Study of Optimal Relocation Time

Bruno Silva

Instituto Superior Técnico, Mathematics Department, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal
brunosilva22@gmail.com

Abstract. The main purpose of this study is the characterization of the moment at which a company should change its location in order to maximize profit, in a random environment, using a deductive methodology of real option analysis in a stochastic context.
The recent unprecedented development of technological innovations (which brings new locations) has given companies many new investment opportunities. However, those innovations are associated to significative uncertainty, and consequently getting to the right location at the correct time has become a great challenge.
The relocation problem for one company may be described as a typical Optimal Stopping Problem, in which the final goal is the determination of the optimal time (in terms of maximization of the expected value of the company's value). In this context, every time a new and more efficient location becomes available, the company has to pick between changing its location (supporting the inherent costs of relocation) and postponing its relocation.
The obtained results are innovative, showing the relevance of the Non-Homogeneous Poisson Process, the Conditional Poisson Process and of the availability rate of new locations in determining the optimal time of relocation.

Keywords: Real Option, Relocation Problem, Efficiency Level, Poisson Process, Optimal Stopping Problem.

1 Introduction

Recently, the unprecedented development of technological innovations and geopolitical changes has given countless opportunities of investment for companies. Therefore, making the right technological innovations at the right time and place has become a challenge.

In the last decades, the growing political and economical interdependency intensified the discussion about globalization. Globalization has become a dominant concept in several public and private domains, and in every corner of the world. One of the observed consequences of globalization is the significant increase of company relocation processes, which creates concerns in the developed countries,as they ate most affected by the general tendency of relocation of industrial units and services to countries where workforce is cheaper and governmental policies are more attractive (see [12, 19]). In particular, Portugal has been largely affected in the last years.

The displacement or relocation processes of companies to more competitive countries (or more competitive regions within the same country) in some areas (like greater efficiency in production levels and/or incentives to investment) has recently been assuming greater importance in ocidental economies, due to a growing number of companies using these solutions, with the aim of improving its position in the market or in the search for new markets.

The geographical movements of companies, along with the formation, expansion, merging and closing of economical units, influence critically the geographical distribution of economical activities. During the last years, we have observed that bigger companies are those that relocate most often. The relocations are, sometimes, the result of acquisitions, fusions and take-overs, which in their turn are consequences of external growth (see [4]). The analysis of company relocation processes contributes also to the orientation of national and regional policies, and has several implications in the level of international policies.

The academic interest centered in the relocation decision process is not recent. However, the focus of the relocation process hasn't been fully explored (see [21]). The few existent works center their attention in the advantages of location where the activities are being transfered in comparison to their original location. The analysis of the impact in the workforce costs, the effects induced by governmental policies and the economical development level of each region are also considered potential factors of influence in the decision of relocation of a unit, which has been a subject of academic interest on a significant segment of literature (see [6, 8, 18]).

In the literature we can find the principal factors that influence the company relocation processes divided in three categories (see [10, 16]): internal factors (e.g. company dimension), external factors (e.g. market evolution) and location factors (e.g. envolving region conditions).

In its turn, these three categories are influenced by three types of relocation motives (see [14]), namely:

- Push factors: which cause the change of the actual location;
- Pull factors: important attractive elements in the decision making process in favor of the new location; and
- Keep factors: which incite the company to stay in the current location.

It is common knowledge that companies traditionally have the tendency to stay in the same location in itťs lifetime, motivated by diverse keep factors. However, once the production capabilities associated with the present location are attained, the companies should change location in order to operate optimally. The adjustment process in terms of location and prodution policy can be explained by several push factors, whose dynamics are often external to the company's production process. Relocation isoften motivated by a cost economy, which makes a certain location more desirable than another. The companies should, in regard to the improvement of production efficiency, take advantage of the favorable cost conditions like, for example, salary gaps, energy price, local incentive, among others, which constitute examples of pull factors (see [20]).

The relocation problem can be described as a decision problem, in which, the company has two options every time a new location becomes available,: choosing to change location and paying the corresponding relocations costs (sunk cost), or keeping the current location, and postponing its relocation process. Therefore, the problem of the company is to decide the instant at which it should change its location.

