A SOCIAL DISCOUNT RATE FOR THE UNITED KINGDOM

by

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and
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Abstract

Discussions about the appropriate discount rate to adopt for public decisions continue unabated. In the United Kingdom, Her Majesty's Treasury has conducted periodic reviews of the theoretical and applied literature in order to assess the appropriateness of 'official' discount rates recommended for public sector decision-making. The last such review was conducted in 1991 and produced two recommended rates: 6% in real terms for public sector projects and 8% in real terms as a required average rate of return. While no official review has taken place since 1991, this paper reports the results of a review of the latest evidence on social discount rates in the context of the United Kingdom. We argue that the 1991 'official' rates are well in excess of any reasonable and defensible discount rate. Our best estimate is 2.4% and a range of 2-4% probably sets the upper and lower bounds of what is a credible social discount rate. Given the very wide disparity between rates of 6-8% and 2-4%, the policy implications are formidable.
1. INTRODUCTION

Discussions about the appropriate discount rate to adopt for public decisions continue unabated. In the United Kingdom, Her Majesty’s Treasury has conducted periodic reviews of the theoretical and applied literature in order to assess the appropriateness of ‘official’ discount rates recommended for public sector decision-making. The last such review was conducted in 1991 and produced two recommended rates: 6% in real terms for public sector projects and 8% in real terms as a required average rate of return. While no official review has taken place since 1991, this paper reports the results of a review of the latest evidence on social discount rates in the context of the United Kingdom. We argue that the 1991 ‘official’ rates are well in excess of any reasonable and defensible discount rate. Our best estimate is 2.4% and a range of 2-4% probably sets the upper and lower bounds of what is a credible social discount rate. Given the very wide disparity between rates of 6-8% and 2-4%, the policy implications are formidable.
2. UK GOVERNMENT DISCOUNT RATE POLICY

UK Government policy on discount rates is set out in a guidance document from HM Treasury (HM Treasury [1991]) and is further elaborated in Spackman [1991]. The essential features of current guidance are:

- the discount rate relevant to returns accruing to the public sector from projects in the public sector is 6% real terms;

- this rate reflects the opportunity cost of public sector investment in terms of the rate of return to the marginal private sector investment displaced;

- it is also argued to be close to the ‘time preference rate’;

- public sector agencies selling commercially should use 8% real as a required average rate of return (RRR) since average rates of return are thought to be above marginal rate of return in the private sector;

- a partial exception is made for forestry where management decisions involve the 6% rate, but the Forestry Enterprise is permitted to use 3% as an explicit subsidy.
3. SETTING DISCOUNTS RATES: CONVENTIONAL CRITERIA

A discount rate is the rate of fall of the social value of public sector income or consumption over time. Note that there are two numeraires here - public sector income and the consumption of the public. Hence the discount rate chosen will depend on which numeraire is chosen. Two approaches are usually considered when determining social discount rates:

a) the 'social time preference' (STP) approach; and
b) the 'social opportunity cost' (SOC) approach.

The ‘social time preference rate’ is the rate of fall in the social value of consumption by the public, as opposed to public sector income. It is as well known in the literature as the ‘consumption rate of interest’ (CRI). SOC is usually identified with the real rate of return earned on a marginal project in the private sector. (Strictly, is the social return on that project, but this is usually ignored, or some attempt is made to account for it through the estimation of the social costs and benefits of the project rather than through the discount rate). In Spackman (1991) this rate is referred to as the opportunity cost rate.

Which is the correct rate? There are four observations to make.

The first is that, in an economy without any distortions (such as taxes), the two rates are the same. Given that distortions exist in any economy, it would be extremely surprising if the two rates were the same (Baumol (1986)). Hence there is an apparent problem of choosing between them.

