Material Balance Calculations

The unfashionable tool

Victor Alcobia  April 1st, 2014
OBJECTIVES

• Practical recommendations for Material Balance calculations, MB
• Only case of black oil reservoirs
• Not considered the cases of volatile oil, gas or condensate gases reservoirs
• The derivation of the MB equation not presented here
  Schilthuis (1936)
Introduction
• MB calculations lost importance following the increasing use of numerical simulation

• **BUT Only** MB calculations allow for the
  - Validation of the Oil In Place calculation (STOOIP)
  - Estimation of the volumes of water influx, $W_e$, & gas-cap volume, $m$ (if any)
  - Identification of the dominant Production Mechanisms
“History matching by simulation can hardly be regarded as an investigative technique but one that merely reflects the input assumptions of the engineer performing the study“ (L. Dake)

- MB calculations are a complement to numerical simulation
Introduction

• For the MB calculations, we need to know
  ✓ The production (oil, gas and water),
  ✓ Average pressures &
  ✓ PVT Data (oil, gas & water) + Rock Compressibility

• With these data, we can estimate
  ✓ The STOIP,
  ✓ The water influx (We),
  ✓ The gas-cap fractional volume (m) &
  ✓ Identify the active production mechanisms
Introduction

• MB results + geology + geophysics + petrophysics + SCAL $\Rightarrow$ numerical simulation model.

• Numerical model validation = matching of observed pressures & productions (saturations).
  • Once model matched $\Rightarrow$ production and pressure forecasts

• Numerical model is not a diagnostic tool of production mechanisms, STOOIP or water influx (We).
  • These are input of the simulator
Material Balance Equation
Basic Form MB Equation

- The expanded form – equation of Schiltuis (1936)

\[
\text{Production (oil + gas + water)} = \text{Oil expansion, Free gas, pores and water Initially in place} + \text{Water Influx}
\]

- Havlena y Odeh (1963)

\[
F = N (Eo + m*Eg + Efw) + We*Bw
\]

\[
F = Np [Bo + (Rp – Rs) Bg] + Wp*Bw
\]

\[
N = \text{STOOIP}
\]

\[
Eo = (Bo – Boi) + (Rsi – Rs) Bg
\]

\[
m = \frac{\text{[gas-cap HCPV]}}{\text{[Oil HCPV]}}
\]

\[
Eg = Boi [(Bg / Bgi) – 1]
\]

\[
Efw = (1 + m) Boi [(cw*Sw + cf) \Delta p] / (1 – Swc)
\]

\[
We = \text{Water influx volume}
\]
• F represents the production at reservoir conditions,

• Sometimes Bt & Bti are used instead of Bo & Boi.
  • Bt = Bo + Bg * (Rsi – Rs)
  • Bti = Boi

• Eo => oil + solution gas expansion

• Eg => gas-cap expansion and

• Efw => Pores & interstitial water expansion

• Eo, Eg y Efw represent the drive indices.
• Eo, Eg, Efw & the water influx (We) can be normalized so that they sum up to 1

• 4 “Drive Indices” can then be defined:

  1. “Depletion Drive Index”

      \[
      DDI = \frac{N \cdot E_o}{F - Wp \cdot Bw}
      \]

  2. “Segregation (Gas-cap) Drive Index”

      \[
      SDI = \frac{N \cdot m \cdot E_g}{F - Wp \cdot Bw}
      \]
Basic Form MB Equation

• 4 “Drive Indices” then can be defined:

3. “Water Drive Index”

\[
WDI = \frac{W_e - Wp \ast B_w}{F - Wp \ast B_w}
\]

4. “Formation & Connate Water Drive Index”

\[
CDI = \frac{N \ast (1 + m) \ast E_{fw}}{F - Wp \ast B_w}
\]

\[
DDI + SDI + WDI + CDI = 1
\]
• MB equation is simplified if there is no active aquifer nor initial gas-cap.

