Some exercises of Chap. 9

Exercise 9.9

```
n = 100;
p0 = 0.08;
UCL = 16.1;
dist = BinomialDistribution[n, p0];
probfalsealarm = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
incontrolARL = 1 / probfalsealarm
```

```
0.00240911

415.09
```

```
p1 = 0.2;
dist = BinomialDistribution[n, p1];
probvalidsignal = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
```

```
0.807662
```

```
dist = GeometricDistribution[probvalidsignal] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
CDF[dist, 4 - 1]
```

```
0.998631
```

Exercise 9.10

```
n = 50;
p0 = 0.02;
LCL = Max[0, n × p0 - 3 × Sqrt[n × p0 × (1 - p0)]]
CL = n × p0
UCL = n × p0 + 3 × Sqrt[n × p0 × (1 - p0)]
p1 = 0.04;
dist = BinomialDistribution[n, p1];
probvalidsignal = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
dist = GeometricDistribution[probvalidsignal] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
PDF[dist, 1 - 1]
PDF[dist, 4 - 1]
```

```
0

1.

3.96985

0.139131

0.139131

0.0887636
```

Exercício 9.11

```
n = 100;
p0 = 0.079;
LCL = Max[0, n × p0 - 3 × Sqrt[n × p0 × (1 - p0)]]
CL = n × p0
UCL = n × p0 + 3 × Sqrt[n × p0 × (1 - p0)];
UCL = 16 (* the UCL value was incorrectly approximated in the lecture notes *)
dist = BinomialDistribution[n, p0];
probfalsealarm = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
incontrolARL = 1 / probfalsealarm
```

```
0

7.9

16

0.00211174

473.543
```

```
p1 = 0.1;
dist = BinomialDistribution[n, p1];
probvalidsignal = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
outofcontrolARL = 1 / probvalidsignal
```

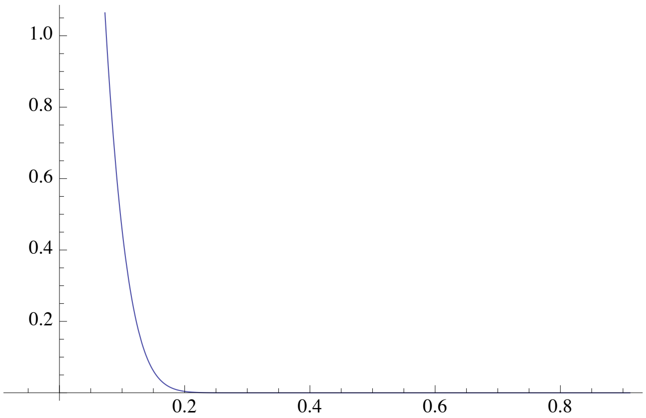
```
0.0205988

48.5465
```

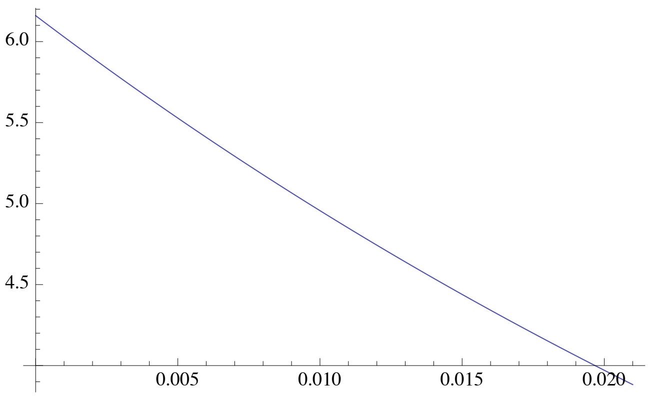
```
p2 = 0.05;
dist = BinomialDistribution[n, p2];
probvalidsignal = 1 - CDF[dist, UCL] (* because LCL is equal to zero! *)
outofcontrolARL = 1 / probvalidsignal
```

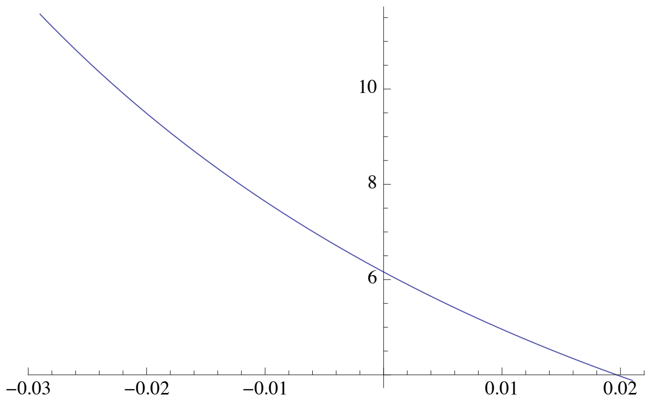
9.42903×10^{-6}

106 055.

```
ξ[δ_] = 1 - CDF[BinomialDistribution[n, p0 + δ], UCL];
Plot[Log[1/ξ[δ]], {δ, -p0 + 0.01, 1 - p0 - 0.01}]
Plot[Log[1/ξ[δ]], {δ, 0, p1 - p0}]
Plot[Log[1/ξ[δ]], {δ, p2 - p0, p1 - p0}]
```





