SOME HINTS for a few exercises from Chap. 10

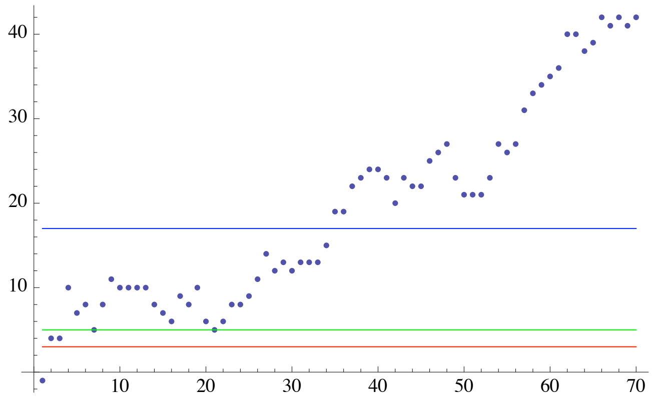
Exercise 10.8

```
data = {4, 10, 5, 11, 2, 6, 2, 8, 8, 4, 5, 5, 5, 3, 4, 4, 8, 4, 7, 1,
   4, 6, 7, 5, 6, 7, 8, 3, 6, 4, 6, 5, 5, 7, 9, 5, 8, 6, 6, 5,
   4, 2, 8, 4, 5, 8, 6, 6, 1, 3, 5, 5, 7, 9, 4, 6, 9, 7, 6, 6,
   6, 9, 5, 3, 6, 8, 4, 6, 4, 6};

n = 100;
p0 = 0.05;
k = n × p0;
LCL = 3;
CL = n × p0;
UCL = 17;

z[0] = 0;
Do[z[i] = z[i - 1] + data[[i]] - k, {i, 1, Length[data]}]
cusum = Table[z[i], {i, 1, Length[data]}];

list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

TableForm[Transpose[{data, list1, cusum, list3}],
 TableHeadings → {Automatic, {"yN", "LCL", "zN", "UCL"}}]

G1 = ListPlot[cusum, PlotStyle → PointSize[0.009], DisplayFunction → Identity];
G2 = ListPlot[list1, PlotStyle → RGBColor[1, 0, 0],
   Joined → True, DisplayFunction → Identity];
G3 = ListPlot[list2, PlotStyle → RGBColor[0, 1, 0],
   Joined → True, DisplayFunction → Identity];
G4 = ListPlot[list3, PlotStyle → RGBColor[0, 0, 1],
   Joined → True, DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[cusum] ×
   Mean[Table[If[cusum[[i]] < LCL || cusum[[i]] > UCL, 1, 0], {i, 1, Length[cusum]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

	y_N	LCL	z_N	UCL
1	4	3	-1.	17
2	10	3	4.	17
3	5	3	4.	17
4	11	3	10.	17
5	2	3	7.	17
6	6	3	8.	17

7	2	3	5.	17
8	8	3	8.	17
9	8	3	11.	17
10	4	3	10.	17
11	5	3	10.	17
12	5	3	10.	17
13	5	3	10.	17
14	3	3	8.	17
15	4	3	7.	17
16	4	3	6.	17
17	8	3	9.	17
18	4	3	8.	17
19	7	3	10.	17
20	1	3	6.	17
21	4	3	5.	17
22	6	3	6.	17
23	7	3	8.	17
24	5	3	8.	17
25	6	3	9.	17
26	7	3	11.	17
27	8	3	14.	17
28	3	3	12.	17
29	6	3	13.	17
30	4	3	12.	17
31	6	3	13.	17
32	5	3	13.	17
33	5	3	13.	17
34	7	3	15.	17
35	9	3	19.	17
36	5	3	19.	17
37	8	3	22.	17
38	6	3	23.	17
39	6	3	24.	17
40	5	3	24.	17
41	4	3	23.	17
42	2	3	20.	17
43	8	3	23.	17
44	4	3	22.	17
45	5	3	22.	17
46	8	3	25.	17
47	6	3	26.	17
48	6	3	27.	17
49	1	3	23.	17
50	3	3	21.	17
51	5	3	21.	17
52	5	3	21.	17
53	7	3	23.	17
54	9	3	27.	17
55	4	3	26.	17

56	6	3	27.	17
57	9	3	31.	17
58	7	3	33.	17
59	6	3	34.	17
60	6	3	35.	17
61	6	3	36.	17
62	9	3	40.	17
63	5	3	40.	17
64	3	3	38.	17
65	6	3	39.	17
66	8	3	42.	17
67	4	3	41.	17
68	6	3	42.	17
69	4	3	41.	17
70	6	3	42.	17



There is (are) 37 observation(s) beyond the control limits.

Exercise 10.10

```
n = 10;
p = 0.5;
χ[z_] = PDF[BinomialDistribution[n, p], z];
k = 6;
LCL = 2;
UCL = 10;