• Efw *could* be dismissed if there is a gas-cap but this hypothesis must be verified => Significant errors.

• Gas or water injection (if any) must be deducted from the produced volumes

• Pressure appears *explicitly* in the Efw index, but pressure is required to estimate volumetric factors and water influx.

• **Representing the MB equation as a straight line facilitates the identification of the production mechanisms**

• And allows for the calculation of unknown parameters as the STOOIP or the water influx from an aquifer
Solution of the MB Equation
Solution of the MB Equation

- The MB parameters can be divided into two categories

<table>
<thead>
<tr>
<th>Known parameters</th>
<th>Unknown parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Np</td>
<td>N</td>
</tr>
<tr>
<td>Rp</td>
<td>We</td>
</tr>
<tr>
<td>Wp</td>
<td>m</td>
</tr>
<tr>
<td>cw</td>
<td>pressure</td>
</tr>
<tr>
<td>Swc</td>
<td>Bo, Rs, Bg</td>
</tr>
<tr>
<td>Bw</td>
<td>cf</td>
</tr>
</tbody>
</table>

- Np is well known but Rp (produced gas) and Wp are not always rigorously measured especially if they do not have commercial value.

- In this case, these two parameters will move to unknown parameters and the MB calculations become impossible.
• The volumetric estimate of the STOOIP is a reference, but MB estimates the “effective” STOOIP (drained by current wells).
• The STOOIP of an isolated block without any wells will never be "seen" or detected by the MB.
• Parameter with greater uncertainty = influx of water, $We$. Can be detected but its size is rarely known.
• Same for $m$ (ratio of gas-cap to oil volumes).

Often however the presence of a gas-cap can be detected and the gas-oil contact can be determined (with RFT/MDT, well tests, electrical logs and PVT).
• Pressures are measured (flowing or closed wells) = known parameters

• But the average pressures used in the calculations are estimated in two steps,
  ✓ Well test interpretation for each well to determine the average pressure in the drainage area and the conversion of that average pressure to the reservoir Datum.
  ✓ Estimation of the reservoir (all wells) average pressure at the Datum from the average pressures of each well
Solution of the MB Equation

• The same can be applied to PVT properties (Bo, Rs y Bg) as a result of a synthesis of several laboratory well tests at a Datum.

• Pore compressibility is often neglected (Schiltuis did it) but this can lead to significant errors, particularly above bubble point pressure with no free gas.

(Note: cf ↓ when pore pressure ↓)
There are two ways to "solve" the MB equation

1. Calculate pressures from production data + PVT + Assumed Unknowns and compare them with the measured pressures (iterative process. Better to use a spreadsheet)

2. Calculate the unknown parameter from the measured pressures + production data + PVT and compare with the appropriate Havlena & Odeh graphics shown later (*LP Dake*)

- There is no “conventional” solution for the MB equation, considering the number of unknown parameters.
- All depends on the parameter(s) to be estimated (STOOIP, We, m, cf or pressure).
Necessary conditions for MB Application
Necessary Conditions

• There are no sufficient conditions to apply the MB equation but there are two necessary conditions,
  1. Existence of data (pressure, production and PVT) adequate in terms of amount and quality
  2. To be able to define an average pressure decline for the reservoir.
• A reservoir with high hydraulic diffusivity (high $k/\phi \mu c$) presents more uniform values of pressure. Low diffusivity implies greater differences of pressure between wells.
• Different pressures in several regions do not prevent MB calculations. But including isolated blocks with different pressure “regimes” should not be done.
Data Validation and Calculation of Averages

