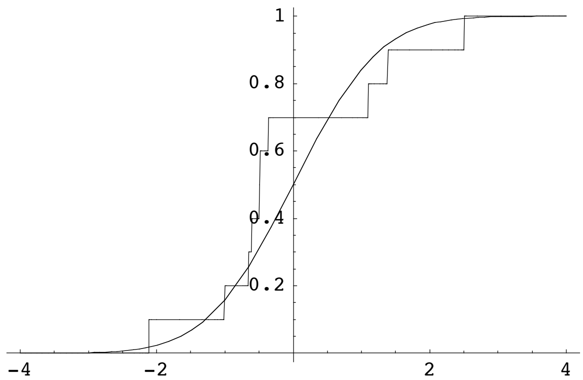
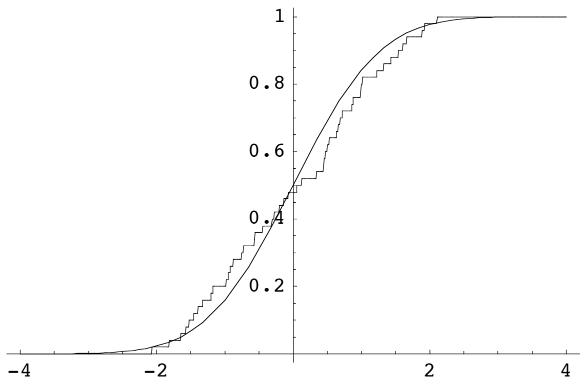
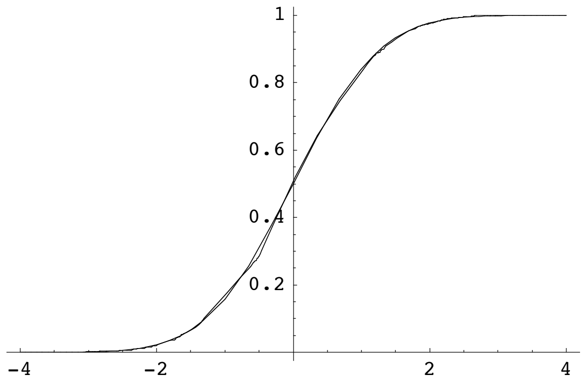
Exercícios
Cap. 5 — Notas apoio 0607

■ Exercício 5.1

```
<< Statistics`ContinuousDistributions`

nsim = {10, 50, 1000};
j = 0;
While[(j = j + 1) ≤ Length[nsim],
 data = RandomArray[NormalDistribution[0, 1], nsim[[j]]];
 n = Length[data];
 empiricalcdf[x_] = 1/n × Sum[If[data[[i]] ≤ x, 1, 0], {i, 1, n}];
 Plot[{empiricalcdf[x], CDF[NormalDistribution[0, 1], x]}, {x, -4, 4}]]
```





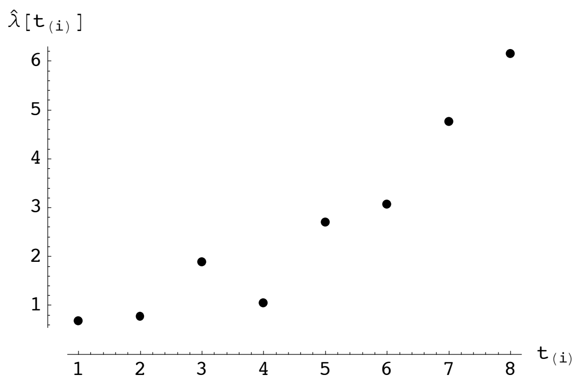


```
(* The empirical c.d.f. is a consistent estimate (tor) of the true c.d.f.! *)
```

■ Exercício 5.2

```
t = {0.41, 0.58, 0.75, 0.83, 1, 1.08, 1.17, 1.25, 1.35};
n = Length[t];
TableForm[Table[{i,
   t[[i]],
   If[i < n, t[[i + 1]] - t[[i]], Null],
   If[i < n, 1/((n + 0.25) × (t[[i + 1]] - t[[i]])), Null],
   (n - i + 0.625)/(n + 0.25),
   If[i < n, 1/((n - i + 0.625) × (t[[i + 1]] - t[[i]])), Null]}, {i, 1, n}],
 TableHeadings -> {None, {"i", "t(i)", "t(i+1)-t(i)", "f̂[t(i)]", "R̂[t(i)]", "λ̂[t(i)]"}}]
ListPlot[Table[1/((n - i + 0.625) × (t[[i + 1]] - t[[i]])), {i, 1, n - 1}],
 PlotStyle → PointSize[0.02], AxesLabel → {"t(i)", "λ̂[t(i)]"}, AxesOrigin → {0.5, 0}]
```

i	$t_{(i)}$	$t_{(i+1)}-t_{(i)}$	$\hat{f}[t_{(i)}]$	$\hat{R}[t_{(i)}]$	$\hat{\lambda}[t_{(i)}]$
1	0.41	0.17	0.63593	0.932432	0.682012
2	0.58	0.17	0.63593	0.824324	0.771456
3	0.75	0.08	1.35135	0.716216	1.88679
4	0.83	0.17	0.63593	0.608108	1.04575
5	1	0.08	1.35135	0.5	2.7027
6	1.08	0.09	1.2012	0.391892	3.06513
7	1.17	0.08	1.35135	0.283784	4.7619
8	1.25	0.1	1.08108	0.175676	6.15385
9	1.35	Null	Null	0.0675676	Null



