

## Exercícios

### Cap. 4 — Notas apoio 0607

```
<< Statistics`ContinuousDistributions`
<< Statistics`DiscreteDistributions`
<< Graphics`MultipleListPlot`
```

#### ■ Exercício 4.6

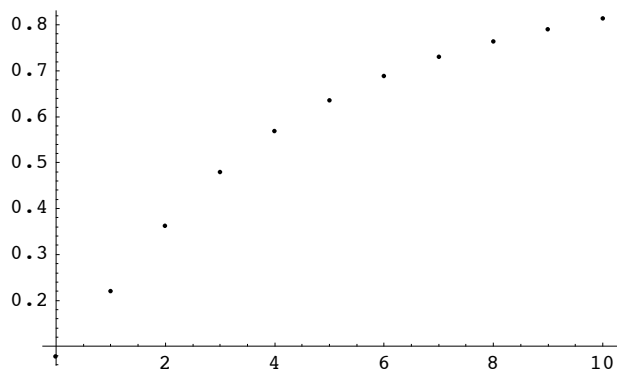
```
f[x_, n_, p_] := PDF[BinomialDistribution[n, p], x];
F[x_, n_, p_] := CDF[BinomialDistribution[n, p], x];
λ[x_, n_, p_] =  $\frac{f[x, n, p]}{1 - F[x - 1, n, p]}$ ;

Mean[BinomialDistribution[50, 0.05]]
Variance[BinomialDistribution[50, 0.05]]

F[14, 50, 0.05]

list1 = Table[{x, N[λ[x, 50, 0.05], 5]}, {x, 0, 10, 1}]
ListPlot[list1]

2.5
2.375
1.
{{0, 0.076945}, {1, 0.219366}, {2, 0.362355},
 {3, 0.478543}, {4, 0.567528}, {5, 0.635424}, {6, 0.687993},
 {7, 0.729491}, {8, 0.762891}, {9, 0.790256}, {10, 0.813033}}
```



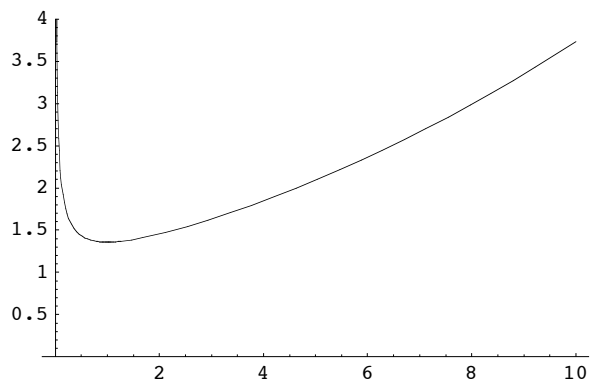
- Graphics -

### ■ Exercício 4.7

```
0.994  
1 - (1 - 0.99)4  
0.992  
1 - (1 - 0.99)3  
  
0.960596  
  
1.  
  
0.9801  
  
0.999999
```

### ■ Exercício 4.17

```
 $\mu = 1;$   
 $\theta = .5;$   
 $\lambda[t_] = \theta \mu (\mu t)^{\theta-1} \text{Exp}[(\mu t)^\theta];$   
Plot[ $\lambda[t]$ , {t, .001, 10}, PlotRange -> {0, 4}]
```

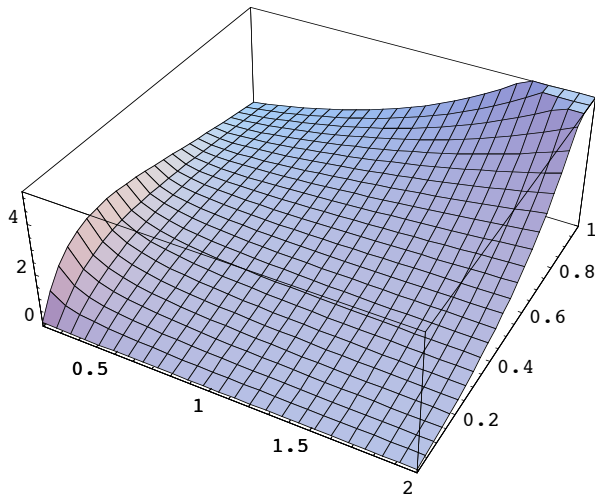


- Graphics -

```

μ = 1;
Clear[θ];
g[t_, θ_] = θ μ (μ t)θ-1 Exp[-(μ t)θ];
Plot3D[g[t, θ], {t, .1, 2}, {θ, 0.001, 1}]

