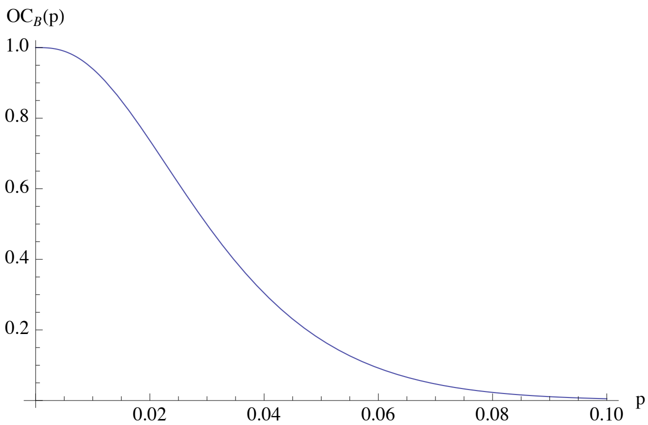
A few exercises from Chap. 13

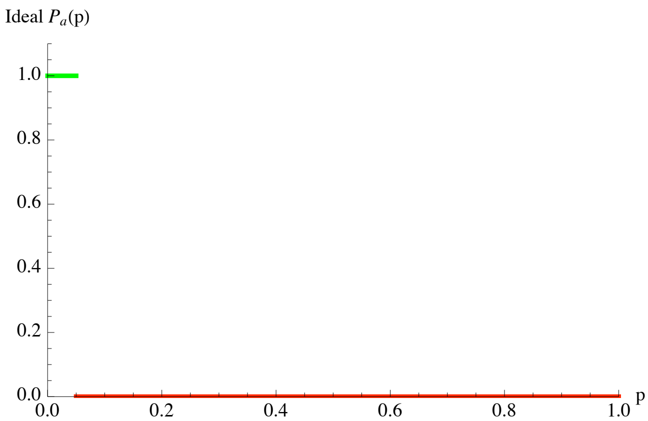
Exercise 13.2

```
n = 89;
c = 2;
OCB[p_] = CDF[BinomialDistribution[n, p], c];

TableForm[Table[{p, OCB[p]}, {p, 0.01, 0.05, 0.01}],
 TableHeadings → {None, {"p", "OCB(p)"}}]
Plot[OCB[p], {p, 0, .1}, AxesLabel → {"p", "OCB(p)"}]
(* OC curve type B*)
```

p	$OC_B(p)$
0.01	0.93969
0.02	0.736578
0.03	0.498483
0.04	0.304158
0.05	0.172077




```
p1 = 0.05;
idealOC[p_] = If[p ≤ p1, 1, 0];
G1 = Plot[idealOC[p], {p, 0, p1},
   PlotStyle → {RGBColor[0, 1, 0], Thickness[0.008]}, AxesLabel → {"p", "Ideal Pa(p)"},
   PlotRange → {{0, 1}, {0, 1.1}}, DisplayFunction → Identity];
G2 = Plot[idealOC[p], {p, p1, 1}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.008]},
   PlotStyle → Thickness[0.008],
   PlotRange → {{0, 1}, {0, 1.1}}, DisplayFunction → Identity];
Show[G1, G2, DisplayFunction → $DisplayFunction]
```



Exercise 13.3

```
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2 × (c + 1)], x];
r[c_] = N[Q[c, 1 - β], 5] / N[Q[c, α], 5];
i = 0;
While[r[i] > p2/p1,
 Print["Do not use acceptance number c=", i, " because r(c)=", r[i], ">p2/p1=", p2/p1]; i++]
Print["Use acceptance number c=", i, " because r(c)=", r[i], "≤p2/p1=", p2/p1]

n[c_] = Ceiling[Q[i, 1 - β] / (2 × p2)];
Print["Use the sample size n=", n[i]]
```

Do not use acceptance number c=0 because r(c)=44.8906> $\frac{p_2}{p_1}$ =10.

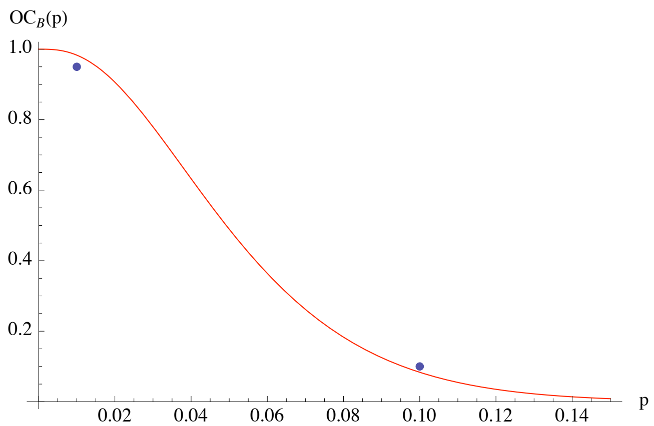
Do not use acceptance number c=1 because r(c)=10.9458> $\frac{p_2}{p_1}$ =10.

Use acceptance number c=2 because r(c)=6.50896≤ $\frac{p_2}{p_1}$ =10.

Use the sample size n=54

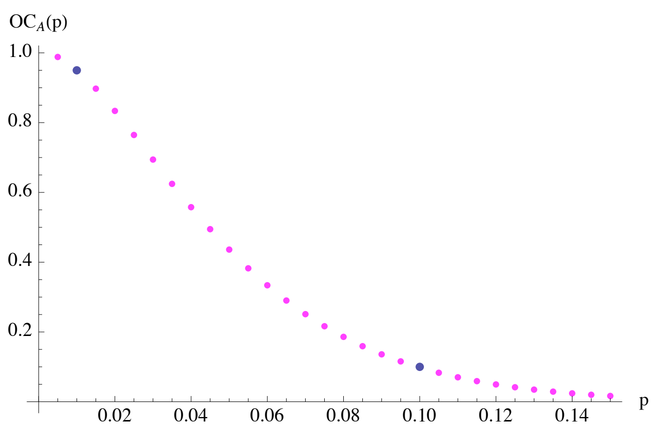
```
OC_B[p_] = CDF[BinomialDistribution[n[i], p], i];

TableForm[Table[{p, OC_B[p]}, {p, 0.005, 0.15, 0.005}],
 TableHeadings → {None, {"p", "OC_B(p)"}}]

G1 = Plot[OC_B[p], {p, 0, .15}, AxesLabel → {"p", "OC_B(p)"},
   PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity];
riskpoints = {{p₁, 1 - α}, {p₂, β}};
G2 = ListPlot[riskpoints, PlotStyle → PointSize[0.014],
   PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
Show[G1, G2, DisplayFunction → $DisplayFunction]
(* OC curve type B obtain via Wetherill and Brown's method;
producer's risk point (left) and consumer's risk point (right). *)
```