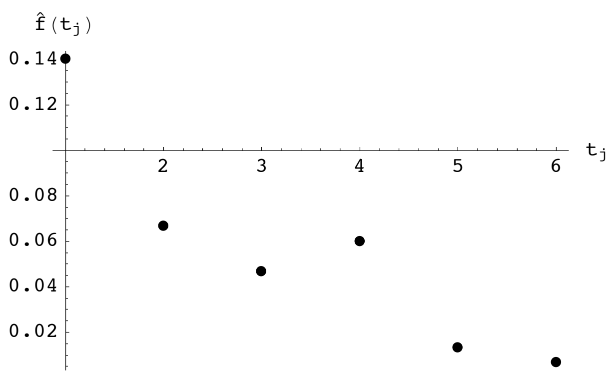
- Graphics -

```
(* The graph suggests an IHR distribution *)
```

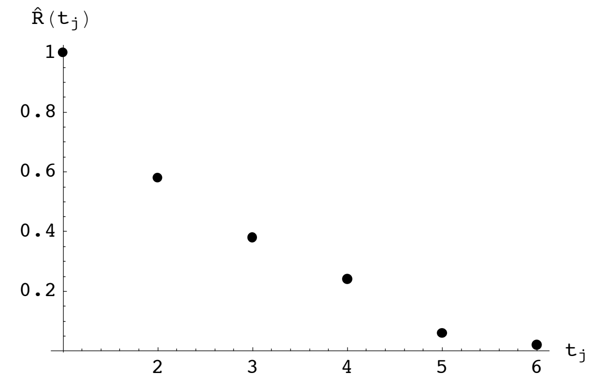
■ Exercício 5.3

```
t = Table[i, {i, 0, 18, 3}];
n = 50;
nbins = Length[t] - 1;
numberfailures = {21, 10, 7, 9, 2, 1};
survivors = Table[If[j == 1, n, n - Sum[numberfailures[[i]], {i, 1, j-1}]], {j, 1, nbins}];
f = Table[N[numberfailures[[j]] / (n × (t[[j + 1]] - t[[j]])), 5], {j, 1, nbins}];
R = Table[N[survivors[[j]] / n, 5], {j, 1, nbins}];
λ = Table[N[numberfailures[[j]] / (survivors[[j]] × (t[[j + 1]] - t[[j]])), 5], {j, 1, nbins}];
TableForm[Table[{j,
   t[[j]],
   t[[j + 1]],
   survivors[[j]],
   numberfailures[[j]],
   f[[j]],
   R[[j]],
   λ[[j]]}, {j, 1, nbins}],
 TableHeadings ->
  {None, {"j", "tj", "tj+1", "N(tj)", "N(tj)-N(tj+1)", "f̂(tj)", "R̂(tj)", "λ̂(tj)"}}]
ListPlot[Table[f[[j]], {j, 1, nbins}], PlotStyle → PointSize[0.02],
 AxesLabel → {"tj", "f̂(tj)"}]
ListPlot[Table[R[[j]], {j, 1, nbins}],
 PlotStyle → PointSize[0.02], AxesLabel → {"tj", "R̂(tj)"}]
ListPlot[Table[λ[[j]], {j, 1, nbins}],
 PlotStyle → PointSize[0.02], AxesLabel → {"tj", "λ̂(tj)"}]
```

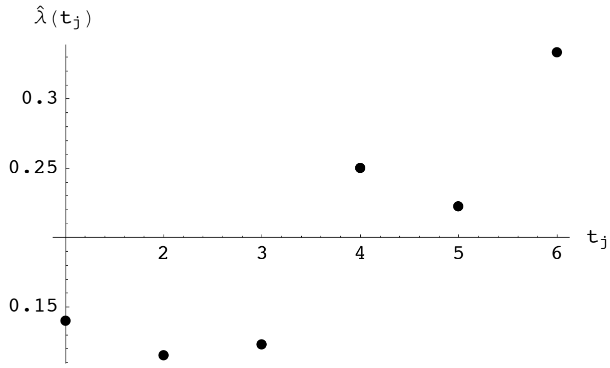
j	t_j	t_{j+1}	$N(t_j)$	$N(t_j)-N(t_{j+1})$	$\hat{f}(t_j)$	$\hat{R}(t_j)$	$\hat{\lambda}(t_j)$
1	0	3	50	21	0.14000	1.0000	0.1400
2	3	6	29	10	0.066667	0.58000	0.1149
3	6	9	19	7	0.046667	0.38000	0.1228
4	9	12	12	9	0.060000	0.24000	0.2500
5	12	15	3	2	0.013333	0.060000	0.2222
6	15	18	1	1	0.0066667	0.020000	0.3333



- Graphics -



- Graphics -



- Graphics -

(* The graph of the h.r.f. suggests an bathtub distribution *)

■ Exercício 5.4

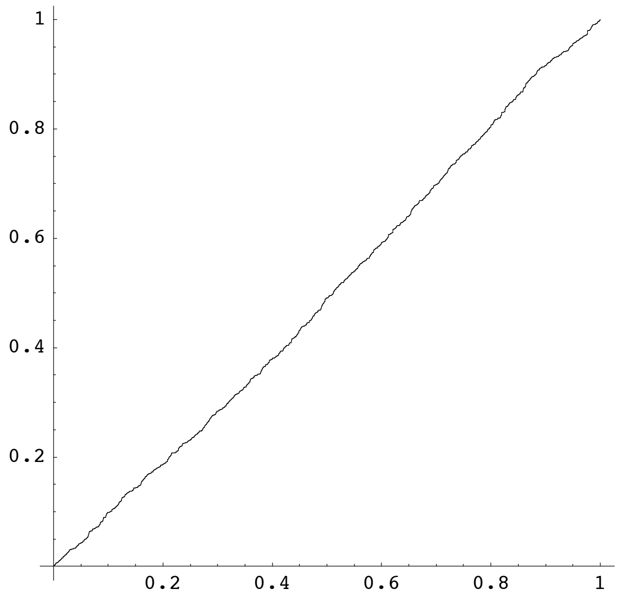
```
t = Table[i, {i, 0, 1000, 100}];
n = 432;
nbins = Length[t] - 1;
numberfailures = {121, 80, 70, 63, 30, 25, 21, 10, 7, 5};
survivors = Table[If[j == 1, n, n - Sum[numberfailures[[i]], {i, 1, j-1}]], {j, 1, nbins}];
f = Table[N[numberfailures[[j]] / (n × (t[[j + 1]] - t[[j]])), 5], {j, 1, nbins}];
R = Table[N[survivors[[j]] / n, 5], {j, 1, nbins}];
λ = Table[N[numberfailures[[j]] / (survivors[[j]] × (t[[j + 1]] - t[[j]])), 5], {j, 1, nbins}];
TableForm[Table[{j,
   t[[j]],
   t[[j + 1]],
   survivors[[j]],
   numberfailures[[j]],
   f[[j]],
   R[[j]],
   100 × λ[[j]]}, {j, 1, nbins}],
 TableHeadings ->
  {None, {"j", "tj", "tj+1", "N(tj)", "N(tj)-N(tj+1)", "f̂(tj)", "R̂(tj)", "10^2×λ̂(tj)"}}]
```