Production Data
Production Data

• Production data of oil, water and gas to be used are the Net values = [Produced values – Injected values]
• Create graphics of productions and gas-oil and water-oil ratios to detect anomalies.
• GOR and WOR tend to increase with time in each well, except if coning occurs.
• Coning can be detected with some basic evaluations,
  ✓ If the GOR or Water Cut depends on oil rate
  ✓ If GOR or Water Cut decrease after a temporary shut in of the well
  ✓ Use of graphics of Chan (water “coning”).
• GOR tend to increase with time except when the wells more affected by gas are shut.
• GOR cannot be larger than solution gas ratio if reservoir pressure is above bubble point pressure.
• GOR cannot also be much lower than solution gas ratio.
• GOR reports can be wrong in cases of wells with artificial gas-lift.
Data Validation and Calculation of Averages

Pressure Data
Pressure Data

• Steps for the calculation of Average Pressures in the reservoir:
- ✓ Calculate average reservoir pressures in the drainage area of each well
- ✓ Convert these pressures to Datum depth
- ✓ Calculate average reservoir pressures at Datum

• Reservoir pressures do not need to be uniform in all the reservoir but all the wells must drain the same block (ensure pressure communication)
Pressure Data

- To evaluate the degree of communication between wells:
  
  Build graphics with well average pressures versus time

![Graphs showing pressure vs. time for cases (a) and (b)]

Case (a) = good balance, case (b) lack of balance.

- The lack of balance does not prevent the application of MB but it is important to ensure than all wells belong to the same block.
Preliminary diagnostic

“The Opening Move” (LP Dake)
Preliminary diagnostic

- Identification of active production mechanisms => parameters to be estimated.
- Plot Pressure versus Np
  When $P > P_b$ (oil expansion) => straight line with a slope proportional to the “connected” STOIP
- First action = determine if there is influence of an aquifer or expansion of a gas-cap.
• If a reservoir has no gas-cap but might be influenced by water influx from an aquifer, the MB equation is simplified as follows,

\[ F = N (E_o + E_{fw}) + W_e * B_w \]

• This can be transformed into

\[ F / (E_o + E_{fw}) = N + W_e * B_w / (E_o + E_{fw}) \]

• Two unknowns \( N \) and \( W_e \) (right side of the equation), but the left side is easy to calculate.

• Plot the left side versus \( N_p \) [or time or \( \Delta p \) (pressure decline)]

Note: the plot originally proposed by Campbell was \( F/E_t \) versus \( F \)
Preliminary diagnostic

Plot $F/(E_o+E_{fw})$ versus $N_p$ => graphic of Campbell

Note: If $E_{fw}$ is negligible => plot $F/E_o$ versus $N_p$ (Beware)
• If no water inflow => red dots on top of horizontal (green) line = STOIP (N).

• STOIP = minimum value of $F/(E_0+E_{fw})$. OK if reservoir produces steadily & if enough pressure measurements at the beginning of field life.

• If points are above the horizontal line => Additional energy factor (aquifer, gas cap expansion and / or variable pore compressibility (compaction).

• The aquifer is strong if calculated values of $F/(E_0+E_{fw})$ diverge from the “N” line. If values increase first & then decrease, it may indicate the presence of a smaller aquifer.
Preliminary diagnostic

- In cases where there is a well-defined gas cap,
  \[ \frac{F}{(E_o + m\cdot E_g + E_{fw})} = N + \frac{W_e \cdot B_w}{(E_o + m\cdot E_g + E_{fw})} \]

- Similar plot, \( \frac{F}{E_t} \), to evaluate “additional” Energy

F/Et Vs. Np

- Observed Pressures
- Et = Eo + mEg + Ef,w
- N (STOIIP)
• Calculated values of $F/(E_0+E_{fw})$ or $F/E_t$ cannot decrease with $N_p$ (or time) if data are reliable.

• If it occurs => Possible communication with another reservoir.

• Decrease of $F/(E_0+E_{fw})$ or $F/E_t$ => production "transferred" to a neighbouring reservoir.