Second, this choice is better understood if we recall that the two rates relate to different numeraires. It is widely accepted in the literature that the proper procedure is to look at each £1 of investment cost and classify the sources of the cost according to whether they come from consumption or investment. Then, the investment component should be converted into consumption-equivalent units through the ‘shadow price of capital’, call it v (Lind (1982), Bradford (1975)). Finally, the resulting consumption equivalent flows should be discounted at the social time preference rate or consumption rate of interest (the CRI). A similar procedure should, in theory, be applied to the resulting benefit flows. Those that accrue as re-investment should be shadow priced at v to convert them to consumption units. Those that accrue as consumption have a consumption value of unity.
However we reconcile the CRI with the SOC there is still in principle no single discount rate that emerges as the CRI. There are two reasons for this. The first is that while interest rates to help ensure an intertemporally efficient allocation of resources by reflecting the rate at which society either can or wants to trade off consumption to-day for consumption in the future, there are infinitely many intertemporally efficient paths. These differ from one another in how they actually transfer resources between people who are alive at different points in time. That is they differ in their inter-temporal equity properties. Using a high interest rate to-day is implicitly to take the view that we do not wish to invest in a lot of capital and other resources which may improve the standard of living of people alive in later years. Which particular inter-temporally efficient path we choose to pursue depends fundamentally on value judgements about inter-temporal equity. But that means these same value judgements will therefore determine the discount rate, and we will see later on exactly how they enter into the various formulae. All economists can do is to give guidance on what seem ‘reasonable’ judgement, and the range of values for discount rates to which they give rise. While this point has long been understood, it has featured prominently in recent work by Howarth and Norgaard (1990; 1991; and 1993).

The second reason why there is in general no such as thing as the CRI is that there is no reason to think that either rate will remain constant over time. For projects of short duration this consideration is unlikely to be important, but for long-lived projects this is important.

What emerges from this procedure is a revised benefit-cost decision rule in which both the CRI and the SOC appear, but in which the CRI is the ‘fundamental’ discount rate. The SOC rate influences the value of v, the shadow price of capital - i.e. the conversion factor that converts investment flows to consumption flows. Estimating the shadow price of capital turns out not to be difficult. A simplified formula is given by Cline (1992). Suppose an investment is made of £1 and it yields an annual payoff of £A over a lifetime of N years. The present value of the consumption stream is:

\[ v = \sum_{t} A(1 + s)^{t} \]

where s is the CRI. If the project has an internal rate of return equal to the rate of return on capital, r, the following equation also has to be satisfied:

\[- I + \sum_{t} A(1 + r)^{t} = 0\]
Solving for the value of A in the second equation and substituting it in the first equation gives:

\[ v = \frac{r}{1 - (1 - r)^{-N}} \cdot \frac{1 - (1 + s)^{-N}}{s} \]

As an example, suppose \( r = 8\% \), \( s = 2\% \) and \( N = 15 \), then:

\[
v = \frac{0.08}{1 - 1.08^{-15}} \cdot \frac{1 - (1.02^{-15})/0.02)}{0.02} = \frac{0.08}{0.68} \cdot \frac{0.26}{0.02} = 1.43
\]

i.e. the shadow price of capital is 1.43. Notice that it is determined solely by the two discount rates, \( s \) and \( r \), and the average lifetime of capital.

The complication with the ‘CRI plus shadow price of capital’ approach is not the shadow price of capital, but determining the likely proportions of public investment expenditures coming from displaced consumption and investment. A common procedure is to use the ratios \( I/GNP \) and \( C/GNP \) where \( I \) is investment, \( C \) is consumption and \( GNP \) is Gross National Product. In the UK these ratios would be 17% and 83%.
4. WHY IS THE SHADOW PRICING APPROACH NOT USED IN THE UK?

Spackman (1991, para 31) acknowledges that the shadow pricing procedure is the theoretically correct one. He also agrees that public investment displaces both investment and consumption, so that it is not legitimate to argue for an opportunity cost rate alone. Spackman's rationale for not pursuing the shadow pricing approach is (a) that 'the problems of quantifying in practice how much a particular public expenditure is financed by diversion from investment and how much directly from consumption are formidable'; (b) that shadow pricing 'would be quite foreign, and not attractive, to most practical managers'; and (c) 'it appears in practice that, even where time preference and the opportunity cost of displaced investment might in principle conflict, this conflict, at least in present UK circumstances is not generally material' (Spackman (1991), paras. 33-35). In short, the justification for not going down this route is a mix of practicability and the belief that $s$ and $r$ in the equations above are in fact very close to each other.

