Design of high-speed track bed with granular and bituminous layers using elastoplastic finite elements models

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Extended abstract

Abstract — A range of section samples for track bed structures is presented. These samples present various combinations of layer materials (granular sub-ballast, granular sub-ballast reinforced with cement, bituminous sub-ballast, soil stabilized with cement, *tout-venant* and four different types of soil), subgrade modulus and layer thickness. The design is made for a high-speed train (static wheel load calculated by Eisenmann's formula), for a life period spawning 50 years and is governed by limiting: 1. the vertical compressive stress on the top of the subgrade to prevent fatigue cracking; 2. the vertical stiffness of the track bed; 3. only for bituminous sub-ballast, maximum extension on sub-ballast layer, also given by fatigue formulas.

For this purpose, various three-dimensional finite element models were studied in order to choose the one that predicts the best approaches in track responses. Depth of system, number of sleepers to be included, the need to consider (or not) materials' weight, mesh's conceptualization, influence of the quality of the subgrade soil and constitutive laws to include (between elastic, elastoplastic and viscoelastic) are discussed.

Keywords: railway, high-speed, analysis by finite elements, elastoplastic models, granular sub-ballast, bituminous sub-ballast, track bed design.

1 Introduction

The great advances in railway technology for the last 40 years, since the beginning of high-speed trains, have highlighted the importance of the rational design in the maintenance costs. Nowadays the finite element method (FEM) is the most used tool for the comprehension of track bed responses.

The slope of this study is to present a range of section samples for track bed structures to aid the designer in his task.

2 Background

The first known studies about stress distribution along the ballast were made by Deharme (1890) and Talbot and his co-workers (1920) [1]. Afterwards, some other methods used for road track design were adopted for railways.

However, all these methods were empirical or semiempirical and when high-speed train appeared, more rigor in the formulations was required. Tracks included for the first time layers between ballast and subgrade and the methods to design them can be divided in two groups: models based on the multilayer elastic Burmister's theory (1945) [1, 2] and finite element models (both elastic and elastoplastic models) [3, 4, 5, 6]. Models that include both elastic multilayer and finite element theories are also available [7].

In the present study a finite element model was developed to carry out numerical analysis and the final design.

3 Developed model

In order to check if the model predicts results accurately, the first model developed was "equal" to the one presented in the *Recomendaciones para el proyecto de plataformas ferroviarias* of the Spanish Public Works Ministry [8].

This is a 3D finite element model that uses 20-noded hexahedral elements.

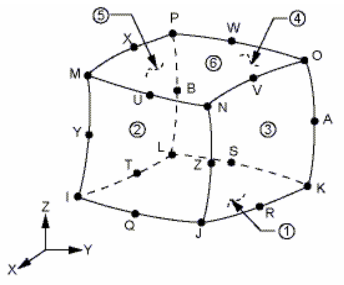

Fig. 1 – 20-noded hexahedral elements.

Due to the symmetry, the model is reduced to one quarter of the studied system. Its dimensions are:

– Ly/2=2,535m (corresponds to 4 ties beyond the loaded one);

– Lx/2=3,9m;

– Lz=4,17m (with 30cm of ballast, 30cm of sub-ballast, 35cm of sub-layer and 3m of subgrade).

Applied boundary conditions are presented in the following table.

Table 1 – Applied boundary conditions

Plane	Restriction
x=0	**u_x=0**
y=0	**u_y=0**
x=Lx/2	**Symmetrical boundary conditions**
y=Ly/2	**Symmetrical boundary conditions**
z=0	**u_z=0**

The rail used is UIC-60 (60kg/m) and the ties PR-90 on concrete with 0,33m spacing.

Rail's height and ties' section were calculated so that their bending resistances were maintained. The same procedure was used in the fasteners height, this time keeping its stiffness.

Elastic constitutive laws were considered for rail, ties and fasteners. For granular materials and soils elastoplastic behaviour is more realistic. For these materials the plasticity was controlled by Drucker-Prager yield function.

Material properties are presented in the next table.

Table 2 – Material properties.