According to [11, 17], in the decision making process the manager not only needs to consider the associated advantages to the efficiency increase in the new locations, but also the inherent costs/losses of the decision of keeping the current location, in face of a new location (e.g., the cost of loosing an opportunity to make big profit).

Every potential location is geographically known. However, for some companies it is not always economically rational to settle down in certain regions, due to political, institutional, geographical (accessibility) and economical reasons or restrictions. Therefore, from here on we use the concept of arrival of new locations, with the objective of referring to the economical possibility by the rational utilization of a new location.

As in [9, 13], the identification and distinction of locations will be made according to the respective efficiency. By arrival of a new location, we mean the arrival of information about a new location whose efficiency level is superior to the efficiency level of the company's best location until that time epoch. The increase of efficiency levels are characterized by the companies' ability to produce greater quantities of output (e.g. due to a greater technical competency of the workforce) and/or by the use of inferior levels of input to generate the same quantity of output (e.g. due to smaller costs of the workforce and smaller costs related to energy resources). We emphasize that, in any of the cases, the result consists in producing at a lesser average unitary cost than the current one. In this context, we consider that the process of arrival of new locations is statistically independent from the company, i.e., the company does not have the option of influencing the generation process of future locations, nor any knowledge about the instant in which those future locations become available.

In this work we tackle the problem of the optimal relocation policy of companies in an uncertain and stochastic environment, using a deductive methodology of real option analysis, for a company that faces two types of uncertainty: the uncertainty about the instants in which the new (and more efficient) locations become available, and the uncertainty about the increase of efficiency level inherent to each one of those new, and still unknown, locations. In particular, we are interested in some properties concerning the instant in which the company/economical unit will proceed to change the location, under an optimal relocation policy. Furthermore, we consider the case in which the company can change, at most, one time its location, and we assume that the increase of efficiency level associated to new locations is constant. Additionally, we assume that the companies operate in a neutral risk environment, and that cash flows are determined only by the decision making of the location for which investment is directed.

The remainder of the paper is organized as follows. In Section 2 we show in detail the theoretical decision model and the considered hypothesis. Taking into account the Real Option Analysis (ROA), we present, in Section 3, a deeper analysis of the decision problem in question, proceeding to the calculation of the optimal efficiency level. In Section 4, we derive the relocation time under the optimal investment policy, considering that the arrival of information about new locations follows a Non-Homogeneous Poisson Process or a Conditional Poisson Process. In Section 5, we proceed to a presentation of comparison results between the theoretical values with those obtained in simulations. Concluding remarks are provided in Section 6.

2 Stochastic Model

In this work we assume that the investment decision in a location in lieu of others is the only factor that influences the change of profit. It is assumed that the company can change its location at most once, and that the cost of investment is kept constant along time, as assumed in [9, 15]. The efficiency of a new location is characterized here by a single parameter. In this case, we are only interested in identifying the locations that make the production of a company more efficient. Finally, it is supposed that after the identification of a new location, the location stays available and always with the same level of efficiency.

In fact, in this work, the value of the project depends only on one variable (the level of efficiency associated to the new location), denoted from now on as $\theta(.)$. This variable is modelled by a Poisson jump process. That said, we consider that the company at time t (> 0) produces with efficiency $\zeta(t)$, and that $\theta(t)$ represents the efficiency of the best available location at instant t. Therefore $0 \leq \zeta(t) \leq \theta(t), \ \forall t \leq 0.$ [1] If $\zeta(t) = \theta(t)$, then at time t the company is located in the best available location.

In this setting we denote by $F(\theta(t))$ the value of the company at instant t when operating in a location with efficiency $\theta(t)$ and when it has not seen relocated (still having the option to do it). This value should include the value

[1] All the equalities and inequalities envolving random variables are to be understood in the almost sure sense.

associated to the option of relocation, which was not yet activated. Furthermore, if at instant t the company opts for relocation to a more efficient location, the functions $\Omega : \mathbb{R} \to \mathbb{R}$ and $\pi : \mathbb{R}_+ \to \mathbb{R}$, are considered, where:

- $\Omega(\theta(t))$ represents the value of the company immediately after the change of location with efficiency $\theta(t)$;
- $\pi(\theta(t^-))$ represents the cash flow immediately before the change of location.