In what follows we will therefore focus on the determination of the CRI in conventional cost-benefit analysis. Later we discuss arguments that might suggest a lower discount rate is appropriate because of the length of life of projects involving long-term welfare concerns e.g. radioactive waste disposal. In all the discussion the presumption will be that the CRI is the appropriate rate.
5. THE CONSUMPTION RATE OF INTEREST

Whatever approach is adopted, it is necessary to estimate the CRI. It is universally accepted that the formula for estimating the CRI is:

\[ s = \delta + \mu \cdot g \]

where \( \delta \) is referred to as the ‘rate of time preference’, i.e. the rate at which utility is discounted; \( \mu \) is the elasticity of the marginal utility of consumption schedule; and \( g \) is the expected rate of growth in average consumption per capita (Pearce and Nash (1981), Lind (1982)). In order to estimate \( s \), then, we need estimates of \( \delta, \mu \) and \( g \).

5.1 The rate of time preference, \( \delta \).

There is a great deal of discussion and controversy over what value (s) \( \delta \) should take. Some of this controversy can be resolved by realising that there are in fact two factors that enter into the rate of time preference. The first is the rate of pure time preference, which we will denote by \( \rho \). This is the rate at which discount the welfare arising to people in the future purely by virtue of this utility arising later. The second is any increase (or decrease) in the risk to life. We will let \( \dot{L} \) be the rate of growth of life chances. If life chances get worse through time, then this makes for a higher rate of time preference, whereas if they get better then this an argument for a lower rate of time preference. Thus we have the following relationship:

\[ \delta = \rho \cdot \dot{L} \]

We discuss the two factors in turn.

5.1.1 Rate of Pure Time Preference, \( \rho \)

Spackman (1991), para. 22) suggests that although these ‘pure time preference’ effects are largely subjective there is no evidence to suggest that they amount to more than a small annual rate of discount. A rate of 1 to 2 per cent a year would amount to discounting marginal utility in 25 years time (roughly one generation) by 20 to 40 per cent.

However a significant number of writers regard zero as the only ethically defensible value for the rate of pure time preference. As Broome (1991) puts it:
“A universal point of view must be impartial about time, and impartiality about time means that no time can count differently from any other. In overall good, judged from a universal point of view, good at one time cannot differently from good at another. Hence.... the [pure time] discount rate... must be nought” (Broome, 1991: p.92).

On Broome’s analysis, then, the focus of debate shifts to whether impartiality is itself justified. His argument here is that the doctrine of utilitarianism, which is often thought to underlie cost-benefit analysis, itself implies impartiality. ‘The doctrine of impartiality – ‘each to count for one, and none for more than one’ - lies at the heart of utilitarianism’ (Broome (1992, p.95)). So, on this view, standard project appraisal, as embraced by HM Treasury, implies utilitarianism; utilitarianism implies impartiality, impartiality implies zero utility discounting. Hence the proper value for the rate of pure time preference, is zero.

However, there are two objections to this view. The first is that all that underlies cost-benefit analysis is a commitment to inter-temporal efficiency, with the particular path that is chosen being decided on some grounds of equity. This by no means commits one to utilitarianism as the particular principle for path selection.

The second is that the utilitarianism argued for by Broome (cardinal utilitarianism) has a major difficulty that was pointed out by Sen (1973). While utilitarianism is perfectly consistent with equality as long as all people have equal productive capabilities, when applied to an economy where people have unequal productivities, pursuing a utilitarian objective will lead to what many would regard as the extremely inequalitarian position of having resources taken from the least productive and given to the most productive. One way to correct this problem is to give greater weight in the social objective to those who are least productive. If we look at this from an intertemporal perspective, then there are two factors that tend to make future generations more productive than current ones: capital accumulation, and technical change. Therefore if we applied a strict utilitarian objective function to inter-temporal resource allocation the effect would be to redistribute heavily away from the current "unproductive" generation to the more "productive" future generation. This would come about through having low interest rates which would encourage a great deal of investment and innovation. So, far from being impartial, utilitarianism would actually lead to re-distribution towards those generations that are likely to be better off. One way to mitigate this problem is to give less weight in the social objective to future generation - which is precisely what a positive rate of pure time preference would do.

Of course this argument is not itself without problems: in particular, it does not tell us what the
appropriate pure rate of time of preference should be. It is also far from obvious that all future
generations will indeed be better off than current ones. Indeed, the more cataclysmic
environmental prognostications would suggest that environmental degradation may be one of
the main factors making future generations worse off than current generations. It remains the
case, then, that there is no clear view about what the pure rate of time preference should be.