p	$OC_B(p)$
0.005	0.997437
0.01	0.98302
0.015	0.952447
0.02	0.906268
0.025	0.84742
0.03	0.779716
0.035	0.706983
0.04	0.632605
0.045	0.559329
0.05	0.489225
0.055	0.423727
0.06	0.363724
0.065	0.309664
0.07	0.26165
0.075	0.219535
0.08	0.182999
0.085	0.151615
0.09	0.124894
0.095	0.102326
0.1	0.0834077
0.105	0.0676558
0.11	0.0546237
0.115	0.0439056
0.12	0.0351398
0.125	0.0280082
0.13	0.022235
0.135	0.0175838
0.14	0.0138534
0.145	0.0108746
0.15	0.00850593



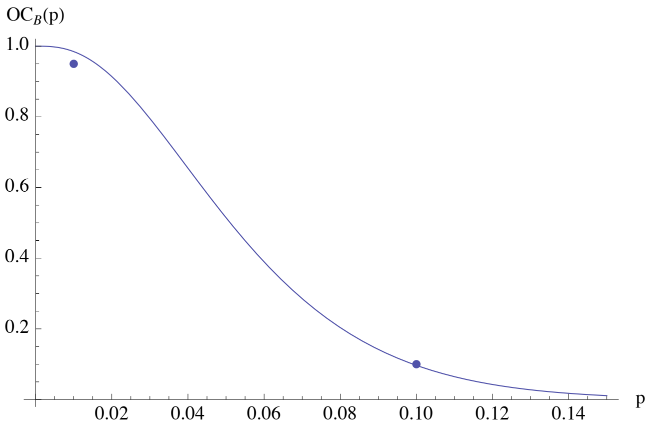
```
bdist[n_, pt_] := BinomialDistribution[n, pt];
pdist[n_, pt_] := PoissonDistribution[n * pt];
hdist[n_, pt_, ng_] := HypergeometricDistribution[n, Round[pt * ng], ng];

bin[x_, n_, pt_, ng_] := PDF[bdist[n, pt], x];
poi[x_, n_, pt_, ng_] := PDF[pdist[n, pt], x];
hip[x_, n_, pt_, ng_] := PDF[hdist[n, pt, ng], x];

Pa[p_, {n_, c_, ng_}, f_] := Sum[f[d, n, p, ng], {d, 0, c}];

Paatr[p_, {{a_, b_}, {e_, d_}}, ng_, f_] :=
 Pa[p, {planoamosatrib[{{a, b}, {e, d}}, ng, f][[1]],
   planoamosatrib[{{a, b}, {e, d}}, ng, f][[2]], ng}, f]

planoamosatrib[{{a_, b_}, {e_, d_}}, ng_, f_] :=
 Module[{n, c},
  j = 0;
  t = 0;
  While[t == 0,
   i = 2;
   While[i ≤ ng && Pa[a, {i, j, ng}, f] ≥ 1 - b,
         i = i + 1]
    If[Pa[e, {i - 1, j, ng}, f] ≤ d, t = 1, t = 0];
   j = j + 1];
  While[Pa[a, {i - 1, j - 1, ng}, f] ≥ 1 - b && Pa[e, {i - 1, j - 1, ng}, f] ≤ d, i = i - 1];
  {n = i, c = j - 1}]

ntot = 800; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

Print["Use the sample size and acceptance number, (n,c)=",
 planoamosatrib[{{p1, α}, {p2, β}}, ntot, hip]]
(* By using the exact distribution, both n and c are smaller than
 the one obtained via the Poisson approximation *)

listpOCA = Table[{p, N[CDF[HypergeometricDistribution[37, Round[ntot × p], ntot], 1], 5]},
   {p, 0.005, 0.15, 0.005}];
TableForm[listpOCA, TableHeadings → {None, {"p", "OCA(p)"}}];

G3 = ListPlot[listpOCA, AxesLabel → {"p", "OCA(p)"},
   PlotStyle → RGBColor[1, 0, 1], DisplayFunction → Identity];
riskpoints = {{p1, 1 - α}, {p2, β}};
G2 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];
Show[G3, G2, DisplayFunction → $DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)
```

Use the sample size and acceptance number, (n,c)={37, 1}

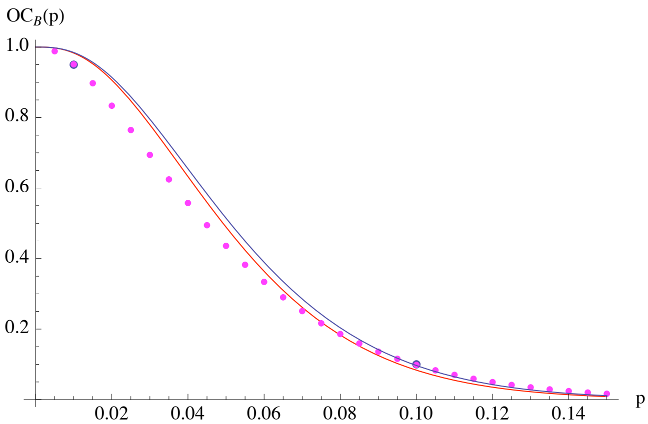


```
Print["Use the sample size and acceptance number, (n,c)=",
 planoamosatrib[{{p₁, α}, {p₂, β}}, n_tot, bin]]
(* Both n and c are smaller than the one obtained via the Poisson approximation *)

OC_B[p_] = CDF[BinomialDistribution[52, p], 2];

TableForm[Table[{p, OC_B[p]}, {p, 0.01, 0.15, 0.01}],
  TableHeadings → {None, {"p", "OC_B(p)"}}];
G4 = Plot[OC_B[p], {p, 0, .15}, AxesLabel → {"p", "OC_B(p)"}, DisplayFunction → Identity];

riskpoints = {{p₁, 1 - α}, {p₂, β}};
G2 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];
Show[G4, G2, DisplayFunction → $DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)
```

```
Use the sample size and acceptance number, (n,c)={52, 2}
```