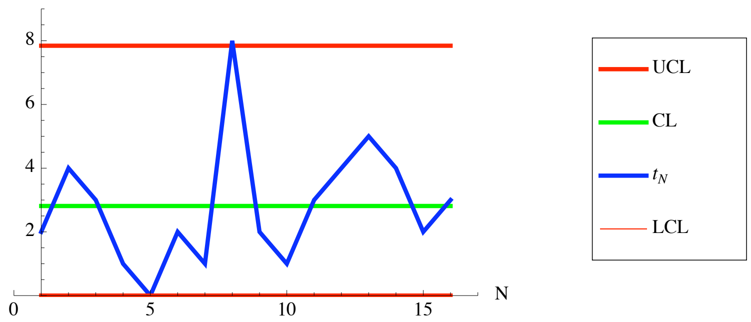


Exercise 9.16

```
Needs["PlotLegends`"]
```

```
data = {2, 4, 3, 1, 0, 2, 1, 8, 2, 1, 3, 4, 5, 4, 2, 3};
λ0 = N[Mean[data], 5];
LCL = Max[0, λ0 - 3 Sqrt[λ0]]
CL = λ0
UCL = λ0 + 3 Sqrt[λ0]
list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];
ListPlot[
 {list3, list2, data, list1},
 AxesOrigin -> {1, 0},
 AxesLabel → {"N", ""},
 PlotStyle → {Directive[Red, Thick],
   Directive[Green, Thick], Directive[Blue, Thick], Directive[Red]},
 PlotRange → {{0, Length[data] + 1}, {0, Max[Max[data], UCL] + 1}},
 PlotLegend → {"UCL", "CL", "tN", "LCL"}, LegendPosition → {1.1, -0.4},
 Joined → {True, True, True, True}, LegendShadow → False]
```

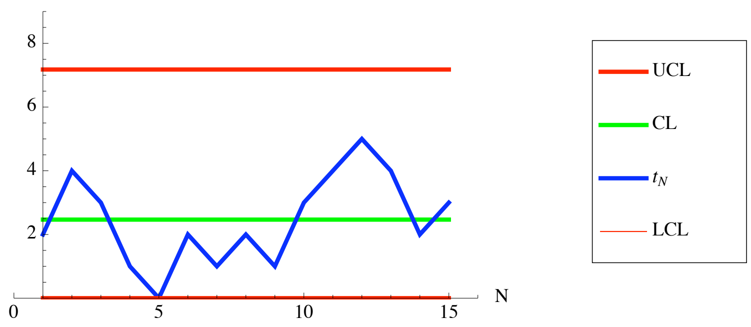
```
0

2.8125

7.8437
```



```
numberobsoutofcontrol = Length[data] ×
   Mean[Table[If[data[[i]] < LCL || data[[i]] > UCL, 1, 0], {i, 1, Length[data]}]];
If[numberobsoutofcontrol > 0, Print["There is (are) ",
  numberobsoutofcontrol, " out of control observation(s),
    thus we need to update the data and control limits."],
 Print["There are no out of control observations."]]

newdata = {};
Table[If[data[[i]] ≥ LCL && data[[i]] ≤ UCL,
   newdata = Append[newdata, data[[i]]]], {i, 1, Length[data]}];
newdata

λ0 = N[Mean[newdata], 5];
LCL = Max[0, λ0 - 3 Sqrt[λ0]]
CL = λ0
UCL = λ0 + 3 Sqrt[λ0]
list1 = Table[LCL, {i, 1, Length[newdata]}];
list2 = Table[CL, {i, 1, Length[newdata]}];
list3 = Table[UCL, {i, 1, Length[newdata]}];

ListPlot[
 {list3, list2, newdata, list1},
 AxesOrigin -> {1, 0},
 AxesLabel → {"N", ""},
 PlotStyle → {Directive[Red, Thick],
   Directive[Green, Thick], Directive[Blue, Thick], Directive[Red]},
 PlotRange → {{0, Length[newdata] + 1}, {0, Max[Max[data], UCL] + 1}},
 PlotLegend → {"UCL", "CL", "tN", "LCL"}, LegendPosition → {1.1, -0.4},
 Joined → {True, True, True, True}, LegendShadow → False]
```

```
There is (are) 1
  out of control observation(s), thus we need to update the data and control limits.
```

```
{2, 4, 3, 1, 0, 2, 1, 2, 1, 3, 4, 5, 4, 2, 3}

0

2.4667

7.1784
```




```
λ1 = 5.;
dist = PoissonDistribution[λ1];
probvalidsignal = 1 - (CDF[dist, UCL] - CDF[dist, Ceiling[LCL] - 0.001]);
outofcontrolARL = 1 / probvalidsignal
outofcontrolSDRL = Sqrt[1 - probvalidsignal] / probvalidsignal
```

```
7.49784

6.97996
```

$$\text{Ceiling}\left[\frac{\text{Log}[1 - 0.5]}{\text{Log}[1 - \text{probvalidsignal}]}\right]$$ (* median of ARL *)

$$(1 - \text{probvalidsignal})^{\text{Floor}[\text{outofcontrolARL}]}$$

(* probability of more than outofcontrolARL observations until a valid signal*)

```
5
```

```
0.367139
```

