INFLUENCE OF INCORPORATING A BITUMINOUS SUB-BALLAST LAYER ON THE DEFORMATIONS OF RAILWAY TRACKBED

Tiago Ferreira

ABSTRACT

This paper analyses theoretically the performance of a bituminous sub-ballast against environmental actions as an alternative to the conventional sub-ballast layers. The comparison between both designs is made in terms of the vertical displacements and its seasonal variation as well as the capability of maintaining the moisture content along the year.

From the different approaches developed, it is found that the bituminous sub-ballast may allow an important reduction in the seasonal vertical displacements, up to 50% or more under poor drainage conditions.

INTRODUCTION

The maintenance needs of a railway track are highly influenced by the behaviour of its substructure along its life cycle. The use of sand and gravel layers on conventional high-speed lines aims to fulfill an accurate protection of the formation not only against traffic load, but also against weather effects. However, it is found that along the track lifetime those layers tend to loose their filter characteristics.

Being almost completely water-resistant, the use of a bituminous sub-ballast may offer important comparative advantages from the point of view of long term deterioration of the subgrade, when compared to the granular solutions.

The aim of this study is to assess the impact of using a bituminous sub-ballast layer on the track substructure deformation process due to variations of relative humidity along a given period.

2. SUCTION DUE TO WEATHER EFFECTS

Railway infrastructure is composed by different soil layers, which are generally exposed to seasonal moisture changes due to weather actions. Rainfall, changes in the relative humidity and in temperature lead to water infiltration/evaporation into the ground, resulting in soil moisture fluctuations. Moreover, the seasonal alternate wetting-drying cycles controlled by atmospheric actions, are responsible for strong changes in suction. These variations in suction due to the evolution of the water content of the soil during the year are associated with volume changes (swelling and shrinking) which are responsible for vertical displacements.

The mechanics of partially saturated soils is considered to be appropriate to perform the calculation of displacements due to environmental changes.

Suction can be defined as the free energy state of soil water and can be measured in terms of the relation between the partial vapour pressure and the saturation pressure of the soil water (Fredlund and Rahardjo, 1974), this relation is generally referred as relative humidity. The thermodynamic relationship between total suction and the relative humidity, *RH*, is given by the Psychometric Law and can be written as follows:

$$\psi = -\frac{RT\rho_w}{w_v}\ln(RH) \qquad (1)$$

ψ - soil suction or total suction (kPa);
R - universal (molar) gas constant [i.e., 8.31432 J/(mol K)] ;
T - absolute temperature [i.e., T = (273.16 + t^0) (K) with temperature t^0 in ºC];
ρ_w - density of water [i.e., 998 kg/m3 at t^0 = 20 ºC] - $\rho_w = 1007{,}9\,\exp^{-4{,}573x10^{-4}T}$;
w_v - molecular mass of water vapour (i.e., 18.016 kg/kmol);
RH - relative humidity on the soil voids;

Soil and the surrounding air exchange water in the gas (vapour) and/or in the liquid phase but the amounts exchanged depend on the soil's water retention capability. In fact, the water in the liquid phase present in the soil depends on many factors such as soil grading size and soil density (pore dimensions and geometry). To use Equation (1) in a realistic way, the relative humidity considered must be the one of the air from the soil voids (in equilibrium with the water in the liquid phase of the soil), and not the one of the atmosphere.

3. CALCULATION OF SOIL DEFORMATIONS USING BBM

Suitable unsaturated soils constitutive models are required for the calculation of soil deformations due to suction changes. The model used in this work is the Barcelona Basic Model, BBM, proposed by Alonso et al. (1990). BBM is a hardening elastoplastic constitutive model appropriated to model the behaviour of slightly or moderately expansive soils and is based in two independent sets of stress variables: the excess of total stress over air pressure, *p*, and suction, *s*. It provides the mathematical formulation to calculate the soil deformations due to suction changes and/or stress changes.

3.1 Application of BBM to the case study

According to the formulation of BBM, an Elastic domain is enclosed in the (*p, s*) plane by the LC (after loading collapse) and SI (after suction increase) yield surfaces. Loading paths that lead to mean stresses or suction values out of the Elastic domain will lead to Plastic Deformations, therefore to a new definition of the LC and/or SI yield curves according to the hardening laws.