(* The (infinite) probability transition matrix of the original Markov
   chain has entries P=[p_ij]_{i,j=...,LCL,...,UCL,...} = [P_Bin(n,p)(k+j-i)]_{i,j=...,LCL,...,UCL,...} *)

(* Q block of the probability transition matrix of the absorbing Markov chain *)
Q = Table[χ[k + j - i],
   {i, LCL, UCL}, {j, LCL, UCL}];
MatrixForm[Round[N[Q, 5] × 10000] * 0.0001]
```

0.2051	0.1172	0.0439	0.0098	0.001	0	0	0	0
0.2461	0.2051	0.1172	0.0439	0.0098	0.001	0	0	0
0.2051	0.2461	0.2051	0.1172	0.0439	0.0098	0.001	0	0
0.1172	0.2051	0.2461	0.2051	0.1172	0.0439	0.0098	0.001	0
0.0439	0.1172	0.2051	0.2461	0.2051	0.1172	0.0439	0.0098	0.001
0.0098	0.0439	0.1172	0.2051	0.2461	0.2051	0.1172	0.0439	0.0098
0.001	0.0098	0.0439	0.1172	0.2051	0.2461	0.2051	0.1172	0.0439
0	0.001	0.0098	0.0439	0.1172	0.2051	0.2461	0.2051	0.1172
0	0	0.001	0.0098	0.0439	0.1172	0.2051	0.2461	0.2051

Exercise 10.11

```
n = 80;
λ = 80 × 0.04;
dist = PoissonDistribution[λ];
χ[z_] = PDF[dist, z];
β[z_] = CDF[dist, z];
k = 2;
LCL = 0;
UCL = 2;

Q = Table[If[j == 0, β[k - i], χ[k + j - i]],
    {i, LCL, UCL}, {j, LCL, UCL}]; (* Check Equation (10.8). *)
MatrixForm[Round[N[Q, 5] × 10000] * 0.0001]
```

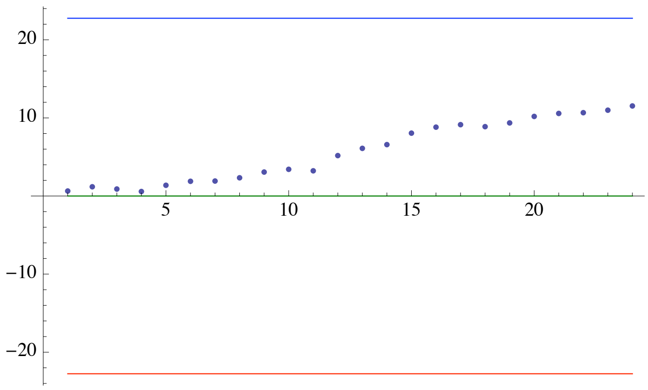
$$\begin{pmatrix} 0.3799 & 0.2226 & 0.1781 \\ 0.1712 & 0.2087 & 0.2226 \\ 0.0408 & 0.1304 & 0.2087 \end{pmatrix}$$

Exercise 10.17

```
data = {34.5, 34.2, 31.6, 31.5, 35.0, 34.1, 32.6, 33.8, 34.8, 33.6, 31.9, 38.6,
    35.4, 34.0, 37.1, 34.9, 33.5, 31.7, 34.0, 35.1, 33.7, 32.8, 33.5, 34.2};
Length[data]

n = 5;
μ0 = 32.5;
σ0 = √49 ;
k = μ0;
h = 22.7610;(* The in-control ARL of this standard CUSUM chart for
   μ is approximately 500. Its control statistic is a deviation between the sample
   mean and the target value (measured in in-control standard deviations of X̄). *)
LCL = -h;
CL = 0;
UCL = h;

z[0] = 0;
Do[z[i] = z[i - 1] + (data[[i]] - k)/(σ0/√n), {i, 1, Length[data]}]
cusum = Table[z[i], {i, 1, Length[data]}];

list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

TableForm[Transpose[{data, list1, cusum, list3}],
 TableHeadings → {Automatic, {"yN", "LCL", "zN", "UCL"}}]