Exercise 9.18

```
data = {0, 1, 1, 0, 2, 1, 1, 3, 2, 1, 0, 3, 2, 5, 1, 2, 1, 1};
λ0 = N[Mean[data], 5];
```

$\text{LCL} = \text{Max}\left[0,\ \lambda_0 - 3\sqrt{\lambda_0}\right]$

$\text{CL} = \lambda_0$

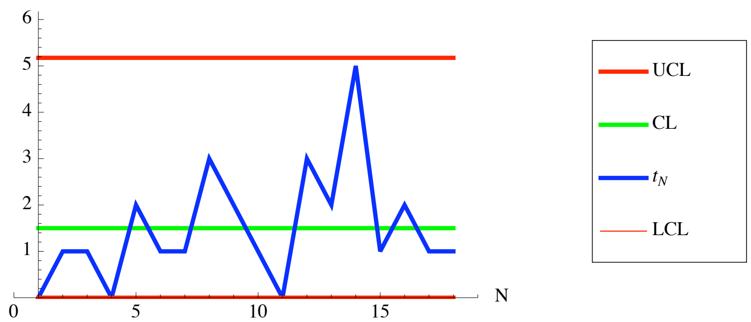
$\text{UCL} = \lambda_0 + 3\sqrt{\lambda_0}$

```
list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

ListPlot[
 {list3, list2, data, list1},
 AxesOrigin -> {1, 0},
 AxesLabel → {"N", ""},
 PlotStyle → {Directive[Red, Thick],
   Directive[Green, Thick], Directive[Blue, Thick], Directive[Red]},
 PlotRange → {{0, Length[data] + 1}, {0, Max[Max[data], UCL] + 1}},
 PlotLegend → {"UCL", "CL", "tN", "LCL"}, LegendPosition → {1.1, -0.4},
 Joined → {True, True, True, True}, LegendShadow → False]
```

```
0
```

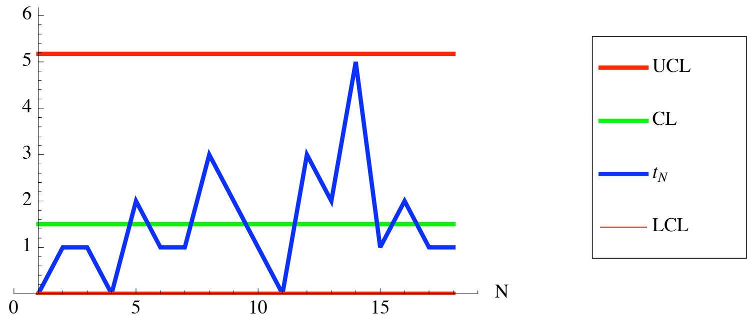
```
1.5000
```

```
5.1742
```



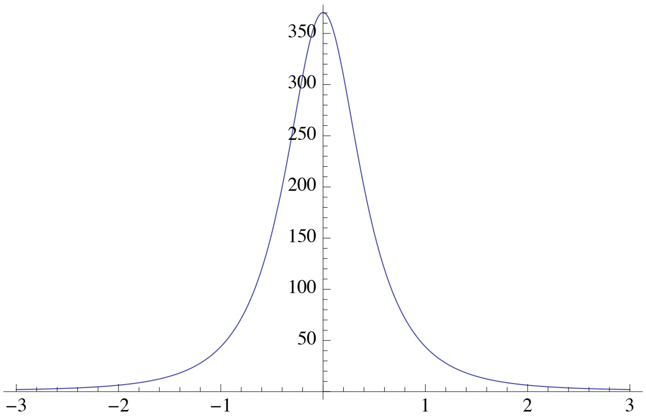
```
numberobsoutofcontrol = Length[data] ×
   Mean[Table[If[data[[i]] < LCL || data[[i]] > UCL, 1, 0], {i, 1, Length[data]}]];
If[numberobsoutofcontrol > 0, Print["There is (are) ",
  numberobsoutofcontrol, " out of control observation(s),
    thus we need to update the data and control limits."],
 Print["There are no out of control observations."]]

newdata = {};
Table[If[data[[i]] ≥ LCL && data[[i]] ≤ UCL,
   newdata = Append[newdata, data[[i]]]], {i, 1, Length[data]}];
newdata;

λ0 = N[Mean[newdata], 5];
LCL = Max[0, λ0 - 3 Sqrt[λ0]]
CL = λ0
UCL = λ0 + 3 Sqrt[λ0]
list1 = Table[LCL, {i, 1, Length[newdata]}];
list2 = Table[CL, {i, 1, Length[newdata]}];
list3 = Table[UCL, {i, 1, Length[newdata]}];

ListPlot[
 {list3, list2, newdata, list1},
 AxesOrigin -> {1, 0},
 AxesLabel → {"N", ""},
 PlotStyle → {Directive[Red, Thick],
   Directive[Green, Thick], Directive[Blue, Thick], Directive[Red]},
 PlotRange → {{0, Length[newdata] + 1}, {0, Max[Max[data], UCL] + 1}},
 PlotLegend → {"UCL", "CL", "tN", "LCL"}, LegendPosition → {1.1, -0.4},
 Joined → {True, True, True, True}, LegendShadow → False]
```

```
There are no out of control observations.

0

1.5000

5.1742
```