j	t_j	t_{j+1}	$N(t_j)$	$N(t_j)-N(t_{j+1})$	$\hat{f}(t_j)$	$\hat{R}(t_j)$	10
1	0	100	432	121	0.0028009	1.0000	0.
2	100	200	311	80	0.0018519	0.71991	0.
3	200	300	231	70	0.0016204	0.53472	0.
4	300	400	161	63	0.0014583	0.37269	0.
5	400	500	98	30	0.00069444	0.22685	0.
6	500	600	68	25	0.00057870	0.15741	0.
7	600	700	43	21	0.00048611	0.099537	0.
8	700	800	22	10	0.00023148	0.050926	0.
9	800	900	12	7	0.00016204	0.027778	0.
10	900	1000	5	5	0.00011574	0.011574	1.

■ Exercício 5.6: Exponential(1) is CHR → TTT plot should resemble a straight line

```
<< Statistics`ContinuousDistributions`
```

```
i = 1;
n = 1000;
t = Sort[RandomArray[ExponentialDistribution[1], n]];
t[[0]] = 0;
ListPlot[
 Prepend[Table[{i / n, (Σ_{j=1}^{i} (n - j + 1) × (t[[j]] - t[[j - 1]])) / (Σ_{j=1}^{n} (n - j + 1) × (t[[j]] - t[[j - 1]]))}, {i, 1, n}], {0, 0}],
 PlotJoined → True, AspectRatio → 1]
```



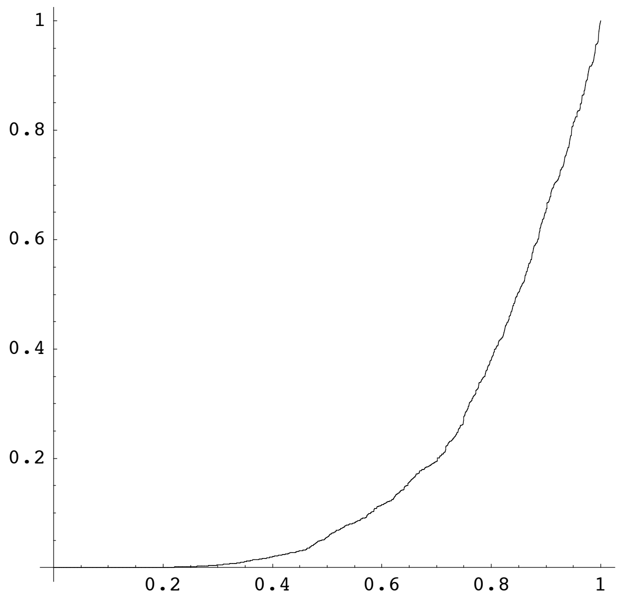
- Graphics -

- **Exercício 5.6': Gamma(0.2,1) is DHR → TTT plot should resemble a convex line**

```
<< Statistics`ContinuousDistributions`
```

```
i = 1;
n = 1000;
t = Sort[RandomArray[GammaDistribution[0.2, 1], n]];
t[[0]] = 0;
```

$$\text{ListPlot}\left[\text{Prepend}\left[\text{Table}\left[\left\{i/n, \frac{\sum_{j=1}^{i} (n-j+1)\times(t[[j]]-t[[j-1]])}{\sum_{j=1}^{n} (n-j+1)\times(t[[j]]-t[[j-1]])}\right\}, \{i, 1, n\}\right], \{0, 0\}\right], \text{PlotJoined}\to\text{True}, \text{AspectRatio}\to 1\right]$$

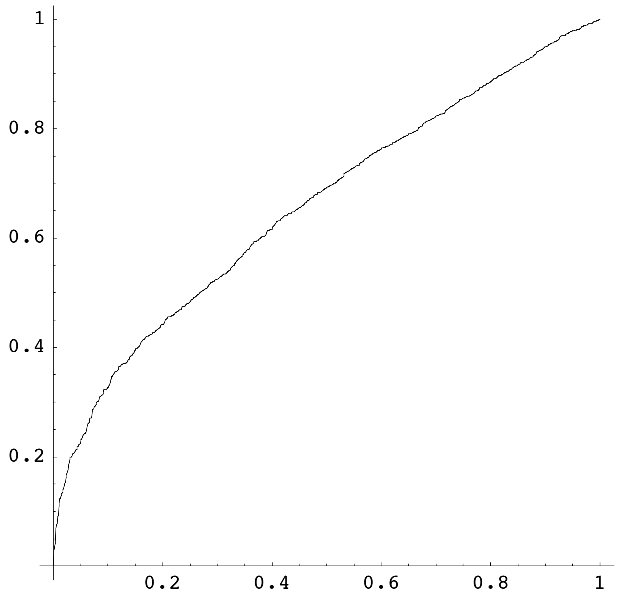


- Graphics -

- **Exercício 5.6'': Gamma(2.5,1) — IHR → TTT plot should resemble a concave line**

```
<< Statistics`ContinuousDistributions`
```

```
i = 1;
n = 1000;
t = Sort[RandomArray[GammaDistribution[2.5, 1], n]];
t[[0]] = 0;
ListPlot[
 Prepend[Table[{i / n, (Sum_{j=1}^{i} (n - j + 1) × (t[[j]] - t[[j - 1]])) / (Sum_{j=1}^{n} (n - j + 1) × (t[[j]] - t[[j - 1]]))}, {i, 1, n}], {0, 0}],
 PlotJoined → True, AspectRatio → 1]
```