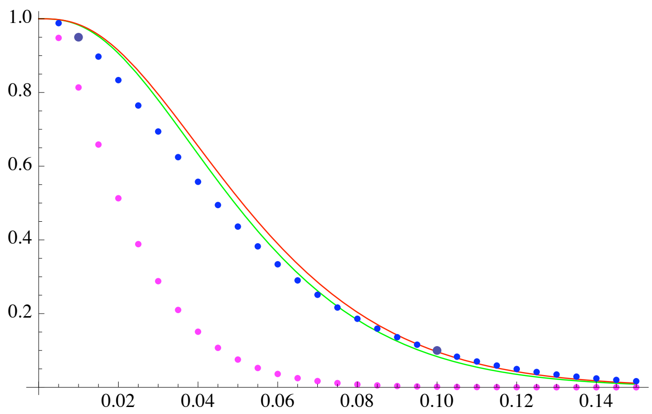



```
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]
```



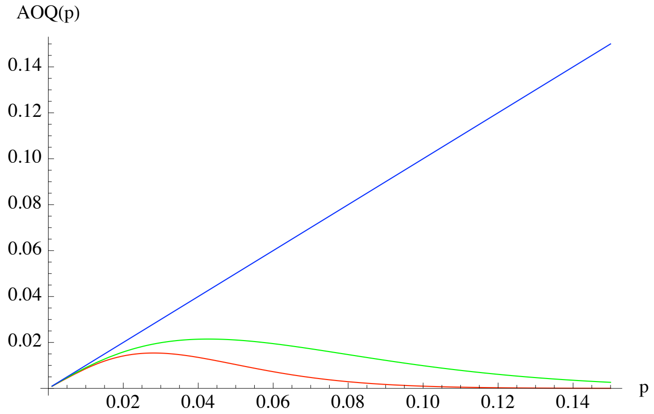
Exercise 13.4

```
n_tot = 800; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

(* Single sampling plans (n,c) according to: *)
(* (54,2), Binomial Distribution approximation + Wetherhill and Brown method RED *)
(* (52,2), Binomial Distribution approximation GREEN*)
(* (37,1), exact HyperGeometric Distribution BLUE *)
(* (80,2), Norm ANSI/ASQC Z1.4-1981 MAGENTA *)

OC_Wetherill[p_] = CDF[BinomialDistribution[54, p], 2];
OC_B[p_] = CDF[BinomialDistribution[52, p], 2];
listpOCA = Table[{p, N[CDF[HypergeometricDistribution[37, Round[n_tot × p], n_tot], 1], 5]},
    {p, 0.005, 0.15, 0.005}];
listpOCAnorm = Table[{p, N[CDF[HypergeometricDistribution[80, Round[n_tot × p], n_tot], 1], 2]},
    {p, 0.005, 0.15, 0.005}];

riskpoints = {{p1, 1 - α}, {p2, β}};

G1 = Plot[OC_Wetherill[p], {p, 0, .15},
    PlotStyle → {RGBColor[0, 1, 0], Thickness[0.002]}, DisplayFunction → Identity];
G2 = Plot[OC_B[p], {p, 0, .15}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.002]},
    AxesLabel → {"p", "OC_B(p)"}, DisplayFunction → Identity];
G3 = ListPlot[listpOCA, PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]},
    DisplayFunction → Identity];
G4 = ListPlot[listpOCAnorm, PlotStyle → {RGBColor[1, 0, 1], Thickness[0.01]},
    DisplayFunction → Identity];
G5 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G2, G3, G4, G5, DisplayFunction → $DisplayFunction]
(* OC curve associated to the Norm far from riskpoints *)
```

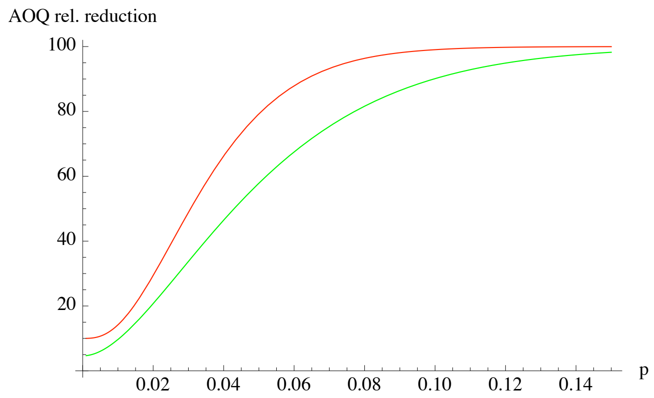


Exercise 13.6

```
ntot = 800; (* lot size *)

AOQ[n_, c_, p_] = ((ntot - n) × p × CDF[BinomialDistribution[n, p], c]) / ntot;
(* Average Outgoing Quality (AOQ) or percentage of defective
 due to rectifying inspection in a single sampling plan and using
 the binomial approximation to the acceptance probability *)

Plot[{AOQ[80, 2, p], AOQ[37, 1, p], p}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 AxesLabel → {"p", "AOQ(p)"}]
(* AOQ of single sampling plan using the Norm RED *)
(* AOQ of single sampling plan obtained by solving
 (13.5) with the exact Hypergeometric distribution GREEN *)

FindMaximum[AOQ[80, 2, p], {p, 0.001, 1}]
FindMaximum[AOQ[37, 1, p], {p, 0.001, 1}]

Plot[{(1 - AOQ[80, 2, p]/p) × 100, (1 - AOQ[37, 1, p]/p) × 100},
 {p, 0.001, 0.15}, AxesLabel → {"p", "AOQ rel. reduction"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}]
(* Associated relative reduction of the percentage of defective *)
```