Exercise 9.22

```
dist = NormalDistribution[0, 1];
ξ[δ_] = 1 - (CDF[dist, 3 - δ] - CDF[dist, -3 - δ]);
ARL[δ_] = 1/ξ[δ];
Plot[ARL[δ], {δ, -3, 3}]
```



Exercise 9.24

```
n = 4;
μ0 = 200;
σ = 6.;
LCL = 191;
UCL = 209; (* these are the 3-sigma control limits *)
dist = NormalDistribution[0, 1];
```

```
μ1 = 188;
```

$$\text{probvalidsignal} = 1 - \left(\text{CDF}\left[\text{dist}, \frac{\text{UCL} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right] - \text{CDF}\left[\text{dist}, \frac{\text{LCL} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right]\right)$$

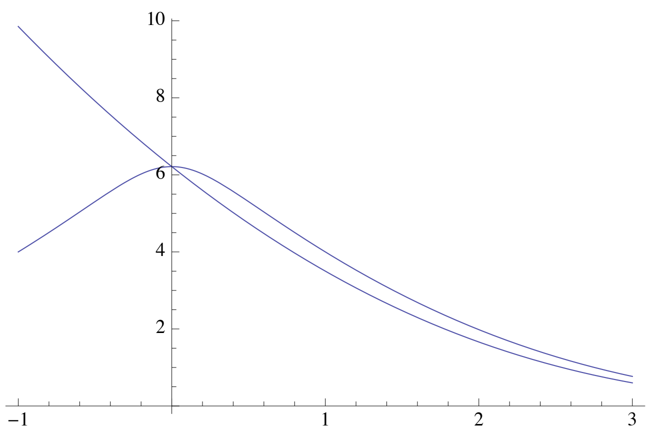
```
μ1 = 212;
```

$$\text{probvalidsignal} = 1 - \left(\text{CDF}\left[\text{dist}, \frac{\text{UCL} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right] - \text{CDF}\left[\text{dist}, \frac{\text{LCL} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right]\right)$$

```
0.841345

0.841345
```

Exercise 9.25

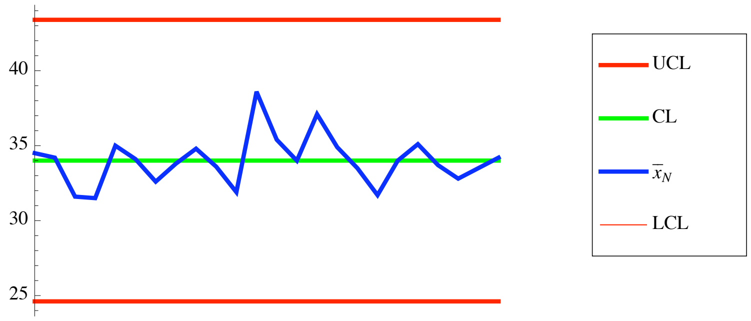
```
dist = NormalDistribution[0, 1];
γ = Quantile[dist, 1 - 1/500];
ξ[δ_] = 1 - CDF[dist, γ - δ];
ARLU[δ_] = 1/ξ[δ];
g1 = Plot[Log[ARLU[δ]], {δ, -1, 3}];

β = Quantile[dist, 1 - (1/500)/2];
ς[δ_] = 1 - (CDF[dist, β - δ] - CDF[dist, -β - δ]);
ARLS[δ_] = 1/ς[δ];
g2 = Plot[Log[ARLS[δ]], {δ, -1, 3}];

Show[g1, g2]
```