5.1.2 Changing Life Chance, $L$
As pointed out above, one reason for a positive rate of time preference could be a belief that life
chances get smaller over time. This factor seems more amenable to empirical investigation,
and less prone to fundamental disputes about value judgements. Nevertheless, there is still
disagreement about what precise risks are being discussed, and the various attempts to
produce estimates of changing life risks differ in both methodology and in the particular risks
being estimated. Thus some authors, such as Kula (1985, 1987), look at the increasing risk of
death for an individual as they get older. While this will certainly be an important risk of death
for an individuals to favour early consumption over later, it is far from clear what role this should
play in discussions about the discount rate. There are three problems: (i) there are factors other
than increased probability of death that come into play in an individual's weighting of
consumption in different periods - e.g. increased dependence on medical treatment in old age
could operate in opposite direction; (ii) if individuals have adequate opportunity to smooth out
their consumption over their lives, it is far from clear why this should lead governments to weight
consumption arising at different times in different ways; (iii) if we are dealing with very long-lived
projects then the risks that are appropriate are not so much the increasing probability of death of
a single individual as they age, but what is happening to the life chances of whole generations.
This is the changing risk that Newbery (1992) tries to measure. Newbery's value of $L = 1.0$ can
be compared to a value of 1.1 that can be obtained by looking at the UK death rate for 1991 and
dividing it by population, i.e.

$$
L = \frac{\text{Total Deaths}}{\text{Total Population}} = \frac{6.466 \text{ mm}}{57.56m} = 0.011
$$

5.1.3 Overall Values for $\delta = \rho - L$
Table 5.1 brings together various estimates for $\delta$ which is further decomposed into separate
values for $L$ and $\rho$. The highest estimate for $L$ is 2.2% (Kula (1985)) based on individual
survival probabilities averaged over a long period of time. For the reasons given above we do
not regard this as the right interpretation of $L$. Kula's own measure comparable to that of
Newbery (1992) is 1.2% (Kula (1987)), while we derived a value of $L$ of 1.1%. Thus we take a range of $L$ as being zero to 1.2%, with a best estimate of 1.1%. For $\rho$ Table 5.1 shows that only Scott (1989) derives an estimate independent on the value of $L$ and this is set at $\rho = 0.5$. Scott's overall estimate of $\delta$ is a fusion of $L$ and $\rho$. Thus, we adopt a range for $\rho$ of 0.5 and a central estimate of 0.3. Table 5.1 also records authors who regard the overall value of $\delta$ as being zero, but inspection of this literature suggest that some of them are rejecting positive values of $\rho$ rather than positive values of $L$.

### Table 5.1: Estimates and Views of the Rate of Time Preference

<table>
<thead>
<tr>
<th>Time Preference</th>
<th>Estimate</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>negative time preference</td>
<td>$\delta &lt; 0$</td>
<td>Arises when costs (benefits) are viewed as a sequence. Phenomena such as ‘savouring’ the best until last, and ‘getting it over with’ - costs preferred now to later, imply negative time preference. Methodology - surveys.</td>
</tr>
<tr>
<td>Lowenstein [1987]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowenstein and Prelec [1991]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>zero time preference</td>
<td>$\delta = 0$</td>
<td>$\delta &gt; 0$ 'ethically indefensible’ and due to 'weakness of the imagination' (Ramsey); a ‘defect of telescopic faculty’ (Pigou); $\delta = 0$ justified by impartiality (Broome), etc.</td>
</tr>
<tr>
<td>Ramsey [1928]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pigou [1932]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Broome [1992]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and others</td>
<td></td>
<td></td>
</tr>
<tr>
<td>positive time preference</td>
<td>$\delta = \rho - L$</td>
<td>$\rho = 0.5$ fits data on UK savings behaviour 1855-1974, raised to 1.5 because of ‘risk of total destruction of our society’</td>
</tr>
<tr>
<td>Scott [1977]</td>
<td>$L = 1.0$</td>
<td></td>
</tr>
<tr>
<td>Scott [1989]</td>
<td>$\delta = \rho - L = 1.3$</td>
<td>See above.</td>
</tr>
<tr>
<td>Newbery [1992]</td>
<td>$L = 1.0$</td>
<td>Consistent with ‘perceived risk of the end of mankind in 100 years’</td>
</tr>
<tr>
<td>Kula [1985]</td>
<td>$L = -2.2$</td>
<td>Based on survival probabilities in the UK 1900-1975, but revised in Kula [1987].</td>
</tr>
<tr>
<td>This paper [1994]</td>
<td>$L = 1.1$</td>
<td>Average probability of death in 1991.</td>
</tr>
</tbody>
</table>
5.2 The value of the elasticity of marginal utility of consumption, $\mu$.