Let us note that $\pi(\theta(t)) > \pi(\theta(t^-))$, because the cash flow should increase immediately after the moment of relocation. r (> 0) is the rate of actualization off the cash flow. Furthermore, we assume that π is increasing on $\theta(.)$.

In the work of [9, 15], the arrival of new information regarding the availability of new locations with improved levels of efficiency follows an Homogeneous Poisson process. In the present work we assume that the arrival of new locations can be modeled by a Non-Homogeneous Poisson Process (NHPP) or by a Conditional Poisson Process (CPP).

Let $N = \{N(t),\ t \geq 0\}$ be the stochastic process that models the arrival of new locations that becomes available, where $N(t)$ represents the number of new (and more efficient) available locations up to time t. In addition, $\{T_i,\ i \in \mathbb{N}\}$ denotes the times of the events of the process N and $\{X_i,\ i \in \mathbb{N}\}$ the times between consecutive events, with $X_i = T_i - T_{i-1}$.

When an event of N occurs the level of efficiency θ jumps to

$$\theta(T_i) = \theta(T_i^-) + u_i, \tag{1}$$

where u_i denotes the range of the jump at T_i.

In the current work we assume that

$$u_i = u,\ \forall\ i \tag{2}$$

,

where u is a deterministic value. Therefore in this work, we only deal with an efficiency process which is deterministic and constant.

If $T_0 = 0$ and $u_0 = 0$, then the efficiency of the best available location in t is given by

$$\begin{aligned} \theta(t) &= \theta_0 + \sum_{i=1}^{N(t)} u_i \\ &= \theta_0 + N(t)u, \end{aligned} \tag{3}$$

in the case $u_i = u$.

3 Decision Problem: Determination of Optimal Efficiency Level

The dilemma that the company faces, every time a new location becomes available, is a decision problem: either it changes its current location (paying the investment costs), or keeps the current location, postponing its relocation decision. In the context of decision problems, this is a problem usually referred as Optimal Stopping Problem. In this setting, the problem may have two extreme solutions:

1. The company changes location every time a new location becomes available.
   - **Advantage:** The company produces in the most efficient location.
   - **Disadvantage:** A great financial capacity is needed to support big investments in sucessive location changes.
2. The company never changes location.
   - **Advantage:** The company has no irreversible relocation costs.
   - **Disadvantage:** The company stays in a location thatbecomes inefficient, in long–term, and does not take advantage of potential profits which it could have by opting for a location change.

The optimal solution is a compromise between of these two extreme solutions. This way, the challenge of a company consists in deciding the time at which it should make a location change.

Following [11], we call the value θ^* the optimal switching level or critical value, which delimits the zone between continuation (i.e., to stay in the location in which the company is located) and stopping (i.e., proceeding to relocate the company) in the following way:

- if at t, $\theta(t) < \theta^*$, continue;
- if at t, $\theta(t) \geq \theta^*$, relocate.

In the context presented in this work, as said before, the company maximizes its expected value when it implements this decision rule.

One question of great importance concerns the existence and unicity of this critical value. Note that both existence and unicity have a significative impact in the implementation of an optimal policy of investment, in mid-term as well as in long-term. This question, that goes beyond the scope of the present work, is addressed [11]. In this work necessary conditions for existence and unicity of the critical level are assumed.

In order to infer about the determination of the critical level, it is necessary to introduce the following variables:

- T^*: optimal relocation instant, i.e.,

$$T^* = \inf\{t \geq 0 : \ \theta(t) \geq \theta^*\}; \tag{4}$$

– $\theta(T^*)$: level of efficiency at time T^*;
– $V : \mathbb{R}_+ \to \mathbb{R}$, value of the company, if it stays forever in its current location, thus if the efficiency of the company is ζ, then this value is

$$V(\zeta) = \int_{t=0}^{\infty} \pi(\zeta) e^{-rt} dt = \frac{\pi(\zeta)}{r}, \tag{5}$$

where, in the context of the relocation problem with only a single change, we have

$$\Lambda(t) = \begin{cases} a(t-s), & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s, 2s+1[\\ a(s+1), & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s+1, 2s+2[\end{cases} \tag{6}$$

$$\pi(\zeta) = \begin{cases} \pi(\zeta_0), & \text{if } \zeta < \theta(T^*) \\ \pi(\theta(T^*)), & \text{if } \zeta \geq \theta(T^*) \end{cases}. \tag{7}$$

For simplicity, we drop time-subcripts when no ambiguities arises.