```



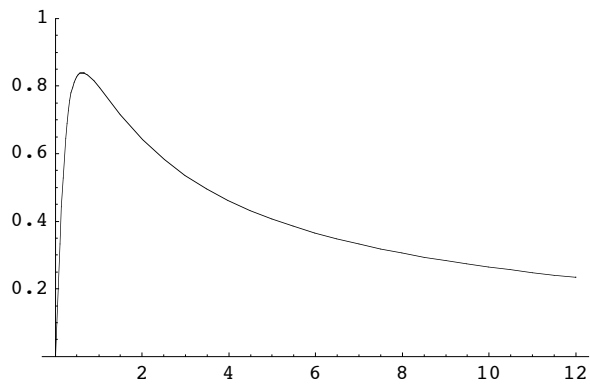
- SurfaceGraphics -

#### ■ Exercício 4.20

```

f[t_, μ_, σ_] = PDF[LogNormalDistribution[μ, σ], t];
R[t_, μ_, σ_] = 1 - CDF[LogNormalDistribution[μ, σ], t];
λ[t_, μ_, σ_] = f[t, μ, σ] / R[t, μ, σ];
Plot[λ[t, 0, 1], {t, 0, 12}, PlotRange -> {0, 1}]

```



- Graphics -

```

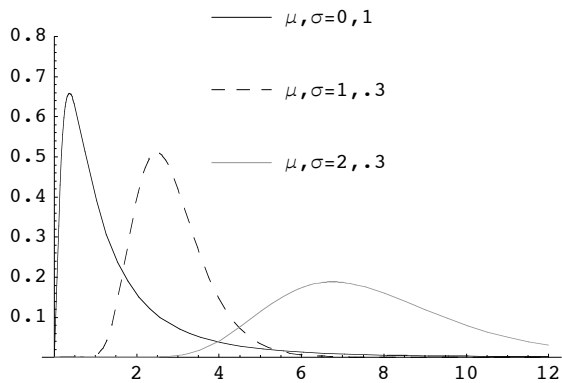
f[t_, μ_, σ_] = PDF[LogNormalDistribution[μ, σ], t];
R[t_, μ_, σ_] = 1 - CDF[LogNormalDistribution[μ, σ], t];
h[t_, μ_, σ_] =  $\frac{f[t, \mu, \sigma]}{R[t, \mu, \sigma]}$ ;

Plot[{f[t, 0, 1], f[t, 1, 0.3], f[t, 2, .3]}, {t, 0, 12},
  PlotRange -> {0, .8}, PlotLegend -> {"μ,σ=0,1", "μ,σ=1,.3", "μ,σ=2,.3"},
  PlotStyle -> {GrayLevel[0], {GrayLevel[0], Dashing[{.03]}}, GrayLevel[.5]},
  LegendPosition -> {-.3, -.0}, LegendShadow -> None]

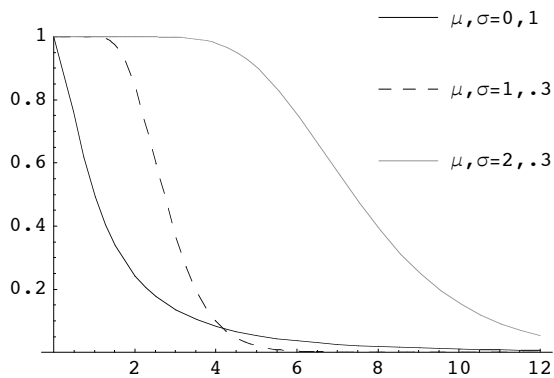
Plot[{R[t, 0, 1], R[t, 1, 0.3], R[t, 2, .3]}, {t, 0, 12}, PlotRange -> {0, 1},
  AxesOrigin -> {0, 0}, PlotLegend -> {"μ,σ=0,1", "μ,σ=1,.3", "μ,σ=2,.3"},
  PlotStyle -> {GrayLevel[0], {GrayLevel[0], Dashing[{.03]}}, GrayLevel[.5]},
  LegendPosition -> {.3, -.0}, LegendShadow -> None]

Plot[{h[t, 0, 1], h[t, 1, 0.3], h[t, 2, .3]}, {t, 0, 12}, PlotRange -> {0, 1.8},
  AxesOrigin -> {0, 0}, PlotLegend -> {"μ,σ=0,1", "μ,σ=1,.3", "μ,σ=2,.3"},
  PlotStyle -> {GrayLevel[0], {GrayLevel[0], Dashing[{.03]}}, GrayLevel[.5]},
  LegendPosition -> {.3, -.0}, LegendShadow -> None]

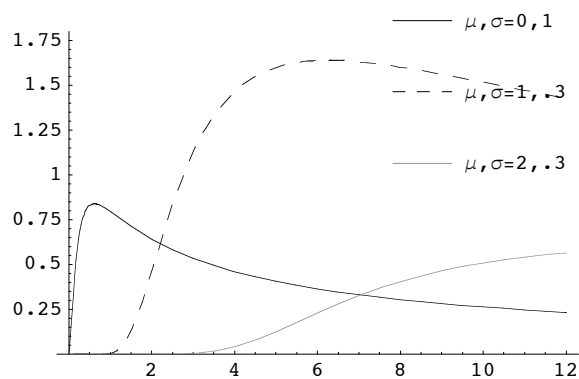
```