Exercise 9.26

```
n = 5;
σ = √49 ;
data = {34.5, 34.2, 31.6, 31.5, 35.0, 34.1, 32.6, 33.8, 34.8, 33.6, 31.9,
   38.6, 35.4, 34.0, 37.1, 34.9, 33.5, 31.7, 34.0, 35.1, 33.7, 32.8, 33.5, 34.2};
μ0 = N[Mean[data], 5];
LCL = μ0 - 3 × σ/√n
CL = μ0
UCL = μ0 + 3 × σ/√n
list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

ListPlot[
 {list3, list2, data, list1},
 AxesOrigin -> {1, 0},
 AxesLabel → {"N", ""},
 PlotStyle → {Directive[Red, Thick],
   Directive[Green, Thick], Directive[Blue, Thick], Directive[Red]},
 PlotRange → {{0, Length[data] + 1}, {Min[Min[data], LCL] - 1, Max[Max[data], UCL] + 1}},
 PlotLegend → {"UCL", "CL", "x̄N", "LCL"}, LegendPosition → {1.1, -0.4},
 Joined → {True, True, True, True}, LegendShadow → False]
```

```
24.6127

34.0042

43.3957
```




```
numberobsoutofcontrol = Length[data] ×
   Mean[Table[If[data[[i]] < LCL || data[[i]] > UCL, 1, 0], {i, 1, Length[data]}]];
If[numberobsoutofcontrol > 0, Print["There is (are) ", numberobsoutofcontrol,
  " out of control observation(s), thus we need to update the data and
   control limits."], Print["There are no out of control observations."]]
```

```
There are no out of control observations.
```

```
dist = NormalDistribution[0, 1];
ξ[δ_] = 1 - (CDF[dist, 3 - δ] - CDF[dist, -3 - δ]);
ξ[0.]
```

```
0.0026998
```

μ_1 = 45;

shift = $\frac{\mu_1 - \mu_0}{\frac{\sigma}{\sqrt{n}}}$;

probvalidsignal = ξ[shift]

dist = GeometricDistribution[probvalidsignal] ;

(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)

PDF[dist, 5 - 1]

0.695846

0.00595508

dist = NormalDistribution[0, 1];

fractiondefective = 1 - $\left(\text{CDF}\left[\text{dist}, \frac{(30+1)-\mu_0}{\frac{\sigma}{\sqrt{n}}}\right] - \text{CDF}\left[\text{dist}, \frac{(30-1)-\mu_0}{\frac{\sigma}{\sqrt{n}}}\right]\right)$

0.886347

Exercise 9.27

n = 5;

$\sigma_0 = \sqrt{4}$;

incontrolARL = 200;

probfalsealarm = $\frac{1}{\text{incontrolARL}}$;

dist = ChiSquareDistribution[n - 1];

γ = Quantile$\left[\text{dist}, \frac{\text{probfalsealarm}}{2.}\right]$;

β = Quantile$\left[\text{dist}, 1 - \frac{\text{probfalsealarm}}{2.}\right]$;

LCL = $\frac{\sigma_0{}^2}{n-1} \times \gamma$

CL = $\sigma_0{}^2$

UCL = $\frac{\sigma_0{}^2}{n-1} \times \beta$

$\xi[\theta\_]$ = 1 - $\left(\text{CDF}\left[\text{dist}, \frac{\beta}{\theta^2}\right] - \text{CDF}\left[\text{dist}, \frac{\gamma}{\theta^2}\right]\right)$;

1 / ξ[1]

0.144867

4

16.4239

200.

```
σ1 = √6 ;
shiftinsigma = σ1/σ0;
probvalidsignal = ξ[shift]
dist = GeometricDistribution[probvalidsignal] ;
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
PDF[dist, 1 - 1]
```

0.856075

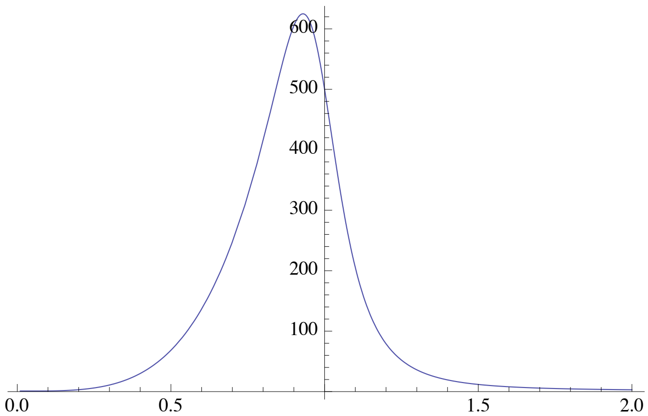
0.856075

Example 9.28

```
σ0 = 1;
probfalsealarm = 0.002;
θ = {0.5, 0.75, 0.8, 0.9, 0.95, 1, 1.1, 1.2};
n = {4, 5, 7, 10, 15, 100};
For[i = 1, i ≤ Length[θ], i++,
 For[j = 1, j ≤ Length[n], j++, ξ[i, j] = 1 - (CDF[ChiSquareDistribution[n[[j]] - 1],
       Quantile[ChiSquareDistribution[n[[j]] - 1], 1 - probfalsealarm/2.] / θ[[i]]^2] -
      CDF[ChiSquareDistribution[n[[j]] - 1],
       Quantile[ChiSquareDistribution[n[[j]] - 1], probfalsealarm/2.] / θ[[i]]^2])]]
MatrixForm[Table[ξ[i, j], {i, 1, Length[θ]}, {j, 1, Length[n]}]]
```