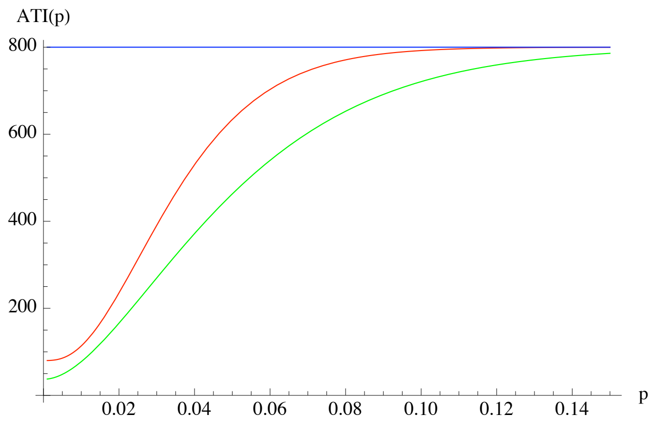



```
{0.0154001, {p → 0.0280931}}

{0.0214757, {p → 0.0427029}}
```



Exercise 13.7

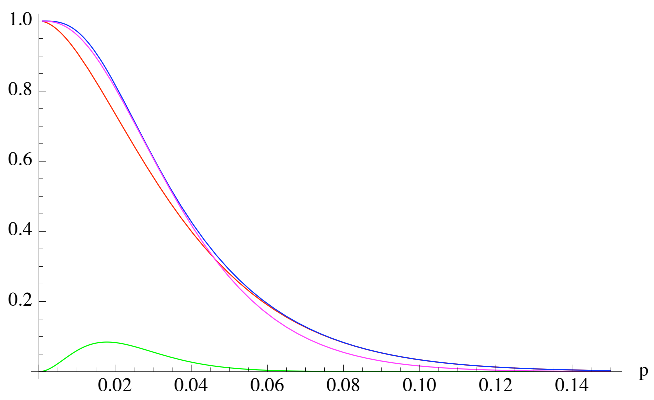
```
n_tot = 800; (* lot size *)

ATI[n_, c_, p_] = n_tot + (n - n_tot) × CDF[BinomialDistribution[n, p], c];
(* Average Total Inspection (ATI) or percentage of defective due to rectifying inspection in a
 single sampling plan and using the binomial approximation to the acceptance probability *)

Plot[{ATI[80, 2, p], ATI[37, 1, p], n_tot}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 AxesLabel → {"p", "ATI(p)"}]
(* ATI of single sampling plan using the Norm RED *)
(* ATI of single sampling plan obtained by solving (13.5) with the exact Hypergeometric distribution GREEN *)
```



Exercise 13.8

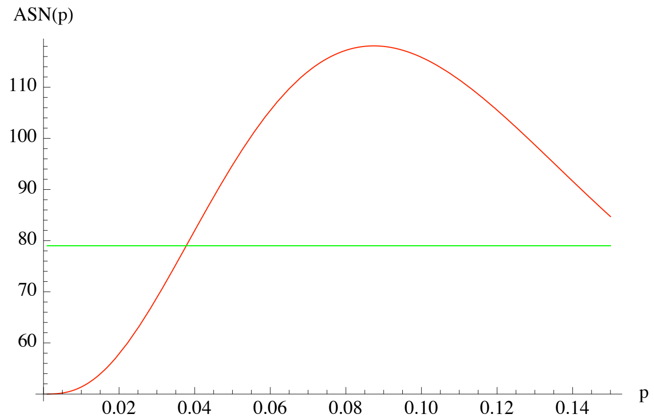
```
n1 = 50; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 100; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1]; (* RED *)

pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] ×
    CDF[BinomialDistribution[n2, p], c2 - k], {k, c1+1, c2}]; (* GREEN *)

pa[p_] = pI[p] + pII[p]; (* BLUE *)

OCB[p_] = CDF[BinomialDistribution[75, p], 2]; (* MAGENTA *)

Plot[{pI[p], pII[p], pa[p], OCB[p]}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1], RGBColor[1, 0, 1]},
 AxesLabel → {"p", ""}]
Null
```

$$pII(p) = \sum_{k=c_1+1}^{c_2} PDF[BinomialDistribution[n_1, p], k] \times CDF[BinomialDistribution[n_2, p], c_2 - k]$$



Exercise 13.9

```
n1 = 50; (* Collect a first sample of size n1 *)
c1 = 2; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 100; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 6; (* Accept the lot if D1+D2≤c2, reject otherwise *)

ASN[p_] =
  n1 + n2 × (CDF[BinomialDistribution[n1, p], c2] - CDF[BinomialDistribution[n1, p], c1]);
(* RED *)
(* Average Sample Number *)

Plot[{ASN[p], 79}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}, AxesLabel → {"p", "ASN(p)"}]
```



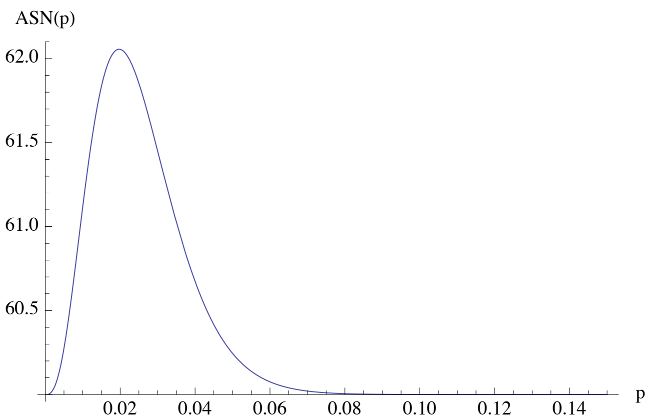
Exercise 13.11

```
n1 = 60; (* Collect a first sample of size n1 *)
c1 = 2; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 120; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject AS SOON AS D1+D2>c2, *)
```

$$ASN[p\_] = n_1 + \sum_{j=c_1+1}^{c_2} PDF[BinomialDistribution[n_1, p], j] \times \left(n_2 \times CDF[BinomialDistribution[n_2, p], c_2 - j] + \frac{c_2 - j + 1}{p} \times PDF[BinomialDistribution[n_2 + 1, p], c_2 - j + 2]\right);$$

```
(* Average Sample Number - double sampling plan with curtailment *)

TableForm[Table[{p, ASN[p]}, {p, 0.005, 0.15, 0.005}],
 TableHeadings → {None, {"p", "ASN(p)"}}]

Plot[ASN[p], {p, 0.001, 0.15}, AxesLabel → {"p", "ASN(p)"}]
```

