Error Analysis in GNSS receivers
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Abstract
This paper studies the most common errors that affect GNSS (global navigation satellite system) receivers. First a study of each GNSS system and their main characteristics is made. The most important systems are GPS (global positioning system) and Galileo but some other local systems are also mentioned like Glonass, Beidou and QZSS (quasi-zenith satellite system).

After knowing each system we are able to see what are the main sources of errors and how do they affect the quality of the information sent to the receiver. The main errors can be divided into two categories: pseudo-distance and receiver errors.

Some important measures like dilution of precision and precision metrics are also considered in this paper as well as the availability of the GNSS systems.

The last part is about error mitigation and some algorithms and methods that allow the reduction of some types of errors. Other techniques that will allow better results are studied like differential GPS and augmentation systems (WAAS (wide area augmentation system) and EGNOS (European Geostationary Navigation Overlay Service)). Finally the RAIM (Receiver Autonomous Integrity Monitoring) algorithm used for checking the system integrity is analyzed.

1. Introduction to GNSS Systems
The importance and diffusion of GNSS systems has increased in the last years. The main goal of these systems is to determine with high accuracy the user position, velocity and time. To do this the information of at least four satellites is needed in order to solve the navigation equation.

\[ \rho_c = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2 + \epsilon \times t_u + \epsilon \times t} \]

The GPS is the American military system, also available to civil users, and is the only one fully functional at this moment. Its constellation has twenty four satellites and they use two frequencies to broadcast the information, L1 and L2. The carriers are modulated using a BPSK (binary phase shift keying) scheme by pseudo-random codes, also known as Gold codes that have a spectral behavior similar to noise but has good autocorrelation and cross correlation properties. There are two types of codes used by GPS satellites, the C/A that modulates the carrier L1 and is available to civil users and the P code that modulates both L1 and L2 and is accessible only to military users. Because different and almost uncorrelated codes are used for each satellite, the signals can be separated and detected using the CDMA (code division multiple access) technique. New signals are being studied to a updated version of GPS, including a modernized L1C signal, compatible with QZSS, a second civil frequency L2C that will allow better corrections of ionospheric errors and a L5 and M signals, the last one exclusively military.

Galileo is the European GNSS system that is currently being developed and will provide a secure and accurate positioning service under civil control [6], [7]. One main characteristic of this system is the use of at least two frequencies that will lead to results with a high degree of accuracy, not available yet in GPS for civil users. The constellation has thirty satellites, twenty seven actives and three active spares. The distribution of the satellites will provide a better coverage even for higher latitudes that are not well covered by GPS. There will be five services available, the open service for all users, the commercial service that transmits other kind of information like weather or traffic information, the safety-of-life will be used by aviation or maritime users, the public regulated service will only be available for users authorized by the governmental authorities and finally the search and rescue service is used to distress messages. The Galileo signals are transmitted into three frequency bands called E5, E6 and E2-L1-E1 and also a special band dedicated to search and rescue service. A representation of the bands used by GPS and Galileo can be seen in Figure 1. In each band two
navigation signals can be transmitted. We can have three channel signals or four channel signals. The E2-L1-E1 is a three channel signal. Channel A is a simple navigation signal modulated by a flexible BOC(m,n) sub-carrier, while B and C are respectively the data and pilot channel that are modulated by a MBOC(6,1;1/11) sub-carrier.

The signal transmitted in the E6 band is similar to the one of the E2-L1-E1 band. Channel A is modulated by a BOC(10,5) sub-carrier and channels B and C don’t have any digital sub-carrier.

In the band E5 we have a four channel signal using the AltBOC(15,10) modulation. The Russian Glonass has already been active but already is being recovered. Its constellation will have twenty one satellites and three active spares. As already seen GPS uses the CDMA format to transmit the signal. Glonass uses FDMA (frequency division multiple access). Some problems related with this technology are the receiver’s size that has to be bigger in order to process different frequencies. The advantage is the fact that an interference in a band will only corrupt the information of one FDMA signal while in this case all CDMA signals will be corrupted. Glonass uses two frequencies in the L band to transmit the signal and as in GPS there are two levels of accuracy, one for civil and other for military users.

