FORMULÁRIO

Teoria de erros e Representação de números no computador

Erro absoluto e erro relativo: $(x, \tilde{x} \in \mathbb{R}, \quad x \approx \tilde{x})$

$$e_{\tilde{x}} = x - \tilde{x}, \qquad \delta_{\tilde{x}} = \frac{e_{\tilde{x}}}{x},\ x \neq 0$$
$$\text{erro absoluto} : |e_{\tilde{x}}|, \qquad \text{erro relativo} : |\delta_{\tilde{x}}|,\ x \neq 0$$

Erros de arredondamento: $(x = \sigma(0.a_1 a_2 ...)_\beta \beta^t,\ a_1 \neq 0; \quad \tilde{x} = fl(x) \in FP(\beta, n, t_1, t_2))$

$$|e_{\tilde{x}}| \leq \beta^{t-n}, \quad |\delta_{\tilde{x}}| \leq \beta^{1-n} := U, \quad \text{(arredondamento por corte)}$$
$$|e_{\tilde{x}}| \leq \frac{1}{2}\beta^{t-n}, \quad |\delta_{\tilde{x}}| \leq \frac{1}{2}\beta^{1-n} := U, \quad \text{(arredondamento simétrico)}$$

Métodos iterativos para equações não-lineares

Método da bissecção: $x_{k+1} = \dfrac{a_k + b_k}{2}, \quad f(a_k)f(b_k) < 0$

$$|x - x_{k+1}| \leq |x_{k+1} - x_k|, \qquad |x - x_k| \leq \frac{b-a}{2^k}$$

Método da secante: $x_{k+1} = x_k - f(x_k)\dfrac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

$$x - x_{k+1} = -\frac{f''(\xi_k)}{2f'(\eta_k)}(x - x_k)(x - x_{k-1}),\ \text{com } \xi_k, \eta_k \text{ num intervalo que contém } z, x_k \text{ e } x_{k-1}$$

$$|x - x_{k+1}| \leq \mathbb{K}\,|x - x_k|\,|x - x_{k-1}|\,, \quad \mathbb{K} = \frac{\max|f''|}{2\min|f'|}$$

Método de Newton: $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

$$x - x_{k+1} = -\frac{f''(\xi_k)}{2f'(x_k)}(x - x_k)^2,\ \text{com } \xi_k \text{ entre } z \text{ e } x_k$$

$$|x - x_k| \leq \frac{1}{\mathbb{K}}\left(\mathbb{K}|x - x_0|\right)^{2^k}$$

$$e_k = x - x_k \simeq x_{k+1} - x_k$$

Método do ponto fixo: $x_{k+1} = g(x_k)$

$$|x - x_{k+1}| \leq \frac{L}{1-L}|x_{k+1} - x_k|,$$

$$|x - x_k| \leq L^k|x - x_0|, \qquad |x - x_k| \leq \frac{L^k}{1-L}|x_1 - x_0|$$

Normas e Condicionamento

$$\|\mathbf{A}\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}| \qquad \text{cond}(\mathbf{A}) = \|\mathbf{A}\|\,\|\mathbf{A}^{-1}\|$$

$$\|\mathbf{A}\|_1 = \max_{1\le j\le n} \sum_{i=1}^{n} |a_{ij}| \qquad \|\delta_{\tilde{\mathbf{x}}}\| \le \frac{\text{cond}(\mathbf{A})}{1-\text{cond}(\mathbf{A})\,\|\delta_{\tilde{\mathbf{A}}}\|}\,(\|\delta_{\tilde{\mathbf{A}}}\| + \|\delta_{\tilde{\mathbf{b}}}\|), \text{ sistema } \mathbf{Ax}=\mathbf{b}$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T\mathbf{A}))^{1/2}$$

Métodos iterativos para sistemas lineares

$$\mathbf{Ax}=\mathbf{b} \Leftrightarrow \mathbf{x}=\mathbf{Cx}+\mathbf{d} \quad \to \quad \mathbf{x}^{(k+1)} = \mathbf{Cx}^{(k)} + \mathbf{d}$$

$$\|\mathbf{x}-\mathbf{x}^{(k)}\| \le \|\mathbf{C}\|^k \|\mathbf{x}-\mathbf{x}^{(0)}\|, \qquad \|\mathbf{x}-\mathbf{x}^{(k)}\| \le \frac{\|\mathbf{C}\|^k}{1-\|\mathbf{C}\|}\|\mathbf{x}^{(1)}-\mathbf{x}^{(0)}\|$$

$$\|\mathbf{x}-\mathbf{x}^{(k+1)}\| \le \frac{\|\mathbf{C}\|}{1-\|\mathbf{C}\|}\|\mathbf{x}^{(k+1)}-\mathbf{x}^{(k)}\|$$