In railway infrastructures, an adequate compaction is necessary to prevent the development of plastic deformations in service. A value for preconsolidation greater than the design loads in service (train, rails, sleepers, track support etc.) must be achieved to ensure that plastic deformations will not occur. Considering that the construction of the railway infrastructure was designed in order to avoid the future development of irrecoverable (plastic) deformations, it was assumed that the paths in the (*p, s*) space associated only to suction changes (under constant stress) to be considered in this work were in the Elastic domain. The comparison between the granular and bituminous sub-ballast considering their effects in the moisture content changes of the soil along the year can be done by evaluating the amplitude of the displacements calculated for each case. To allow this procedure it is necessary to relate the moisture content of the soil with the suction in it and then, using BBM, calculate the associated deformations.

3.2 Cross sections design

The design of the cross sections, defined to compare both trackbed solutions, was made in order to simulate a 5 m high embankment and was based in typical geometry, material and thicknesses of trackbed layers. To reach an equivalent structural behaviour of a granular sub-ballast layer with 30 cm of thickness, the sub-ballast layer was design with 12 cm (as proposed by Teixeira et al., 2006). Granular and Bituminous cross sections design is presented in Fig. 1.

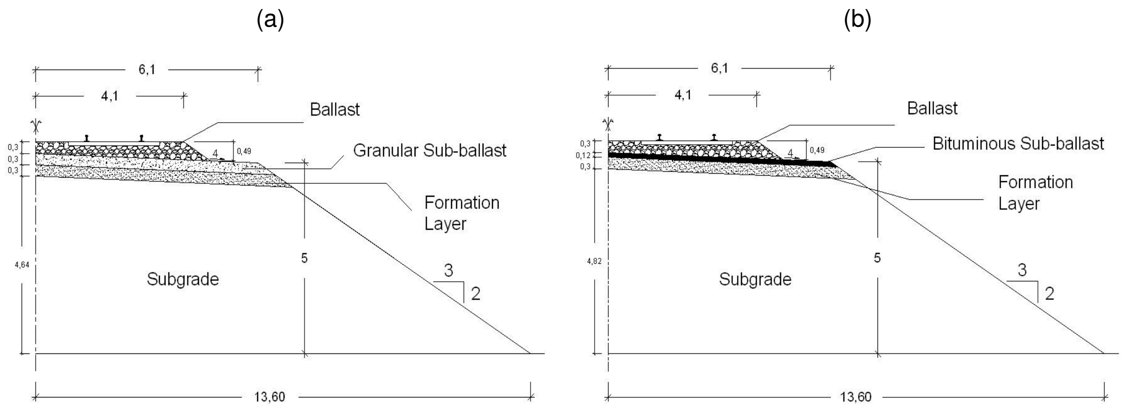


Fig. 1 – Cross sections: (a) Granular sub-ballast; (b) Bituminous sub-ballast.

3.3 Characteristics of Materials

The initial state of the formation layer and of the subgrade is controlled by the compaction conditions of the materials that compose each one of them. For the calibration of BBM it is required the definition of the parameters for the materials (p^c, $\lambda(0)$, k, r, β and k_s). These parameters are related only with the elastic volumetric behavior of the soil. Table 1 shows the parameters adopted for the soils considered in the cross sections presented before.

Table 1 – Model Parameters for different types of soil (from *Alonso, 1998*)

	Model Parameters						
	$\lambda(0)$	k	r	β (MPa^{-1})	p^c (MPa)	λ_s	k_s
(1) Reference Soil	0.200	0.020	0.75	12.5	0.100	0.080	0.008
(2) Compacted silty soil	0.140	0.015	0.26	16.4	0.043	0.050	0.010
(3) Lower Cromer till	0.066	0.008	0.25	20.0	0.012	-	0.001
(4) Compacted silty soil	0.065	0.011	0.75	20.0	0.010	0.025	0.005

3.4 Weather actions

In this work, three reference climates have been simulated (from *Alonso, 1998*):

- A **Tropical climate (Moa, Cuba)** characterized by an uniform high Temperature (T_{av}=25.1ºC) and high Relative Humidities (RH_{av} = 83.2%);
- A **Mediterranean climate (Tarragona, Spain)** characterized by warm and dry summers and moderate winters (T_{av}=18ºC). The Relative Humidity has an irregular distribution (RH_{av}=71.3%). High Temperatures in summer enhance evaporation;
- A **Subalpine climate (Camprodón, Spain)** a Pre- Pre-Pyrenean city. The average Relative Humidity (RH_{av}=74.5%) is relatively high and the mean Temperature is moderate (T_{av}=11.5ºC).