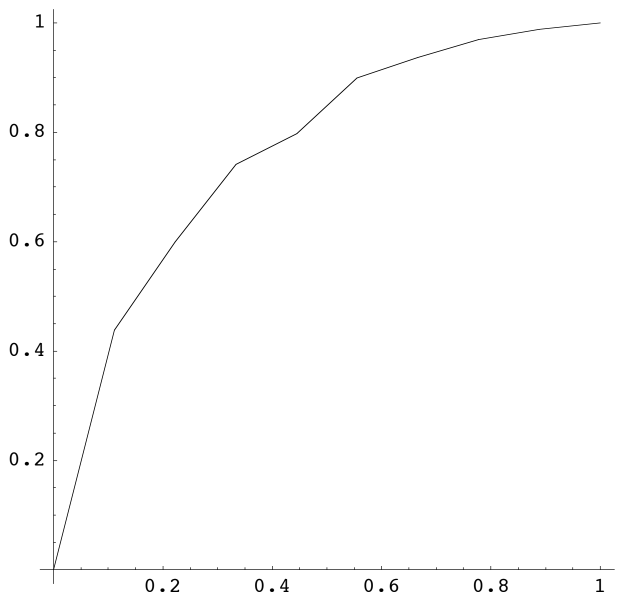


- Graphics -

■ Exercício 5.7

```
t = {0.41, 0.58, 0.75, 0.83, 1, 1.08, 1.17, 1.25, 1.35};
n = Length[t];
t[[0]] = 0;
TableForm[Table[{i,
   t[[i]],
   t[[i]] - t[[i - 1]],
   n - i + 1,
   (n - i + 1) × (t[[i]] - t[[i - 1]]),
   Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}],
   Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}],
 TableHeadings -> {None, {"i", "t(i)", "t(i)-t(i-1)",
   "n-i+1", "(n-i+1)×[t(i)-t(i-1)]", "τ[t(i)]", "τ[t(i)]/τ[t(n)]"}}]

ListPlot[Prepend[Table[{i / n, Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}], {0, 0}],
 PlotJoined → True, AspectRatio → 1]
```

i	$t_{(i)}$	$t_{(i)}-t_{(i-1)}$	n-i+1	$(n-i+1)\times[t_{(i)}-t_{(i-1)}]$	$\tau[t_{(i)}]$	$\frac{\tau[t_{(i)}]}{\tau[t_{(n)}]}$
1	0.41	0.41	9	3.69	3.69	0.438242
2	0.58	0.17	8	1.36	5.05	0.599762
3	0.75	0.17	7	1.19	6.24	0.741093
4	0.83	0.08	6	0.48	6.72	0.7981
5	1	0.17	5	0.85	7.57	0.89905
6	1.08	0.08	4	0.32	7.89	0.937055
7	1.17	0.09	3	0.27	8.16	0.969121
8	1.25	0.08	2	0.16	8.32	0.988124
9	1.35	0.1	1	0.1	8.42	1.



- Graphics -

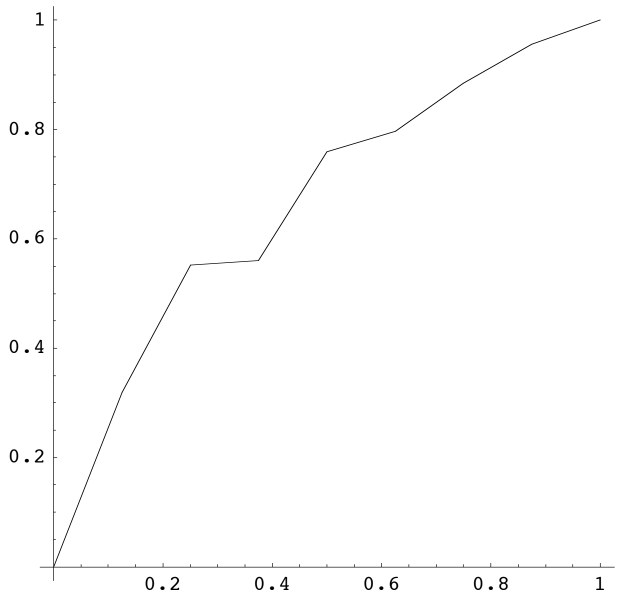
```
(* TTT plot resembles a concave line, thus, suggesting a IHR model *)
```

■ Exercício 5.9 (* Data from Exercise 5.10 *)

```
t = {0.30, 0.55, 0.56, 0.86, 0.93, 1.15, 1.42, 1.75};
n = Length[t];
t[[0]] = 0;
TableForm[Table[{i,
   t[[i]],
   t[[i]] - t[[i - 1]],
   n - i + 1,
   (n - i + 1) × (t[[i]] - t[[i - 1]]),
   Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}],
   Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}],
 TableHeadings -> {None, {"i", "t(i)", "t(i)-t(i-1)",
    "n-i+1", "(n-i+1)×[t(i)-t(i-1)]", "τ[t(i)]", "τ[t(i)]/τ[t(n)]"}}]

ListPlot[Prepend[Table[{i / n, Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}], {0, 0}],
 PlotJoined → True, AspectRatio → 1]
```

$$\sum_{j=1}^{i} (n - j + 1) \times (t[[j]] - t[[j - 1]]), \qquad \frac{\sum_{j=1}^{i} (n - j + 1) \times (t[[j]] - t[[j - 1]])}{\sum_{j=1}^{n} (n - j + 1) \times (t[[j]] - t[[j - 1]])}$$