• Increase of $F/(E_0+E_{fw})$ or $F/E_t$, if there is no initial gas-cap, neither the water influx is credible => "transfer" of fluids from another reservoir.
Preliminary diagnostic

Plot Drive indices as a function of Np to determine their importance throughout the reservoir life & check if they sum up to unity (if not => possible data inconsistency). Beware Commercial software do normalise indices.
Production Mechanisms

Reservoirs with Volumetric Depletion
• **NO** initial gas-cap or aquifer influx.

• When Reservoir Pressure > Bubble Point Pressure

• MB equation is simplified,

\[ Np \ Bo + Wp \ Bw = N Boi \ c_{eff} \ \Delta p \]

Where \( c_{eff} \) (Effective compr.) = \( \frac{Co \cdot So + Cw \cdot Sw + Cf}{1 - Swc} \)

• If \( Wp = 0 \) the Recovery Factor from initial pressure to \( Pb \) can be estimated as follows,

\[ \frac{Np}{N} = \text{Recovery Factor} = \frac{Boi \cdot c_{eff} \cdot (Pi - Pb)}{Bo} \]

\[ FR \approx c_{eff} \cdot (Pi - Pb) \approx Co \cdot (Pi - Pb) \]
• **NO** initial gas cap or water influx.

• Reservoir Pressure > Bubble Pressure

• It can also be written

\[ P \approx P_i - \left[ \frac{1}{(Co\cdot N)} \right] \cdot N_p \]

A graphic of pressure versus \( N_p \) is a straight line (assuming that \( Co \) is constant) with a gradient inversely proportional to \( Co \cdot N \).

The larger \( Co \) (oil compressibility) or \( N \) (STOIP) the lower the pressure drop gradient.
• **NO** initial gas cap or water influx.

• Reservoir Pressure < Bubble Pressure

• The MB equation can be "solved" using the methods of Muskat or Tracy - Turner (See L Dake).

• The equation for this production mechanism is

\[ F = N (E_o + E_{fw}) \]

• If \( E_{fw} \) is negligible, the equation is simplified

\[ F = N E_o \]
Reservoirs with Volumetric Depletion (4)

• NO initial gas cap or water influx.
• Reservoir Pressure < Bubble Pressure
• Plot $F$ versus $(E_o + E_{fw})$

[+ F vs Eo]

For Fluids Expansion
$F = N(E_o+E_{fw})$
• NO initial gas cap or water influx.
• Reservoir Pressure < Bubble Pressure
• The STOIP corresponds to the gradient of the straight line.
• In the previous figure, the points (red dots) are above the line because there was a water influx.
• If the water influx had not been detected, the calculated value for the STOIP would have been higher than it actually is.

This shows the importance of identifying the active production mechanisms in a reservoir.
Mechanisms of Production

Reservoirs with Water Influx
Reservoirs with Water Influx (1)

- If the reservoir has a water influx and NO initial gas cap, the equation is the following

\[ F = N (E_0 + E_{fw}) + W_e B_w \]

- If \( E_{fw} \) is negligible the equation is the following

\[ F = N E_0 + W_e B_w \]

- \( B_w \) is often negligible (value \( \sim 1 \))

- The validity of neglecting \( E_{fw} \) should be evaluated in each case (it can generate significant errors in calculations).
• Plot \( \frac{F}{(E_0 + E_{fw})} \) versus \( \frac{W_e B_w}{(E_0 + E_{fw})} \)

✓ More simplified, \( \frac{F}{E_0} \) versus \( \frac{W_e}{E_0} \)

For water drive
\( \frac{F}{(E_0 + E_{fw})} = N + \frac{W_e}{(E_0 + E_{fw})} \)

Reservoirs with Water Influx (2)
If We estimate is correct, the relationship between the two variables is a straight line,

- If We is under-estimated => Line goes up and
- If We is over-estimated => Line goes down.

If the geometry is incorrect (radial or linear aquifer for example) the line could have the shape shown in the figure.