3.5 Granular *vs.* Bituminous: parametric analysis

Using BBM, it is possible to calculate the volumetric strains within the Elastic domain associated to changes in suction from s_0 to s_1 in distinct time intervals t_0 and t_1. Their calculation depends on the soil parameter k_s according to Equation (2) where p_{at} is the atmospheric pressure.

$$\varepsilon_s^e(t_1) = k_s \ln\left(\frac{s_0 + p_{at}}{p_{at}}\right) - k_s \ln\left(\frac{s_1 + p_{at}}{p_{at}}\right) = \kappa_s \ln\left(\frac{s_0 + p_{at}}{s_1 + p_{at}}\right) \qquad (2)$$

The comparison between the granular and the bituminous solutions was made by comparing the maximum amplitude of the displacements, Δ: the difference between the maximum shrinking (volume decrease) and the maximum swelling (volume increase) observed in one year. These displacements were calculated with the volumetric strain occurred due to suction changes (Eq. 2) on control points located over the rail axis line between the sub-ballast and the formation layers.

To allow a theoretical approach, a simplified method was adopted, where for each layer (Figure 1) it is assumed a given reduction of the relative humidity (RH) of the air from the atmosphere. This reduction is dependent of the depth of the layer and increases with it.

Considering that the granular sub-ballast layer provides a given reduction in the relative humidity, 10.5% (this value was computed by the finite elements program CODE_BRIGHT), a parametric analysis in terms of the reduction in the relative humidity provided by the bituminous sub-ballast layer (this variable is named λ) and its effects on maximum vertical displacements (amplitude), Δ, was performed. A parameter named Bituminous reduction ($Bit._{red}$) was defined according to Equation (3) to relate the displacements (amplitudes) measured in both cross sections.

$$Bit._{red} = \frac{\Delta Gran. - \Delta Bit.}{\Delta Gran} \qquad (3)$$

The results from the analysis have shown that:

- lower displacements are associated to low values of suction compressibility, k_s;
- generally, maximum amplitude of vertical displacements decreases as the bituminous reduction for relative humidity, λ, increases;
- the adoption of bituminous sub-ballast, designed in order to guarantee a relative humidity reduction superior to 15%, is a better solution than the granular sub-ballast layers;
- a 50% reduction in the displacements (amplitude) performed by the bituminous solution might be reached if considering a reduction in the relative humidity higher than 50% for Tarragona climate and 55% for Moa and Camprodón;
- for each climate it is possible to define a characteristic curve of reduction provided by the bituminous sub-ballast layer calculated with Equation (3) (Figure 2). This curve evidences the loss of efficiency for high values of relative humidity reduction performed by the bituminous sub-ballast;

- the percentage of Bituminous reduction associated to the vertical displacements (amplitude) is similar to the one associated to the increased of relative humidity reduction;
- the efficiency loss may lead to an inversion of the slope in the associated characteristic curve. The existence of an inflexion point as observed for Tarragona climate indicates that it may exist an optimum value for the relative humidity reduction associated to the maximum efficiency of the design solution.

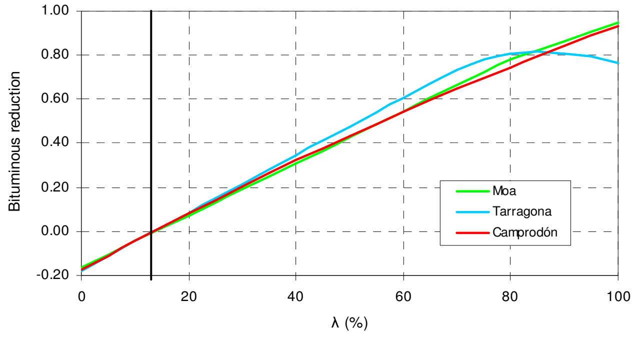


Fig. 2 – Bituminous reduction for maximum vertical displacements

4. SIMULATION OF BITUMINOUS PERFORMANCE WITH CODE_BRIGHT

The use of adequate formulation is necessary to rule the exchanges of water in both vapour and liquid phases between soil and the environment. Once this calculation is mathematically complex, an analysis of the bituminous sub-ballast performance was carried out by recurring to the finite elements program CODE_BRIGHT (Olivella et al., 1994, 1996; DIT-UPC, 2000). Its formulation considers the hydraulic constitutive equations, the conductive, heat, diffusive and dispersive fluxes as well as the intrinsic permeability. Using this program, a simulation exercise concerning a period of 5 years was performed to allow the study of the cross sections long term behaviour.