0.00782755	0.0146238	0.0421337	0.132929	0.406761	1.
0.00235933	0.00308864	0.00503566	0.00931304	0.0206716	0.76245
0.00195778	0.00240876	0.00352773	0.0057508	0.011016	0.419837
0.00153265	0.00165234	0.00192614	0.00239141	0.00327391	0.0377238
0.0016004	0.00162849	0.00169852	0.0018186	0.00203525	0.00694909
0.002	0.002	0.002	0.002	0.002	0.002
0.00452223	0.00487444	0.00555329	0.00656903	0.00832324	0.0547606
0.0108084	0.0126543	0.0164466	0.0225299	0.0338476	0.373172

```
n = 5;
σ0 = 1;
probfalsealarm = 0.002;
dist = ChiSquareDistribution[n - 1];
γ = Quantile[dist, probfalsealarm/2.];
β = Quantile[dist, 1 - probfalsealarm/2.];
ξ[θ_] = 1 - (CDF[dist, β/θ^2] - CDF[dist, γ/θ^2]);
ARL[θ_] = 1/ξ[θ];
Plot[ARL[θ], {θ, 0.01, 2}, AxesOrigin → {1, 0}]
```



Exercise 9.29

```
σ0 = 1;
probfalsealarm = 0.002;
θ = {0.5, 0.75, 0.8, 0.9, 0.95, 1, 1.1, 1.2};
n = {4, 5, 7, 10, 15, 100};
For[i = 1, i ≤ Length[θ], i++,
 For[j = 1, j ≤ Length[n], j++,

   r = FindRoot[
     {1 - (CDF[ChiSquareDistribution[n[[j]] - 1], b] - CDF[ChiSquareDistribution[n[[j]] - 1],
           a]) == probfalsealarm, a × PDF[ChiSquareDistribution[n[[j]] - 1], a] -
        b × PDF[ChiSquareDistribution[n[[j]] - 1], b] == 0},
     {{a, Quantile[ChiSquareDistribution[n[[j]] - 1], probfalsealarm/2.]},
      {b, Quantile[ChiSquareDistribution[n[[j]] - 1], 1 - probfalsealarm/2.]}}];
   NQ = {a, b} /. Dispatch[Flatten[r]]; (* Non balanced quantiles *)
   γ[j] = NQ[[1]];
   β[j] = NQ[[2]];
   LCL[j] = σ0^2 × γ[j] / (n[[j]] - 1); (* New lower control limits *)
   UCL[j] = σ0^2 × β[j] / (n[[j]] - 1); (* New upper control limits *)
   ξ[i, j] = 1 - (CDF[ChiSquareDistribution[n[[j]] - 1], β[j] / θ[[i]]^2] -
       CDF[ChiSquareDistribution[n[[j]] - 1], γ[j] / θ[[i]]^2])]]
MatrixForm[Table[LCL[j], {j, 1, Length[n]}]] (* New lower control limits *)
MatrixForm[Table[UCL[j], {j, 1, Length[n]}]] (* New upper control limits *)
MatrixForm[Table[ξ[i, j], {i, 1, Length[θ]}, {j, 1, Length[n]}]] (* New prob. signal *)
```

0.0116342
0.0293589
0.0744406
0.141288
0.230551
0.622052

6.30712
5.20764
4.07774
3.28875
2.68582
1.50672

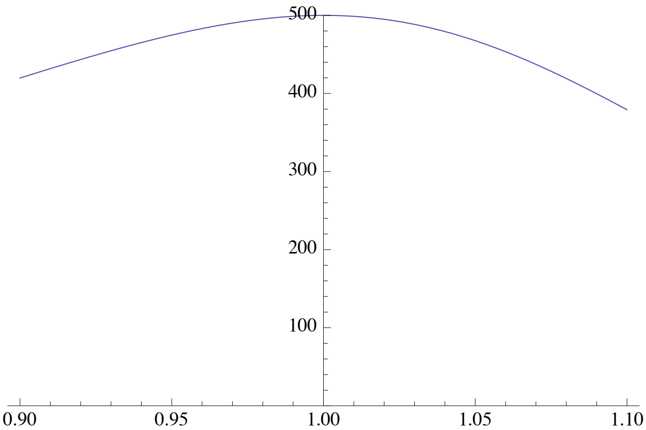
0.0133069	0.0236211	0.0617561	0.173284	0.466445	1.
0.0040353	0.00508381	0.00776738	0.0133554	0.0273342	0.778485
0.00333397	0.00396143	0.00546432	0.00832683	0.0147993	0.439725
0.00238236	0.00253998	0.00288047	0.00343723	0.00446717	0.0417415
0.00210643	0.00214889	0.00223806	0.00237744	0.00261838	0.0078643
0.002	0.002	0.002	0.002	0.002	0.002
0.00263722	0.00289534	0.00344314	0.00431697	0.00587911	0.0496898
0.00534009	0.00675704	0.00987162	0.0151182	0.0252165	0.356332

```
MatrixForm[Table[1/ξ[i, j], {i, 1, Length[θ]}, {j, 1, Length[n]}]]
(* ARL values: all ≤ incontrolARL=500 *)
```