i	$t_{(i)}$	$t_{(i)}-t_{(i-1)}$	n-i+1	$(n-i+1)\times[t_{(i)}-t_{(i-1)}]$	$\tau[t_{(i)}]$	$\frac{\tau[t_{(i)}]}{\tau[t_{(n)}]}$
1	0.3	0.3	8	2.4	2.4	0.319149
2	0.55	0.25	7	1.75	4.15	0.551862
3	0.56	0.01	6	0.06	4.21	0.55984
4	0.86	0.3	5	1.5	5.71	0.759309
5	0.93	0.07	4	0.28	5.99	0.796543
6	1.15	0.22	3	0.66	6.65	0.884309
7	1.42	0.27	2	0.54	7.19	0.956117
8	1.75	0.33	1	0.33	7.52	1.



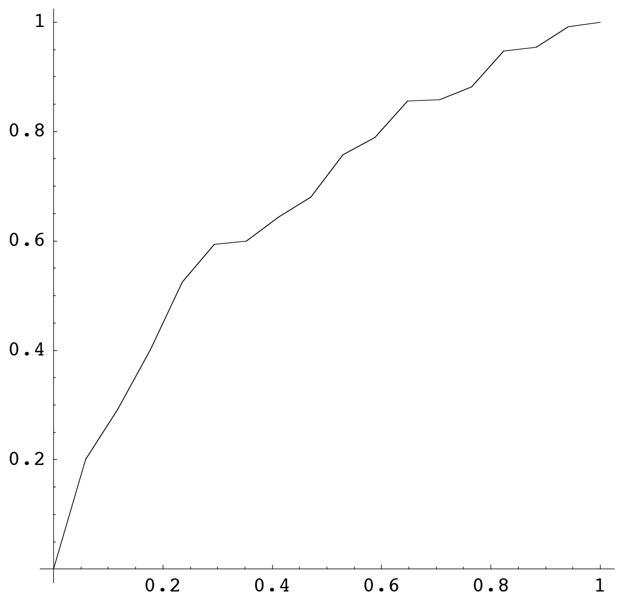
- Graphics -

```
(* TTT plot is close to a concave line, thus, suggesting a IHR model *)
```

■ Exercício 5.9 (* Data from Exercise 5.12 *)

```
t = {49, 73, 103, 140, 162, 164, 181, 196, 232, 248, 288, 290, 309, 377, 388, 464, 500};
n = Length[t];
t[[0]] = 0;
TableForm[Table[{i,
   t[[i]],
   t[[i]] - t[[i - 1]],
   n - i + 1,
   (n - i + 1) × (t[[i]] - t[[i - 1]]),
   Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}],
   Sum[(n - j + 1.) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}],
 TableHeadings -> {None, {"i", "t(i)", "t(i)-t(i-1)",
   "n-i+1", "(n-i+1)×[t(i)-t(i-1)]", "τ[t(i)]", "τ[t(i)]/τ[t(n)]"}}]
ListPlot[Prepend[Table[{i / n, Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}] / Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}]}, {i, 1, n}], {0, 0}],
 PlotJoined → True, AspectRatio → 1]
```

i	$t_{(i)}$	$t_{(i)}-t_{(i-1)}$	n−i+1	$(n-i+1)\times[t_{(i)}-t_{(i-1)}]$	$\tau[t_{(i)}]$	$\frac{\tau[t_{(i)}]}{\tau[t_{(n)}]}$
1	49	49	17	833	833	0.200048
2	73	24	16	384	1217	0.292267
3	103	30	15	450	1667	0.400336
4	140	37	14	518	2185	0.524736
5	162	22	13	286	2471	0.59342
6	164	2	12	24	2495	0.599183
7	181	17	11	187	2682	0.644092
8	196	15	10	150	2832	0.680115
9	232	36	9	324	3156	0.757925
10	248	16	8	128	3284	0.788665
11	288	40	7	280	3564	0.855908
12	290	2	6	12	3576	0.85879
13	309	19	5	95	3671	0.881604
14	377	68	4	272	3943	0.946926
15	388	11	3	33	3976	0.954851
16	464	76	2	152	4128	0.991354
17	500	36	1	36	4164	1.



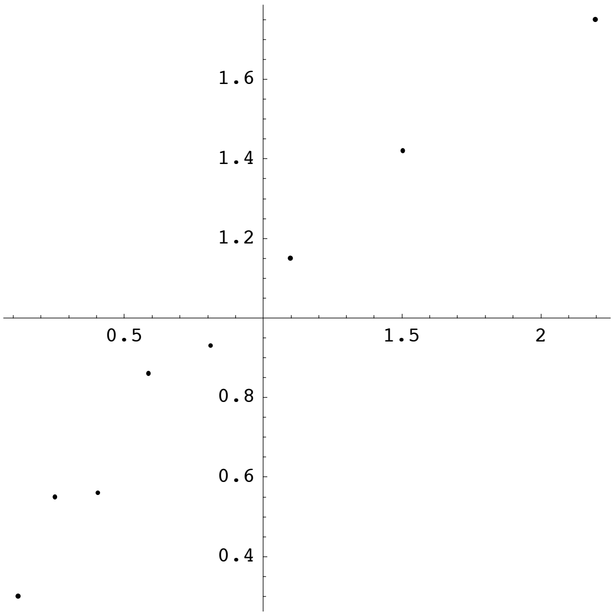
- Graphics -

(* TTT plot is close to a concave line, thus, suggesting a IHR model *)