4.1 Weather actions

The Tarragona climate data was the one simulated in this exercise (Figure 3).

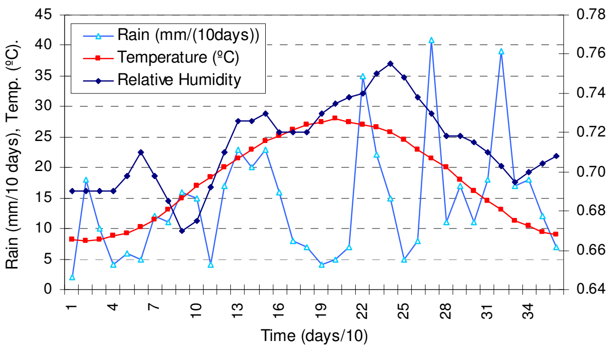


Fig. 3 – Atmospheric variables of average representative year in Tarragona, Spain.

4.2 Material constitutive models calibration

The trackbed materials were described by constitutive models and parameters given in Table 2 and assumed to be adapted to the type of materials usually adopted in practice.

Table 2 – Material models and parameters

PROPERTY	MODEL	BALLAST	GRANULAR SUB-BALLAST	BITUMINOUS SUB-BALLAST	FORMATION LAYER	SUBGRADE	Legend
Retention curve	$S_e = \frac{S_l - S_{rl}}{S_{ls} - S_{rl}} = \left(1 + \left(\frac{P_g - P_l}{P_0}\right)^{\frac{\lambda}{1-\lambda}}\right)$	$\lambda = 0.50$ $P_0 = 0.01\,MPa$ $S_{rl} = 0.0001$ $S_{ls} = 0.98$	$\lambda = 0.55$ $P_0 = 0.15\,MPa$ $S_{rl} = 0.05$ $S_{ls} = 0.80$	$\lambda = 0.50$ $P_0 = 0.35\,MPa$ $S_{rl} = 0.01$ $S_{ls} = 0.80$	$\lambda = 0.55$ $P_0 = 0.15\,MPa$ $S_{rl} = 0.05$ $S_{ls} = 0.80$	$\lambda = 0.55$ $P_0 = 0.15\,MPa$ $S_{rl} = 0.05$ $S_{ls} = 0.80$	S_r : degree of sat. of liquid phase P_l : liquid pressure P_g : gas pressure
Liquid Flow	$q_l = \frac{k k_{rl}}{\mu_l}(\nabla P_l - \rho_l g)$ $k_{rl} = \sqrt{S_e}\left(1 - \left(1 - S_e^{\frac{1}{\lambda}}\right)^{\lambda}\right)^2$ or, $k_{rl} = A(S_e)^{\alpha}$	$k = 1.0\times10^{-12}(m^2)$	$\lambda = 0.55$ $k = 0.5\times10^{-15}(m^2)$ $S_{rl} = 0.15$ $S_{ls} = 0.98$	$A = 1.0;\ \alpha = 3.0$ $k = 0.1\times10^{-19}(m^2)$ $S_{rl} = 0.01$ $S_{ls} = 0.98$	$\lambda = 0.55$ $k = 0.5\times10^{-15}(m^2)$ $S_{rl} = 0.15$ $S_{ls} = 0.98$	$\lambda = 0.55$ $k = 0.5\times10^{-15}(m^2)$ $S_{rl} = 0.15$ $S_{ls} = 0.98$	μ_l : liquid viscosity ρ_l : liquid density q_l : liquid advective g : gravity
Conductive Heat Flow	$i_e = -\lambda \nabla T$	$\lambda = 2(W/mK)$	$\lambda = 2(W/mK)$	$\lambda = 2(W/mK)$	$\lambda = 2(W/mK)$	$\lambda = 2(W/mK)$	T : Temperature
Diffusion of Vapour	$i_g^w = -(n\rho_g(1 - S_r)D_m^w I)\nabla\omega_g^w$ $D_m^w = \tau D_0^w\left(\frac{T^t}{P_g}\right); S_g = 1 - S_r$	$t = 2.3;\ \tau = 1$ $D_0^w = 5.9\times10^{-6}$	$t = 2.3;\ \tau = 0.4$ $D_0^w = 5.9\times10^{-6}$	$t = 2.3;\ \tau = 1$ $D_0^w = 5.9\times10^{-6}$	$t = 2.3;\ \tau = 1$ $D_0^w = 5.9\times10^{-6}$	$t = 2.3;\ \tau = 1$ $D_0^w = 5.9\times10^{-6}$	i_g^w : non-advective water mass flux in gas phase n : porosity ω_g^w : mass fraction of water in gas phase
Deformation	Linear elastic (E, ν) Strain due to suction and temperature changes: $\Delta\varepsilon_v = 3a\Delta(P_l - P_g) + 3b\Delta T$	$E = 150\,MPa$ $\upsilon = 0.33$ $a = 0.00$ $b = 0.55\times10^{-5}$	$E = 50\,MPa$ $\upsilon = 0.33$ $a = 0.0025$ $b = 0.55\times10^{-5}$	$E = 3000\,MPa$ $\upsilon = 0.33$ $a = 0.00$ $b = 0.55\times10^{-5}$	$E = 50\,MPa$ $\upsilon = 0.33$ $a = 0.0025$ $b = 0.55\times10^{-5}$	$E = 50\,MPa$ $\upsilon = 0.33$ $a = 0.0025$ $b = 0.55\times10^{-5}$	υ : Poisson Coefficient E : Young Modulus