- Graphics -



- Graphics -



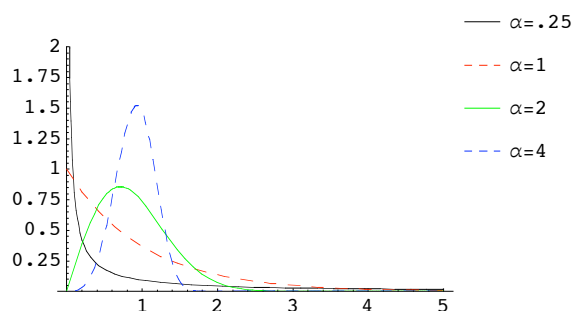
- Graphics -

## ■ Exercício 4.22

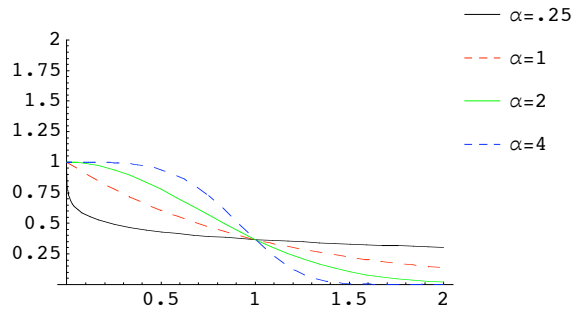
```
f[t_, α_, δ_] = PDF[WeibullDistribution[α, δ], t];
R[t_, α_, δ_] = 1 - CDF[WeibullDistribution[α, δ], t];
h[t_, α_, δ_] = f[t, α, δ] / R[t, α, δ];
Plot[{f[t, 0.25, 1], f[t, 1, 1], f[t, 2, 1], f[t, 4, 1]}, {t, 0, 5},
  PlotRange -> {0, 2}, PlotLegend -> {"α=.25", "α=1", "α=2", "α=4"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {1, -.0}, LegendShadow -> None]

Plot[{R[t, 0.25, 1], R[t, 1, 1], R[t, 2, 1], R[t, 4, 1]}, {t, 0, 2},
  PlotRange -> {0, 2}, PlotLegend -> {"α=.25", "α=1", "α=2", "α=4"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {1, -.0}, LegendShadow -> None]

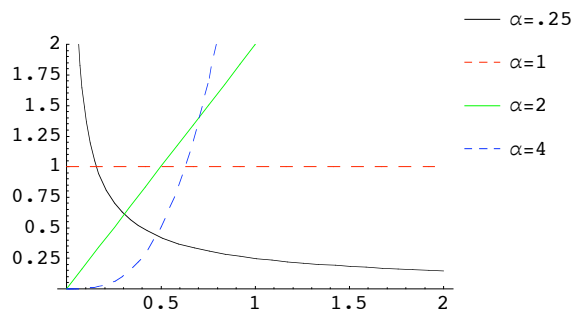
Plot[{h[t, 0.25, 1], h[t, 1, 1], h[t, 2, 1], h[t, 4, 1]}, {t, 0, 2},
  PlotRange -> {0, 2}, PlotLegend -> {"α=.25", "α=1", "α=2", "α=4"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {1, -.0}, LegendShadow -> None]
```