Beidou is a regional navigation satellite system that has actually three satellites in orbit over China. The goal will be a regional or global system with a constellation with fourteen to thirty satellites. The first two satellites existing use the L band for downloading information and S band to upload. The third satellite uses GPS L1 and L2 frequencies which indicate that this system can be used as an augmentation system for GPS and Glonass.

Other regional system is the Japanese QZSS that has a three satellite constellation with an eight shaped orbit over Japan and Australia. This will allow to have during the 24h at least one satellite on the zenith. This system was developed as an augmentation for GPS due to the large number of mountains and densely populated cities that leads to problems with the GPS coverage in 80% of the territory. Along with the position and time information this system will also transmit audio and video communications and the diffusion of several types of information.

2. Common error sources in GNSS systems

The main error sources that affect GNSS systems can be divided into two, the pseudo-distance and the receiver errors. Pseudo-distance errors include satellites and atmospheric errors and the receiver error can be receiver structure errors, code or multipath errors or problems related with the solution of the navigation equation.

Satellite errors can be due to ephemeris errors that are sent in the navigation message and contain information needed to the estimation of the satellites position. This error can be decomposed in three directions being the radial error the smallest one but also the only one that affects the final results. This error increases with the information age and typical values are 1m to 6m. If the error value is above a threshold, the navigation message is sent again in order to minimize the error. Also satellite clock errors can be a source of errors. In these systems atomic clocks are used due to their stability. In GPS are used rubidium and cesium clocks while in Galileo we have rubidium and hydrogen clocks. Figure 2 shows the stability of several clocks used by GPS and foreseen for Galileo.

Rubidium clocks are less stable and accurate but lighter and cheaper and have a ten years live time. Also the age of the information has influence in the error due to satellite clocks. For zero old information the error values can be about 0.8m while for
24h old information the same error can be from 1m to 4m.
The atmospheric errors are due to two layers, the ionosphere and the troposphere.
The ionosphere is a layer that starts at 50 km high and has no end defined. This layer has free electrons and ions produced by ultra-violet light and x rays from the sun. During the night time these ions and electrons are slowly recombined but at sunrise there are still available ions and electrons. The error will be determined by the number of electrons that the signal will find between the satellite and the receiver. This can be expressed by the TEC (total electron content). Since the ionosphere is a dispersive mean, the code is delayed and the phase is advanced. If we have a two frequency receiver this error is easily mitigated using the pseudo-distance concept. For single frequency receivers other methods will have to be used to mitigate the error.
The troposphere goes from the Earth surface to a high of about 40 km and is a non dispersive mean, so code and phase are affected the same way and a two frequencies receiver can’t mitigate this error. The tropospheric delay depends on the atmospheric pressure, temperature and relative humidity and can be divided into two components a wet and a dry. The dry component represents about 80% to 90% of the total tropospheric error but it can be calculated with a low error. The wet component is most difficult to predict depending on the pressure and temperature and knowledge of the refractivity profile is desired in order to obtain a good estimation. There are several models to correct tropospheric errors depending on the kind of receiver. If it has access to meteorological data the dry component can be calculated with great accuracy and the wet component is calculated using one of the several existing models. For users with no access to meteorological data we need to use models based in statistical values.
Receiver errors can be due to the receiver structure, code and phase errors and errors in the resolution of the navigation equation but the most important receiver error is the multipath. Along with the signal, several replicas are received, usually with lower amplitude, due to reflections of the direct ray in the ground or other obstacle. Comparing with the direct ray, these replicas travel different distances and have different polarizations and delays. This error is one of the most important for the total error along with the atmosphere errors. This error doesn’t allow the correct alignment between the local code and the received signal, which corresponds to a deformation in the correlation peak and a phase error. This error will depend of several factors like the kind of signal and the modulation scheme, filter characteristics, code chip rate, power of the replicas, number of replicas and their geometry and others. In an ideal situation the code is received and correlated with the replicas P (prompt), E (early) and L (late) generated locally. The DLL function is to maintain the code replica P aligned with the code received.
The real precision for the pseudo-distance value is called UERE (user equivalent range error). For a certain satellite this parameter is obtained by adding the statistical contribution of all error sources for that specific satellite (see Table 1).