In case a relocation occurs, with associated cost I (which we assume to be constant), then the liquid return of the investment on the change of location is equal to

$$V(\theta) - I. \tag{8}$$

We note that the critical level θ^* can be found using the following equation

$$F(\theta^*) = \Omega(\theta^*) = V(\theta^*) - I, \tag{9}$$

which states that, for $\theta = \theta^*$, it is indifferent to the company to stay in actual location or to change to this new location.

As an example, if the arrival of new and more eficient locations follows an Homogeneous Poisson Process, with rate λ, then equation (9) is equivalent

$$\frac{\pi(\zeta_0)}{r+\lambda} + \frac{\lambda}{r+\lambda}\left[\frac{\pi(\theta^*+u)}{r} - I\right] = \frac{\pi(\theta^*)}{r} - I. \tag{10}$$

4 Optimal Relocation Time

In this section we describe in more detail the way of determining the optimal relocation time, T^* (i.e., the instant in which it is advisable to proceed to a change of location under the optimal investment policy). From the operational point of view, it is important for the company to know (or characterize in some way, for example in terms of expected values) the optimal time for relocation.

Since we assumed that the efficiency jumps are constant, we can calculate the number of arrivals of information about new locations until the company is in conditions of getting benefit from a change of location, n^*, which is equal to

$$n^* = \left\lfloor \frac{\theta^* - \zeta_0}{u} \right\rfloor + 1, \tag{11}$$

where $\lfloor x \rfloor$ is the integer part of $x \geq 0$.

Note that in the present work n^* is deterministic, but T^* is random, because the arrival process is regulated by a stochastic mechanism.

It comes from the definition of $N(t)$ given before that

$$T^* = \inf\{t \geq 0 : \ \theta(t) \geq \theta^*\} \tag{12}$$

and therefore

$$P(T^* \leq t \ |n^*) = P(N(t) \geq n^* \ |n^*) = \sum_{n=n^*}^{+\infty} P(N(t) = n \ |n^*). \tag{13}$$

In the remaining of the paper we use the notation $E[T^* \ |n^*]$ in order to emphasize that the results that we derive can also be applied in the more general settings of random jumps (i.e., u_i is a random variable) given n^*.

In what follows we derive properties concerning the expected value of T^*, for different processes of arrivals.

We note that the instances of the arrival process that we considered in this work cannot encompass the complexity of the reality. In fact we only present situations for which we can derive in a closed form expressions for the expected value of T^*. It should be mentioned that in general this cannot be done, and therefore in most applications one should rely on numerical/ simulation results.

We note that [15] has already derive some properties concerning $E[T^* \ |n^*]$ when N is a Poisson Process with rate λ. In fact, in that situation:

$$E[T^* \ |n^*] = \frac{n^*}{\lambda}. \tag{14}$$

Now consider particular cases of Non-Homogeneous Poisson process and Conditional Poisson process.

4.1 Non-Homogeneous Poisson Process

In this subsection we explore the fact that the arrival of new locations is governed by Non-Homogeneous Poisson Process. In this context, we consider two scenarios,and several intensity rates functions $\lambda(.)$.

First note that for a Non-Homogeneous Poisson Process we let the following results

$$\begin{aligned} E[T^* \ |n^*] &= \int_0^{+\infty} P(T^* \geq t \ |n^*) \ dt \\ &= \int_0^{+\infty} \sum_{n=0}^{n^*-1} P(N(t) = n \ |n^*) \ dt \\ &= \int_0^{+\infty} \sum_{n=0}^{n^*-1} e^{-\Lambda(t)} \ \frac{(\Lambda(t))^n}{n!} \ dt \\ &= \sum_{n=0}^{n^*-1} \int_0^{+\infty} e^{-\Lambda(t)} \ \frac{(\Lambda(t))^n}{n!} \ dt \end{aligned} \tag{15}$$

where

$$\Lambda(t) = \int_0^t \lambda(s) \ ds. \tag{16}$$

– **Case 1**

Here we assume that arrival rate of new locations is a monomial of degree $b-1$, i.e.,

$$N \sim \ PPNH(\lambda(t) = at^{b-1}), \tag{17}$$

with $a > 0$, $b \neq 0$ and therefore

$$\Lambda(t) = \int_0^t ax^{b-1} dx = \frac{a \ t^b}{b}. \tag{18}$$

Next we present a result involving the gamma function used in the proof of Proposition 2. We ommit the proof but it can be found in [22]. We note that this is a seemingly new result, which we could not found in the related literature.