The traditional assumption underlying the CRI is that the utility to be gained from the stream of consumption $C = \{C_0, C_1, \ldots, C_t, \ldots\}$ takes the additively separable form:

$$W(C) = \sum_{t=0}^{\infty} (1 + \delta)^t U(C_t)$$

where $\delta$ is, as before, the discount factor, and $U(C_t)$ is the flow rate of utility accruing in period $t$ from consumption in period $t$. The marginal utility of consumption is then:

$$\frac{dU}{dC}$$

We normally assume that this is positive but strictly decreasing in consumption (diminishing marginal utility). Formally, we assume:

$$U'(C) = \frac{dU}{dC} > 0,$$

but

$$U''(C) = \frac{d^2U}{dC^2} < 0.$$ 

The elasticity of the marginal utility of consumption, $\mu$, then measures the percentage rate at which the marginal utility falls for every percentage increase in consumption. Formally:

$$\mu = -\frac{C.U''(C)}{U(C)} > 0.$$ 

In general, the value of $\mu$ will depend on the level of consumption $C$. However, a widely used form of utility function is one for which $\mu$ is independent of the level of $C$. This is the iso-elastic utility function:

$$U(C) = \frac{a}{1 - \mu} C^{1-\mu},$$

for which the marginal utility is:
\[ U'(C) = aC^\mu. \]

There are two approaches to obtaining estimates of \( \mu \).

The first is to regard \( W(.) \) as reflecting the views of individuals about how they wish to transfer consumption across time. In this case we try to infer values of \( \mu \) from observations on individual savings behaviour.

The second is to regard \( W(.) \) as reflecting society's judgement about how we should transfer consumption across people at different times. In this case, we think of \( \mu \) as telling us about how much more worthwhile it is to carry out transfer of income from a rich person to a poor person depending on how well off the two are.

We now briefly discuss each approach in turn. But before doing so we need to note that a number of economists regard the marginal utility of income as unmeasurable. In the next section we discuss the relevance of this observation for the estimates of \( \mu \).

5.2.1 Estimates of \( \mu \) from observations on individual behaviour

As pointed out above, we can try to estimate \( \mu \) from the observations on individual savings behaviour. We first explain how this can be done, and then point out how this discussion relates to the view that \( \mu \) is unmeasurable.

An immediate point to note here is that if we adopt the widely used iso-elastic form of single-period utility function then the intertemporal utility function \( W(C) \) is essentially a CES utility function, and \( \mu \) can also be interpreted as the inter-temporal elasticity of substitution. Given the definition of the elasticity of substitution, this means that we can infer values of \( \mu \) from observations on how sensitive an individual's relative levels of consumption in different periods are to changes in the relative prices of consumption in different periods - i.e. to the rate of interest. Formally, if an individual maximises life-time utility subject to an intertemporal budget constraint, then the optimal consumption path satisfies the condition that at all times:

\[ \frac{\mu}{\delta + r_t} \frac{dC}{dt} = r_t, \]

where \( r_t \) is the (instantaneous) rate of interest at time \( t \). So, assuming that we know the factors
determining \( \delta \), we can in principle determine the value of \( \mu \) from data which allow us to look at the relationship between the rate of growth of individual consumption and the rate of interest.

The advent of data sets and techniques that allow us to estimate savings behaviour from panel data or pseudo-panel data has enabled recent work on estimating \( \mu \). We report later in more detail on the findings by Blundell et al., (1994) which give the most authoritative UK estimates.

Now of course there is not a single consumption good in each period, but a whole vector of consumption goods, so consumption at time \( t \) is to be thought of as total consumption expenditure, and the single-period utility function is essentially an indirect utility function giving the maximum utility available from a given level of expenditure. We know from consumer theory that individual consumption behaviour within each period depends only on individual preferences and not on the particular utility function used to represent these preferences. Thus consumption behaviour within each period is completely unaffected by any monotonic transformation of \( U(.) \). Such transformations will, in general, give very different values for the elasticity of the marginal utility of consumption. Thus \( \mu \) cannot in principle be estimated from observations on individual consumption behaviour within each period, and, in this sense, \( \mu \) is unmeasurable from observed consumption behaviour. Early attempts to estimates \( \mu \) this way only `work' because they impose a particular functional form for the single-period utility function. This explains the caution of Stern (1977) and others in interpreting empirical estimates of \( \mu \).