- Graphics -



- Graphics -



- Graphics -

### ■ Exercício 4.23

<< `Statistics`DescriptiveStatistics``

```

amostra = {.41, .58, .75, .83, 1., 1.08, 1.17, 1.25, 1.35};
n = Dimensions[amostra][[1]]
Mean[amostra]
VarianceMLE[amostra] + Mean[amostra]^2

h = FindRoot[{Mean[amostra] -  $\delta \times \text{Gamma}\left[\frac{1}{\alpha} + 1\right] == 0,$ 
  VarianceMLE[amostra] + Mean[amostra]^2 -  $\delta^2 \times \text{Gamma}\left[\frac{2}{\alpha} + 1\right] == 0$ }, { $\delta$ , 1}, { $\alpha$ , 1}]
raiz = { $\delta$ ,  $\alpha$ } /. Dispatch[h];

FindRoot[{ $\delta - \left(\frac{\sum_{i=1}^n (\text{amostra}[[i]])^\alpha}{n}\right)^{\frac{1}{\alpha}} == 0,$ 
   $\frac{n}{\alpha} - n \times \text{Log}[\delta] + \sum_{i=1}^n \text{Log}[\text{amostra}[[i]]] + \delta^{-\alpha} \times \text{Log}[\delta] \times \sum_{i=1}^n (\text{amostra}[[i]])^\alpha$ 
  -  $\delta^{-\alpha} \times \sum_{i=1}^n \text{Log}[\text{amostra}[[i]]] (\text{amostra}[[i]])^\alpha == 0$ }, { $\delta$ , raiz[[1]]}, { $\alpha$ , raiz[[2]]}]

9

0.935556

0.964022

{ $\delta \rightarrow 1.04018$ ,  $\alpha \rightarrow 3.47575$ }

{ $\delta \rightarrow 1.04077$ ,  $\alpha \rightarrow 3.68246$ }

```

## ■ Exercício 4.25

```

<< Statistics`ContinuousDistributions`
<< Graphics`MultipleListPlot`

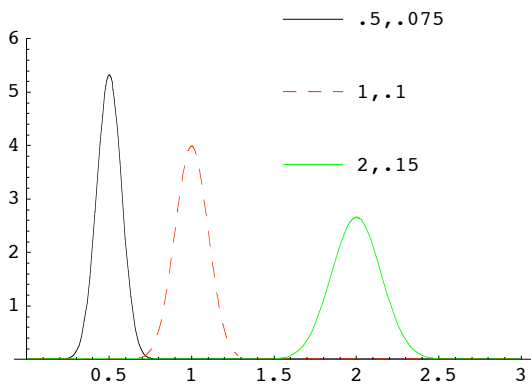
f[t_,  $\mu$ _,  $\sigma$ _] = PDF[NormalDistribution[ $\mu$ ,  $\sigma$ ], t];
R[t_,  $\mu$ _,  $\sigma$ _] = 1 - CDF[NormalDistribution[ $\mu$ ,  $\sigma$ ], t];
 $\lambda[t_, \mu_, \sigma_] = \frac{f[t, \mu, \sigma]}{R[t, \mu, \sigma]}$ ;

Plot[{f[t, .5, .075], f[t, 1, .1], f[t, 2, .15]}, {t, 0, 3},
  PlotRange -> {0, 6}, PlotLegend -> {".5, .075", "1, .1", "2, .15"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition -> {0, -0}, LegendShadow -> None]

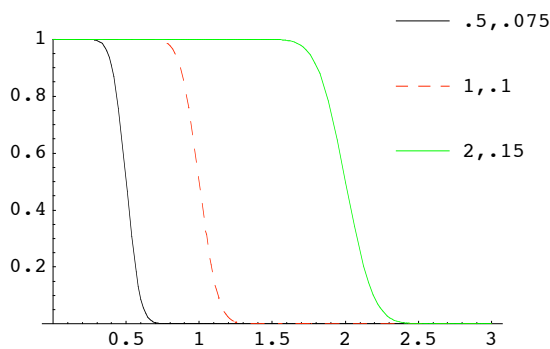
Plot[{R[t, .5, .075], R[t, 1, .1], R[t, 2, .15]}, {t, 0, 3},
  PlotRange -> {0, 1}, PlotLegend -> {".5, .075", "1, .1", "2, .15"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition -> {0.5, -0}, LegendShadow -> None]

Plot[ $\{\lambda[t, .5, .075], \lambda[t, 1, .1], \lambda[t, 2, .15]\}$ , {t, 0, 3},
  PlotRange -> {0, 6}, PlotLegend -> {".5, .075", "1, .1", "2, .15"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition -> {1, -0}, LegendShadow -> None]

```



- Graphics -



- Graphics -

Power::infy : Infinite expression  $\frac{1}{0.}$  encountered. More...