Proposition 1 *If* $n \in \mathbb{N}$ *and* $a \neq 0$ *then*

$$\sum_{k=0}^{n-1} \frac{\Gamma(k+a)}{\Gamma(k+1)} = \frac{\Gamma(n+a)}{a \ \Gamma(n)}. \tag{19}$$

The expression for the expected value of the instant of change of location under optimal policy is given by the following proposition

Proposition 2 *If* $\Lambda(.)$ *is given by (18), then*

$$E[T^* \ |n^*] = \left(\frac{b}{a}\right)^{\frac{1}{b}} \ \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)}. \tag{20}$$

Proof:

From (15) we have:

$$E[T^* \; |n^*] = \sum_{n=0}^{n^*-1} \left(\int_0^{+\infty} \frac{e^{-\frac{a\; t^b}{b}} (\frac{a\; t^b}{b})^n}{n!} \; dt \right). \tag{21}$$

By making the change of variable, $z = \frac{a\; t^b}{b}$, we have:

$$\begin{aligned} E[T^* \; |n^*] &= \sum_{n=0}^{n^*-1} \left(\int_0^{+\infty} e^{-z} \frac{z^n}{n!} (b\; z)^{\frac{1-b}{b}} a^{-\frac{1}{b}} \; dz \right) \\ &= \sum_{n=0}^{n^*-1} \left(\left(\frac{b}{a}\right)^{\frac{1}{b}} \frac{1}{b\; n!} \int_0^{+\infty} e^{-z} z^{n+\frac{1}{b}-1} \; dz \right) \\ &= \left(\frac{b}{a}\right)^{\frac{1}{b}} \frac{1}{b} \sum_{n=0}^{n^*-1} \left(\frac{\Gamma(n+\frac{1}{b})}{\Gamma(n+1)} \right) \\ &= \left(\frac{b}{a}\right)^{\frac{1}{b}} \sum_{n=0}^{n^*-1} \left(\frac{\Gamma(n+\frac{1}{b})}{b\; \Gamma(n+1)} \right). \end{aligned} \tag{22}$$

Finally, by using (19) and (22), we let

$$E[T^* \; |n^*] = \left(\frac{b}{a}\right)^{\frac{1}{b}} \; \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)}. \tag{23}$$

□

It follows from Proposition 2 the following result.

Corollary 1 *The effect of the monomial, a, on the expected value of the instant T^* is*

$$E[T^* \; |\lambda(t) = a t^{b-1}, n^*] = \frac{E[T^* \; |\lambda(t) = t^{b-1}, n^*]}{a^{\frac{1}{b}}}. \tag{24}$$

The result expressed in the corollary above is interesting, because it gives us a clear idea of the effect of the coeficient of the monomial in this context. More precisely, it describes the relation between the expected value of the optimal relocation time, whose intensity rate is a monome, and the expected value of that time associated with a intensity rate described by a monic monomial of the same degree.

– **Case 2**

Let

$$N \sim \ PPNH(\lambda(t)) \tag{25}$$

with

$$\lambda(t) = \begin{cases} a, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s, 2s+1[\\ 0, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s+1, 2s+2[\end{cases} \tag{26}$$

where $t,\ a \in \mathbb{R}_{+}$. Therefore arrrivals can only take place in intervals with origin in even times. Note that the symmetrical case (i.e. arrivals can only take place in intervals with origin in odd times) can also be analysed in a similar fashion.

From (26) we have:

$$\Lambda(t) = \begin{cases} a(t-s), & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s, 2s+1[\\ a(s+1), & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s+1, 2s+2[\end{cases} . \tag{27}$$

Note that this case corresponds roughly to a kind of a on-off process, where the intervals on-off are deterministically set.