G1 = ListPlot[cusum, PlotStyle → PointSize[0.009], DisplayFunction → Identity];
G2 = ListPlot[list1, Joined → True,
    PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity];
G3 = ListPlot[list2, Joined → True, PlotStyle → RGBColor[0, 1, 0],
    DisplayFunction → Identity];
G4 = ListPlot[list3, Joined → True, PlotStyle → RGBColor[0, 0, 1],
    DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[cusum] ×
    Mean[Table[If[cusum[[i]] < LCL || cusum[[i]] > UCL, 1, 0], {i, 1, Length[cusum]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
   numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
  Print["There are no observations beyond the control limits."]]
```

24

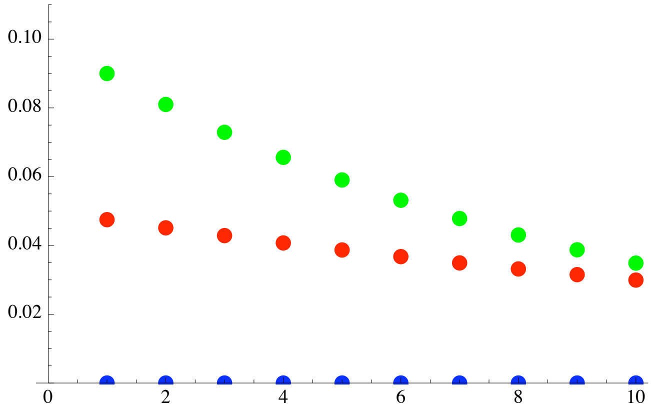
	y_N	LCL	z_N	UCL
1	34.5	-22.761	0.638877	22.761
2	34.2	-22.761	1.18192	22.761
3	31.6	-22.761	0.894427	22.761
4	31.5	-22.761	0.574989	22.761
5	35.	-22.761	1.37358	22.761
6	34.1	-22.761	1.88469	22.761
7	32.6	-22.761	1.91663	22.761
8	33.8	-22.761	2.3319	22.761
9	34.8	-22.761	3.06661	22.761
10	33.6	-22.761	3.41799	22.761
11	31.9	-22.761	3.22633	22.761
12	38.6	-22.761	5.1749	22.761
13	35.4	-22.761	6.10127	22.761
14	34.	-22.761	6.58043	22.761
15	37.1	-22.761	8.04984	22.761
16	34.9	-22.761	8.8165	22.761
17	33.5	-22.761	9.13593	22.761
18	31.7	-22.761	8.88038	22.761
19	34.	-22.761	9.35954	22.761
20	35.1	-22.761	10.1901	22.761
21	33.7	-22.761	10.5734	22.761
22	32.8	-22.761	10.6692	22.761
23	33.5	-22.761	10.9887	22.761
24	34.2	-22.761	11.5317	22.761



There are no observations beyond the control limits.

Exercise 10.21

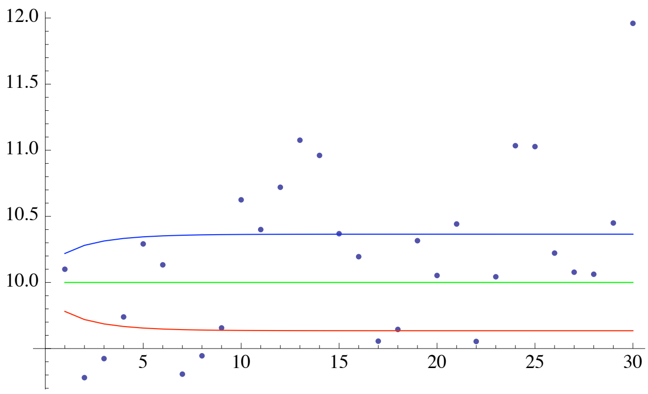
```
λ = {0.05, 0.1, 1.};
G1 = ListPlot[Table[λ[[1]] × (1 - λ[[1]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[1, 0, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G2 = ListPlot[Table[λ[[2]] × (1 - λ[[2]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 1, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G3 = ListPlot[Table[λ[[3]] × (1 - λ[[3]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 0, 1], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
Show[G1, G2, G3, DisplayFunction → $DisplayFunction]
```



Exercise 10.25

```
μ0 = 10.0;
w0 = μ0;
λ = 0.2;
σ0 = 2;
γ = 3;
n = Length[data];

data = {10.5, 6, 10, 11, 12.5, 9.5, 6, 10, 10.5, 14.5, 9.5, 12,
  12.5, 10.5, 8, 9.5, 7, 10, 13, 9, 12, 6, 12, 15, 11, 7, 9.5, 10, 12, 18};

W = Function[N, (1 - λ)^N × w0 + Sum[λ × (1 - λ)^j × data[[N - j]], {j, 0, N - 1}]]; (* Check Equation (10.19). *)

ewma = Table[W[i], {i, 1, Length[data]}];

LCLexact = Table[μ0 - γ × σ0 × Sqrt[(λ × (1 - (1 - λ)^(2 N)))/((2 - λ) × n)], {N, 1, Length[data]}];
CL = Table[μ0, {N, 1, Length[data]}];
UCLexact = Table[μ0 + γ × σ0 × Sqrt[(λ × (1 - (1 - λ)^(2 N)))/((2 - λ) × n)], {N, 1, Length[data]}];

TableForm[Transpose[{data, LCLexact, ewma, UCLexact}],
 TableHeadings → {Automatic, {"x̄N", "LCLN", "wN", "UCLN"}}]