The following comments complete Table 2:

- Retention curve for ballast exhibits very small air entry pressure values (this property is controlled by parameter P_0). Bituminous sub-ballast has the highest value for air entry pressure to simulate its impermeability;
- Intrinsic permeability matches the grain size distribution expected for the materials;
- Water vapour flow is simulated by means of Fick's law. A diffusion vapour molecular coefficient, depending on temperature, controls the intensity of vapour mass transfer;
- A linear elastic stress-strain relationship is adopted. The Ballast and the Bituminous are very stiff against suction changes. The subgrade and the granular materials can change their volume under suction changes (swells or shrinks);
- All the trackbed layers dilate or contract when subjected to temperature changes.

4.3 Boundary Conditions

The initial water content corresponds to an equilibrium situation consistent with the water level located at the lowest boundary of the discretized domain. Boundary conditions are given in Figure 4. Different percentages of rain infiltration, α, were considered in the upper boundary for both solutions (bituminous and granular sub-ballast).

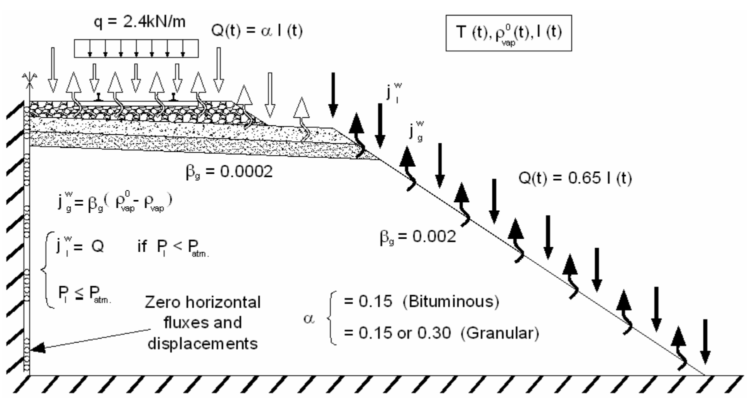


Fig. 4 – Boundary flow rate conditions.

In order to understand the influence of cross section geometry in the hydraulic boundary conditions, the Original design was also compared with alternative geometric solutions: Extended and Lower.

4.3 Conclusions – Performance of the Original, Lower and Extended designs

The comparison between the bituminous and granular sub-ballast solutions and their different designs was made by recurring to the evolution of liquid saturation, vertical displacements and their amplitude in several points of control. Figure 5 presents the Liquid Saturation inside the embankments for the Bituminous and Granular cross sections at the end of the 5 years.

