Sound Fields Generated by Footsteps on Stairs of Residential Buildings

Extended Abstract

JOANA MARIA COSTA MARIANO

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Jury
President: Prof. Jorge Manuel Caliço Lopes de Brito
Supervisor: Prof. Albano Luís Rebelo da Silva das Neves e Sousa
Examiner: Prof. Luís Manuel Coelho Guerreiro

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INTRODUCTION

Impact sound transmission at frequencies below 200 Hz is nowadays an issue in dwellings. Although human beings are able to detect sounds from 20 to 20000 Hz, current standards apply only to frequencies in the range 100 – 3150 Hz. Recently, this interval was extended to 5000 Hz in some standards. An extension to 50 Hz is also suggested in standards dealing with sound insulation measurements [N.1, N.2] but results have been unsatisfactory because the standard methods are not adequate for low frequencies [8]. Standards for measurement and prediction of sound transmission are based on classical methods which assume diffuse structural and sound fields. However, at low frequencies, construction elements exhibit a modal behaviour and resonances may occur. Mechanical devices are generally the most problematic sources of low frequency noise, but footsteps are identified as the most common source of low frequency impact sound in dwellings. Thus, it is important to develop prediction methods for low frequency impact sound transmission. The alternatives are the finite element method (FEM) and analytical methods based on natural mode analysis. Neves e Sousa [8] uses the latter to characterise impact sound transmission through different types of floor of current dimensions in dwellings. The method is limited to rectangular plates and rooms and therefore it is not adequate for direct application to the prediction of room sound fields generated by impact forces acting on stairs clamped to one of the room’s walls.

FEM will be used to model the vibration field of a stair/wall structural system. Unfortunately, at the moment, computer programs using FEM to model sound fields are not available at the Department of Civil Engineering and Architecture of IST and therefore analytical methods will have to be used. Thus, analytical predictions of the structural interaction between the stair and the wall and then of the sound field in the room will have to be performed. The method will then be applied to describe sound fields generated by human footsteps on stairs.

Predictions of the sound field generated in rooms by impact forces applied on stairs adjacent to one of the room’s walls can be obtained in two steps. The first step describes the vibration field induced in the wall by an impact force applied on the stair. The second step describes the sound field generated in the room by the vibrating wall.

STAIR/WALL STRUCTURAL SYSTEM VIBRATION MODEL

The wall/stair structural system was modelled analytically by natural mode analysis. The model assumes that the impedance of the stair is much higher than that of the wall, which means that the wall/stair structural system can be divided into two independent structures: the stair subjected to an impact force; and the wall subjected to the edge reactions of the vibrating stair. According to Cremer [4], the acceleration field $a_x(y, z)$ generated by a point impact force of amplitude $F$ acting at the coordinates $(y_0, z_0)$ on a simply supported wall can be analytically predicted by
\[
a_x(y, z) = -\frac{4\omega^2F}{m^\prime bc}\sum_{m,n=1} \frac{\varphi_{m\prime n\prime}(y,z) \varphi_{m\prime n\prime}(y_n, z)}{\omega^2_{m\prime n\prime}(1 + \frac{\eta}{\omega^2}) - \omega^2}
\]

where: \(b\) (m) and \(c\) (m) are the wall dimensions; \(m^\prime\) (kg/m\(^2\)) is the mass per unit surface area; \(\eta\) is the loss factor, which can be obtained as indicated by Craik [3]; \(\omega\) (rad/s) is the angular frequency; \(\varphi_{m\prime n\prime}(y,z) = \sin(m\pi y/b) \cdot \sin(n\pi z/c)\) are the eigenfunctions which satisfy the boundary conditions of simply supported plates; and \(\omega_{m\prime n\prime} = \sqrt{B^2/m^\prime}\left[\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{c}\right)^2\right]\) are the corresponding eigenfrequencies.

As a clamped connection is assumed between the stair and the wall, which is true for the small amplitude vibration fields generated by dynamic actions of human origin, reaction forces and moments in the stair/wall connection can be calculated either numerically or analytically. In this thesis, the computer program SAP2000 [5] was used to calculate those reactions, which were then transformed into couples of forces according to

\[F(z, t) = M(z, t)/h_e\]

where \(h_e\) (m) is the thickness of the stair.

The couples of forces obtained numerically for each joint of the FEM stair/wall model can then be inserted into the analytical model described by equation (1) making use of the superposition principle, which is valid for elastic vibrations.

In order to assess the validity of the procedure described above, the three cases illustrated in Fig. 1 were analysed.

Fig. 1 – Analysis cases: a) floor connecting the wall at half-height; b) floor connecting the wall at 2/3 of the height; c) stair/wall system.

The floors and stair considered in Fig. 1 correspond to a 15 cm thick concrete plate with a width of 1 m. The wall is a 20 cm thick masonry wall, with a length of 5 m and a height of 3 m.