G1 = ListPlot[ewma, PlotStyle → PointSize[0.009], DisplayFunction → Identity];
G2 = ListPlot[LCLexact, Joined → True,
   PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity];
G3 = ListPlot[CL, Joined → True, PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
G4 = ListPlot[UCLexact, Joined → True,
   PlotStyle → RGBColor[0, 0, 1], DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[ewma] ×
   Mean[Table[If[ewma[[i]] < LCLexact[[i]] || ewma[[i]] > UCLexact[[i]], 1, 0],
     {i, 1, Length[ewma]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

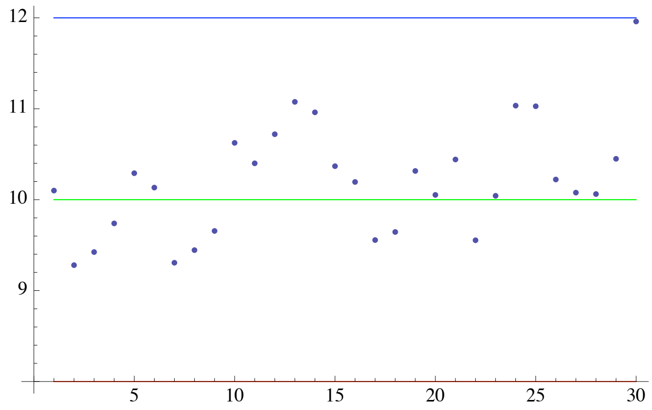
	$\overline{x}_N$	LCL_N	w_N	UCL_N
1	10.5	9.78091	10.1	10.2191
2	6	9.71943	9.28	10.2806
3	10	9.68634	9.424	10.3137
4	11	9.66689	9.7392	10.3331
5	12.5	9.65501	10.2914	10.345
6	9.5	9.64762	10.1331	10.3524
7	6	9.64297	9.30647	10.357
8	10	9.64003	9.44518	10.36
9	10.5	9.63816	9.65614	10.3618
10	14.5	9.63696	10.6249	10.363
11	9.5	9.6362	10.3999	10.3638
12	12	9.63571	10.7199	10.3643
13	12.5	9.6354	11.076	10.3646
14	10.5	9.6352	10.9608	10.3648
15	8	9.63508	10.3686	10.3649
16	9.5	9.635	10.1949	10.365
17	7	9.63494	9.55591	10.3651
18	10	9.63491	9.64473	10.3651
19	13	9.63489	10.3158	10.3651
20	9	9.63488	10.0526	10.3651
21	12	9.63487	10.4421	10.3651
22	6	9.63486	9.55368	10.3651
23	12	9.63486	10.0429	10.3651
24	15	9.63486	11.0344	10.3651
25	11	9.63485	11.0275	10.3651
26	7	9.63485	10.222	10.3651
27	9.5	9.63485	10.0776	10.3651
28	10	9.63485	10.0621	10.3651
29	12	9.63485	10.4497	10.3651
30	18	9.63485	11.9597	10.3651



There is (are) 17 observation(s) beyond the control limits.

```
LCLasympt = Table[μ0 - γ × s × Sqrt[λ/(2 - λ)], {N, 1, Length[data]}];
CL = Table[μ0, {N, 1, Length[data]}];
UCLasympt = Table[μ0 + γ × s × Sqrt[λ/(2 - λ)], {N, 1, Length[data]}];

TableForm[Transpose[{data, LCLasympt, ewma, UCLasympt}],
 TableHeadings → {Automatic, {"x̄N", "LCLa", "wN", "UCLa"}}]

G1 = ListPlot[ewma, PlotStyle → PointSize[0.009], DisplayFunction → Identity];
G2 = ListPlot[LCLasympt, Joined → True,
   PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity];
G3 = ListPlot[CL, Joined → True, PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
G4 = ListPlot[UCLasympt, Joined → True,
   PlotStyle → RGBColor[0, 0, 1], DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[ewma] ×
   Mean[Table[If[ewma[[i]] < LCLasympt[[i]] || ewma[[i]] > UCLasympt[[i]], 1, 0],
     {i, 1, Length[ewma]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

	$\overline{x}_N$	LCL_a	w_N	UCL_a
1	10.5	8.	10.1	12.
2	6	8.	9.28	12.
3	10	8.	9.424	12.
4	11	8.	9.7392	12.
5	12.5	8.	10.2914	12.
6	9.5	8.	10.1331	12.
7	6	8.	9.30647	12.
8	10	8.	9.44518	12.
9	10.5	8.	9.65614	12.
10	14.5	8.	10.6249	12.
11	9.5	8.	10.3999	12.
12	12	8.	10.7199	12.
13	12.5	8.	11.076	12.
14	10.5	8.	10.9608	12.
15	8	8.	10.3686	12.
16	9.5	8.	10.1949	12.
17	7	8.	9.55591	12.
18	10	8.	9.64473	12.
19	13	8.	10.3158	12.
20	9	8.	10.0526	12.
21	12	8.	10.4421	12.
22	6	8.	9.55368	12.
23	12	8.	10.0429	12.
24	15	8.	11.0344	12.
25	11	8.	11.0275	12.
26	7	8.	10.222	12.
27	9.5	8.	10.0776	12.
28	10	8.	10.0621	12.
29	12	8.	10.4497	12.
30	18	8.	11.9597	12.