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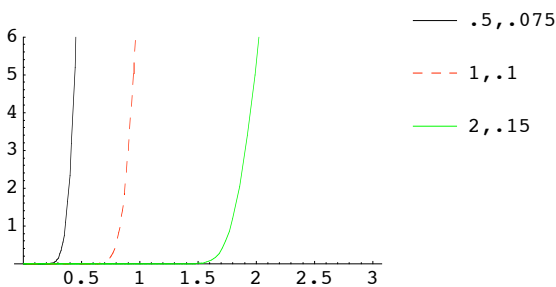
General::stop : Further output of Power::infy will be suppressed during this calculation. More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 1.1324058236399117^{\wedge}$ . More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 1.1252401365755285^{\wedge}$ . More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 1.1212636449958993^{\wedge}$ . More...

General::stop : Further output of Plot::plnr will be suppressed during this calculation. More...



- Graphics -



## ■ Exercício 4.26

```
<< Statistics`ContinuousDistributions`
<< Graphics`MultipleListPlot`

f[t_, μ_, σ_] = 
$$\frac{\text{PDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]}{1 - \text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]};$$

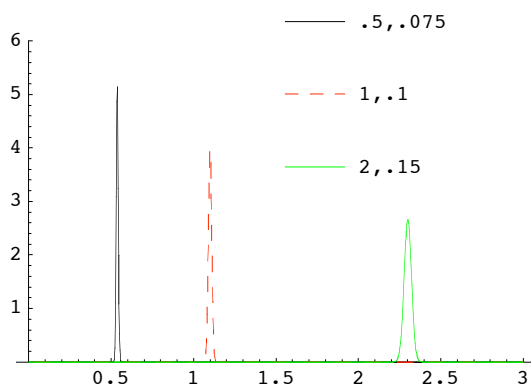

R[t_, μ_, σ_] = 
$$\frac{1 - \text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]}{1 - \text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{-\mu}{\sigma}]};$$


λ[t_, μ_, σ_] = 
$$\frac{f[t, \mu, \sigma]}{R[t, \mu, \sigma]};$$

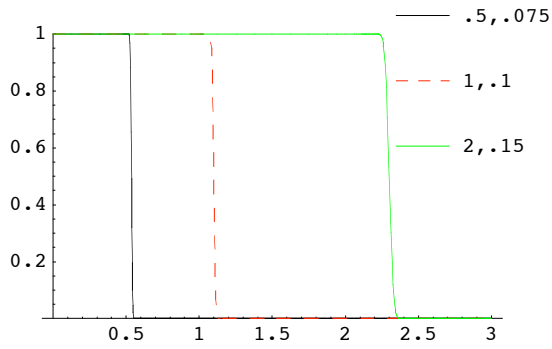

Plot[{f[t, .5, .075], f[t, 1, .1], f[t, 2, .15]}, {t, 0, 3},
  PlotRange → {0, 6}, PlotLegend → {".5, .075", "1, .1", "2, .15"},
  PlotStyle → {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition → {0, -0}, LegendShadow → None]

Plot[{R[t, .5, .075], R[t, 1, .1], R[t, 2, .15]}, {t, 0, 3},
  PlotRange → {0, 1}, PlotLegend → {".5, .075", "1, .1", "2, .15"},
  PlotStyle → {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition → {0.5, -0}, LegendShadow → None]

Plot[{λ[t, .5, .075], λ[t, 1, .1], λ[t, 2, .15]}, {t, 0, 3},
  PlotRange → {0, 6}, PlotLegend → {".5, .075", "1, .1", "2, .15"},
  PlotStyle → {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03} ]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03} ]}},
  LegendPosition → {1, -0}, LegendShadow → None]
```



- Graphics -



- Graphics -

Power::infy : Infinite expression  $\frac{1}{0.}$  encountered. More...

Power::infy : Infinite expression  $\frac{1}{0.}$  encountered. More...

Power::infy : Infinite expression  $\frac{1}{0.}$  encountered. More...

