PhD Program in Transportation

Simulation of Land Use-Transportation Systems

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Lecture 2
Location Choice Theories
Location Choice Theories

- **Introduction**
- The Urban Activity and Transportation Systems

**Spatial Competition Theories**
- Simplifying Assumptions
- Central Place Theory (*Christaller*)
- Classic Firm Location Theory (*Hotelling*)
- Agglomeration and Central Place

**Economic Models of Location Choice**
- Problem definition
- Bid Rent (*Alonso*)
- Household location choice (*Lerman*)
- Stochastic Bid Rent (*Ellickson*)
- Bid Choice (*Martinez*)
Transportation and urban form are fundamentally linked.

- How we build our city directly determines travel needs, viability of alternative travel modes, etc.

- Transportation, in turn, influences land development and location choices of people & firms.
Introduction

Accessibility to Activities / Mobility

Participation in Social, Recreational & Economic Activities
- Economic Productivity

Greenhouse Gas Emissions
Air Pollution
Urban Sprawl
Congestion
Accidents
Sedentary Lifestyle

Global Climate Change
Respiratory & Other Diseases
Loss of Farmland, Natural & Urban Habitat
Loss of Productivity & Leisure Time; Stress
Injuries/Deaths
Productivity/Property Loss
Obesity, Other Health Problems

QUALITY OF LIFE

PhD in Transportation / Simulation of Land Use-Transportation Systems
Introduction

r **Mobility:**
- “the quality or state of being mobile” (Picket et al. 2000);
- “the ability to move or to be moved; facility of movement” (Oxford English Dictionary);
- “the potential for movement, the ability to get from one place to another” (Hansen 1959; Handy 1994).

r **Accessibility:**
- “the potential of interaction” (Hansen 1959);
- “is the ability to reach desired goods, services, activities and destinations – together called opportunities” (TDM web-page).

r Just as an automobile is a machine for mobility, a city is a machine for accessibility (Levinson, Krizek and Gillen 2005).
The Urban Activity and Transportation Systems

The Urban Activity and Transportation Systems

LONG-RUN ACCESSIBILITY INTERACTIONS
- Land Development
- Transportation Network
- Location Choice
- Automobile Ownership

SHOT-RUN ACTIVITY/TRAVEL INTERACTIONS
- Activity Schedules
- Travel Demand
- Activity Patterns
- Network Flows

The Urban Activity and Transportation Systems

Key System Elements
T – transport system
A – activity system
F – flows & transport system performance

System Interactions/Feedbacks
I  Market demand-supply interactions determine flows & system performance
II Government, public & private service providers respond system demand & performance
III System performance (accessibility) influences activity system markets

Firms can compete along a number of dimensions:

- Price
- Service
- Quality
- Location

One theme in urban economics is the examination of how firms compete spatially; i.e., how choice of location can be used to maximize sales/profits.
Spatial Competition Theories

- Simplifying Assumptions
- Central Place Theory (Christaller)
- Classic Firm Location Theory (Hotelling)
- Agglomeration and Central Place
Simplifying Assumptions

- A single homogeneous good, \( g \), is being sold.
- The good is sold at a fixed unit price \( p \).
- Travel in any direction is possible at uniform cost \( r \) [€/km].
- The total cost to the buyer is the price of the good plus the transportation required to travel to the store to purchase the good:

\[
C = p + r \times d
\]

\( d \) = distance travelled
\( r \) = transport cost per unit distance

Assumptions:

- Stores are free to locate anywhere on a flat, featureless plane.
- Uniform population density over the plane.
- Free entry/exit of firms into/out of the market.
- Production functions (costs) are the same for all firms at all locations.
Simplifying Assumptions

- A key behavioral assumption is that consumers will always travel to the nearest store to purchase the good. (Since the good is the same quality and price everywhere, it would be irrational to travel farther)

- Similar purchasing power of all consumers;

- Also, “home-based”, “single-purpose” trips/purchases only (i.e., no trip chaining our multi-purpose, multi-stop trips).
Central Place Theory

- A classical theory of spatial competition is provided by Central Place Theory (Christaller, 1964; Lösch, 1957).

  A central place can be defined as:
  “a place whose prime function is the provision of a wide range of goods and services to a dispersed population around it.”