```
There are no observations beyond the control limits.
```

Exercise 10.35

- **Upper one-sided Shewhart chart**

```
Φ[z_] = CDF[NormalDistribution[0, 1], z];
n = 5;
pfa_μ = 1 / 500.0;
γS-μ = Quantile[NormalDistribution[0, 1], 1 - pfa_μ];
```

$$\text{ARL}_{S-\mu}[\delta\_, \theta\_] = \frac{1}{1 - \Phi\left[\frac{\gamma_{S-\mu} - \delta}{\theta}\right]};$$

```
ARLS-μ[0, 1]
500.
```

- **Upper one-sided EWMA chart** (0% Head Start)

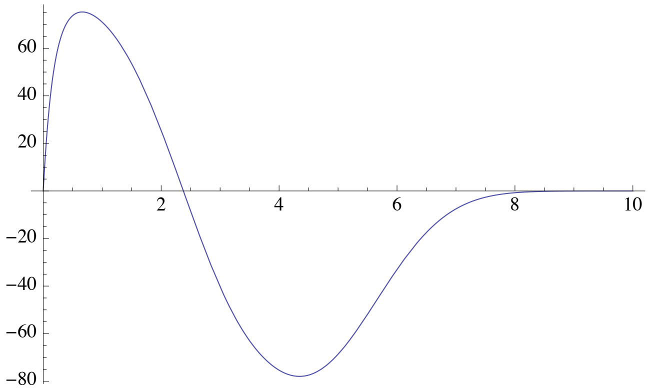
```
γE-μ = 2.8116;
λμ = 0.134;
xμ = 40;

Aμ[δ_, θ_] =
  Table[If[j == -1, 0, Φ[1/θ ((γE-μ ((j + 1) - (1 - λμ) (i + 1/2)))/((xμ + 1) Sqrt[λμ (2 - λμ)]) - δ)]], {i, 0, xμ}, {j, -1, xμ}];

Tμ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, xμ + 1}, {j, 0, xμ}];

Qμ[δ_, θ_] = Aμ[δ, θ].Tμ;

IDμ = IdentityMatrix[xμ + 1];
Mμ[δ_, θ_] := Inverse[IDμ - Qμ[δ, θ]];
umμ = Table[1, {i, 0, xμ}];
ARLE-μ[i_, δ_, θ_] := Table[If[j == i, 1, 0], {j, 0, xμ}] . Mμ[δ, θ] . umμ;
ARLE-μ[0, 0, 1]

shift = {0, 0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};

TableForm[Table[{shift[[i]], ARLE-μ[0, shift[[i]], 1], ARLS-μ[shift[[i]], 1]},
  {i, 1, Length[shift]}], TableHeadings → {Automatic, {"δ", "ARLE-μ", "ARLS-μ"}}]
(* ARL values for the EWMA and Shewhart charts *)
```

500.047

	δ	$ARL_{E-\mu}$	$ARL_{S-\mu}$
1	0	500.047	500.
2	0.05	342.199	427.203
3	0.1	239.007	365.849
4	0.2	124.546	270.17
5	0.3	71.1556	201.354
6	0.4	44.4944	151.445
7	0.5	30.1927	114.948
8	0.6	21.9546	88.04
9	0.7	16.8793	68.0411
10	0.8	13.5571	53.0582
11	0.9	11.2646	41.7445
12	1.	9.61043	33.1351
13	1.5	5.52942	11.8939
14	2.	3.92261	5.26515
15	3.	2.55816	1.8232

- **EWMA vs. Shewhart chart** (percentual reduction in the ARL when the Shewhart chartis replaced by a EWMA chart)

```
Plot[(1 - ARL_{E-μ}[0, δ, 1] / ARL_{S-μ}[δ, 1]) × 100, {δ, 0, 10}]
```



Exercise 10.37

- **Upper one-sided EWMA chart** (0% Head Start)

```
n = 5;
sigma_0 = 1;
λ_σ = 0.05;
LCL_{E-σ} = 0;
UCL_{E-σ} = 0.157079;
```

$$\gamma_{E-\sigma} = \frac{UCL_{E-\sigma} - \text{Log}\left[(\text{sigma}_0)^2\right]}{\sqrt{\text{PolyGamma}\left[1, \frac{n-1}{2}\right] \times \frac{\lambda_\sigma}{2-\lambda_\sigma}}};$$