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite clock and ephemeris parameters</td>
<td>$\sigma \approx 3m$</td>
</tr>
<tr>
<td>Atmosphere propagation model</td>
<td>$\sigma \approx 5m$</td>
</tr>
<tr>
<td>Receiver noise and multipath</td>
<td>$\sigma \approx 1m$</td>
</tr>
<tr>
<td>UERE</td>
<td>$\sigma \approx 6m$</td>
</tr>
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</table>

There are several methods to mitigate this error and will be studied in this paper.

3. Dilution of precision
DOP (dilution of precision) or GDOP (geometric dilution of precision) is a measure that allows describing the influence of the satellites geometry in the accuracy of the obtained measures. When the visible satellites for a certain receiver are too close we are in the presence of a large DOP and a weak geometry. So the lower the DOP value is the better. The factors that affect DOP values are the satellite orbits and the obstacles that don’t allow the use of certain satellites. This problem is relevant in highly populated regions or in places with mountains.
Other DOP parameters are also used to evaluate the accuracy of the several components in the solution time/position. We have PDOP (position dilution of precision), HDOP (horizontal dilution of precision), VDOP (vertical dilution of precision) and TDOP (time dilution of precision).
This is a very powerful concept used by GPS receivers to select the best set of satellites among a maximum of eleven with line of site. Other metrics can be used to describe the system accuracy like the circular error probable.

The availability of a system is the percentage of time in which it is available for the users. This measure gives us an idea of the system capacity to provide the service in a certain area. The capacity of obtaining availability values through the navigation equation depends on the satellite geometry for a certain place and the time of the day. We have first to verify what the visible satellites for that place and time are which can be done recurring to the information in the almanac. In the GPS case the coverage by the system is evaluated between latitudes 90ºN and 90ºS with sample points separated of 5º and the same spacing is used to the longitude. The grid obtained is refreshed every 5 minutes during a 12 hour period. The availability depends also on the used receiver mask, lower angles allow better availability but there is the risk that the signal is blocked by a building or a mountain, or bigger atmospheric errors common for low angles. The GPS 24 satellite constellation is available only 72% of the time, during 98% of the time, 21 satellites are operational. The impacts of an incomplete constellation depend on the geometry and can be small, medium or very big impacts.

4. Error Mitigation

4.1 Ionosphere Errors

The algorithm used by GPS to mitigate the errors due to the ionosphere in single frequency receivers is the Klobuchar algorithm. This algorithm results from a compromise between the computational complexity, the knowledge of daily TEC variations, the number of coefficients in the satellite message to ionospheric corrections and the probability that a single frequency user is used in a certain region. With this model a 50% error correction is achieved. This model has a cosine shape for the day period and is represented as a constant for the night period.

This method can be resumed in 7 steps where some of the expressions used are approximations that reduce the computational complexity and don’t add a big error.

Figure 3 – Klobuchar model [6]

The initial values known are user latitude (ΦU), user longitude (λU), satellite elevation angle (E) and azimuth (A). The steps are the following:

1 – Earth angle determination
\[ \Psi = \frac{E - 0.022}{E + 0.11} \]

2 – Sub-ionospheric latitude determination
\[ \Phi_I = \Phi_U + \Psi \times \cos A \]
If \( \Phi_I > +0.416 \) then \( \Phi_I = +0.416 \)
If \( \Phi_I < -0.416 \) then \( \Phi_I = -0.416 \)

3 - Sub-ionospheric longitude determination
\[ \lambda_I = \lambda_U + \frac{\Psi \times \sin A}{\cos \Phi_I} \]