- Originally developed to explain the size and spacing of towns & cities in terms of their role as suppliers of goods/services to a rural hinterland;
- CPT has also been used to explain the size and spacing of centers within large metropolitan regions.
Central Place Theory

Demand Function Assumption

CPT assumes that an individual’s demand for \( g \) is a function of its total cost, \( C \).

\[
C = p + r \times d
\]

- \( X \) = maximum demand for \( g \) (occurs when \( d=0 \))
- \( Y \) = demand for \( g \) at distance \( d' \) from the store
- \( C^* \) = total cost at which demand goes to zero
- \( D = (C^* - p)/r \) = maximum distance that a consumer is willing to travel to purchase \( g \) = range of good \( g \)

\( Q = \text{demand for } g \)
Sales can thus be expressed as a function of distance:

\[ Q = h \left[ C = p + r \times d \right] = f \left[ d = \frac{(C - p)}{r} \right] \]
Central Place Theory

Given a uniform population density, $\rho$, and travel in all directions at constant unit cost, one can integrate this distance-based demand function to compute the total sales at a given location:

$$S = \text{sales} = \int_{\theta=0}^{2\pi} \int_{x=0}^{D} f(d)\rho \, dx \, d\theta$$

For a store to remain in business:
$$S > S_{\text{min}} = \text{sales threshold for good } g$$
Central Place Theory

Multiple Firms & Market Equilibrium

- Given free entry into the market, firms will continue to enter the market as long their sales exceed the threshold $S_{\text{min}}$.
- CPT assumes that firms will only locate as close to one another as they need to in order to maximize their profit.
- As a result, firms “pack together” until they form a hexagonal lattice, each with a market area that generates sales of exactly $S_{\text{min}}$.
- According to the CPT, this is the market equilibrium distribution.
- It also minimizes the average travel distance of the consumers.
**Central Place Theory**

**Market Equilibrium**

Firm with maximum market area & sales

Unfulfilled demand: Induces additional firms to enter the market.

As more firms enter the market, they locate more closely together. Unfulfilled demand, however, still exists with circular (non-overlapping) market areas.

Eventually, firms push closer together, encroaching on each other’s market areas. Sales per firm go down, but firms will continue to enter the market until $S = S_{min}$ for all stores. At this point, no incentive exists for stores to enter the market and an equilibrium is established. With customers always going to the closest store, this results in hexagonal market areas.
Multiple Goods

- Many goods, \( g = 1, \ldots, G \) are actually sold in an urban region.

- Each good \( g \) will have a range \( D_g \) and threshold \( S_{\min,g} \).

- Goods can be arranged in a hierarchy in terms of their range, with low order goods having small ranges and thresholds and high order goods have large ranges and thresholds.

- Each good is assumed to have an equilibrium hexagonal lattice distribution of stores.
Hierarchical Distribution of Central Places

The final major assumption of CPT is that any given center not only sells goods corresponding to its own rank in the hierarchy, but it also sells all goods of lower order.

As a result, the hexagonal lattices overlap with one another, with the centers of the hexagons at one level defining the vertices of the hexagons at the next highest level.
Central Places Theory

Hierarchical Distribution of Central Places

- Low order market boundary
- Medium order market boundary
- High order market boundary
  - Low order center
  - Medium order centre
  - High order centre
Critique of CPT

- A hierarchy of centers obviously exists within an urban area.

- Retail location obviously depends upon access to a local market.

- Particularly for lower-order goods, a “uniform” spacing of stores can occur:
  - Local supermarkets
  - Newspapers stands
  - Pharmacies
  - …
Critique of CPT

- However:
  - Hexagonal lattice is clearly highly abstracted.
  - Why should multiple levels of stores co-locate?
  - Strong behavioural and other assumptions referent to both consumer and retailer behaviours.
  - Model of structure, not process: how do the central places actually form?
Classic Firm Location Theory

Hotelling’s Ice-Cream Salesmen on a Beach

- Hotelling (1929) argued that stores compete spatially very differently than is assumed in CPT.

- Same starting assumptions as CPT, but a different locational behavior of stores is assumed.

- To illustrate, consider a one-dimensional problem of two ice-cream salesmen with moveable carts deciding where to locate along a beach on a hot summer’s day.

- Also, to begin, assume that demand does not depend on distance.
Classic Firm Location Theory

Beach of length $D$
Uniform distribution of customers
Customers visit the closest salesman.
Demand inelastic with respect to distance

Where will the two salesman located their carts?
The CPT solution would locate the salesman at the $\frac{1}{4}$ & $\frac{3}{4}$ points along the beach. Each salesman gets 50% of the market and the average customer walk distance is minimized ($D/8$).