```
χ[z_] = CDF[ChiSquareDistribution[n - 1], z];
x_σ = 20;
A_σ[θ_] = Table[If[j == -1, 0,
    χ[(n - 1)/θ^2 ×
      Exp[(γ_{E-σ} × Sqrt[PolyGamma[1, (n - 1)/2]] ((j + 1) - (1 - λ_σ) (i + 1/2))) / ((x_σ + 1) Sqrt[λ_σ (2 - λ_σ)])]
    ]], {i, 0, x_σ}, {j, -1, x_σ}];

T_σ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, x_σ + 1}, {j, 0, x_σ}];

Q_σ[θ_] = A_σ[θ].T_σ;

ID_σ = IdentityMatrix[x_σ + 1];
M_σ[θ_] := Inverse[ID_σ - Q_σ[θ]];
um_σ = Table[1, {i, 0, x_σ}];
b_σ[i_] = Table[If[j == i, 1, 0], {j, 0, x_σ}];
ARL_{E-σ}[i_, θ_] := b_σ[i] . M_σ[θ] . um_σ;

ARL_{E-σ}[0, 1]
```

370.411

Upper one-sided Shewhart chart

$UCL_{S-\sigma} = 4.06286;$

$\gamma_{S-\sigma} = UCL_{S-\sigma} \times \frac{n-1}{1};$

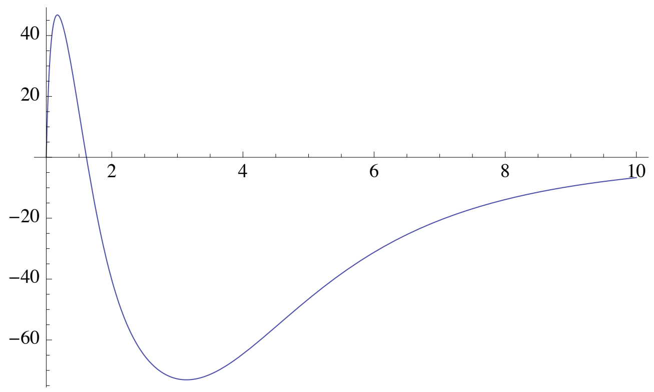
$ARL_{S-\sigma}[\theta\_] = \frac{1}{1 - \chi\left[\frac{\gamma_{S-\sigma}}{\theta^2}\right]};$

$ARL_{S-\sigma}[1]$

370.415

- **EWMA vs. Shewhart chart** (percentual reduction in the ARL when the Shewhart chartis replaced by a EWMA chart)

$\text{Plot}\left[\left(1 - \frac{ARL_{E-\sigma}[0, \theta]}{ARL_{S-\sigma}[\theta]}\right) \times 100, \{\theta, 1, 10\}, \text{AxesOrigin} \to \{1, 0\}\right]$



Exercise 10.38

```
n = 5;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfaμ = 1 / 1000.;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfaμ];

pfaσ = 1/1000.;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfaσ];
```

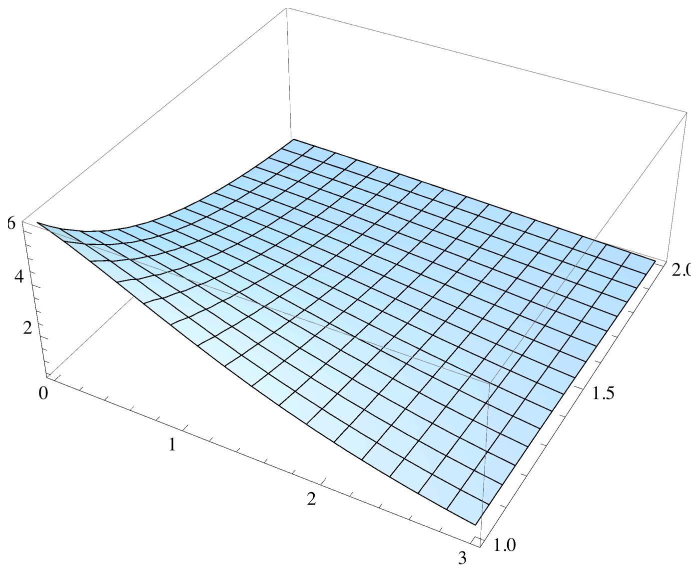
$$ARL_\mu[\delta\_, \theta\_] = \frac{1}{1 - \Phi\left[\frac{\gamma_\mu - \delta}{\theta}\right]};$$

$$ARL_\sigma[\theta\_] = \frac{1}{1 - \chi\left[\frac{\gamma_\sigma}{\theta^2}\right]};$$

$$ARL_{\mu,\sigma}[\delta\_, \theta\_] = \frac{1}{1 - \Phi\left[\frac{\gamma_\mu - \delta}{\theta}\right] \times \chi\left[\frac{\gamma_\sigma}{\theta^2}\right]};$$

```
ARLμ[0, 1]
ARLσ[1]
ARLμ,σ[0, 1]
Plot3D[Log[ARLμ,σ[δ, θ]], {δ, 0, 3}, {θ, 1, 2}]
```

```
1000.