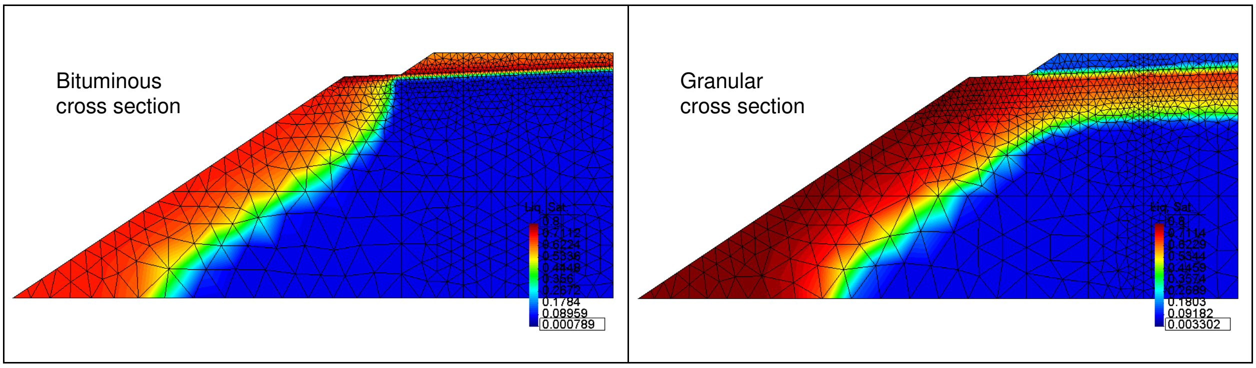


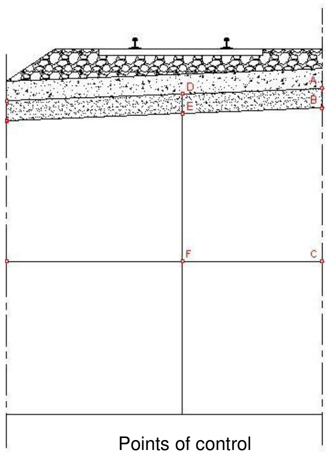
Fig. 5 – Liquid Saturation at the end of 5 years (Original cross sections ($\alpha = 15\%$))

Figure 5 shows the Bituminous sub-ballast working as a barrier against water infiltration. In terms of liquid saturation, bituminous sub-ballast helps maintaining low levels of the moisture content along the year in opposite to the granular solution.

Table 3 presents the ranges for qualitative reduction performed by the bituminous sub-ballast cross section taking the results from the granular one for each type of section as reference. As before, for evaluate the efficiency of the bituminous solution it was used the parameter named Bituminous reduction defined by Eq. (3).

Table 3 – Ranges for Qualitative reduction performed by bituminous sub-ballast Displacements

	Points	Vertical Displacement 1800 days (mm)		Average Amplitude of the Vertical Displacements (mm)	
		$\alpha = 15\%$	$\alpha = 30\%$	$\alpha = 15\%$	$\alpha = 30\%$
Original	A D	65 – 70	80	**40 – 70**	**50 – 70**
	B E	65 – 70	80	**30 – 60**	**40 – 70**
	C F	≈ 0	≈ 0	**60**	**60**
Extended	A D	65 – 70	80	**40 – 70**	**50 – 70**
	B E	60 – 70	80	**50 – 70**	**60 – 80**
	C F	0 – 60	0 – 65	**30 – 60**	**20 – 60**
Lower	A D	80 – 90	90 – 95	**40 – 75**	**60 – 80**
	B E	85 – 90	95	**55 – 70**	**60 – 75**



Points of control
(Granular cross section)

The results registered in Table 3 allow the following comments:

- considering the same percentage of rain infiltration, 15%, for both sub-ballast solutions (bituminous and granular), it is possible to assume a range of reduction in the amplitude of vertical displacements from 45% to 70% (average values) performed by the bituminous sub-ballast;
- if the differences in drainage behaviour between both sub-ballast solutions are taking into account (bituminous and granular percentages of rain infiltration of 15% and 30% respectively), then a range of reduction from 50% to 75% (average values) is obtained by using the bituminous sub-ballast;
- analyzing the ranges of reductions performed by bituminous sub-ballast in terms of displacements at the end of the 5 years (under poor drainage conditions), it can be assumed reductions from 60% to 90% or 80% to 95% (average values) on the upper points for rain's infiltration percentages of 15% and 30% respectively;

The efficiency of the Bituminous sub-ballast solution also depends on the geometry of the embankment. A relation ($\Delta_{Extended}/\Delta_{Original}$ or $\Delta_{Lower}/\Delta_{Original}$) between the displacements found for each case ($\Delta_{Extended}$ and Δ_{Lower}, respectively) with those found in the original embankment ($\Delta_{Original}$) allows the comparison of their performance (Figure 6).