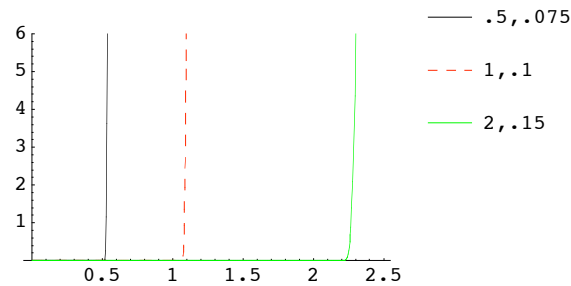
General::stop : Further output of Power::infy will be suppressed during this calculation. More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 0.6131818981088902^`$ . More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 0.5853488771584009^`$ . More...

Plot::plnr :  $\lambda[t, 0.5, 0.075]$  is not a machine-size real number at  $t = 0.5844523919214953^`$ . More...

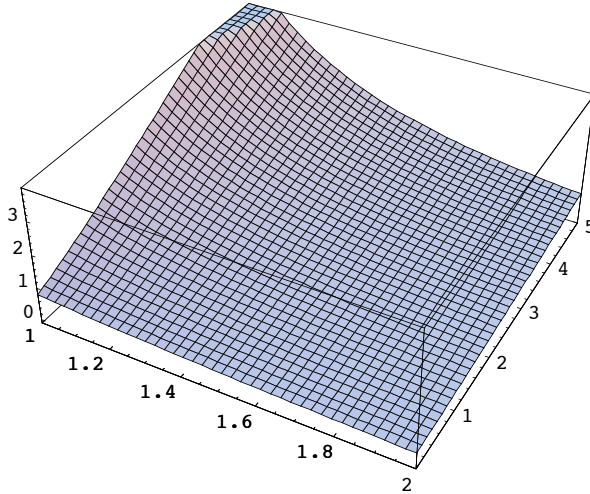
General::stop : Further output of Plot::plnr will be suppressed during this calculation. More...



- Graphics -

$$\lambda[t_, \mu_, \sigma_] = \frac{\frac{\text{PDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]}{1-\text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]}}{\frac{1-\text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{t-\mu}{\sigma}]}{1-\text{CDF}[\text{NormalDistribution}[\mu, \sigma], \frac{-\mu}{\sigma}]}};$$

`Plot3D[λ[t, 0, σ], {σ, 1, 2}, {t, 0.00001, 5}, PlotPoints → 40]`



- SurfaceGraphics -

## ■ Exercício 4.28

```
<< Statistics`ContinuousDistributions`
<< Graphics`MultipleListPlot`
```

`PDF[GammaDistribution[α, 1/λ], t]`

$$\frac{e^{-t\lambda} t^{-1+\alpha} \left(\frac{1}{\lambda}\right)^{-\alpha}}{\text{Gamma}[\alpha]}$$

$$f[t\_ , \lambda\_ , \alpha\_ ] = \text{PDF}[\text{GammaDistribution}[\alpha, \frac{1}{\lambda}], t];$$

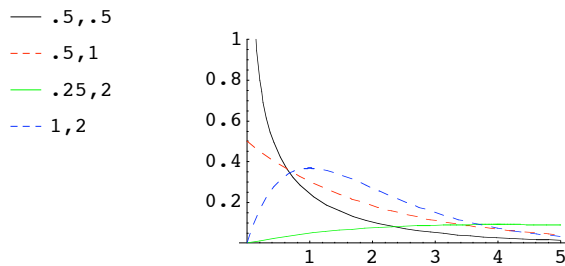
$$R[t\_ , \lambda\_ , \alpha\_ ] = 1 - \text{CDF}[\text{GammaDistribution}[\alpha, \frac{1}{\lambda}], t];$$

$$h[t\_ , \lambda\_ , \alpha\_ ] = \frac{f[t, \lambda, \alpha]}{R[t, \lambda, \alpha]};$$