■ Exercício 5.10

```
data = {0.30, 0.55, 0.56, 0.86, 0.93, 1.15, 1.42, 1.75};
sorteddata = Sort[data];
n = Length[sorteddata];

abcordExpProbPlot = Table[{Log[(n + 1) / (n - i + 1)], sorteddata[[i]]}, {i, 1, n}];
ListPlot[abcordExpProbPlot, AspectRatio → 1]
```

$$\left(\frac{\text{sorteddata[[n]]} - \text{sorteddata[[1]]}}{\text{Log[(n + 1) / (n - n + 1)]} - \text{Log[(n + 1) / (n - 1 + 1)]}}\right)^{-1}$$

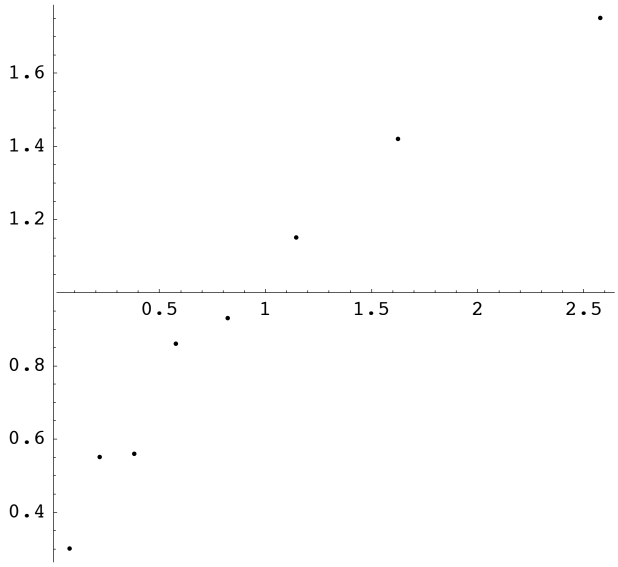


- Graphics -

1.4341

```
data = {0.30, 0.55, 0.56, 0.86, 0.93, 1.15, 1.42, 1.75};
sortedddata = Sort[data];
n = Length[sorteddata];

abcordExpProbPlot = Table[{Log[(n + 0.25) / (n - i + 0.625)], sorteddata[[i]]}, {i, 1, n}];
ListPlot[abcordExpProbPlot, AspectRatio → 1]
```

$$\left(\frac{\text{sorteddata[[n]] - sorteddata[[1]]}}{\text{Log[(n + 0.25) / (n - n + 0.625)] - Log[(n + 0.25) / (n - 1 + 0.625)]}}\right)^{-1}$$




```
- Graphics -

1.72513
```

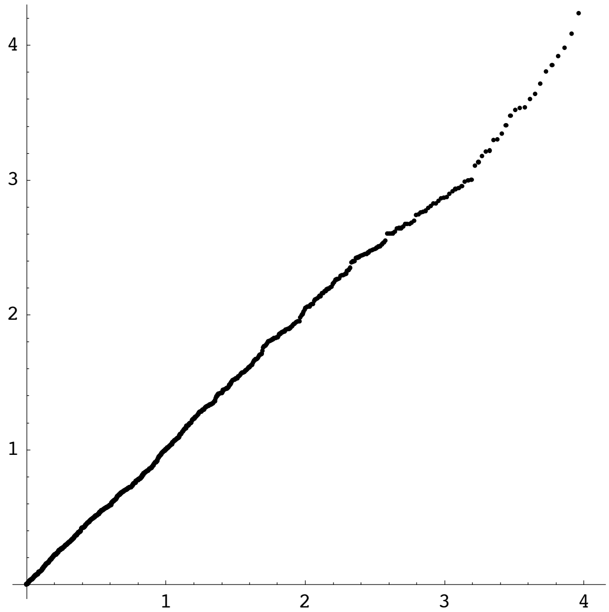
▪ Exercício 5.10': exponential probability plot with simulated exponential data

```
<< Statistics`ContinuousDistributions`
```

```
data = RandomArray[ExponentialDistribution[1.], 1000];
sorteddata = Sort[data];
n = Length[sorteddata];

abcordExp = Table[{Log[(n + 1) / (n - i + 1)], sorteddata[[i]]}, {i, 1, n}];
ListPlot[abcordExp, AxesOrigin → {0, 0}, AspectRatio → 1]
```

$$\left(\frac{\text{sorteddata[[n]] - sorteddata[[1]]}}{\text{Log[(n + 1) / (n - n + 1)] - Log[(n + 1) / (n - 1 + 1)]}}\right)^{-1}$$

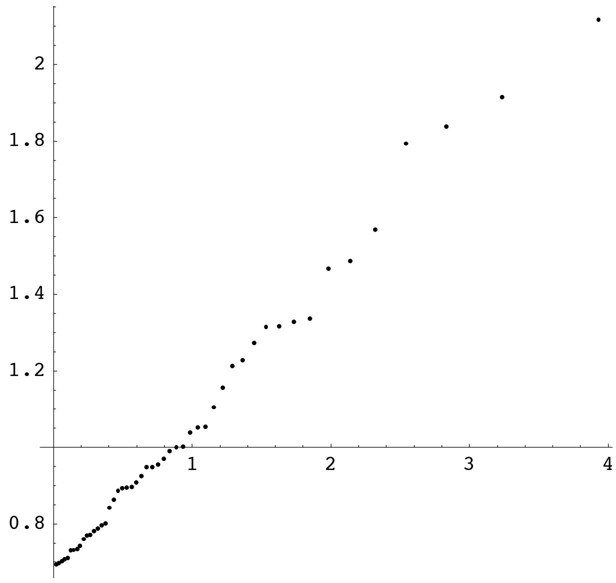
$$\left(\frac{\text{sorteddata[[Floor[0.9}\times\text{n]]] - sorteddata[[Floor[0.1}\times\text{n]]]}}{\text{Log[(n + 1) / (n - Floor[0.9}\times\text{n] + 1)] - Log[(n + 1) / (n - Floor[0.1}\times\text{n] + 1)]}}\right)^{-1}$$




```
- Graphics -

0.644146

0.992656

(* Unsurprisingly, the probability plot is very close to a straight line! *)
(* The first and last ordered obs. should
   be not used in the estimation of the scale parameter... *)
```