This is the outcome of a *cooperative game* in game theory terms.
Classic Firm Location Theory

- **Stackelberg Competition:**

  - Each salesman will try to capture a larger portion of the market;
  - This will continue until they reach the middle of the beach – no move can increase market share.
  - This is the outcome of a *non-cooperative game* in game theory (*Nash Equilibrium*).
  - Customer average travel distance is increased by this outcome (D/4).

![Diagram of Classic Firm Location Theory](image)
Classic Firm Location Theory

- Hotelling argues that this is a far more common locational strategy.

- Clearly we see this strategy in many instances:
  - Fast food restaurants
  - Shoe stores, etc.
  - Many other specialized store types (e.g., gas stations)
  - Food trucks
  - …
Agglomeration and Central Places

- How and why do central places arise?

- In addition to spatial competition, *agglomeration economies* often exist:
  - Greater visibility to customers if “centrally” located.
  - Better/shared infrastructure
  - Better accessibility
  - Comparison shopping
  - …

- Three types of sources to the foundation of *agglomeration economies*, according to Marshall (1920):
  - labour market pooling;
  - spillovers in the knowledge;
  - sharing inputs.
Transportation plays a role in creating central places by creating points of high accessibility:

- Ports / harbors
- Rail terminals / stations
- Freeway interchanges
- Airports
- Intermodal terminals
- Metro (or rapid transit) stations
- ...
Positive feedback effects exist that tend to encourage attractive places to grow and unattractive places to decline.

Activities tend to locate at points of competitive advantage. Many factors can define such an advantage, including:

- Points of high accessibility
- Co-locating with other activities

Central places emerge due to:

- Non-uniform distributions of people, jobs, etc.
- Non-uniform transport costs (accessibility).
- Non-linear, dynamic feedbacks
Economic Models of Location Choice

- Problem definition
- Bid Rent (*Alonso*)
  - Household location choice (*Lerman*)
  - Stochastic Bid Rent (*Ellickson*)
- Bid Choice (*Martinez*)
Problem Definition

- Firms and households can be assumed to locate in space so as to maximize profits and utility, respectively.

- They are not, however, free to locate wherever they want: they must compete with other potential occupants of a given location.

- All land is owned by someone, who also can be assumed to be profit/utility maximize and, hence, can be assumed to want to sell/rent the land to the highest bidder.
Bid Rent Theory

Assumptions (Alonso, 1964)

- Plane – flat and with uniform features.
- Travel in all directions feasible at uniform unit cost.
- A “city center” exists, with respect to which distances can be measured.
- All locations equidistant from the city center are evaluated as being the same.

A monocentric, radially symmetric city, which can be analyzed based on the single dimension of distance from the center.
Bid Rent Theory

Monocentric City

\[ d = \text{distance from city center} \]
\[ \theta = \text{angle from arbitrary reference radius} \]
\[ \rho(d) = \text{population density at distance } d \]
\[ \text{(assumed constant for all } \theta \text{)} \]

\[ P(d) = \text{population living distance } d \text{ from centre} \]
\[ = \int_{\theta=0}^{2\pi} \rho(d) \, d\theta = \rho(d) 2\pi \]

\[ P = \text{total population of the city} \]
\[ = 2\pi \int_{d=0}^{D} \rho(d) \, dd \]

\[ D = \text{distance of city boundary from the centre} \]
One-Dimensional City

Since all locations at the same distance from the center are equal, we can collapse our analysis down to the single dimension of distance, $d$, from the center.
Activity Size

- Firms and households do not occupy points in space, they occupy a finite amount of land.

- Thus, the location decision of a firm or household involves not just where to locate ($d$), but how much land to occupy at this location ($q$).

Note: The concept of consumption of land can be extended to the consumption of floorspace → a 3-dimensional city.
Bid Rent Theory

Firm Location

It is assumed that an individual firm will choose its location \((d)\) and size \((q)\), so as to maximize its profit \((\pi)\), where:

\[
\pi(d, q) = V(d, q) - C(V, d, q) - R(d, q)
\]  

[1]

Where:

\(V(d, q)\) = Volume of sales at location \(d\) with size \(q\)

\(C(V, d, q)\) = Operating and annualized capital costs of a store of size \(q\) at location \(d\) with sales volume \(V\)

\(R(d, q)\) = Rent (cost of land) at location \(d\) for a store of size \(q\)

\[= q \times Y(d)\]  

[2]