From (27) we let

$$\sum_{n=0}^{n^*-1} \frac{\Lambda(t)^n}{n!} = \begin{cases} e^{a(t-s)} \ \frac{\Gamma(n^*,\ a(t-s))}{\Gamma(n^*)}, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s, 2s+1[\\ e^{a(s+1)} \ \frac{\Gamma(n^*,\ a(s+1))}{\Gamma(n^*)}, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s+1, 2s+2[\end{cases} . \tag{28}$$

Proposition 3 *The expected value of the instant T^* is equal to*

$$E[T^* \ |n^*] = \frac{n^*}{a} + \sum_{s=1}^{+\infty} \frac{\Gamma(n^*, as)}{\Gamma(n^*)}. \tag{29}$$

Proof:

From (15) we have:

$$
\begin{aligned}
E[T^* \ |n^*] &= \frac{1}{(n^*-1)!}\sum_{s=0}^{+\infty}\left(\int_{2s}^{2s+1} \Gamma(n^*, a(t-s)) \ dt \ + \Gamma(n^*, a(s+1))\ \right) \\
&= \frac{1}{\Gamma(n^*)}\sum_{s=0}^{+\infty}\left[\frac{1}{a}\Big(-as\Gamma(n^*, as) + a(s+1) \ \Gamma(n^*, a(s+1)) + \Gamma(n^*+1, as) - \right. \\
&\left. - \Gamma(n^*+1, a(s+1))\Big) + \Gamma(n^*, a(s+1)) \ \right] \\
&= \frac{\Gamma(n^*+1)}{a \ \Gamma(n^*)} + \sum_{s=0}^{+\infty} \frac{\Gamma(n^*, a(s+1))}{\Gamma(n^*)} \\
&= \frac{n^*}{a} + \sum_{s=1}^{+\infty} \frac{\Gamma(n^*, as)}{\Gamma(n^*)}.
\end{aligned}
\tag{30}
$$

□

4.2 Conditional Poisson Process

In this subsection we assume that the arrival process of new locations can be modelled as a Conditional Poisson process. We assume that the rate of the process is a function of a random variable, that we denote by X, so that:

$$N|X = x \ \sim NHPP(\lambda(t, x)).$$

Next we present particular instances of the distribution of X.

– **Case 3**

In this scenario, the rate of arrival of new locations is similar to that of scenario no. 1 of the previous section with the coeficient of the monomial, a, being a random variable:

$$N|X = x \sim \ PPNH(\lambda(t, x) = x \ t^{b-1}). \tag{31}$$

One of the greatest contributions of this work consists in the result expressed by the following proposition:

Proposition 4 *In this context, if $E[|T^*| \ |n^*] < \infty$, then $E[T^* \ |n^*]$ depends of X through $E[X^{-\frac{1}{b}}]$.*

Proof:

Let D e $F(.)$ be the domain and the distribution function of the r.v. X, respectively. Then, we have:

$$\begin{aligned} E[T^* \mid n^*] &= E[E[T^* \mid X = x,\ n^*]] \\ &= \int_{x \in D} E[T^* \mid X = x,\ n^*]\ dF(x). \end{aligned} \tag{32}$$

From (20) and (32) we have:

$$\begin{aligned} E[T^* \mid n^*] &= E[E[T^* \mid X = x,\ n^*]] \\ &= \int_{x \in D} \left(\frac{b}{x}\right)^{\frac{1}{b}} \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)}\ dF(x) \\ &= b^{\frac{1}{b}} \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)} \int_{x \in D} x^{-\ \frac{1}{b}}\ dF(x) \\ &= b^{\frac{1}{b}} \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)}\ E[X^{-\ \frac{1}{b}}]. \end{aligned}$$

□

If for instance $X \sim Gamma(m, \alpha)$ ($m > \frac{1}{b}$ and $E[X] = \frac{m}{\alpha}$), then:

$$E[T^* \mid n^*] = (\alpha b)^{\frac{1}{b}} \left(\frac{\Gamma(n^* + \frac{1}{b})\ \Gamma(m - \frac{1}{b})}{\Gamma(n^*)\Gamma(m)}\right). \tag{33}$$