■ Exercício 5.11

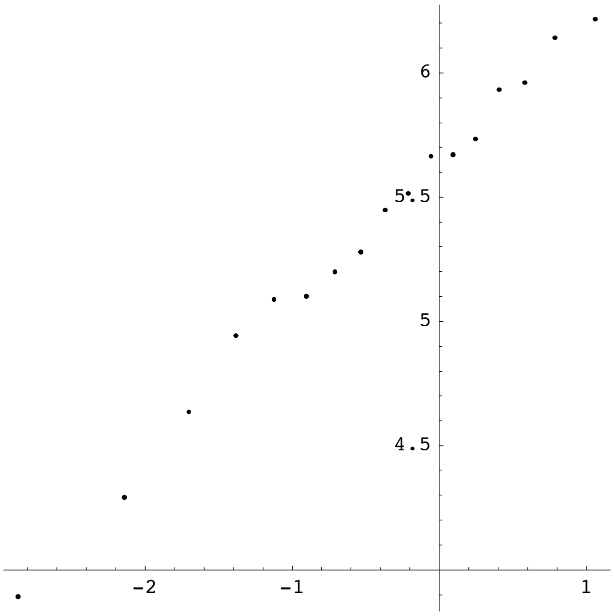
```
data = {2.001, 2.007, 2.017, 2.026, 2.036, 2.075, 2.077, 2.082, 2.101, 2.137,
   2.156, 2.161, 2.181, 2.196, 2.214, 2.227, 2.320, 2.367, 2.424, 2.443,
   2.444, 2.449, 2.478, 2.520, 2.579, 2.581, 2.598, 2.637, 2.691, 2.715,
   2.720, 2.825, 2.863, 2.867, 3.016, 3.176, 3.360, 3.413, 3.567, 3.721,
   3.727, 3.769, 3.803, 4.329, 4.420, 4.795, 6.009, 6.281, 6.784, 8.305};
sorteddata = Sort[data];
n = Length[sorteddata];

abcordParetoProbPlot =
  Table[{Log[(n + 1) / (n - i + 1)], Log[sorteddata[[i]]]}, {i, 1, n}];
ListPlot[abcordParetoProbPlot, AspectRatio → 1]
```




```
- Graphics -

(* The probability plot is approximately linear, thus,
 the data set seems to have been generated by a Pareto model *)
```

■ Exercício 5.12

```
data = {49, 73, 103, 140, 162, 164, 181, 196, 232, 248, 288, 290, 309, 377, 388, 464, 500};
sorteddata = Sort[data];
n = Length[sorteddata];

abcordParetoProbPlot =
  Table[{Log[Log[(n + 1) / (n - i + 1)]], Log[sorteddata[[i]]]}, {i, 1, n}];
ListPlot[abcordParetoProbPlot, AspectRatio → 1]
```

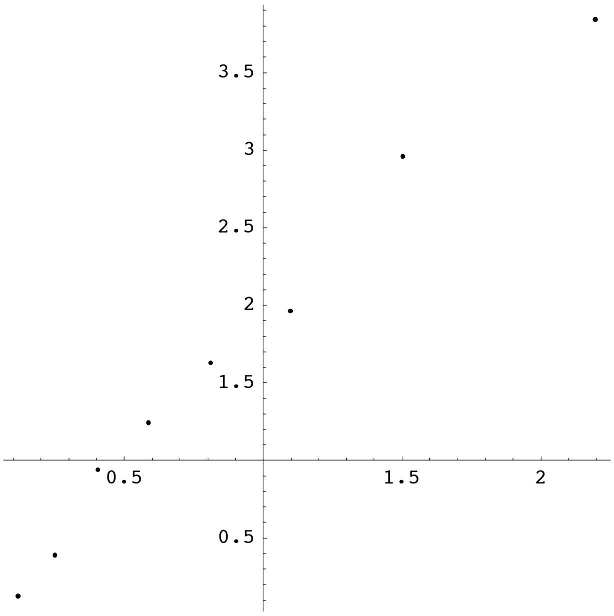



```
- Graphics -

(* The probability plot is approximately linear, thus,
 the data set seems to have been generated by a Weibull model *)
```

■ Exercício 5.13

```
data = {1.242, 1.626, 0.123, 2.957, 0.388, 3.841, 1.961, 0.938};
sorteddata = Sort[data];
n = Length[sorteddata];

abcordParetoProbPlot = Table[{Log[(n + 1) / (n - i + 1)], sorteddata[[i]]}, {i, 1, n}];
ListPlot[abcordParetoProbPlot, AspectRatio → 1]
```

$$\left(\frac{\text{sorteddata[[n]]} - \text{sorteddata[[1]]}}{\text{Log[(n + 1) / (n - n + 1)]} - \text{Log[(n + 1) / (n - 1 + 1)]}}\right)^{-1}$$



- Graphics -

0.55929

■ Exercício 5.14

```
data = Sort[{1.242, 1.626, 0.123, 2.957, 0.388, 3.841, 1.961, 0.938}];
n = Length[data];
fde = Table[N[i / n, 5], {i, 1, n}];
fdeatras = Table[N[(i - 1) / n, 5], {i, 1, n}];
fdconjecturada = Table[1 - Exp[-0.56 × data[[i]]], {i, 1, n}];
fdemenosfdc = fde - fdconjecturada;
fdcmenosfdeatras = fdconjecturada - fdeatras;
values = TableForm[
  Transpose[{data, fde, fdeatras, fdconjecturada, fdemenosfdc, fdcmenosfdeatras}],
  TableHeadings →
   {Automatic, {"t(i)", "i/n", "(i - 1)/n", "F0[t(i)]", "i/n-F0[t(i)]", "F0[t(i)]-(i - 1)/n"}}]
dplus = Max[Max[fdemenosfdc], 0]
dminus = Max[Max[fdcmenosfdeatras], 0]
Max[dplus, dminus]
```