\(Y(d)\) = Rent function
Some Differences from CPT

- Amount of land consumed is a decision variable, along with location.
- Consumers not evenly distributed in space
- Stores are not equal: differentiated by size, volume sales and costs.
- Sales depend upon size as well as location.
- Land is not merely occupied, it must be purchased/rented.
Bid Rent Theory

Optimal Location for a Single Store

Given a distribution of existing stores, the optimal location and size of a single new store \((d^*, q^*)\) is given by solving the first-order optimality conditions:

\[
\pi' d = 0 \iff V_d - C_V V_d - C_d - q \left( \frac{Y}{d} \right) = 0 \tag{3}
\]

\[
\pi' q = 0 \iff V_q - C_V V_q - C_q - Y = 0 \tag{4}
\]

Note: subscripts denote partial derivatives

Note that in this case, \(Y(d)\) is assumed to be known and unchanged by the individual store’s decision (the store is a price-taker).
Bid Rent Theory

Bid Rent Function

- Alternatively, eqn. [4] can be solved for the optimal store size at any given location \( d \Rightarrow q^*(d) \).
- This expression can be substituted into [1] to yield a profit function \( \pi(d) \) which is only a function of location.

If we fix profit at some arbitrary amount \( \pi_0 \), we can rearrange [1] to solve for the rent the store would be willing to pay (or “bid”) for a given location so as to achieve profit level \( \pi_0 \):

\[
Y(d|\pi_0) = \frac{[V(d, q^*) - C(V, d, q^*) - \pi_0]}{q^*}
\]

This is the store’s bid rent function: the amount that it is willing to pay at various locations in order to be indifferent among locations, since they will all return the same profit level \( \pi_0 \).
A bid rent curve will exist for each level of desired profit.

The shape of the bid rent curve will vary depending on the sales and cost functions ($V$ and $C$).

Generally, however, for a monocentric city it is assumed that they will be decreasing functions of distance from the center.
Bid Rent Theory

Bidders:

- A store will always be happy to pay less than $Y(d|\pi)$, since this will mean higher profit; the bid rent curve represents the maximum it will pay so as to maintain a given profit level.

- Different firms will have different bid rent functions, reflecting their different sales ($V$) and cost ($C$) functions.

- Note that the bid rent function characterizes the demand for land at a given location.
**Bid Rent Theory**

**Land Owners**

- All land is owned by economic agents who are profit maximizers.
- In order to occupy a given parcel of land, it must be purchased or rented from the previous owner/occupant.
- Since purchase costs can always be annualized into an equivalent “rent”, we will simply speak of rent and landlords.
- In order to maximize profit, landlords will rent to the highest bidder.
- This represents the *supply* side of the land market.
Bid Rent Theory

Bidding Process

- Since different firms exist who are willing to pay varying amounts for a given location, the bidder that values the land the most will be willing to pay more than others and so will “out bid” the others for the land.
- Firms can also compete for a given location by lowering their target profit level (and so being able to increase their bid given fixed sales and operation costs).
- As a result, in a perfect market, landlords will be able to induce demanders to “bid up” the price of the land until they are all operating at the $\pi = 0$ level.
- This will be an equilibrium since no firm can bid any higher and stay in business.

Note: same assumption as in CPT
Bid Rent Theory

Equilibrium Bid Rent Surface

\[ Y(d) \]

- \( \pi=0 \) bid rent surface, activity 1
- \( \pi=0 \) bid rent surface, activity 2
- \( \pi=0 \) bid rent surface, activity 3
- Price of agricultural/undeveloped land
**Equilibrium Bid Rent Surface**

\[ Y(d) \]

- Activity 1 is high bidder & occupies these locations.
- Activity 2 is high bidder & occupies these locations.
- Activity 3 is high bidder & occupies these locations.

City boundary

Rural hinterland: no-one is willing to pay more for the land than it is worth for agricultural purposes.
2-D View of the Monocentric City

The equilibrium outcome of an Alonso-type monocentric city is homogeneous rings of activity. Each ring is occupied by the activity type that is willing to pay the most for locating at this distance from the city center.
Bid Rent Theory

The Monocentric City

- The city consists of concentric circles of homogeneous activity within each circle.
- “Higher order” activities (i.e., ones that are willing to pay more) occupy more central locations.
- Activities requiring more land will tend to locate at the periphery, where the unit price of land is cheaper.
- The city expands only to the point where uses other than agricultural are economically viable.
Bid Rent Theory