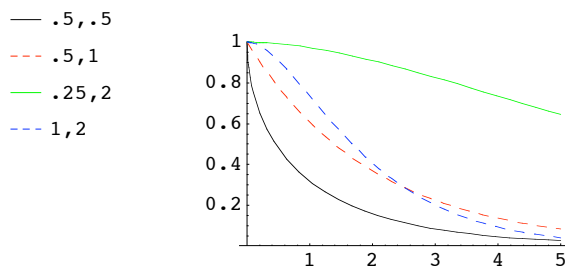
```
Plot[{f[t, .5, .5], f[t, .5, 1], f[t, .25, 2], f[t, 1, 2]}, {t, 0, 5},
  PlotRange -> {0, 1}, PlotLegend -> {".5, .5", ".5, 1", ".25, 2", "1, 2"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {-2, -0},
  LegendShadow -> None]
```

```
Plot[{R[t, .5, .5], R[t, .5, 1], R[t, .25, 2], R[t, 1, 2]}, {t, 0, 5},
  PlotRange -> {0, 1}, PlotLegend -> {".5, .5", ".5, 1", ".25, 2", "1, 2"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {-2, -0},
  LegendShadow -> None]
```

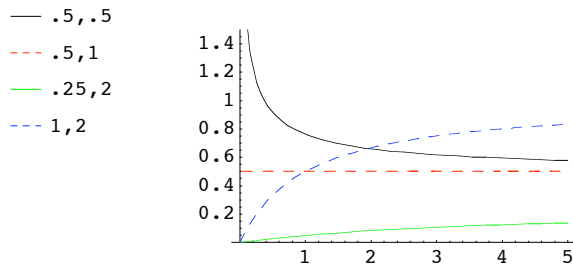
```
Plot[{h[t, .5, .5], h[t, .5, 1], h[t, .25, 2], h[t, 1, 2]}, {t, 0, 5},
  PlotRange -> {0, 1.5}, PlotLegend -> {".5, .5", ".5, 1", ".25, 2", "1, 2"},
  PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
  RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
  LegendPosition -> {-2, -0},
  LegendShadow -> None]
```



- Graphics -



- Graphics -



- Graphics -

### ■ Exercício 4.30

```
<< Statistics`ContinuousDistributions`

n = 50;
α = 0.05;

h = FindRoot[{
  PDF[ChiSquareDistribution[2 × n], a] - PDF[ChiSquareDistribution[2 × n], b] == 0,
  (CDF[ChiSquareDistribution[2 × n], b] - CDF[ChiSquareDistribution[2 × n], a]) -
  (1 - α) == 0}, {a, Quantile[ChiSquareDistribution[2 × n],  $\frac{\alpha}{2}$ ]},
  {b, Quantile[ChiSquareDistribution[2 × n],  $1 - \frac{\alpha}{2}$ ]}}]
raiz = {a, b} /. Dispatch[h];


$$\frac{\text{raiz}[[1]] \times 1.89}{2 \times n}$$


$$\frac{\text{raiz}[[2]] \times 1.89}{2 \times n}$$

{a → 73.0212, b → 128.114}

1.3801

2.42135

raiz[[2]] - raiz[[1]]
Quantile[ChiSquareDistribution[2 × n],  $1 - \frac{\alpha}{2}$ ] -
Quantile[ChiSquareDistribution[2 × n],  $\frac{\alpha}{2}$ ]

55.0924

55.3393
```

### ■ Exercício 4.32

```
<< Statistics`DescriptiveStatistics`
<< Statistics`ContinuousDistributions`
<< Graphics`MultipleListPlot`
```

```

data = {152.7, 172.0, 172.5, 173.3, 193.0, 204.7, 216.5, 239.9, 262.6, 422.6};
n = Dimensions[data][[1]]
miuinic = Mean[data]
lambdainic = 
$$\frac{(\text{Mean}[\text{data}])^3}{\text{VarianceMLE}[\text{data}]}$$


```

```

h = FindRoot[{

$$\frac{n}{2 \lambda} - \frac{1}{2 \mu^2} \sum_{i=1}^n \frac{(\text{data}[[i]] - \mu)^2}{\text{data}[[i]]} == 0,$$


$$\frac{\lambda}{\mu^3} \sum_{i=1}^n \frac{(\text{data}[[i]] - \mu)^2}{\text{data}[[i]]} + \frac{\lambda}{\mu^2} \sum_{i=1}^n \frac{(\text{data}[[i]] - \mu)}{\text{data}[[i]]} == 0\},
\{\mu, \text{miuinic}\}, \{\lambda, \text{lambdainic}\}]$$

```

```
10
```

```
220.98
```

```
1944.53
```

```
{miu -> 220.98, lambda -> 2698.7}
```

```

f[t_, μ_, λ_] = 
$$\sqrt{\frac{\lambda}{2 \times \pi \times t^3}} \text{Exp}\left[-\frac{\lambda (t - \mu)^2}{2 \mu^2 t}\right];$$