Material	E kg/cm^2	υ –	c kg/cm^2	φ °	Stiffness kN/mm
Subgrade	**60,0**	**0,35**	**0,15**	**25,0**	**–**
QS1 soil	**125,0**	**0,40**	**0,15**	**10,0**	**–**
Tout-venant	**200,0**	**0,30**	**0,00**	**35,0**	**–**
Sub-ballast	**1200,0**	**0,30**	**0,00**	**35,0**	**–**
Ballast	**1300,0**	**0,20**	**0,00**	**45,5**	**–**
Ties	**$5,0x10^5$**	**0,25**	**–**	**–**	**–**
Rail	**$2,1x10^6$**	**0,30**			**–**
Fasteners	**–**	**0,35[1]**	**–**	**–**	**244,0**

The wheel load is static and values 10ton (or 20ton/axle).

4 Evaluation of model predictions

In order to validate the model, rail displacements and subgrade stresses were compared with those values predicted by the Spanish recommendations models.

Although the built model is based on the Spanish one, there are some differences:

– Rigid interconnection between surface contacts are assumed;

– Ballast thickness (25cm in the original model);

[1] This value was admitted by the author by reasons of not being specified in the bibliography.

– Rail width (developed model uses international rail width - 1,435m);

– A few mesh details.

Taking into account these differences, the model was considered to predict accurately both stresses and displacements.

5 Model improvements

The Spanish model was intended to be improved in order to perform a more sophisticate analysis. Displacements and stresses are compared to original model.

5.1 Ballast shoulder

Ballast shoulder is included on the model. The new model is represented in fig. 2. This element is bent and meshing problems can occur.

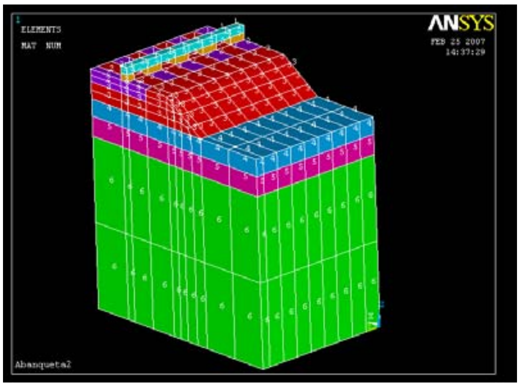

Fig. 2 – Model with ballast shoulder.

Ballast is the only layer that presents differences on results (only on stresses). Anyway, differences are too small (5%) to consider the necessity of this additional element in the final model.

5.2 Depth of subgrade

Two other models were studied, with depth of subgrade of 4,5m and 6m.

In its inferior horizontal plane models are connected to the bedrock by rigid bearings. This means that when the depth of subgrade is increased, models' stiffness is decreases. Is well-known that stiff elements "absorb" more force and allow less displacements. For this reason displacements are very different in these cases and can not be used for comparison.

Stresses on top of the subgrade in the three models (3m, 4,5m and 6m depth) do not vary (differences lower than 1%), so that 3m depth is sufficient.

5.3 Number of ties

New models were developed, with 8, 12 and 16 ties. For these models, the system length is:

– Ly/2=4,95m (8 ties);

– Ly/2=7,35m (12 ties);

– Ly/2=9,75m (16 ties).

Measuring the displacement in the non loaded top of the rail is verified that rail has a vertical and positive (up) movement. This does not correspond to reality, due to rail continuity. To avoid this deficiency, in models including more than 4 ties two changes should be done:

– Inclusion of the ties and rail weight;

– Addition of a new boundary condition in non loaded rail top: uz=0.

Results predicted by these models present considerable variations, especially in the ballast layer. System length was chosen according to the slope of this study - track beds design by limiting vertical stress on top of the subgrade. Model with 8 ties provides a lower value than the original model and for this reason is excluded. Models with 12 and 16 ties predict nearly the same stress for subgrade and higher than the 4 ties model. Then 12 ties model is preferable to the others.

5.4 Material contacts

A special contact type between ties and ballast was created to substitute rigid interconnection between these two surface contacts. In these areas double nodes were considered, each one belonging to one layer. Displacements on the surface plan are allowed. In the perpendicular direction displacements are restricted.