Figures 2 to 4 show the results, obtained by the analytical model of the wall subjected to a set of force couples corresponding to the action of a unit point impact force on the stair, for the three analysed cases, respectively. The accelerance spectra were also obtained by FEM in order to assess the error associated with the analytical model.

Figures 2 to 4 show that, in general, the numerical and analytical models yield similar amplitudes of the accelerance spectra. The shape of the accelerance spectra is also similar but
the analytically obtained spectra are shifted to lower frequencies. This means that the analytical model yields a more flexible structural system than expected.

An alternative analytical model based on an equivalent wall directly subjected to a unit impact point force was also developed. The mechanical properties of the equivalent wall, such as wall thickness and the elasticity modulus and density of the material, are defined iteratively in order to obtain the first natural frequencies in agreement with those given numerically for the stair/wall system. The iterative procedure makes use of the expression given above for $\omega_{m1n1}$ and therefore it is relatively fast and easy to conduct.

The numerical prediction of the magnitude of the transfer function between the impact force on the stair and the acceleration field of wall are compared in Fig. 4 with the corresponding transfer
function obtained with the analytical equivalent wall model. As occurred with the analytical considering the action of a set of force couples, also the equivalent wall model provides a generally good estimate of the accelerance magnitude. The shift of the spectra visible in Figures 2 to 4 vanishes when the equivalent wall model is used, which means that a better estimate of the coupling between the wall vibration modes and the acoustic modes of the room can be expected.

Fig. 4 – Magnitude of the transfer function between \( F[(b/3, c/3) = (0.33, 1.94) \text{ m}] \) and \( a_1(y, z) = (b/3, 2c/3) = (1.00, 3.33) \text{ m} \) obtained for the stair/wall system modeled numerically and analytically, either by considering a set of force couples equivalent to the stair reactions or by considering an equivalent wall directly subjected to an arbitrary point force.

**SOUND FIELD MODEL**

The sound field generated in the room by a point impact force applied in a wall can be analytically predicted by natural mode analysis as proposed by Neves e Sousa [8] based on the work of Kihlman [7]. In this model, the room is limited by five rigid plates (walls and floors) and one vibrating wall. The sound field generated in the room at coordinates \((x, y, z)\) by a point impact force with amplitude \( F \) acting on the wall at \((y_0, z_0)\) is given by

\[
p(x, y, z, t) = -j\omega\rho_0 \sum_{l,m,n=1}^{\infty} \frac{8c_0^2(-1)^lC_{lmn}\varphi_{lmn}(x, y, z)}{abc\left[\omega_{lmn} + j\delta - \omega^2\right]}e^{j\omega t},
\]

where: \( a \) (m), \( b \) (m) and \( c \) (m) are the dimensions of the room; \( \rho_0 \) (kg/m\(^3\)) is the static value of the air density; \( c_0 \) (m/s) is the speed of sound; \( \delta \) is a temporal absorption coefficient; \( \varphi_{lmn}(x, y, z) = \cos(l\pi x/a)\cos(m\pi y/b)\cos(n\pi z/c) \) are the eigenfunctions of the room;

\[
\omega_{lmn} = c_0\sqrt{\left(l\pi/a\right)^2 + \left(m\pi/b\right)^2 + \left(n\pi/c\right)^2}
\]

are the corresponding eigenfrequencies; and \( C_{lmn} \) is the coupling factor between the wall vibration modes and the room acoustic modes.
Since the vibration field of the stair/wall system was modelled analytically in two different ways, either by considering a set of force couples equivalent to the stair reactions or by considering an equivalent wall directly subjected to an arbitrary point force, the sound field generated in the room by an impact force applied on a stair adjacent to one of the room’s walls should be modelled accordingly.

Fig. 6 shows the magnitude of the predicted transfer functions between the impact force applied on the stairs and the sound pressure in a room with dimensions $abc = 3 \times 5 \times 4 \text{ m}^3$. The transfer functions were obtained with both analytical models near one of the room’s corners, where most acoustic modes can be excited [8].

Fig. 6 – Magnitude of the predicted transfer functions between the impact force applied on the stairs and the sound pressure in a room with dimensions $abc = 3 \times 5 \times 4 \text{ m}^3$. The transfer functions were obtained with both analytical models near one of the room’s corners, where most acoustic modes can be excited [8].