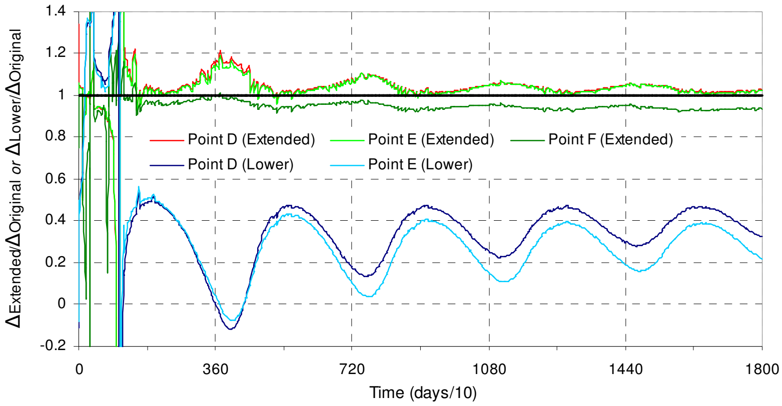


Fig. 6 – Relation between original and alternative designs using bituminous sub-ballast

A comparison in terms of displacements registered at the end of the 5 years between the different designs using the bituminous sub-ballast solution shows that Lower design may perform a reduction higher than 50% on the upper points when compared with the Original one; no significant differences are observed between Original and Extended designs.

5. CONSIDERATIONS REGARDING THE GEOMETRIC QUALITY OF HIGH-SPEED RAILWAY TRACK

Maintenance expenses should be kept as low as possible and it depends on two fundamentally different classes of parameters: on the one hand geometrical parameters, the degradation of which is usually reversible; and on the other, mechanical parameters which in most cases cannot be restored without parts replacement (rails, fastenings, sleepers, welds, etc.). Geometrical parameters, however, degrade much faster than mechanical parameters.

Deviations between the real and theoretical values of geometrical track characteristics are termed *track defects* and their restoration is done through track maintenance mainly tamping works. In terms of comfort and safety, the geometrical quality of the railway track must fulfill some requisites quantified through specific parameters. Among them, in high-speed lines the longitudinal defects (distance defined by the difference of level between the rolling surface and the theoretical plan) are known to be the principal factor in determining the magnitude of the track tamping needs. According to SNCF (Société Nationale des Chemins de Fer) criteria, Table 4, shows the limits/tolerances in terms of Longitudinal defects for corrective operations required to guarantee an adequate geometric quality:

Table 4 – Limits for corrective operations – Influence of speed in Geometric Quality.

Parameter	Conventional	High-Speed		
	Reference Value	Current defect	Isolated defect	Standard Deviation in 300 m (mm)
Longitudinal defect	±5 mm (base 10 m)	±2.5 mm	±5 mm	0.8/1.0

The distribution of the various types of defects is of a stochastic nature and can be approximated with the aid of spectral analysis. Thus, it can be calculated for each class of defects, their frequency of occurrence, the wavelength to which they correspond, their relation to train speed, etc. The control of the displacements and their annual amplitude plays a central rule when high-speed railway lines are considered.

From a trackbed design perspective, the use of a bituminous sub-ballast as an alternative to the granular one, may improve the geometric performance of the railway infrastructure and contribute to an effective reduction of track maintenance needs. Assuming that the bituminous sub-ballast may perform a reduction in the order of 50% in terms of the displacements and their annual amplitude, it would imply a reduction for the longitudinal defects shown by Figure 7. Considering $LD^{gran.}_{current}$ the current value for longitudinal defect registered for the granular sub-ballast and $\Delta_{gran.}$ its seasonal amplitude, the correspondent values registered for the bituminous sub-ballast would be $LD^{bit.}_{current} = 0.5 \times LD^{gran.}_{current}$ and $\Delta_{bit.} = 0.5 \times \Delta_{gran.}$. Lower displacements and amplitudes performed by the bituminous sub-ballast in each cross section would lead to a reduction in Current ($LD_{current}$) and Standard Deviation ($\delta_{bit.} < \delta_{gran.}$) values for the longitudinal defects.