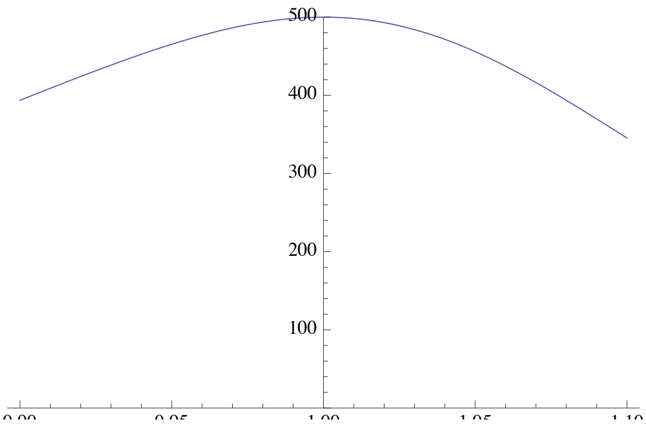
75.149	42.335	16.1927	5.77087	2.14388	1.
247.813	196.703	128.743	74.8761	36.5842	1.28455
299.943	252.434	183.005	120.094	67.5708	2.27415
419.751	393.703	347.166	290.932	223.855	23.957
474.737	465.357	446.816	420.62	381.915	127.157
500.	500.	500.	500.	500.	500.
379.187	345.383	290.432	231.644	170.094	20.1249
187.263	147.994	101.301	66.1454	39.6565	2.80638

```
MatrixForm[Table[{n[[j]], Plot[1/(1 - (CDF[ChiSquareDistribution[n[[j]] - 1], β[j]/δ^2] -
      CDF[ChiSquareDistribution[n[[j]] - 1], γ[j]/δ^2])), {δ, 0.9, 1.1},
    AxesOrigin → {1, 0}, PlotRange → {0, 1/probfalsealarm}]}, {j, 1, Length[n]}]]
```

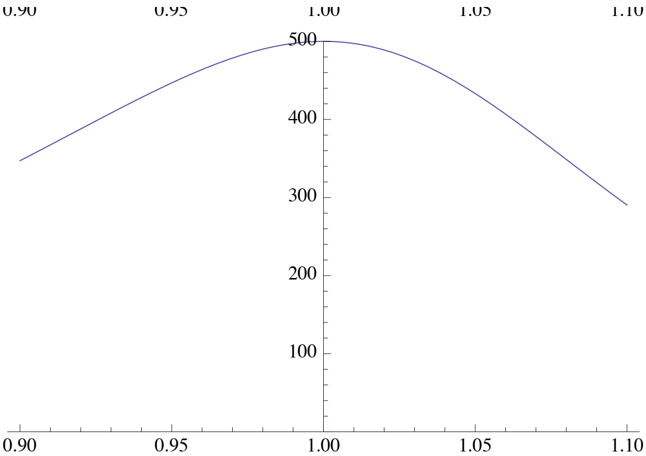
4



5



7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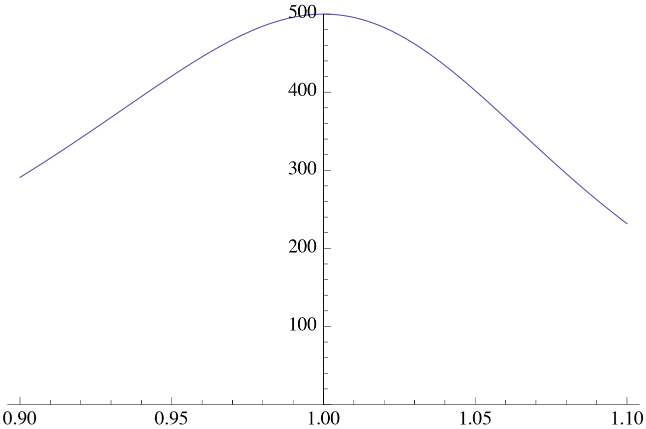
400
300
200
100
0.90
0.95
1.00
1.05
1.10