4 – Geomagnetic latitude determination
\[ \Phi_m = \Phi_I + 0.064 \times \cos(\lambda_I - 1.617) \]

5 – Local time determination
\[ t = 4.32 \times 10^4 \lambda_I + GPS\text{time}(s) \]

6 – Obliquity factor
\[ F = 1.0 + 16.0 \times (0.53 - E)^3 \]

7 – Ionospheric delay determination
\[ T_{IONO} = F \times \left[ 5 \times 10^{-9} + \sum_{n=0}^{\infty} a_n \Phi_m^n \times \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right] \]
with:
\[ x = \frac{2\pi(t - 50400)}{\sum_{n=0}^{\infty} \beta_n \Phi_m^n} \]

There are some constraints regarding this algorithm, we assume that TEC is concentrated in a very thin layer at 350 km high and during the night a constant of 5 ns is considered.

The model that will be used for Galileo is the NeQuick and uses the Epstein
formulation for the lower part of the ionosphere and a semi-Epstein formulation for the higher part with a parameter that increases linearly with the altitude. For this model is required the knowledge of the monthly mean solar flux with a wavelength of 10cm as an initial parameter. When both methods are compared we can see that NeQuick is better adapted to the real ionospheric delay. In the equatorial region none of the methods presents an effective correction.

4.2 Troposphere errors
There are several existing troposphere models. One of them is the Hopfield based on the relation between the refractivity in a dry mean at an h altitude and at the surface [4].

For the dry component we have:

\[ N_d(h) = N_{d0} \left(1 - \frac{h}{h_d}\right)^4 \]

h – Altitude above the antenna

\( N_{d0} \) - Dry refractivity at the surface

\( h_d \) - High above antenna with zero refractivity (43 km)

For the wet component:

\[ N_w(h) = N_{w0} \left(1 - \frac{h}{h_w}\right)^4 \]

\( N_{w0} \) - Surface refractivity

\( h_w \) - 12 km

So:

\[ T_e = 10^{-6} \int N(h)dh = 10^{-6} \int \left[N_d(h) + N_w(h)\right]dh = \]

\[ = \frac{10^{-6}}{5} \left[N_{d0}h_d + N_{w0}h_w\right] = T_{e,d} + T_{e,w} \]

Substituting the wet and dry refractivity expressions we obtain:

\[ T_{e,d} = 77.6 \times 10^{-6} \frac{P_0}{T_0} \frac{h_d}{5} \]

\[ T_{e,w} = 0.373 \frac{e_0}{T_0} \frac{h_w}{5} \]

There are mapping functions that can be used for both troposphere components. The most simple is \(1/\sin E\) that is not a good approach. One other example of a more complex mapping function is:

\[ m_d(E) = \frac{1}{\sin E + \frac{0.00143}{\tan E + 0.0445}} \]

The tropospheric obliquity factor highly increases for smaller elevation angles. The same does not happen for the ionospheric elevation angle.

4.3 Multipath errors
The multipath error represents an important contribution to the total error. There are several methods proposed in the literature to mitigate this error like:

- Narrow Correlator Receiver
- Multipath Estimating DLL (MEDLL)
- Double Delta Receiver (Strobe Correlator, High Resolution Correlator)

The HRC (high resolution correlator) is easy to implement and gives good results. This correlator uses five signal replicas, the three already mentioned, P, E and L and also VE (very-early) and VL (very late) spaced of \(4\delta\) chips. The discriminator response is:

\[ d_{HRC}(\tau_e) = (L - E) - (VL - VE)/2 \]

The use of HRC allows a decrease of multipath errors to values close to zero except in the region with values of \(\tau\) close to zero or around \(\tau = T_c\) (Figure 4).

The HRC reaches a good mitigation at the code level when compared with other correlators with a degradation of noise performance neglectable. From the carrier point of view we get also a good mitigation but there is a considerable degradation of noise performance. This technique can’t be applied when the multipath delays are very small that is the most common case and other techniques must be considered.