The results given by this model are extremely different from those predicted by the original model (stresses: ballast 14,1%, sub-ballast 38,1%, sub-layer 22,5% and subgrade 16,9%; displacement's errors are about 13% for all materials).

This type of contacts should be considered in track bed design.

5.5 Mesh density

This model pretended to evaluate the results' improvement when mesh lines are closer on force's application region. Meshing densification always ameliorates results, for numerical reasons; nevertheless, the associated computing cost becomes important. The mesh in this model is represented in the next figure.

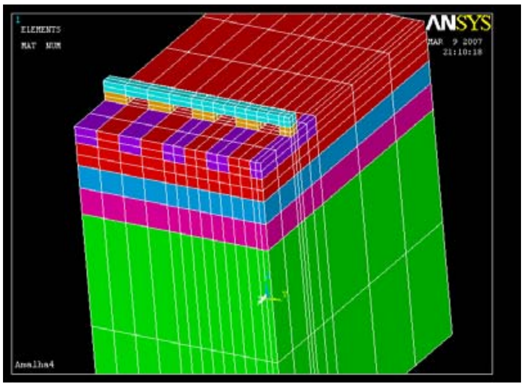

Fig. 3 – Mesh densification.

Differences in the results are important for superior layers (ballast 22,5%, sub-ballast 4,4%), but stress on subgrade is maintained stable. As a consequence, the original mesh is kept in the final model.

5.6 Governing constitutive laws for granular materials

Is known that elastoplastic constitutive laws use to predict better results for granular materials than elastic ones. However, in some special cases elastic models can be a good approach.

Two models – elastic and elastoplastic – were compared. Elastoplastic strains, due to the plastic part of the behaviour, are always higher than elastic strains (differences are between 12% and 14% for every material). Also the stresses show important differences that can not be ignored. As a result, elastic model is not considered an available approach for granular materials in this model.

5.7 Governing constitutive laws for bituminous material

It is common in a finite element analysis to consider viscoelastic laws to the bituminous layer. Experimental data determinated by Mulungye et al. (2007) [9] was used to characterise viscoelastic behaviour. The data were obtained from a bituminous mixture in the following conditions: 20ºC, E=6000MPa and v=0,3.

Two models are analysed. The diference between them consists on the constitutive law considered for the bituminous layer. One of these models consider a viscoelastic bituminous. The other describes bituminous behaviour by elastic constitutive laws. The results predicted by both models do not vary significantly, then elastic behaviour is considered in the final model for bituminous bed.

6 Final model

Based on the considerations above, the model used for design was chosen:

– Ly/2=7,35m (12 ties beyond the loaded one);
– Lx/2=3,9m;
– Depth of subgrade of 3m;
– Ties and rail weight;
– Boundary conditions indicated on table 1 and 5.3;
– Special contacts between ties and ballast (see 5.4);
– Elastic constitutive laws for rail, fasteners, ties, bituminous sub-ballast and granular reinforced with cement sub-ballast;
– Elastoplastic constitutive laws for ballast, granular sub-ballast, sub-layer(s) and subgrade;
– Material properties for ballast, ties, rail and fasteners are indicated on table 2.

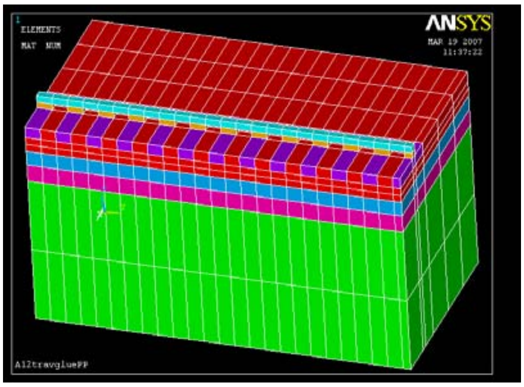

Fig. 4 – Final model.