	$t_{(i)}$	$\frac{i}{n}$	$\frac{i-1}{n}$	$F_0[t_{(i)}]$	$\frac{i}{n}-F_0[t_{(i)}]$	$F_0[t_{(i)}]-\frac{i-1}{n}$
1	0.123	0.12500	0	0.0665613	0.0584387	0.0665613
2	0.388	0.25000	0.12500	0.195295	0.0547046	0.0702954
3	0.938	0.37500	0.25000	0.40861	-0.0336102	0.15861
4	1.242	0.50000	0.37500	0.501185	-0.001185	0.126185
5	1.626	0.62500	0.50000	0.597701	0.0272989	0.0977011
6	1.961	0.75000	0.62500	0.666516	0.0834841	0.0415159
7	2.957	0.87500	0.75000	0.809084	0.0659163	0.0590837
8	3.841	1.0000	0.87500	0.883628	0.116372	0.00862761

0.116372

0.15861

0.15861

■ Exercício 5.22

```
a = {100, 7120, 24110, 36860, 340, 12910, 28570, 38540, 940, 13670,
   31620, 42110, 5670, 19490, 32800, 43970, 6010, 23700, 34910, 64730};
r = Dimensions[a][[1]];
```

$$N\left[\frac{1}{r}\sum_{i=1}^{r} a[[i]],\ 5\right]$$

$$N\left[\frac{1}{r}\sum_{i=1}^{r} \mathrm{Log}[a[[i]]],\ 5\right]$$

$$N\left[\frac{2\times r}{1+\frac{r+1}{6\,r}}\left(\mathrm{Log}\left[\frac{\sum_{i=1}^{r} a[[i]]}{r}\right]-\frac{1}{r}\sum_{i=1}^{r}\mathrm{Log}[a[[i]]]\right),\ 5\right]$$

23409.

9.3748

23.354

■ Exercício 5.23

```
a = {21.2, 26.7, 11.3, 2.8, 12.6, 0.1, 2.1, 7.5,
   6.7, 2.3, 15.3, 4.3, 14.1, 16.9, 7.7, 5.8, 7.3, 32.1, 17.6, 4.5};
r = Dimensions[a][[1]];
```

$$N\left[\frac{1}{r}\sum_{i=1}^{r} a[[i]],\ 5\right]$$

$$N\left[\frac{1}{r}\sum_{i=1}^{r} \mathrm{Log}[a[[i]]],\ 5\right]$$

$$N\left[\frac{2\times r}{1+\frac{r+1}{6\,r}}\left(\mathrm{Log}\left[\frac{\sum_{i=1}^{r} a[[i]]}{r}\right]-\frac{1}{r}\sum_{i=1}^{r}\mathrm{Log}[a[[i]]]\right),\ 5\right]$$

```
10.945

1.94019

15.4108
```

■ Exercício 5.24

```
a = {6700, 4600, 4100, 14000, 850, 5400, 3100, 5000, 2600, 4700};
r = Dimensions[a][[1]];
```

$$N\left[\frac{1}{r}\sum_{i=1}^{r} a[[i]],\ 5\right]$$

$$N\left[\frac{1}{r}\sum_{i=1}^{r} \mathrm{Log}[a[[i]]],\ 5\right]$$

$$N\left[\frac{2\times r}{1+\frac{r+1}{6\,r}}\left(\mathrm{Log}\left[\frac{\sum_{i=1}^{r} a[[i]]}{r}\right]-\frac{1}{r}\sum_{i=1}^{r}\mathrm{Log}[a[[i]]]\right),\ 5\right]$$

```
5105.0

8.3324

3.4753
```

■ Exercício 5.29

$$\lambda = \frac{50.}{5613 \times 8760}$$
$$\lambda L = \frac{74.22}{2 \times 5613 \times 8760}$$
$$\lambda U = \frac{129.6}{2 \times 5613 \times 8760}$$

```
Exp[-λ × 8760]
```
$$\left(1 - \frac{1.}{5613}\right)^{50}$$
```
Exp[-λU × 8760]
Exp[-λL × 8760]
```

$$-\frac{\text{Log}[1 - 0.8]}{\lambda}$$
$$\frac{-\frac{\text{Log}[1-0.8]}{\lambda}}{24 \times 365}$$
$$-\frac{\text{Log}[1 - 0.8]}{\lambda U}$$
$$-\frac{\text{Log}[1 - 0.8]}{\lambda L}$$

1.01688×10^{-6}

7.5473×10^{-7}

1.31788×10^{-6}

0.991132

0.991131

0.988522

0.99341

1.58272×10^{6}

180.676

1.22123×10^{6}

2.13247×10^{6}

$$\begin{pmatrix} 10 & 0.689621 \\ 20 & 0.608942 \\ 30 & 0.564574 \\ 50 & 0.513486 \\ 100 & 0.453652 \end{pmatrix}$$

■ Exercício 5.30

```
r = {10, 20, 30, 50, 100};
ratio = {1.1, 1.2, 1.3, 1.5, 2, 3};
```

$$\text{MatrixForm}\Big[\text{Table}\Big[\text{N}\Big[\frac{\sum_{i=1}^{r[[j]]} \frac{1}{\text{ratio}[[k]]\times r[[j]]-i+1}}{\sum_{i=1}^{r[[j]]} \frac{1}{r[[j]]-i+1}}, 5\Big],$$

```
  {k, 1, Dimensions[ratio][[1]]}, {j, 1, Dimensions[r][[1]]}]]
```

0.689621	0.608942	0.564574	0.513486	0.453652
0.547364	0.470469	0.431681	0.389158	0.34143
0.459821	0.390362	0.356591	0.320289	0.280219
0.353331	0.296302	0.269518	0.241242	0.210506
0.22833	0.18923	0.17144	0.15295	0.13314
0.13563	0.11155	0.10080	0.089750	0.078003