The proof of this result is

$$\begin{aligned} E[T^* \mid n^*] &= E[E[T^* \mid X = x,\ n^*]] \\ &= \int_0^{+\infty} f_X(x)\ E[T^* \mid X = x,\ n^*]\ dx \\ &= \int_0^{+\infty} \frac{\alpha^m}{\Gamma(m)} x^{m-1}\ e^{-\alpha\ x} \left(\frac{b}{x}\right)^{\frac{1}{b}} \frac{\Gamma(n^* + \frac{1}{b})}{\Gamma(n^*)}\ dx \\ &= (\alpha b)^{\frac{1}{b}} \left(\frac{\Gamma(n^* + \frac{1}{b})\ \Gamma(m - \frac{1}{b})}{\Gamma(n^*)\Gamma(m)}\right)) \underbrace{\int_0^{+\infty} \frac{\alpha^{m-\frac{1}{b}}}{\Gamma(m - \frac{1}{b})}\ e^{-\alpha\ x}\ x^{(m-\frac{1}{b})-1}\ dx}_{=1} \\ &= (\alpha b)^{\frac{1}{b}} \left(\frac{\Gamma(n^* + \frac{1}{b})\ \Gamma(m - \frac{1}{b})}{\Gamma(n^*)\Gamma(m)}\right). \end{aligned} \tag{34}$$

□

From Proposition 4 we get the following corollary:

Corollary 2 *Let*

$$N_1 \sim NHPP(\lambda_1(t))$$

with $\lambda_1(t) = \lambda(t, 1)$*, where* $\lambda(t, x) = x\ t^{b-1}$*,* $t \geq 0$*,* $x > 0$ *and let* T_1^* *be the optimal relocation instant when the process of arrival of new locations is modeled by* N_1*. Then:*

$$E[T^*\ |n^*] = E[T_1^*\ |n^*]\ E[X^{-\ \frac{1}{b}}]. \tag{35}$$

This result is interesting due to the fact that it reflects "robustness" of the expected time of a change of location in terms of the distribution function of the variable X.

- **Case 4**

In this scenario, the availability rate of new locations is similar to that of case 2 of the former subsection. In other words,

$$N|X = x \sim\ PPNH(\lambda(t, x)) \tag{36}$$

where,

$$\lambda(t, x) = \begin{cases} x, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s, 2s+1[\\ 0, & \text{if } t \in \bigcup_{s=0}^{+\infty} [2s+1, 2s+2[\end{cases} . \tag{37}$$

We derive the expected value of T^* when $X \sim Unif(a, c)$, $0 < a < c$.

Proposition 5 *Under these conditions the expected value of the optimal relocation time* T^* *is given by*

$$E[T^*\ |n^*] = n^* \frac{log\left(\frac{c}{a}\right)}{c-a} + \\ + \sum_{s=1}^{+\infty} \frac{1}{s(c-a)} \left[\frac{cs\ \Gamma(n^*, cs) - \Gamma(n^*+1, cs) - (as\ \Gamma(n^*, as) - \Gamma(n^*+1, as))}{\Gamma(n^*)} \right]. \tag{38}$$

Proof:

$$
\begin{aligned}
&E[T^* \mid n^*] = E[E[T^* \mid X = x,\ n^*]] \\
&= \int_a^c f_X(x)\ E[T^* \mid X = x,\ n^*]\ dx \\
&= \int_a^c \frac{1}{c-a} \left(\frac{n^*}{x} + \sum_{s=1}^{+\infty} \frac{\Gamma(n^*, xs)}{\Gamma(n^*)} \right) dx \\
&= n^* \left(\frac{log\left(\frac{c}{a}\right)}{c-a} \right) + \sum_{s=1}^{+\infty} \int_a^c \frac{\Gamma(n^*, xs)}{\Gamma(n^*)}\ dx \\
&= n^* \frac{log\left(\frac{c}{a}\right)}{c-a} + \\
&+ \sum_{s=1}^{+\infty} \frac{1}{s(c-a)} \left[\frac{cs\ \Gamma(n^*, cs) - \Gamma(n^*+1, cs) - (as\ \Gamma(n^*, as) - \Gamma(n^*+1, as))}{\Gamma(n^*)} \right].
\end{aligned}
\tag{39}
$$

□

5 Concluding remarks

The problem of a company relocation tackled in this work is a major concern worldwide, given their social and economical consequences. The managers company search for more and more methods and strategical structures that may help them in their decisions and in the assessment of investment opportunities in an uncertainty environment.