```

```

R[t_, λ_, α_] = CDF[NormalDistribution[0, 1],  $\sqrt{\frac{\lambda}{t}} \left(1 - \frac{t}{\mu}\right)$ ] -
Exp[ $\frac{2 \lambda}{\mu}$ ] × CDF[NormalDistribution[0, 1],  $-\sqrt{\frac{\lambda}{t}} \left(1 + \frac{t}{\mu}\right)$ ];

```

```

h[t_, λ_, α_] = 
$$\frac{f[t, \lambda, \alpha]}{R[t, \lambda, \alpha]};$$

```

```

Plot[{f[t, 1, .5], f[t, 1, 1], f[t, 1, 3], f[t, 1, 10]},
{t, 0.1, 5}, PlotLegend -> {"λ=.5", "λ=1", "λ=3", "λ=10"},
PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
LegendPosition -> {-2, -0},
LegendShadow -> None]

```

```

Plot[{R[t, 1, .5], R[t, 1, 1], R[t, 1, 3], R[t, 1, 10]},
{t, 0.1, 5}, PlotLegend -> {"λ=.5", "λ=1", "λ=3", "λ=10"},
PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
LegendPosition -> {-2, -0},
LegendShadow -> None]

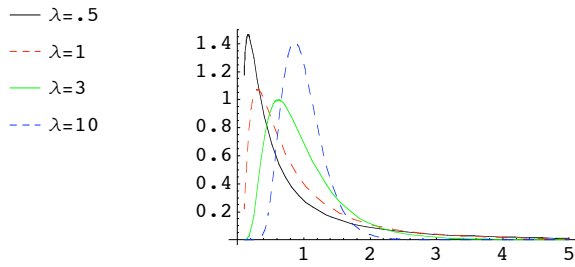
```

```

Plot[{h[t, 1, .5], h[t, 1, 1], h[t, 1, 3], h[t, 1, 10]},
{t, 0.1, 5}, PlotLegend -> {"λ=.5", "λ=1", "λ=3", "λ=10"},
PlotStyle -> {GrayLevel[0], {RGBColor[1, 0, 0], Dashing[ {.03}]},
RGBColor[0, 1, 0], {RGBColor[0, 0, 1], Dashing[ {.03}]},
LegendPosition -> {-2, -0},
LegendShadow -> None]

```

```
Set::write : Tag List in {miu -> 220.98, lambda -> 2698.7}[t_, λ_, α_] is Protected. MORE...
```



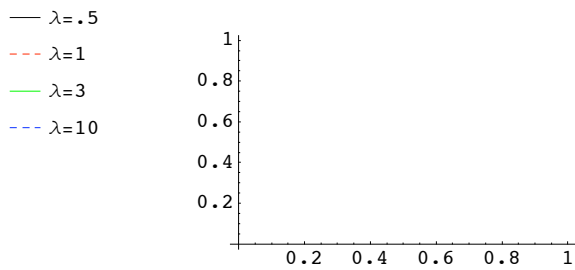
- Graphics -

Plot::plnr : R[t, 1, 0.5] is not a machine-size real number at t = 0.10000020416666668` . More...

Plot::plnr : R[t, 1, 0.5] is not a machine-size real number at t = 0.2987782587072874` . More...

Plot::plnr : R[t, 1, 0.5] is not a machine-size real number at t = 0.5155631193109311` . More...

General::stop : Further output of Plot::plnr will be suppressed during this calculation . More...



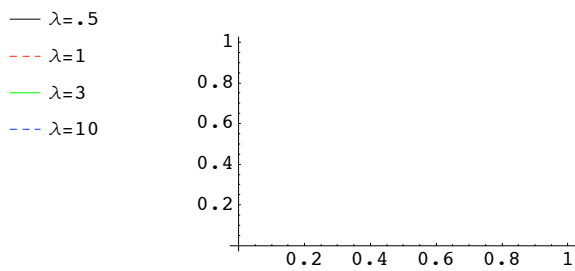
- Graphics -

Plot::plnr : h[t, 1, 0.5] is not a machine-size real number at t = 0.10000020416666668` . More...

Plot::plnr : h[t, 1, 0.5] is not a machine-size real number at t = 0.2987782587072874` . More...

Plot::plnr : h[t, 1, 0.5] is not a machine-size real number at t = 0.5155631193109311` . More...

General::stop : Further output of Plot::plnr will be suppressed during this calculation . More...



- Graphics -