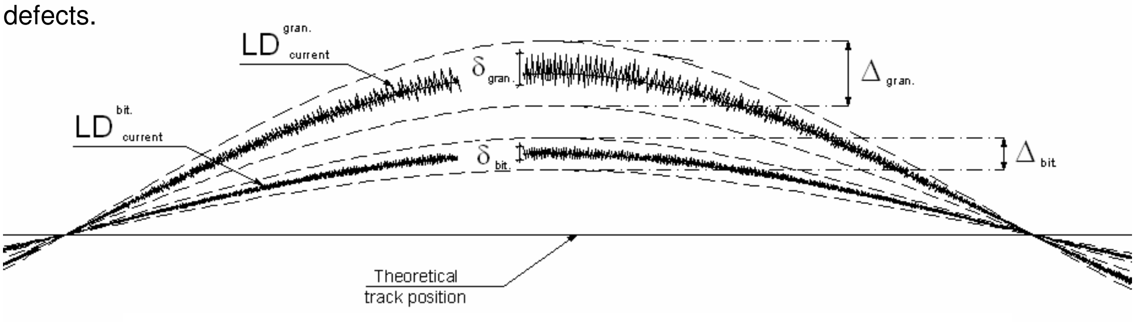


Fig.7 – Register for Longitudinal defects: Granular *vs.* Bituminous

CONCLUSIONS AND CURRENT RESEARCH

This work discussed the possible improvements of conventional high-speed trackbed design. The possible interest of using a bituminous sub-ballast layer was analyzed and, from a theoretical point of view, a comparison between its performance and the granular one was carried out.

From the different approaches developed, the main conclusion is that bituminous sub-ballast may allow an important reduction in the seasonal vertical displacements, up to 50% or more under poor drainage conditions.

The amplitude of the displacements is related with their cyclic nature and leads to fatigue problems in the infrastructure as well as an increase in the number of maintenance operations once these seasonal recoverable displacements may be considered irreversible (plastic) for small periods of time (associated to maintenance intervals).

The results presented in the study correspond to an unfavourable situation of poor drainage conditions. Further research will aim at introducing on the model different drainage assumptions in order to get deepen on the evaluation of the impact of using bituminous sub-ballast on track deformation.

In a parallel way, in-situ measurements on both the variations on relative humidity and the resulting settlement of railway embankments should help to confirm the interest of the bituminous layers for high-speed tracks. In relation to this issue, Spanish Railways are proceeding to the instrumentation of a trial with bituminous sub-ballast in a section of the Barcelona – French Border high-speed line under construction, to evaluate its behaviour and compare it with sections with granular sub-ballast. The interpretation of these results together with ongoing research on the assessment of the geometric quality of tracks with bituminous sub-ballast (in Italy) will help quantifying in what conditions this solution can contribute to an effective reduction of track life cycle costs.

ACKOWLEDGMENTS

The author would like to thank the support and collaboration of Prof. P. F. Teixeira (supervisor). Special thanks to Prof. M. Rafaela C. (co-supervisor) for the constant availability and continuous attendance. The assistance provided by Dr. S. Olivella with the finite elements program CODE_BRIGHT is greatly acknowledged.

BIBLIOGRAPHIC REFERENCES

Alonso, E. E., Gens, A. And Josa, A. (1990) – *A constitutive model for partially saturated soils*, Géotechnique 40, No. 3, pp. 405-430, 1990.

Alonso, E. E. – *Suction and moisture regimes in roadway bases and subgrades*, Simposio Internacional: Drenaje Interno de Firmes y Explanadas, pp. 57-104, 1998.

Fredlund and Rahardjo (1974), Fredlund, D.G. Rahardjo, H., Soil mechanics for unsaturated soils, Wiley, 1974.

Olivella, S. – Manual user for CODE_BRIGHT - thermal-hydro-mechanical finite elements program, 1994, 1996; DIT - Universitat Politécnica de Catalunya, 2000.

Teixeira, P. F., A. López Pita, C. Casas, A. Bachiller and F. Robusté (2006) "Improvements in high-speed ballasted track design: benefits of bituminous sub-ballast layers". Transportation Research Record: Journal of the Transportation Research Board Nº 1943, 2006, pp.43-49, ISSN 0361-1981, ISBN 0-309-09425-9.