Notice, however, the while intertemporal consumption behaviour is also independent of the particular utility function used to represent preferences, the relevant utility function here is \( W(.) \). Monotonic transformation of \( W(.) \) leave \( \mu \) as defined above completely unaffected, since, as we have pointed out, in this inter-temporal setting \( \mu \) is essentially the inter-temporal elasticity of substitution, and this is a property of individual preferences for consumption in different periods. Thus Stern's strictures about the non-measurability of \( \mu \) do not apply to estimates obtained from savings behaviour.

5.2.2 Stern's survey
Stern (1977) is widely quoted in support of `central' and `consensus' estimates of the value of \( \mu \) in the range 1-10. Indeed, his survey appears to have influential in determining the adopted values of \( \mu \) used by Spackman (1991), and hence in determining the official view of discount rates in the United Kingdom. Stern uses three approaches to investigate the value of \( \mu \):

(a) analysis of complete demand systems
(b) von Neumann-Morgenstern utility functions, and
(c) savings behaviour

The theory underlying approach (a) has been addressed above. Assuming some credibility could be given to the estimates, Stern's own survey produces values of $\mu$ as low as 0.4 and a high as 10.3. Indeed, he states that:

"From estimates of demand systems, we have found a concentration of estimates of $(\mu)$ around 2 with a range of roughly 0-10" (Stern, 1977: p.221).

This makes the selection of values such as 1.5 or 2 look reasonable, until it is recognised that the value of 10.3 is for South Korea in 1970. In fact, Stern quotes only two UK estimates -2 from a judgement of Brown and Deaton in 1972, and 2.8 by the same authors based on the 'Rotterdam model'.

Kula (1985, 1987) used an earlier approach of Fellner's. Essentially, this approximates $\mu$ as:

$$\mu = \frac{y_e}{P_e}$$

where $y_e$ is the income elasticity of demand and $p_e$ is the price elasticity of demand. Stern (1977) notes that this is not accurate, and that Fellner's own estimates is over-stated because is its use.

Approach (b) is analyzed by Stern in the context of von Neumann-Morgenstern utility, i.e. behaviour under uncertainty. Once again, additive separability is assumed:

"In this case we have the Ramsey - von Neumann-Morgenstern theorems to support additivity (thus taking the expectation of $U(.)$), but they do not of course distinguish between $U$ and $\phi(U)$ as measures of overall utility for the individual" (Stern, 1977: p.216).

Deaton and Muellbauer (1980) note the same thing. As it happens, it is not clear what we would learn from the uncertainty approach even if it was credible, since the results suggest values of $\mu$ of minus infinity to plus infinity.

Finally, Stern looks at saving behaviour for approach (c). Maximising a lifetime utility function subject to a constraint on saving behaviour, the resulting formula for $\mu$ is (See Hicks, (1965):
\[ \frac{S}{Y} = \frac{1}{\mu} \left[ 1 - \frac{\rho}{r} \right] \]

i.e. \[ \mu = \frac{1 - \rho}{r} \frac{Y}{S} \]

where \( S \) is saving, \( Y \) is income, \( \rho \) is the utility discount rate and \( r \) is the rate of return on investment. Stern selects \( \frac{S}{Y} = 0.1 \), \( r = 5\% \), and \( \rho = 2.5\% \) to obtain \( \mu = 5 \). If \( \rho = 0 \), then \( \mu = 10 \).

Stern's savings behaviour approach thus suggests further evidence for the '0–10' range for \( \mu \), making values of 1.5 or 2 look 'reasonable'. But there are major problems. Scott (1989) notes one problem with the savings formula above. If incomes are expected to grow, then 'the formula used by Stern no longer holds' (Scott, (1989), p.233). The correct formula becomes (keeping to our notation):

\[ \frac{S}{Y} = \left( \frac{1}{\mu} \right) \left( \frac{r - \rho - y}{r - y} \right) \]

or

\[ \mu = \frac{r - \rho}{S/Y \left( r - y \right) + y} \]

where \( y \) is now the expected growth rate of incomes from work. Keeping Stern's figures and putting \( y = 0.025 \) (Scott's estimate) shows that, once this formula is used, the value of \( \mu \) becomes 1 to 1.3, not 5 as in Stern's model. Moreover, even if \( \rho = 0 \) the upper bound on \( \mu \) is 1.82, not 10.