The main idea of this work consists in the characterization (in terms of expected value) of the time at which the company should adopt a new location, in an random environment, with the goal of maximizing its value.

We considered that this random environment has one stochastic level: the arrival of information concerning new locations follows particular stochastic processes, but the jump in the efficienty is constant. In particular, we considered Non-Homogeneous Poisson and Conditional Poisson Processes. In this setting, we studied several scenarios considering various intensity rates and obtained expressions for expected value of the optimal relocation time, T^*.

In the future, this work should be extended in order to comprise random jumps and multiple relocations.

References

1. ABRAMOWITZ, M. e Stegun, I. A. (1964). *Handbook of Mathematmal Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards

Applied Mathematics Series, Vol. 55, U.S. Government Printing Office, Washington, D. C..

2. AMRAM, M. e N. Kulatilaka (2000). Strategy and Shareholder Value Creation: The Real Options Frontier. *Journal of Applied Corporate Finance*, Summer, 13.
3. BRATLEY, P., B.L. Fox, L.E. Schrage (1983). *A Guide to Simulation*. Springer Verlag, NewYork.
4. BROUWNER, A.E., I. Mariotti, J.N. van Ommeren (2002). *The firm relocation decision: a logit model*. Department of Spatial Sciences, University of Groningen, Groningen, The Netherlands.
5. ÇINLAR, E. (1975). *Introduction to Stochastic Processes*. Prentice Hall, New Jersey.
6. COLLIE, D. e H.Vandenbussche (1999). Trade, FDI, and Unions. Working Paper, Cardiff Business School.
7. COPELAND. T. e V. Antikarov (2003). *Real Options: A Practitionert's Guide*. New York: Thomson, Texere.
8. CORDELLA, T. e I. Grilo (1998). Globalization and relocation in a vertically differentiated industry. CEPR discussion paper.
9. COUTO, G. (2006). *Opções Reais e Decisão sob Incerteza no Processo de Relocalização*. Tese de doutoramento, Instituto Superior de Economia e Gestão, Instituto Superior Técnico, Portugal.
10. DIJK, J. van e P.H. Pellenbarg (1999). *The demography of firms: progress and problems in empirical research*. In: Van Dijk, J. and P.H. Pellenbarg *Demography of firms, spatial dynamics of firm behaviour*, University of Groningen, The Netherlands.
11. DIXIT, A.K. e R.S. Pindyck (1994). *Investment Under Uncertainty*, Princeton University Press.
12. EUROPEAN PARLIAMENT (1999). *The Threat and Relocation within the European Community*. Working Paper.
13. FARZIN, Y.H., K.J.M. Huisman, and P.M. Kort (1995). *Optimal Timing of Technology Adoption*. Journal of Economic Dynamics and Control, 22, 779-799.
14. HAYTER, R. (1997). *The dynamics of industrial location: the Factory, the Firm, and the Production system*. Chichester (Wiley).
15. HUISMAN, K. (2000). *Technology Investment: a Game Theoretic Real Options Approch*. CentER Dissertion Series, Tilburg University, Department of Econometrics.
16. LLOYD, P.E. e P. Dicken (1992). *Location in space. A theoretical approach to economic geography*, 2nd ed.. London: Harper & Row.
17. MCDONALD, R. e D.Siegel (1986). *The Value of Waiting to Invest*. The Quarterly Journal of Economics, 101, Nov, 707-728.
18. MOTTA, M. e J.-F. Thisse. (1994). Does environmental dumping lead to delocation?,*European Economic Review*, 38, 563-576.
19. OECD (1995). *Trade and Unemployment*. Paris.
20. PEN, C. J. e P. H. Pellenbarg (1998). *Firm migrations, theory and practice; an analysis of empirical studies concerning the impact of government policies*. Paper presented at the Conference at the University of Lodz, about industrial restructuring on 26 to 27 March 1999 in Lodz.
21. SLEUWAEGEN, L. e E. Pennings (2002). *New Empirical Evidence on the International Relocation of Production*. Working paper, Erasmus University Rotterdam.
22. SILVA, B. (2007). *Estudo do Instante Óptimo de Relocalização*, Dissertação de Mestrado. Universidade Técnica de Lisboa - Instituto Superior Técnico.