Residential Location Choice

- Households are also competing for land.
- In this case, they try to maximize their utility ($U$).
- Household bid rents are the rents that they are willing to pay to achieve a given utility level $U$.
- Results in the same outcome.
Bid Rent Theory

More Differences from CPT

- The demand for and supply of land are both explicit in the model.
- The outcome of the demand-supply interaction determines type and density of activity at each point in space.
- Land prices are explicit and endogenous within the model. Equilibrium prices are those that balance demand and supply.
- “Classic” (Alonso) Bid Rent assumes a single, dominant central place (the city centre). Multiple centres, however, can be generated by relaxing simplifying assumptions.
Bid Rent Theory

Multiple Centers

[Graph showing the relationship between willingness to pay and distance from the CBD for different household types and locations.]
Lerman (1976) : Residential Choice

- We have said that it can be assumed that households choose their residential location so as to maximize their utility, subject to the price they have to pay for a given location.

- This process can be modeled using random utility theory.
Household Location Choice

Random Utility Model of Location Choice

\[ U_{ith} = \text{Utility of a dwelling unit of type } t, \text{ located in zone } i \text{ for a household (hhld) of type } h \quad [i=1,\ldots,I; t=1,\ldots,T; h=1,\ldots,H] \]

\[ = V_{ith} + \Sigma_{ith} \quad [6] \]

\[ V_{ith} = \text{Systematic (observable/average) utility} \]
\[ \Sigma_{ith} = \text{Random (unobservable) utility} \]

\[ V_{ith} = \mathbf{\gamma}' X_{ith} \quad [7] \]

\[ \mathbf{\gamma}' = \text{Vector of parameters} \]
\[ X_{ith} = \text{Vector of explanatory variables (e.g., #people, income, #child, education, #rooms, building age, location)} \]

\[ P_{ith} = \text{Probability that hhld } h \text{ chooses dwelling unit of type } t \text{ in zone } i \]
\[ = P(U_{ith} \geq U_{i't'h}, \quad i' \neq i, j' \neq j; \quad i,i' \quad I; j,j' \quad J) \quad [8] \]
\[ = P(V_{ith} + \Sigma_{ith} \geq V_{i't'h} + \Sigma_{i't'h}, \quad i' \neq i, j' \neq j; \quad i,i' \quad I; j,j' \quad J) \]
\[ = P(\Sigma_{i't'h} - \Sigma_{ith} \leq V_{ith} - V_{i't'h}, \quad i' \neq i, j' \neq j; \quad i,i' \quad I; j,j' \quad J) \quad [9] \]
Household Location Choice

Location Choice

Various models can be derived from [9], depending on the assumption made concerning the distribution of the random error terms.

The simplest & most common assumption is that the Σ’s are all identically and independently distributed (iid) with a Type I Extreme Value distribution (often referred to as a Gumbel distribution).

This yields the well-known multinomial logit model:

\[
P_{ith} = \frac{\exp(V_{ith})}{\sum_{i'} \exp(V_{i't'h})}
\] [10]
Logit Residential Location Choice Models

- Logit residential location choice models of various types have for some time been the bulk of housing market models.
- Note that the price of dwelling type $t$ in zone $i$ is assumed to be one of the explanatory variables.
- This price needs to be endogenously determined, usually through a market clearing process that results in demand matching supply by zone and market sector (dwelling type).
Ellickson (1981)

Ellickson applied random utility theory to Alonso’s bid rent theory.

To simplify notation, let \( z = \{i, t\} \) = dwelling unit characterized by its location and type/attributes; \( z \) is now a vector of attributes characterizing a given dwelling unit.

Also let:

\[
\begin{align*}
  n & = n^{\text{th}} \text{ hhld of type } h \\
  N_h & = \text{number of hhlds of type } h \\
  b_n(z) & = \text{Bid of hhld } n \text{ for a dwelling with attributes } z \\
  & = \int_h(z) + \sum_{nh}
\end{align*}
\]  

[11]

Landlord of dwelling unit \( z \) will sell to the highest bidder.

For hhld type \( h \), this bid will be:

\[
B_h(z) = \text{Max}_n \quad N_h \quad b_n(z) = \text{Max}_n \quad N_h \quad [\int_h(z) + \sum^n_{*h}]
\]  

[12]

Where \( \sum^n_{*h} = \text{Max}_n \quad N_h \quad \sum_{nh} \)  

[13]
Under quite general conditions, $\varepsilon^*_h$ can be assumed to be iid Type I Extreme Value. Given this, the probability that a household of type $h$ will be the highest bidder for a dwelling with attributes $z$ is:

$$P(h|z) = P[B_h(z) \geq B_{h'}(z)] \quad , \quad h' \neq h; h, h' \quad H$$

$$= \exp(\int_h(z)) / \exp(\int_{h'}(z))$$

[14] [15]

i.e., the “landlord of dwelling unit chooses the house” (supply side).