10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100
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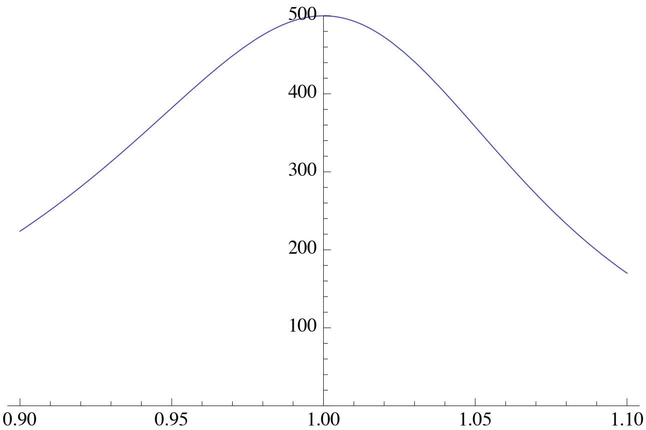
1.05
1.10

15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400
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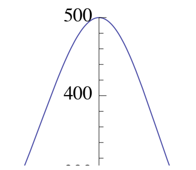
0.90
0.95
1.00
1.05
1.10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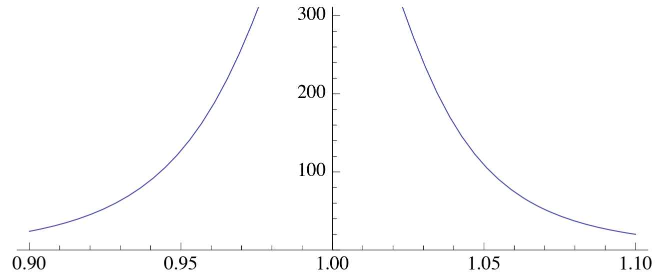

Exercise 9.34

```
(* We should have replaced σ,
in the expressions of the control limits of the standard X̄-
 chart (check Table 9.12) and standard S-chart (check Exercise 9.31),
```

by its unbiased estimate $\frac{\bar{S}}{\frac{E(S)}{\sigma}} = \frac{\bar{S}}{\left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}}$. Check (9.11). *)

```
n = 6;
numberofsamples = 50;
sumofsamplemeans = 1000;
sumofsamplestandarddeviations = 75;
```

$$\text{estimateofmu} = \frac{\text{sumofsamplemeans}}{\text{numberofsamples}};$$

$$\text{unbiasedestimateofsigma} = N\left[\frac{\frac{\text{sumofsamplestandarddeviations}}{\text{numberofsamples}}}{\left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\text{Gamma}\left[\frac{n}{2}\right]}{\text{Gamma}\left[\frac{n-1}{2}\right]}}, 5\right];$$

(* estimates of control limits for the $\overline{X}$-chart *)

$$\text{LCL} = \text{estimateofmu} - 3 \times \frac{\text{unbiasedestimateofsigma}}{\sqrt{n}}$$

$$\text{UCL} = \text{estimateofmu} + 3 \times \frac{\text{unbiasedestimateofsigma}}{\sqrt{n}}$$

```
dist = NormalDistribution[0, 1];
```

$$\text{probfalsealarm} = 1 - \left(\text{CDF}\left[\text{dist}, \frac{\text{UCL} - \text{estimateofmu}}{\frac{\text{unbiasedestimateofsigma}}{\sqrt{n}}}\right] - \text{CDF}\left[\text{dist}, \frac{\text{LCL} - \text{estimateofmu}}{\frac{\text{unbiasedestimateofsigma}}{\sqrt{n}}}\right]\right)$$

$\mu_1 = 25;$
$\sigma_1 = 2;$

$$\text{probvalidsignal} = 1 - \left(\text{CDF}\left[\text{dist}, \frac{\text{UCL} - \mu_1}{\frac{\sigma_1}{\sqrt{n}}}\right] - \text{CDF}\left[\text{dist}, \frac{\text{LCL} - \mu_1}{\frac{\sigma_1}{\sqrt{n}}}\right]\right);$$

```
dist = GeometricDistribution[probvalidsignal] ;
```
(* geometric distribution for the number of trials before the first success, where the probability of success in a trial is p *)
```
CDF[dist, 5 - 1]
```

(* estimates of control limits for the S-chart *)

$$\text{LCL} = \text{unbiasedestimateofsigma} \times \left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\text{Gamma}\left[\frac{n}{2}\right]}{\text{Gamma}\left[\frac{n-1}{2}\right]} - 3 \times \text{unbiasedestimateofsigma} \times \sqrt{1 - \left(\left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\text{Gamma}\left[\frac{n}{2}\right]}{\text{Gamma}\left[\frac{n-1}{2}\right]}\right)^2}$$

$$\text{UCL} = \text{unbiasedestimateofsigma} \times \left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\text{Gamma}\left[\frac{n}{2}\right]}{\text{Gamma}\left[\frac{n-1}{2}\right]} + 3 \times \text{unbiasedestimateofsigma} \times \sqrt{1 - \left(\left(\frac{2}{n-1}\right)^{\frac{1}{2}} \times \frac{\text{Gamma}\left[\frac{n}{2}\right]}{\text{Gamma}\left[\frac{n-1}{2}\right]}\right)^2}$$

```
18.0693

21.9307

0.002700

0.9999999999999999999954956

0.0455
```

2.9545