Método de Jacobi: $C = -\mathbf{D}^{-1}(\mathbf{L}+\mathbf{U})$ $\qquad x_i^{(k+1)} = (b_i - \sum_{j=1,j\ne i}^{n} a_{ij}\, x_j^{(k)})/a_{ii}$

Método de Gauss-Seidel:

$$C = -(\mathbf{L}+\mathbf{D})^{-1}\mathbf{U}$$
$$x_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)})/a_{ii}$$

Método de Newton para sistemas não-lineares

$$\mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \qquad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)}$$

Aproximação de funções

- **Interpolação Polinomial**
  Fórmula de Lagrange:

$$l_i(x) = \prod_{j=0,j\ne i}^{n} (\frac{x-x_j}{x_i-x_j}) \qquad p_n(x) = \sum_{i=0}^{n} y_i\, l_i(x)$$

Fórmula de Newton com dif. divididas:

$$\begin{cases} f[x_j] = f(x_j), \quad j=0,...,n \\ f[x_j,...,x_{j+k}] = \dfrac{f[x_{j+1},...,x_{j+k}] - f[x_j,...,x_{j+k-1}]}{x_{j+k}-x_j}, \quad j=0,...,n-k, \quad k=1,...,n \end{cases}$$

$$p_n(x) = f[x_0] + \sum_{i=1}^{n} f[x_0, ..., x_i](x - x_0) \cdots (x - x_{i-1})$$

Fórmula de erro: $e_n(x) = \dfrac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$

- **Mínimos Quadrados**

$$\begin{bmatrix} (\phi_0, \phi_0) & \dots & (\phi_0, \phi_m) \\ \dots & \dots & \dots \\ (\phi_m, \phi_0) & \dots & (\phi_m, \phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \dots \\ (\phi_m, f) \end{bmatrix}$$

$$(\phi_i, \phi_j) = \sum_{k=0}^{n} \phi_i(x_k)\phi_j(x_k), \quad (\phi_i, f) = \sum_{k=0}^{n} \phi_i(x_k) f_k$$

Integração Numérica

Regra dos trapézios:

$$T_1(f) = T(f) = \frac{b-a}{2}[f(a) + f(b)], \qquad T_N(f) = \frac{h}{2}\left[f(x_0) + f(x_N) + 2\sum_{i=1}^{N-1} f(x_i)\right]$$

$$E_N^T(f) = -\frac{(b-a)\,h^2}{12} f''(\xi) \qquad \xi \in (a, b)$$

Regra de Simpson:

$$S_2(f) = S(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right], \qquad S_N(f) = \frac{h}{3}\left[f(x_0) + f(x_N) + 4\sum_{i=1}^{N/2} f(x_{2i-1}) + 2\sum_{i=1}^{N/2-1} f(x_{2i})\right]$$

$$E_N^S(f) = -\frac{(b-a)\,h^4}{180} f^{(4)}(\xi) \qquad \xi \in (a, b)$$

Métodos numéricos para equações diferenciais

- Métodos de Euler explícito $y_{i+1} = y_i + hf(t_i, y_i)$

$$|y(t_i) - y_i| \leq \frac{hM}{2L}\left[e^{L(t_i - t_0)} - 1\right], \qquad |y''(t)| \leq M, \, t \in [t_0, t_i]$$

- Métodos de Taylor de ordem k: $y_{i+1} = y_i + hf(t_i, y_i) + ... + \dfrac{h^k}{k!} f^{(k-1)}(t_i, y_i)$

- Métodos de Runge-Kutta de ordem 2: $y_{i+1} = y_i + \left(1 - \dfrac{1}{2\alpha}\right) hf(t_i, y_i) + \dfrac{1}{2\alpha} hf(t_i + \alpha h, y_i + \alpha hf(t_i, y_i))$

$\alpha = \frac{1}{2}$ - Método de Euler modificado $y_{n+1} = y_n + hf(x_n + \dfrac{h}{2}, y_n + \dfrac{h}{2} f(x_n, y_n))$

$\alpha = 1$ - Método de Heun: $y_{n+1} = y_n + \dfrac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))$

- Método de Runge-Kutta de ordem 4 clássico: $y_{i+1} = y_i + \dfrac{h}{6}(V_1 + 2V_2 + 2V_3 + V_4)$

$$V_1 = f(t_i, y_i), \quad V_2 = f(t_i+\tfrac{h}{2}, y_i+\tfrac{h}{2}V_1), \quad V_3 = f(t_i+\tfrac{h}{2}, y_i+\tfrac{h}{2}V_2), \quad V_4 = f(t_i+h, y_i+hV_3)$$

- Método de Euler implícito $y_{i+1} = y_i + hf(t_{i+1}, y_{i+1})$