Scott's own savings approach simultaneously estimates values of \( \rho \) and \( \mu \). For the period 1951 to 1973 in the UK Scott obtains \( \rho = 1.3\% \) and \( \mu = 1.5 \). This is based on an assumed fit of these values to the equation:

\[ r = \rho + \mu g \]

where \( r = 4.4\% \) and \( g = 2.2\% \). Notice that Scott's \( \mu = 1.5\% \) is nowhere near Stern's value of 5, but is in keeping with the maximum of 1.8\% (see above).

Stern himself is very cautious about the uses to which his estimates of \( \mu \) can be put, but declares that he prefers, 'if forced', the savings behaviour estimates to the demand system estimates, provided better models of savings behaviour are used. As already indicated, those
models are now available and $\mu$ can be correctly measured within the context of such models.

5.2.3 Empirical estimates of $\mu$: recent work

The range of estimates for $\mu$ for the UK would appear to be 0.7 (Kula) to 1.5 (Scott). However, both studies have their problems. As noted above, only studies adopting savings behaviour models are relevant. Thus, Kula's work cannot be used to justify estimates of $\mu$. Scott's work is more relevant, but recent work by Blundell, Browning and Meghir (1994) is the most sophisticated analysis yet. For the UK it suggests $\mu = 0.83$ (see Annex). On this basis a value of $\mu = 1.5$ is unquestionably at the upper end of the range. A value of $\mu = 1$ is defensible and a value relevant to current UK conditions is $\mu = 0.8-0.9$.

5.2.4 A `Thought Experiment'

The other approach to estimating $\mu$ is to regard it as an ethical evaluator, i.e. an indicator of the rate at which consumption should be allocated through time. It is possible to sketch out the implications of different values of $\mu$ for egalitarian judgements. Consider two households, one with a consumption twice that of the other, i.e. $C_1 = 2C_2$. Then the ratio of the two marginal utilities of consumption will be:

$$\frac{aC_1^\mu}{aC_2^\mu} = 2^{-\mu}$$

This says if we were to transfer 1 unit of consumption from household 1 to household 2, then the loss to household 1 would be worth only a fraction $2^{-\mu}$ of the gain to household 2. This indicates what views we implicitly hold about the desirability of such a redistribution. The figures below set out the implications.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0.5</th>
<th>0.7</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^\mu$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.35</td>
<td>0.25</td>
<td>0.03</td>
<td>negligible</td>
</tr>
</tbody>
</table>

A value of $\mu = 5$ or above effectively means that the social value of an extra £1 to the higher income group is zero or close to zero, underlining the implausibility of such values which, in any event, we show below to have no foundation anyway. A value of $\mu = 1.5$, which is the one selected by Spackman (1991), still implies that the higher income group's extra £1 is valued at only 35% of £1 to the lower income group. As Newbery (1992) remarks, this is a strongly egalitarian judgement.
5.3 The expected rate of growth of per capita consumption, g.

The final component of the STPR is g, the expected rate of growth of per capita consumption. There are several considerations relevant to determining g. First, if the population choose to substitute leisure for consumption, a value of g based on real per capita consumption growth will understate the relevant magnitude. Second, real consumption per capita may fail to reflect rising social costs of consumption, in which case g will be overstated. One way to ‘smooth out’ such considerations is to take vary long-run rates of growth in real capita consumption. The relevant data are set out below for the UK:

<table>
<thead>
<tr>
<th>Year</th>
<th>Real per capita consumption at 1985 prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1885</td>
<td>£1185</td>
</tr>
<tr>
<td>1895</td>
<td>£1331</td>
</tr>
<tr>
<td>1951</td>
<td>£1891</td>
</tr>
<tr>
<td>1992</td>
<td>£4660</td>
</tr>
</tbody>
</table>

With 1885 or 1895 as base year the resulting value of g is 1.3% p.a.

If the focus is shifted to the post-war period 1951-1992, then the value of g is 2.2% p.a.

