using a CPU Pentium III Intel inside.

		Objective function (u.m.)	Total number of variables	Binary number of variables	Number of restrictions	Gap (%)	Iteration number	Precessing time (s)
Present	2006	2.09E+07	5048	3	1728	0.5	2488	1.9
	2008	2.78E+07	5048	3	1728	0.5	2259	1.9
	2010	3.59E+07	5048	3	1728	0.5	2322	1.8
Model	2006	1.76E+07	290818	173	33197	0.5	45544	2113.3
	2008	2.30E+07	290818	173	33197	0.5	115537	5517.3
	2010	2.85E+07	290818	173	33197	0.5	28834	43396.4

4. Conclusions

The objective of this paper was to develop a model that could optimize the location of warehouses and ports in a situation with two transport steps and having the possibility of using two different transport means. We think all the objectives were achieved and maybe even exceeded.

The model was developed having per base some papers and works that were mentioned in the introduction. Even though in many situations models to locate warehouses were developed, it seems to us that this model has made significant improvements as some specificities were modeled. The model developed here is a mixed integer formulation, meaning that is impossible to find any better solution if the same restrictions are respected and the same objective function is optimized. In spite of this we are aware that there are some aspects that can be improved in the model. In particular, the modeling of a third mean of transportation like, the train, could also be tested.

It is important to say that there are some aspects that were not used on the case study, for example the possibility of having a warehousing fixed cost. But those not used aspects remain available for future utilizations in other situations.

Finally, for future developments we think that two big different areas remain for possible future improvements of this article. The first is to model even with more precision the reality taking into account detailed considerations of the operation. And the second is to apply this model to different situations that may appear.

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