This is the opposite perspective of the Lerman model [10], which could be written as:

$$P(z|h) = \exp(V(z, h)) / \exp(V(z', h))$$

[16]

i.e., the “(systematic) utility of bidders chooses the house” (demand side).
Bid Choice Theory

- Bid Choice Theory derives from the seminal bid-rent model of Alonso (1964) and Ellickson’s extension of bid-rent to the discrete choice case (Ellickson, 1981).

- It is best elaborated by Martinez in a series of papers. The following is largely derived from:

Bid Choice Theory

Define:

$I$ = household income
$\mathbf{r}_s$ = rent value at location $s$ for a dwelling of type $t$
$T$ = available time after accounting for compulsory activities
$z_s$ = neighborhood characteristics at $s$
$\mathbf{acc}_s$ = accessibility at $s$ to activities
$P$ = price of composite good
$\beta_h$ = parameters for household of type $h$
Bid Choice Theory

Then:

\[ V_h (I - r_s, t, T, z_s, acc_s, P, \beta_h) \]  \[17\]

= indirect utility of dwelling of type \( t \) at location \( s \) for household of type \( h \)

(e.g., as in Lerman)
Bid Choice Theory

Expression [17] can be inverted to obtain the amount household $h$ would be willing to pay at location $s$ for a dwelling of type $t$ in order to achieve a given utility level, $u$:

$$ r_{s,\text{max}} = V^{-1}(u, \ldots) = \text{“willingness to pay”} $$

$$ = WP_{s,h}(I, t, T, z_s, acc_s, P, u, \beta_h) \quad [18] $$

Expression [18] converts attributes into monetary units (€) – often referred to as a “hedonic price function”.

Note this is Ellickson, but now with an explicit link to Lerman.
Expression [18], in practice, is always linear in income; can therefore be re-written as:

\[ WP_{s,h} = I - f(t, T, z_s, acc_s, P, u^*, \beta_h) \]  \[19\]

where

- \( u^* \) is the maximum utility achievable subject to exogenous prices \( P \);
- \( f(\ldots) \) is an expenditure function (i.e., the inverse of utility expressed in monetary terms).
Bid Choice Theory

The household’s consumer’s surplus for a given dwelling/location is the difference between what they would be willing to pay, and the actual price of the dwelling ($r_s$):

$$CS_{hs} = WP_{h,s} - r_s$$  \[20\]

The optimal location for the household is then the one which maximizes CS:

$$CS^* = \text{Max}_{s,s}(WP_{h,s} - r_s)$$  \[21\]

$S$ = choice set of feasible locations
Bid Choice Theory

What a household is willing to bid for a dwelling is:

\[ B_{h,s} = WP_{h,s} - w_{h,s} \] \[ \text{[22]} \]

where:

\[ w_{h,s} = \text{“speculation factor”} \]
\[ = w_{h,s}(A, N) \geq 0 \]
\[ A = \text{“auction conditions”} \]
\[ N = \text{number of bidders} \]
Bid Choice Theory

Owners (landlords) will sell (rent) to the highest bidder; the price is the winning bid:

$$r_{s}^{*} = \text{Max}_{h,N}(WP_{h,s} - w_{h,s})$$  \[23\]

Substituting Equation [18] into [23] and maximizing yields the equilibrium rent function:

$$r_{s}^{*} = r(I_{h}, t_{s}, T, z_{s}, acc_{s}, P, u_{h}^{*}, \beta_{h}, w_{h,s}(A, N), N)$$  \[24\]
Implications of Equation [24]:

1. Rents depend **only** on consumer behavior, and only indirectly on supply. This result differs from conventional models which determine prices by equilibrating supply and demand.

2. The rent function is directly defined by the utility (or, equivalently, $WP$) function and the speculation function $w(\ldots)$.

3. Model parameters should be estimated so that models simultaneously replicate location choices and rents.
Bid Choice Theory

Equilibrium conditions:

1. Every household must be located somewhere.

2. Supply adjusts to demand = f(rents,…).

3. Allocated land can not exceed total land available.
Bibliography