If the calculations of water influx are made

- separating the constant of influx $C = 1.119 f \phi h c r_o^2$
- from the solution of Van Everdingen and Hurst for the diffusivity equation $[\Sigma \Delta p Q_D(\Delta p, r_D)]$
- the slope of the straight line is $C$ and not $45^\circ$ (grad. = 1).
• Calculations of water influx = Method proposed by Hurst and Van Everdingen.
• There is an approximate solution, for the case of the unsteady-state regime, proposed by Fetkovitch.
• Fetkovitch calculations are simple and easy to apply while the calculations of Van Everdingen and Hurst are tedious.
• Fetkovitch calculations are implemented in most commercial numerical simulators.
• The two methods of calculation are described in many reference texts of reservoir engineering.
Mechanisms of Production

Reservoirs with Gas Cap
Reservoirs with Gas Cap

• If the reservoir HAS initial gas cap but NO water influx
  \[ F = N (E_o + m \cdot E_g + E_{fw}) \]

• If E_{fw} is negligible (reasonable as \( c_{gas} \gg c_w \) and \( c_{gas} \gg c_o \))
  \[ F = N (E_o + m \cdot E_g) \]

• Alternatively if you want to determine \( m \) and \( N \)
  \[ \frac{F}{E_o} = N + m \cdot N \cdot E_g / E_o \]

• These two equations provide two alternatives for the calculations of reservoirs with gas cap.
Reservoirs with Gas Cap

- Plot $F$ versus $[E_o + m*E_g + E_{fw}]$
  - Simpler form => $F$ versus $[E_o + m*E_g]$
Reservoirs with Gas Cap

• The straight line gradient => STOOIP (N).
• Straight line always passes through the origin of the axes (important for the analysis).
• If the estimation of "m" is correct, the relationship between both variables is a straight line
  ✓ If “m” is under-estimated, the line goes up and
  ✓ If “m” is over-estimated, the line goes down
  ✓ If the reservoir has a water influx, the points are going up regardless of the value of "m".
• This shows again the importance of a proper evaluation of the active production mechanisms.
Reservoirs with Gas Cap

- Odeh and Havlena considered that only had limited success in determining the volume of the gas cap that needs exceptional accuracy of data used (pressures in particular).

- Alternative graphic is $F / E_o$ versus $E_g / E_o$.
  Plot slope = $m*N$. Value of ordinate = $N$

- The first plot has the advantage that the line has to pass necessarily through the origin

- While in this case, the ordinate is the value of $N$ (unknown).
Mechanisms of Production

Reservoirs with Gas Cap and Water Influx
• This is the most complex of all.
  \[ F = N (E_o + m*E_g + E_{fw}) + W_e*B_w \]
• If \( E_{fw} \) is removed (this assumption should always be verified before being applied).
• Plot \( \frac{F}{(E_o + m*E_g)} \) versus \( \frac{N + W_e*B_w}{(E_o + m*E_g)} \)
• \( B_w \) can be neglected if it is close to 1.
• The interpretation of this graphic is similar to the graphic used for the water influx with the significant difference that there is an additional parameter to determine \( m \).
Havlena and Odeh recommend differentiating MB equation with respect to pressure and manipulate the equation in order to eliminate “m”

\[
\frac{FE'_g - F'E_g}{E_o E'_g - E'_o E_g} = N + \frac{W_e E'_g - W'_e E_g}{E_o E'_g - E'_o E_g}
\]

\( \dot{\cdot} \) = derivative with respect to pressure

Plot the left side of the equation versus the second part of the right side of the equation. Same interpretation as for water influx.

Once N and We determined in this figure, m can be calculated from the MB equation.
Conclusion
Conclusions / recommendations

• Identify production mechanisms prior to numerical simulation.

• To identify mechanisms => use Havlena and Odeh plots

• Dake thought difficult to develop a software to cover all situations for which MB calculations are applied.
THANK YOU VERY MUCH

(b)

F (MMrb)

(E_o + mE_g) (rb/stb)

x m = .4
○ m = .5
• m = .6

correct straight line for N = 115 MMstb.