1000.

500.25
```



$$\text{ARL}_{\mu,\sigma}[\delta\_,\ \theta\_] = \frac{\text{ARL}_{\mu}[\delta,\ \theta] \times \text{ARL}_{\sigma}[\theta]}{\text{ARL}_{\mu}[\delta,\ \theta] + \text{ARL}_{\sigma}[\theta] - 1};$$

```
(* Relating the ARL of the joint scheme and ssthe ones of the individual charts... *)
```

$\text{ARL}_{\mu}[0,\ 1]$
$\text{ARL}_{\sigma}[1]$
$\text{ARL}_{\mu,\sigma}[0,\ 1]$

```
1000.

1000.

500.25
```

Exercise 10.39

```
n = 10;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];
```

data_{μ} = {1.04, 1.06, 1.09, 1.05, 1.07, 1.06, 1.05, 1.10, 1.09,
1.05, 0.99, 1.06, 1.05, 1.07, 1.11, 1.04, 1.03, 1.05, 1.06, 1.04};
data_{σ} = {0.87, 0.85, 0.90, 0.85, 0.73, 0.80, 0.78, 0.83, 0.87, 0.860 .86,
0.79, 0.82, 0.75, 0.76, 0.89, 0.91, 0.85, 0.83, 0.79, 0.85};

$\text{pfa}_{\mu} = 0.002;$
$\gamma_{\mu} = \text{Quantile}\left[\text{NormalDistribution}[0, 1], 1 - \text{pfa}_{\mu}\right];$
$\text{pfa}_{\sigma} = 0.002;$
$\gamma_{\sigma} = \text{Quantile}\left[\text{ChiSquareDistribution}[n - 1], 1 - \text{pfa}_{\sigma}\right];$

$\mu_0 = \text{Mean}[\text{data}_{\mu}];$
$\sigma_0 = \sqrt{\text{Mean}[\text{data}_{\sigma}]};$
$\text{LCL}_{\mu} = 0$
$\text{UCL}_{\mu} = \mu_0 + \gamma_{\mu} \times \frac{\sigma_0}{\sqrt{n}}$
$\text{LCL}_{\sigma} = 0$
$\text{UCL}_{\sigma} = \frac{\sigma_0^2}{n - 1} \times \gamma_{\sigma}$

```
0

1.88368

0

2.38266
```

$$\text{probsignal}_{\mu,\sigma}[\delta\_,\ \theta\_] = 1 - \Phi\left[\frac{\gamma_{\mu} - \delta}{\theta}\right] \times \chi\left[\frac{\gamma_{\sigma}}{\theta^2}\right];$$

$\text{probsignal}_{\mu,\sigma}[0,\ 1]$

```
0.003996
```

$\text{probvalidsignal} = \text{probsignal}_{\mu,\sigma}[0.1,\ 1.1];$

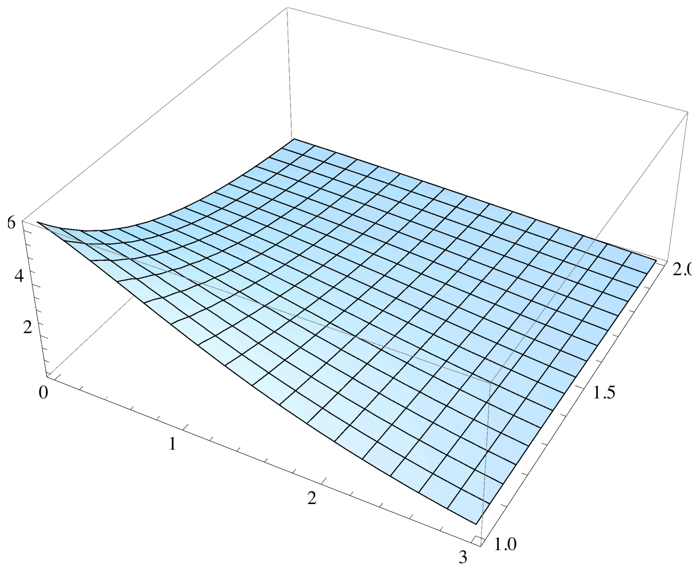
```
dist = GeometricDistribution[probvalidsignal] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
CDF[dist, 10 - 1]

0.150624
```

```
ARLμ[0, 1]
ARLσ[1]
ARLμ,σ[0, 1]
Plot3D[Log[ARLμ,σ[δ, θ]], {δ, 0, 3}, {θ, 1, 2}]
```