![Figure 4 – Multipath error envelope](image)
5. Differential GPS

DGPS is a way to increase the time or position accuracy using one or more reference base stations. These stations give information to the users like pseudo-distance and time corrections, satellite integrity and ephemeris information and also auxiliary data about the local, state and weather conditions of the stations.

There are several DGPS techniques that can be divided into local, regional and wide area techniques. Local DGPS covers areas from 10 to 100 km, regional DGPS covers 1000km areas and wide DGPS covers bigger areas.

6. GNSS augmentation

6.1 WAAS

The WAAS (wide area augmentation system) has as its main goal the augmentation of the GPS in air navigation. There are obtained three services, first the satellites send spread spectrum signals. Then the receiver will add these signals to the ones from the GPS constellation. This will make the system less sensitive to failures of individual components, increasing the continuity and availability of the service. Second, is used a ground station that monitors the satellite state and sends a warning when there is a situation that may affect the flight security. Third, the ground network will be used to estimate corrections for the errors that limit the system accuracy without augmentation.

There are two methods used by WAAS to get a higher accuracy, the reduction of the pseudo-distance error and improves the satellite geometry adding new signals to the needed measures [2].

6.2 EGNOS

As seen previously DGPS has been developed to increase the GPS accuracy. Although, there are some limitations like the distance between the user and the ground station, the need to use common satellites and new algorithms in the receiver, so a new receiver will be needed.

The EGNOS (European Geostationary Navigation Overlay Service) system was developed to increase the accuracy, availability and integrity of the GPS system in European territory. This system can also be used with Galileo and Glonass. Along all Europe, thirty four reference stations monitors the GPS signals and send their data to four main stations. These generate a signal of integrity data and corrections of the type WADGPS for Europe. The data is modulated in a signal similar to the GPS and sent to all users from three geostationary satellites. The users can benefit from a higher availability without needing a new receiver. The EGNOS signal complies with all international requisites regarding GNSS augmentation and this allows interoperability with the Japanese MSAS or the Canadian CWAS [6].

7. Integrity

The integrity is a measure of the confidence that we can have on the information provided by the system. This permits to determinate if the system can be used in a moment and this is very important especially for air and maritime navigation. Although there is a constant monitoring of the satellite status it can take several minutes until the user is informed of the existence of problems. The integrity must be checked locally by the user that needs a redundant source of information to validate a position. The RAIM (receiver autonomous integrity monitoring) identifies inconsistencies in the received data early enough to avoid the inclusion of erroneous data in the navigation equation solution. This algorithm will solve the navigation equations and compare the results that must be similar, if not there is no guarantee on integrity. The use of five satellites only allows detecting that there is a failure in one satellite. To detect what is the satellite with problem we will need six satellites.

The fluxogram for the RAIM algorithm is shown in Figure 5.

![Figure 5 – RAIM algorithm](image)
8. Conclusions

This field of investigation is becoming more important with the increasing use of GNSS system for several types of activities. There is always the need to get a system with better accuracy, integrity and availability. Along this paper several sources of errors where discussed and the ionospheric error is one of the most important for the total error. In the following table we can find the contribution of each type of errors for the total error.

<table>
<thead>
<tr>
<th>Type of error</th>
<th>Approximate error values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric effects</td>
<td>± 5 m</td>
</tr>
<tr>
<td>Orbit errors</td>
<td>± 2.5 m</td>
</tr>
<tr>
<td>Satellites clocks</td>
<td>± 2 m</td>
</tr>
<tr>
<td>Multipath effect</td>
<td>± 1 m</td>
</tr>
<tr>
<td>Tropospheric effects</td>
<td>± 0.5 m</td>
</tr>
<tr>
<td>Calculation and rounding errors</td>
<td>± 1 m</td>
</tr>
</tbody>
</table>

Other ways of increasing integrity and accuracy were developed like DGPS, WAAS, EGNOS and the RAIM algorithm. The study of errors and their mitigation in GNSS systems is in constant progress with new results every day.

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