5.4 Estimates of the CRI

Bringing these estimates together, we can derive an estimates for the CRI (STPR). We offer lower and upper bounds:

<table>
<thead>
<tr>
<th>Estimates</th>
<th>ρ</th>
<th>L</th>
<th>μ</th>
<th>g</th>
<th>CRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Bound</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>1.3</td>
<td>0.9</td>
</tr>
<tr>
<td>Best Estimate</td>
<td>0.3</td>
<td>-1.1</td>
<td>0.8</td>
<td>1.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>0.5</td>
<td>-1.2</td>
<td>1.5</td>
<td>2.2</td>
<td>5.0</td>
</tr>
</tbody>
</table>

The best estimate that emerges is CRI = 2.4%. If, however, the post war period is taken as the basis of g, and if, as suggested earlier, a value of μ = 1 is defensible, then a value of CRI = 3.9% emerges (taking the maximum value of ρ - L = 1.7%). A range of 2-4% for the CRI would seem to be appropriate, and our own view would be inclined to the lower end of the range.

While it is certainly possible to arrive at a figure reasonably close (5%) to the UK Treasury figure of 6%, it is very difficult to justify, since it involves taking upper bound figures on each of the components of the formula. To go much above 4.0% one would either have to (i) be very
pessimistic about future survival probabilities for mankind, while at the same time being very
optimistic about prospects for consumption growth in the meantime; or (ii) be prepared to
discount future generations at a very high rate; or (iii) be very much more egalitarian than people
seem to be in terms of the tax policies they are prepared to vote for.

We have seen that there is a great deal of controversy over what is the "correct" CRI. This partly
reflects the point made earlier that there is no conceptually correct single rate. Moreover there
is no good reason to think that a single rate should prevail through time. But, with these caveats
in mind, we find it impossible to support the continued use of rates in the region of 6% for the
UK. Such rates are far too high.
References


Appendix

Recent Savings Models to Estimate $\mu$

Blundell, Browning and Meghir (1994) use a ‘pseudo-panel’ constructed by taking a time-series of repeated cross-sections to estimate a model which simultaneously explains the allocation of household income within and across time-periods. The model thus integrates conventional demand theory with savings behaviour in a completely consistent fashion. The data source is the UK Family Expenditure Survey, and the sample consists of over 70,000 households over a 17 year period from 1970-1986.

An important feature of this period is that there is a very obvious structural break in the data at the start of the 80s. In the 70s the real interest rate was negative and the average real consumption growth was higher than the real interest rate but tracked the behaviour of the interest rate fairly accurately. At the start of the 80s the real interest rate rose sharply and was now higher than real consumption growth, moreover real consumption growth no longer tracked the real interest rate. Obviously, people’s inflationary expectations dramatically shifted at the start of the 80s with the arrival of the Thatcher government, and it is important to allow for this.

Abstracting from details to do with the intra-period consumption patterns, the underlying single-period utility function used by Blundell et al., (1994) is

$$u(c) = F[V(c)]$$

where $c$ is real consumption, and:

$$V(c) = \frac{C^\rho - 1}{1 - \theta}; \quad \theta > 0; \quad F(V) = \frac{V^{1+\rho} - 1}{1+\rho}$$

This implies:

$$\mu \equiv \frac{c u''(c)}{u'(c)} = (1 + \theta) \frac{\rho \theta}{C^\rho - 1}$$

The implications of this are that: if $\rho \to 0$ then we get the conventional iso-elastic formulation; as $c \to \infty$, $\mu \to 1 + \theta$; if $\rho > 0$ then $\mu$ decreases with $c$; if $\rho < 0$ then $\mu$ increases with $c$. Blundell et al., estimate both $\theta$ and $\rho$, allowing the latter to depend on demographics.
Their findings are:

1) It is extremely important to allow for a behavioural shift at the start of the 80’s through the inclusion of a 1980 dummy - the model gives a much better fit to the data when this is done.

2) It is very important to allow \( \mu \) to vary. Restricting the utility function to be iso-elastic results in much more poorly determined equation.

3) Although demographics have a significant effect on inter-temporal consumption allocation, their effect is small.

For their preferred equation including the 1980 dummy estimates, \( \theta = 0.54 \) and \( \rho \) is negative. The estimated value of \( \mu \) at the sample means is 0.83, and the estimates at different percentiles of the income distribution (setting demographics at sample mean) are given in table below.

<table>
<thead>
<tr>
<th>percentile</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.34</td>
<td>0.53</td>
<td>0.71</td>
<td>0.88</td>
<td>1.04</td>
</tr>
</tbody>
</table>

If we look at percentiles of the entire sample we get essentially the same table - figures change only in the second decimal, which illustrates the fact that demographics are not playing a key role.