Comparison of the magnitude of the transfer functions showed in Fig. 5 yields similar conclusions as those drawn from Fig. 4. Below 170 Hz, the accelerance predicted with the wall subjected to a set of equivalent force couples is generally higher than that obtained numerically and with the equivalent wall. Thus, at those frequencies, the sound pressure also exhibits higher magnitude when the wall is modelled considering the force couples. In this frequency interval, a better prediction of the sound field is obtained with the equivalent wall although an uncertainty always remains regarding modal coupling. Indeed, although the equivalent wall model gives wall vibration modes occurring at the same frequencies as those obtained numerically, the modes themselves are certainly different and therefore modal coupling should also occur differently. In this thesis, these differences were assumed to be small, but only experimental or validated numerical results can confirm this assumption.
DYNAMIC LOAD INDUCED BY HUMAN FOOTSTEPS

In community areas of residential buildings, human motion can occur as walking, running or jumping. These actions are composed by two horizontal force components (lateral and longitudinal) and another one vertical which exhibits higher amplitude than those of the horizontal components and therefore controls the human dynamic action on floors. Dynamic forces due to human footsteps have been studied by several authors [1, 2, 6, 9]. According to them, the weight of the moving person and the pacing rate are the parameters which control the human dynamic action. Thus, the time function of the vertical force component, \( F(t) \), exerted by human footsteps is given by

\[
F(t) = \alpha_1 \cdot \sin(2 \cdot \pi \cdot f_s \cdot t) + \alpha_2 \cdot \sin(4 \cdot \pi \cdot f_s \cdot t - \phi_2) + \alpha_3 \cdot \sin(6 \cdot \pi \cdot f_s \cdot t - \phi_3),
\]

where: \( G \) (N) is the static weight of the person; \( \alpha_i \) are the coefficients of amplitude of the harmonic force components; and \( \phi_i \) are the phase angle differences of the \( n \)th harmonic force component [1, 2, 6].

Applying a FFT procedure yields the impact force spectra induced by a person. These spectra are plotted in Fig. 6 for an 80 Kg person walking, running and jumping on a floor or stair. Fig. 6 shows that the highest magnitudes of the impact force are obtained for a person running on a stair.

![Dynamic vertical force spectra induced by different types of human motion on floors and stairs.](image)

Fig. 6 – Dynamic vertical force spectra induced by different types of human motion on floors and stairs.

CASE STUDY

The analytical model developed for the prediction of sound fields generated in rooms by impact forces applied on contiguous stairs is used to assess the sound field generated by human motion on stairs. The case study applies to the same stair, wall and room considered before.

The envelope of the magnitude of the sound pressure in the room is obtained by simple multiplication of the transfer functions between a point impact force acting on the stair and the
sound field in the room (Fig. 5) by the dynamic force spectrum induced by a person walking and running on the stair (Fig. 6). The analytically predicted sound pressure level spectra are showed in Fig. 8 for both analytical models of the stair/wall structural system.

![Sound level spectra](image)

Fig. 7 – Sound level spectra at \((x, y, z) = (3.73, 1.00, 1.67)\) m, obtained, for one running footstep on the stair at \((b/3, c/3) = (0.33, 1.94)\) m, for the stair/wall system modeled either by considering a set of force couples equivalent to the stair reactions or by considering an equivalent wall directly subjected to an arbitrary point force.

The highest sound pressure levels are below 60 dB, which is not annoying for frequencies below 90 Hz [10].

**CONCLUSIONS AND FURTHER WORKS**

Modelling low frequency impact sound transmission through stairs adjacent to rooms requires numerical methods, such as FEM, in order to deal with a complex geometry. However, commercial computer programs using FEM to model sound field in rooms are too expensive. A home-made computer program using FEM is now being constructed in the Department of Civil Engineering and Architecture of IST although it is not finished yet. Therefore, analytical models using natural mode analysis are the remaining alternative for modelling sound fields. They have the advantage of being much faster than FEM and adequate for most common rooms in dwellings, i.e., rectangular rooms.

Conversely, the vibration field of the stair/wall structural system cannot be accurately predicted by analytical methods. However, reasonable estimates can be obtained by considering a wall with mechanical characteristics carefully chosen in order to obtain a vibration field described by similar modes occurring at the same natural frequencies as those provided by FEM. This vibration field should be generated by a single unit point impact force. Poorer estimates can be obtained by considering the effect of the stair on the wall as a set of force couples.

In order to assess the accuracy of the sound field model when the analytical models of the vibration field of the stair/wall system are used, experimental or numerical validated results are
required. The ongoing FEM programming aims to fulfil this requirement. This computer program should then be used to model different types of stair and connection to the wall.

For lightweight structures, such as wooden stairs, the human dynamic forces presented in this thesis cannot be used because the impedance of the structure is no longer much higher than that of the moving person. Little work has been done on characterising human induced vibration on lightweight structures and thus numerically or analytically obtained transfer functions between impact force and sound pressure level are of great use for future comparison with experimental results.

These studies are also important for subjective noise evaluation of enclosed spaces and for the so-called source auralisation.

REFERENCES

[1] Bachmann, H.; Ammann, W. – Vibrations in structures induced by man and machines, Structural Engineering Documents, International Association for Bridge and Structural Engineering, Zurich, Switzerland, 1987;