6.1 Wheel load

Wheel load was calculated using Eisenmann's high-speed formula. Some assumptions were made:
– Both freight and passengers trains can circulate;
– Vehicle type: train circulating on Madrid-Sevilla track (17,21ton/axle);
– Circulation speed: 300km/h;
– High statistical reliability (t=3);
– Very good state of the track (s=0,1);

Consequently, load is

$$Q_D = Q_E(1 + t \cdot \bar{s} \cdot \varphi) = 25{,}63 ton/axle$$

With $$\varphi = 1 + \frac{V - 60}{380} = 1{,}632$$

6.2 Soil types

Track beds are proposed for various types of subgrade soil. They are classified by UIC (International Union of Railways), depending on their quality:
– QS1 – subgrade of bad quality;
– QS2 – subgrade of medium quality;
– QS3 – subgrade of good quality;
– Rock.

Sub-layers included on track bed solutions are soil QS3, *tout-venant* (type of gravel) or a soil-cement mixture.
The section samples also include three different types of sub-ballast: granular, granular reinforced with cement and bituminous.
Mixing cement with soil or granular sub-ballast improves the resistance of these materials, thus of the whole structure and is specially important for poor subgrades (QS1 and QS2).

Soil parameters are presented in the following table.

Table 3 – Soil properties.

Material	E MPa	υ	c kg/cm²	φ °
QS1	**12,25**	**0,40**	**0,15**	**10**
QS2	**24,50**	**0,30**	**0,10**	**20**
QS3	**78,40**	**0,30**	**0,00**	**35**
Rock	**2940**	**0,20**	**–**	**–**
Soil-cement	**500**	**0,30**	**–**	**–**
Tout-venant	**19,60**	**0,30**	**0,00**	**35**
Granular sub-ballast	**117,60**	**0,30**	**0,00**	**35**
Granular reinforced with cement sub-ballast	**1000**	**0,30**	**–**	**–**
Bituminous sub-ballast	**6000**	**0,35**	**–**	**–**

The possible combinations of material type and thickness in order to choose the best track bed design are the following:

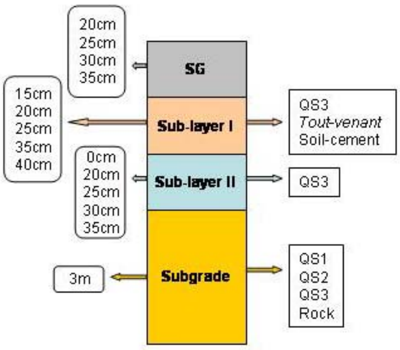

Fig. 5 – Combinations of material type and thickness for granular sub-ballast.

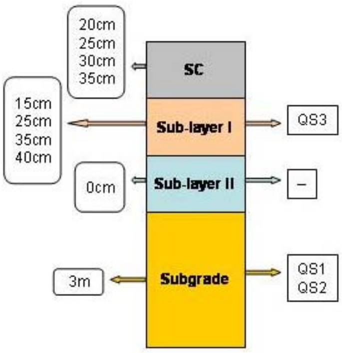

Fig. 6 – Combinations of material type and thickness for granular reinforced with cement sub-ballast.

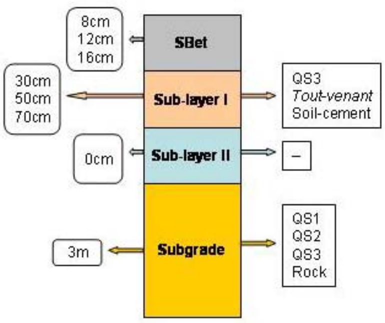

Fig. 7 – Combinations of material type and thickness for bituminous sub-ballast.

6.3 Subgrade fatigue

The fatigue law used for subgrade soil is the Heukelom's formula:

$$\sigma = \frac{a \cdot Ed}{1 + 0{,}7\log(N)}$$

Where

σ – maximum vertical stress (Kg/cm^2);

E_d –Young dynamic modulus (Kg/cm^2)≈2,2 a 2,5Ev2;

a – parameter valuing 0,006 or 0,008. In this study is used a = 0,006 to be more conservative;

N – number of load cycles.