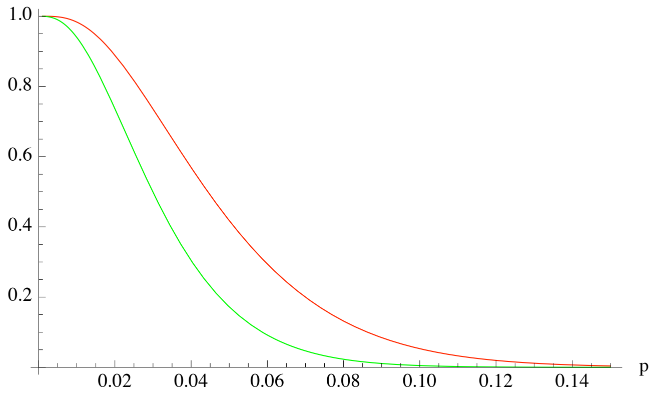
p	ASN(p)
0.005	60.2756
0.01	61.1169
0.015	61.8346
0.02	62.0549
0.025	61.8529
0.03	61.4504
0.035	61.0265
0.04	60.6734
0.045	60.4161
0.05	60.2449
0.055	60.1384
0.06	60.0756
0.065	60.04
0.07	60.0206
0.075	60.0104
0.08	60.0051
0.085	60.0025
0.09	60.0012
0.095	60.0005
0.1	60.0003
0.105	60.0001
0.11	60.0001
0.115	60.
0.12	60.
0.125	60.
0.13	60.
0.135	60.
0.14	60.
0.145	60.
0.15	60.



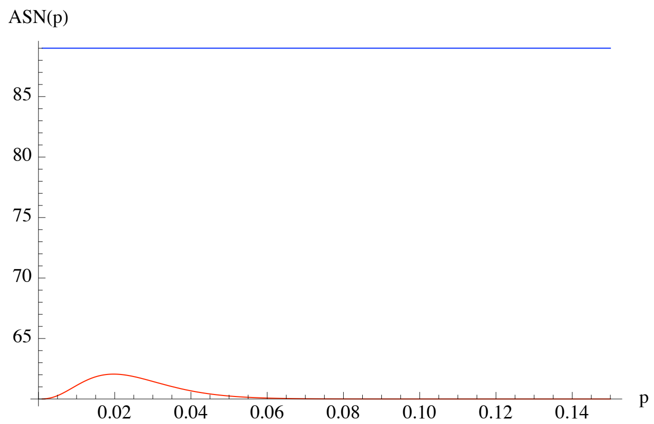
```
pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] =
```

$$\sum_{k=c_1+1}^{c_2} \text{PDF[BinomialDistribution}[n_1, p], k] * \text{CDF[BinomialDistribution}[n_2, p], c_2 - k];$$

```
pa[p_] = pI[p] + pII[p]; (* RED *)

OCB[p_] = CDF[BinomialDistribution[89, p], 2]; (* GREEN *)

Plot[{pa[p], OCB[p]}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}, AxesLabel → {"p", ""}]
```

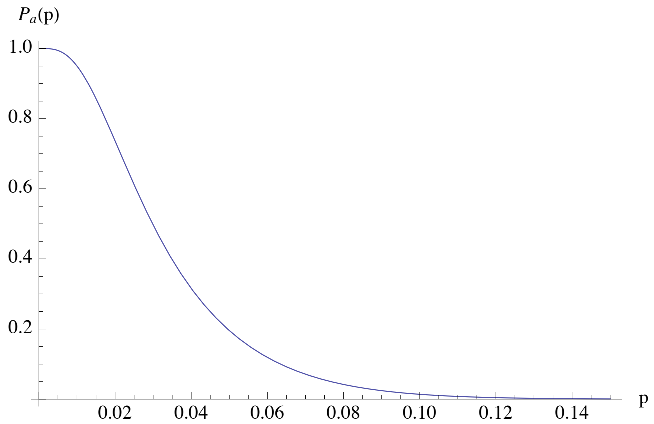


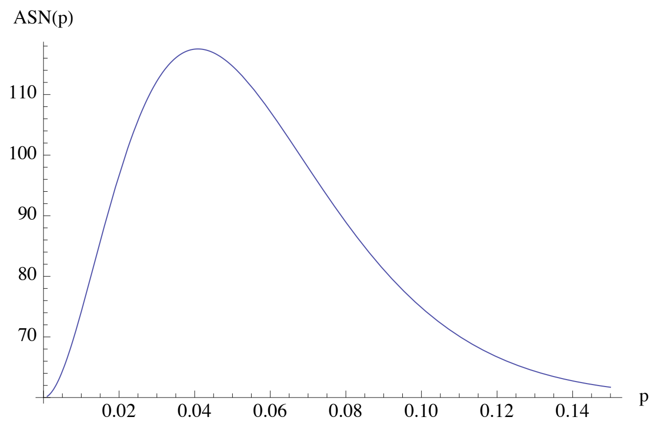

```
Plot[{ASN[p], 89}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}, AxesLabel → {"p", "ASN(p)"}]
```



Exercise 13.12

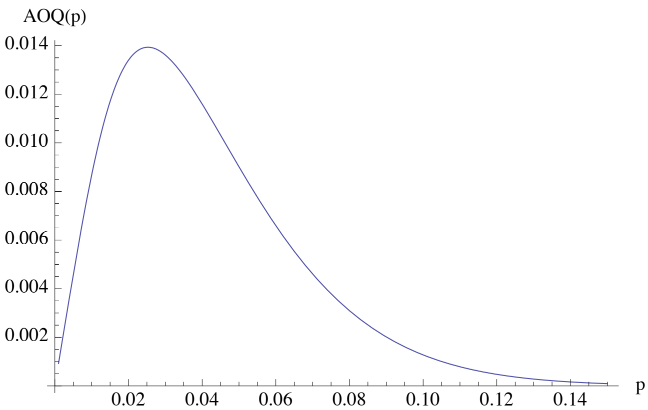
```
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

n1 = 60; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 2 × n1; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] =
  Sum[PDF[BinomialDistribution[n1, p], k] * CDF[BinomialDistribution[n2, p], c2 - k], {k, c1+1, c2}];
pa[p_] = pI[p] + pII[p];

Plot[pa[p], {p, 0.001, 0.15}, AxesLabel → {"p", "Pa(p)"}]
(* Primary OC curve of type B of a double sampling
 plan (without rectifying inspection or curtailment) *)

ASN[p_] =
  n1 + n2 × (CDF[BinomialDistribution[n1, p], c2] - CDF[BinomialDistribution[n1, p], c1]);
(* RED *)
(* Average Sample Number *)

Plot[ASN[p], {p, 0.001, 0.15}, AxesLabel → {"p", "ASN(p)"}]
```