```
1000.
```

```
1000.
```

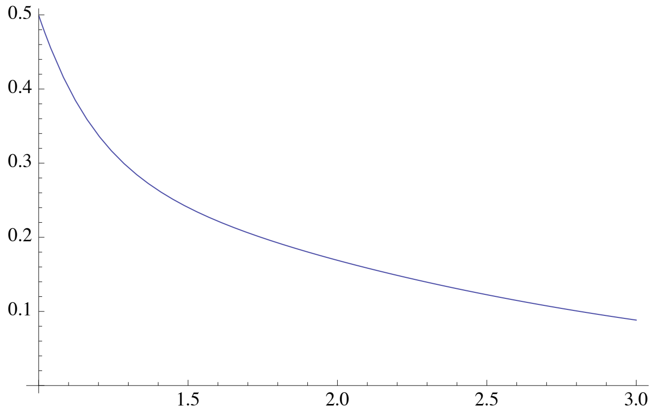
```
500.25
```



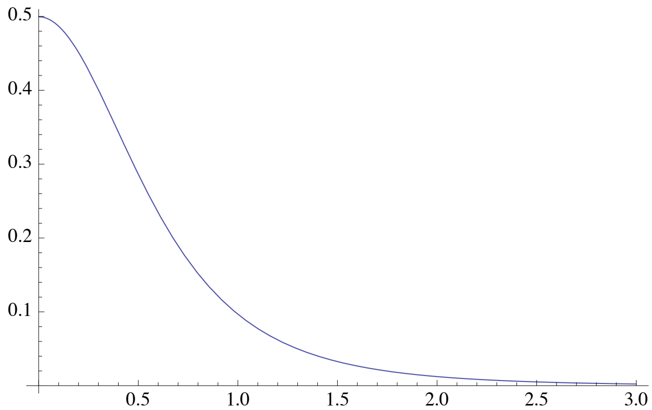
Exercise 10.44

```
n = 5;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfa_μ = 1 / 500.0;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfa_μ / 2];

pfa_σ = 1 / 500.0;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfa_σ];

PMS_III[θ_] = (1 - (Φ[γμ/θ] - Φ[-γμ/θ])) / (1/χ[γσ/θ^2] - (Φ[γμ/θ] - Φ[-γμ/θ]));

theta =
  {1.01, 1.03, 1.05, 1.10, 1.20, 1.30, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 3.00};
TableForm[Table[{theta[[i]], PMS_III[theta[[i]]]}, {i, 1, Length[theta]}],
 TableHeadings → {Automatic, {"δ", "PMS_III"}}]
Plot[{PMS_III[θ]}, {θ, 1.001, 3}, AxesOrigin -> {1, 0}]


PMS_IV[δ_] = (1 - χ[γσ]) / (1/(Φ[γμ-δ] - Φ[-γμ-δ]) - χ[γσ]);
delta = {0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};
TableForm[Table[{delta[[i]], PMS_IV[delta[[i]]]}, {i, 1, Length[delta]}],
 TableHeadings → {Automatic, {"δ", "PMS_IV"}}]

Plot[{PMS_IV[δ]}, {δ, 0.001, 3}, AxesOrigin -> {0, 0}]
```

	δ	PMS_{III}
1	1.01	0.487829
2	1.03	0.465842
3	1.05	0.445584
4	1.1	0.401783
5	1.2	0.337471
6	1.3	0.294136
7	1.4	0.2634
8	1.5	0.240238
9	1.6	0.221722
10	1.7	0.206146
11	1.8	0.192512
12	1.9	0.18023
13	2.	0.16895
14	3.	0.0883101



	δ	PMS_{IV}
1	0.05	0.496258
2	0.1	0.48673
3	0.2	0.451344
4	0.3	0.400673
5	0.4	0.343289
6	0.5	0.286308
7	0.6	0.234262
8	0.7	0.189271
9	0.8	0.151773
10	0.9	0.121258
11	1.	0.0967965
12	1.5	0.0326784
13	2.	0.0123586
14	3.	0.00230455



Exercise 10.47

```
n = 10;
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfa_σ = 1 / 100.0;
γ_σ = Quantile[ChiSquareDistribution[n - 1], 1 - pfa_σ];
1 - χ[γ_σ / .9^2]
```

0.00153939

```
pfaσ = 1 / 100.0;
a = Quantile[ChiSquareDistribution[n - 1], pfaσ/2];
b = Quantile[ChiSquareDistribution[n - 1], 1 - pfaσ/2];
1 - (χ[b/.9^2] - χ[a/.9^2])
```

0.0115981

```
probmisleadingsignal_upper = 1 - χ[γσ/.9^2];
dist = GeometricDistribution[probmisleadingsignal_upper] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
CDF[dist, 100 - 1]

probmisleadingsignal_standard = 1 - (χ[b/.9^2] - χ[a/.9^2]);
dist = GeometricDistribution[probmisleadingsignal_standard] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
CDF[dist, 100 - 1]
```

0.142778

0.688574