The number of load cycles is obtained assuming the following hypothesis: a frequency of 100 trains per day; the same vehicle type used in load calculation (26 axles); and a period of life of 50 years for the structure.

6.4 Bituminous sub-ballast fatigue

Fatigue cracking is prevented by limiting the maximum extension in the bituminous bed. The formula in use was proposed by the Asphalt Institute (1982):

$$N_a = 0{,}0795\varepsilon_t^{-3.291} E_a^{-0.853}$$

With

N_a – number of load cycles;

ε_t – maximum extension in the bituminous bed;

E_a – bituminous' Young modulus [MPa].

6.5 Design criteria

The design was governed by limiting:

– Vertical stress on the top of the subgrade to its maximal values;

– Track vertical stiffness from 60kN/mm to 200kN/mm;

– Maximum extension on sub-ballast layer (only for sections with bituminous bed).

7 Proposed track beds

Tables 4, 5 and 6 present the results obtained for the different thicknesses of the layers shown in fig. 8, in the case of using granular only sub-ballast (table 4), granular sub-ballast treated with cement (table 5) and bituminous sub-ballast (table 6).

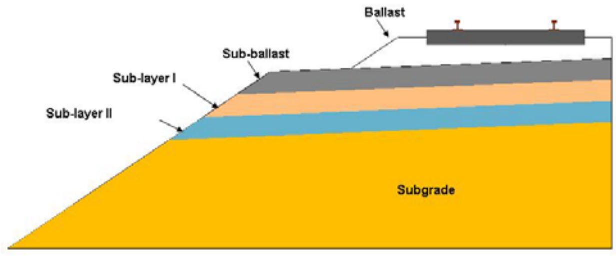

Fig. 8 – General track bed scheme.

Table 4 – Proposed track beds with granular sub-ballast.

Subgrade	Sub-layer II		Sub-layer I		Sub-ballast thick.
	Thick.	Material	Thick.	Material	
QS1	30cm	QS3	35cm	Soil-cem.	35cm
QS2	20cm –	QS3 –	25cm 15cm	QS3 Soil-cem.	25cm 20cm
QS3	–	–	15cm	QS3	20cm
Rock	– –	– –	25cm 15cm	QS3 *Tout-ven.*	20cm 20cm

Table 5 – Proposed track beds with granular reinforced with cement sub-ballast.

Subgrade	Sub-layer II		Sub-layer I		Sub-ballast thick.
	Thick.	Material	Thick.	Material	
QS1	35cm	QS3	40cm	QS3	35cm
QS2	–	–	15cm	QS3	20cm
QS3	For these types of subgrade is not nececessary to reinforce the granular sub-ballast.				
Rock					

Table 6 – Proposed track beds with bituminous sub-ballast.

Subgrade	Sub-layer II		Sub-layer I		Sub-ballast thick.
	Thick.	Material	Thick.	Material	
QS1	70cm –	QS3 –	70cm 50cm	QS3 Soil-cem.	16cm 8cm
QS2	–	–	30cm	QS3	8cm
QS3	–	–	30cm	QS3	8cm
Rock	– –	– –	70cm 30cm	QS3 *Tout-ven.*	8cm 8cm

8 Concluding remarks and further research

It is evident in this paper that FEM is efficient in track bed responses analysis. Parameters can be rapidly changed and many studies are possible. However, it is essential that material parameters are correctly inputted in the model. Soils must be rigorously tested in same conditions that they have *in situ.* Models can only predict results accurately if those requirements are fulfilled, especially for elastoplastic and viscoelastic constitutive laws that require more soil parameters. Railways are very long and cross many types of soils, this fact must be considered too.

Rational design of track beds must take into account questions related to future exploitation of the track. Frequency and type of trains to circulate on the track must be predicted for the entire live period.

It would be important to continue this study adding other variables:

– Consider tracks where both passengers and freight trains can circulate;

– Consider tracks with other frequencies of circulation;

– Consider tracks where other vehicle types can circulate;

– Include other types of materials composing track bed and to stabilize bad quality subgrades;

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