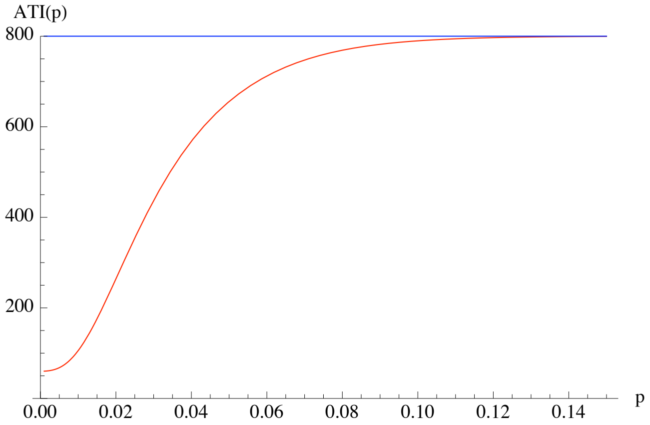
Exercise 13.13

```
ntot = 800; (* lot size *)

p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

n1 = 60; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 2 × n1; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] =
  Sum[PDF[BinomialDistribution[n1, p], k] * CDF[BinomialDistribution[n2, p], c2 - k], {k, c1+1, c2}];
pa[p_] = pI[p] + pII[p];

AOQ[p_] = p × ((ntot - n1) × pI[p] + (ntot - n1 - n2) × pII[p]) / ntot;
Plot[AOQ[p], {p, 0.001, 0.15}, AxesLabel → {"p", "AOQ(p)"}]
(* AOQ of a double sampling (WITH rectifying inspection and NO curtailment) *)

FindMaximum[AOQ[p], {p, 0.001, 1}]
```



{0.0139338, {p → 0.0252902}}

```
ATI[p_] = n₁ × pI[p] + (n₁ + n₂) × pII[p] + n_tot × (1 - pa[p]);
Plot[{ATI[p], 800}, {p, 0.001, 0.15}, PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]},
 AxesLabel → {"p", "ATI(p)"}, PlotRange → {0, n_tot}]
(* ATI of a double sampling plan(WITH rectifying inspection and NO curtailment) *)
```




```
p₁ = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p₂ = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2 × (c + 1)], x];
r[c_] = N[Q[c, 1 - β], 5] / N[Q[c, α], 5];
i = 0;
While[r[i] > p₂/p₁,
 Print["Do not use acceptance number c=", i, " because r(c)=", r[i], ">p₂/p₁=", p₂/p₁]; i++]
Print["Use acceptance number c=", i, " because r(c)=", r[i], "≤p₂/p₁=", p₂/p₁]

n[c_] = Ceiling[Q[i, 1 - β] / (2 × p₂)];
Print["Use the sample size n=", n[i]]

ATISingle[p_] = n_tot + (n[i] - n_tot) × CDF[BinomialDistribution[n[i], p], i];
(* ATI of a single sampling plan(WITH rectifying inspection) *)

Plot[{ATI[p], ATISingle[p], 800}, {p, 0.001, 0.15},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 AxesLabel → {"p", "ASN(p)"}, PlotRange → {0, n_tot}]
```

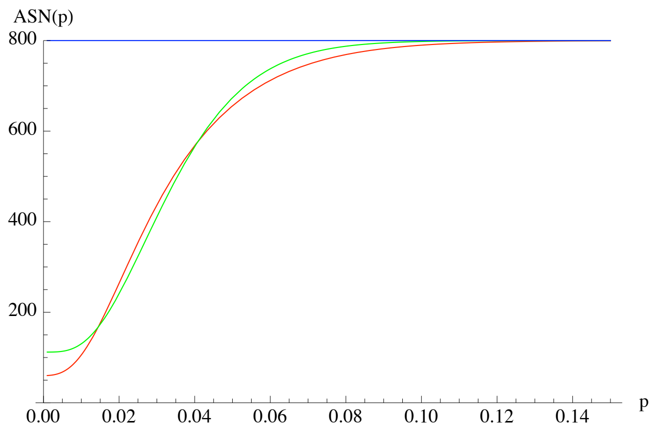
Do not use acceptance number c=0 because r(c)=44.8906> $\frac{p_2}{p_1}$ =6.

Do not use acceptance number c=1 because r(c)=10.9458> $\frac{p_2}{p_1}$ =6.

Do not use acceptance number c=2 because r(c)=6.50896> $\frac{p_2}{p_1}$ =6.

Use acceptance number c=3 because r(c)=4.88962≤ $\frac{p_2}{p_1}$ =6.

Use the sample size n=112



Exercise 13.14

```
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];
```

$$n_\sigma = \text{Floor}\left[\left(\frac{\Omega[1-\alpha] - \Omega[\beta]}{\Omega[p_2] - \Omega[p_1]}\right)^2\right];$$

$$k_\sigma = \frac{\Omega[p_2] \times \Omega[1-\alpha] - \Omega[p_1] \times \Omega[\beta]}{\Omega[\beta] - \Omega[1-\alpha]};$$

$$\text{PVar}[n\_, p\_] = \Phi\left[\sqrt{n} \times (-k_\sigma - \Omega[p])\right];$$

```
i = nσ;
While[PVar[i, p1] < 1 - α || PVar[i, p2] > β, Print["Do not use sample size nσ=", i,
  " because Pa[p1]=", PVar[i, p1], "<", 1 - α , " or Pa[p2]=", PVar[i, p2], ">", β]; i++]

Print["Use sample size nσ=", i, " and acceptance constant", kσ,
 " because Pa[p1]=", PVar[i, p1], "≥", 1 - α , " and Pa[p2]=", PVar[i, p2], "≤", β]
```

Do not use sample size n_σ=11 because $P_a[p_1]$=0.943584<0.95 or $P_a[p_2]$=0.108344>0.1

Use sample size n_σ=12 and acceptance constant
1.84827 because $P_a[p_1]$=0.951149≥0.95 and $P_a[p_2]$=0.0984707≤0.1

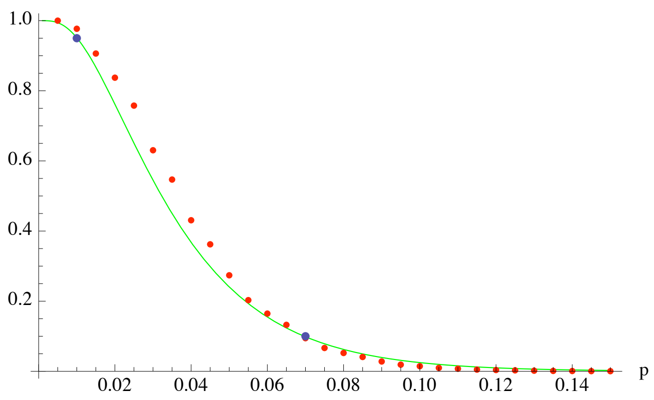
```
bdist[n_, pt_] := BinomialDistribution[n, pt];
pdist[n_, pt_] := PoissonDistribution[n * pt];
hdist[n_, pt_, ng_] := HypergeometricDistribution[n, Round[pt * ng], ng];

bin[x_, n_, pt_, ng_] := PDF[bdist[n, pt], x];
poi[x_, n_, pt_, ng_] := PDF[pdist[n, pt], x];
hip[x_, n_, pt_, ng_] := PDF[hdist[n, pt, ng], x];
```

$$Pa[p\_, \{n\_, c\_, ng\_\}, f\_] := \sum_{d=0}^{c} f[d, n, p, ng];$$

```
Paatr[p_, {{a_, b_}, {e_, d_}}, ng_, f_] :=
 Pa[p, {planoamosatrib[{{a, b}, {e, d}}, ng, f][[1]],
   planoamosatrib[{{a, b}, {e, d}}, ng, f][[2]], ng}, f]

planoamosatrib[{{a_, b_}, {e_, d_}}, ng_, f_] :=
 Module[{n, c},
  j = 0;
  t = 0;
  While[t == 0,
   i = 2;
   While[i ≤ ng && Pa[a, {i, j, ng}, f] ≥ 1 - b,
         i = i + 1]
    If[Pa[e, {i - 1, j, ng}, f] ≤ d, t = 1, t = 0];
   j = j + 1];
  While[Pa[a, {i - 1, j - 1, ng}, f] ≥ 1 - b && Pa[e, {i - 1, j - 1, ng}, f] ≤ d, i = i - 1];
  {n = i, c = j - 1}]

ntot = 500; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

Print["Use the sample size and acceptance number, (n,c)=",
 planoamosatrib[{{p1, α}, {p2, β}}, ntot, hip]]
```

Use the sample size and acceptance number, (n,c)={72, 2}

```
G1 = Plot[PVar[12, p], {p, 0.001, 0.15}, PlotStyle → RGBColor[0, 1, 0],
   AxesLabel → {"p", ""}, DisplayFunction → Identity];
(* GREEN *)

listpOCA = Table[{p, N[CDF[HypergeometricDistribution[72, Round[ntot × p], ntot], 2], 5]},
   {p, 0.005, 0.15, 0.005}];
TableForm[listpOCA, TableHeadings → {None, {"p", "OCA(p)"}}];
G2 = ListPlot[listpOCA, PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity];
(* RED *)

riskpoints = {{p1, 1 - α}, {p2, β}};
G3 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G2, G3, DisplayFunction → $DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)
```



Exercise 13.15

```
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];
```

$$n_\sigma = \text{Floor}\left[\left(\frac{\Omega[1-\alpha]-\Omega[\beta]}{\Omega[p_2]-\Omega[p_1]}\right)^2\right];$$

$$n_s = \text{Floor}\left[\left(1+\frac{3\times n_\sigma\times k_\sigma^2}{6\times n_\sigma-8}\right)\times n_\sigma\right];$$

$$k_\sigma = \frac{\Omega[p_2]\times\Omega[1-\alpha]-\Omega[p_1]\times\Omega[\beta]}{\Omega[\beta]-\Omega[1-\alpha]};$$

$$k_s = \sqrt{\frac{3\times n_s-4}{3\times n_s-3}}\times k_\sigma;$$

$$\text{PVarDesc}[n\_, k\_, p\_] = \Phi\left[\frac{\Omega[1-p]-k\times\sqrt{\frac{3\times n-4}{3\times n-3}}}{\sqrt{\frac{1+\frac{3\times n\times k^2}{6\times n-8}}{n}}}\right];$$

$i = n_s;$

$$\text{While}\left[\text{PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_1\right] < 1-\alpha \;||\; \text{PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_2\right] > \beta,\right.$$

$$\text{Print}\left[\text{"Do not use sample size } n_\sigma\text{=", i, "acceptance constant } k_s\text{=",}\right.$$

$$\sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, \text{" because } P_a[p_1]\text{=", PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_1\right],$$

$$\left.\left.\text{"<", } 1-\alpha\text{, " or } P_a[p_2]\text{=", PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_2\right]\text{, ">", }\beta\right];\ i{+}{+}\right]$$

$$\text{Print}\left[\text{"Use sample size } n_s\text{=", i, " and acceptance constant } k_s\text{=",}\right.$$

$$\sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, \text{" because } P_a[p_1]\text{=", PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_1\right],$$

$$\left.\text{"≥", } 1-\alpha\text{, " and } P_a[p_2]\text{=", PVarDesc}\left[i, \sqrt{\frac{3\times i-4}{3\times i-3}}\times k_\sigma, p_2\right]\text{, "≤", }\beta\right]$$

Do not use sample size n_σ=32acceptance constant k_s= 1.83831 because $P_a[p_1]$=0.954921<0.95 or $P_a[p_2]$=0.115079>0.1

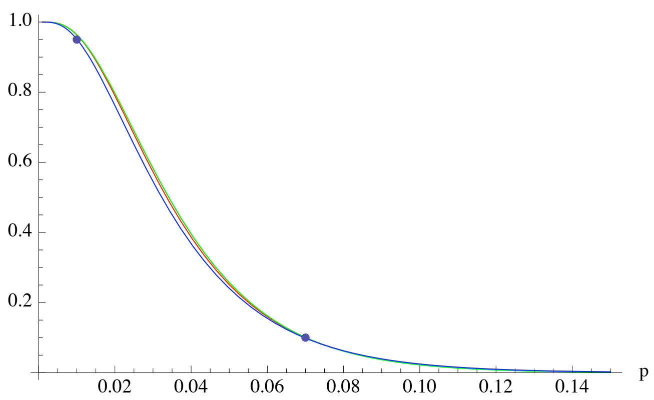
Do not use sample size n_σ=33acceptance constant k_s= 1.83862 because $P_a[p_1]$=0.957213<0.95 or $P_a[p_2]$=0.111027>0.1

Do not use sample size n_σ=34acceptance constant k_s= 1.83891 because $P_a[p_1]$=0.959381<0.95 or $P_a[p_2]$=0.107144>0.1

Do not use sample size n_σ=35acceptance constant k_s= 1.83919 because $P_a[p_1]$=0.96143<0.95 or $P_a[p_2]$=0.103422>0.1

Use sample size n_s=36 and acceptance constant k_s= 1.83945 because $P_a[p_1]$=0.963369≥0.95 and $P_a[p_2]$=0.0998507≤0.1

```
G1 = Plot[PVarDesc[36, 1.83945, p], {p, 0.001, 0.15},
    PlotStyle → RGBColor[1, 0, 0], AxesLabel → {"p", ""}, DisplayFunction → Identity];
(* RED *)

G2 = Plot[CDF[NoncentralStudentTDistribution[36 - 1, √36. × Ω[p]], -√36 × 1.83945],
    {p, 0.001, 0.15}, PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
(* GREEN *)

G3 = Plot[Φ[√12 × (-1.84827 - Ω[p])], {p, 0.001, 0.15},
    PlotStyle → RGBColor[0, 0, 1], DisplayFunction → Identity];
(* BLUE *)

riskpoints = {{p₁, 1 - α}, {p₂, β}};
G4 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]
```

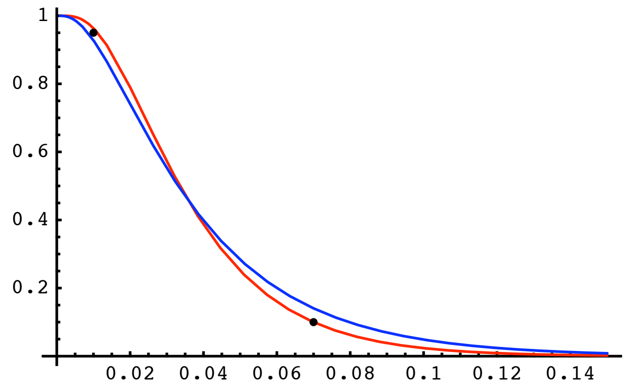


Exercise 13.17

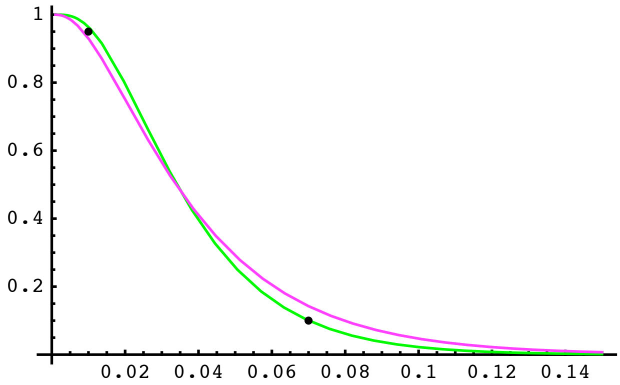
```
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];
```

$$\text{PVarDesc}[n\_, k\_, p\_] = \Phi\left[\frac{\Omega[1-p] - k \times \sqrt{\frac{3\times n-4}{3\times n-3}}}{\sqrt{\frac{1+\frac{3\times n\times k^2}{6\times n-8}}{n}}}\right];$$

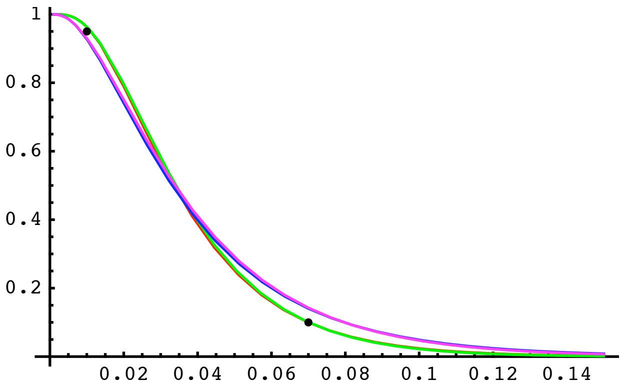
```
G1 = Plot[PVarDesc[36, 1.83945, p], {p, 0.001, 0.15},
   PlotStyle → RGBColor[1, 0, 0], AxesLabel → {"p", ""}, DisplayFunction → Identity];
(* RED *)

G2 = Plot[CDF[NoncentralStudentTDistribution[36 - 1, √36. × Ω[p]], -√36 × 1.83945],
   {p, 0.001, 0.15}, PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
(* GREEN *)

G3 = Plot[PVarDesc[25, 1.85, p], {p, 0.001, 0.15},
   PlotStyle → RGBColor[0, 0, 1], DisplayFunction → Identity];
(* BLUE *)

G4 = Plot[CDF[NoncentralStudentTDistribution[25 - 1, √25 × Ω[p]], -√25 × 1.85],
   {p, 0.001, 0.15}, PlotStyle → RGBColor[1, 0, 1], DisplayFunction → Identity];
(* MAGENTA *)

riskpoints = {{p1, 1 - α}, {p2, β}};
G5 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G3, G5, DisplayFunction → $DisplayFunction]

Show[G2, G4, G5, DisplayFunction → $DisplayFunction]

Show[G1, G2, G3, G4, G5, DisplayFunction → $DisplayFunction]
```



- Graphics -



- Graphics -



- Graphics -