Abstract—This paper presents sliding-mode controlled multilevel converters for reactive power compensation. The reactive power compensation scheme here proposed includes a prototype three-phase multilevel converter and a digital signal processing system, in which sliding-mode and linear controllers are implemented. The instantaneous power theory is presented and used for reactive power compensation, together with the converter models needed for the application of the multilevel sliding-mode controllers and dc voltage capacitor equalization. Simulation and experimental results are shown in order to highlight the system operation and control robustness.

Index Terms— Reactive power, Sliding-mode control, Multilevel converter modeling, power quality enhancement.

I. INTRODUCTION

The operation of electrical power lines at low power factor reduces the distribution capability, since current flow is unnecessarily increased, causing phase lag and voltage drops. Inductive loads such as induction motors and lighting ballasts are the main consumers of reactive power, worsening the regulation of the service voltage.

Strategies to improve voltage regulation include tap changing transformers or the installation of shunt capacitor banks to reduce the current magnitude and shift it to be nearly in phase with the voltage. However, the shunt capacitor solution often produces parallel resonance frequencies near the 5th or 7th harmonic, increasing voltage distortion. Moreover, these compensation schemes are unsuccessful for rapidly varying loads.

To solve these problems, reactive power compensation systems are being proposed, based in two-level three-phase electronic power converters. This work proposes the use of Neutral Point Clamped (NPC) three-phase multilevel converters, suitable for voltages in the kV range. Closed loop control is accomplished using the instantaneous p-q theory [5,6,9], together with sliding-mode for line current control and linear controllers for dc capacitor voltage equalization [7,8]. All the controllers are implemented in a digital signal processing system (DSP). The fully controlled multilevel converter can operate as a source of reactive power (leading or lagging power factor).

This paper presents the converter model (section II), as well as the instantaneous p-q theory to obtain the compensation currents for a linear three-phase inductive load (section III a). The sliding-mode approach, to enforce the line compensation currents, is presented in section III b. The controller for dc capacitor voltage equalization is described in section III c. The dc capacitor voltage control, being slow, uses a linear controller (section III d). Simulation results are presented in order to show the effectiveness of the proposed strategy (section IV). Section V shows experimental results.

II. MULTILEVEL CONVERTER MODELING

The NPC multilevel converter (fig. 1) model in systems coordinates (123) can be obtained using the Kirchhoff voltage and current, and assuming a ternary variable \( \gamma_k(t) \) describing the output of each converter leg \( k \):

\[
\gamma_k(t) = \begin{cases} 
1 & \text{if } (S_{k1} \land S_{k2}) \text{ ON } \land (S_{k3} \land S_{k4}) \text{ OFF} \\
0 & \text{if } (S_{k2} \land S_{k3}) \text{ ON } \land (S_{k1} \land S_{k4}) \text{ OFF} \\
-1 & \text{if } (S_{k3} \land S_{k4}) \text{ ON } \land (S_{k1} \land S_{k2}) \text{ OFF} 
\end{cases} \tag{1}
\]

Using also (2, 3, 4), the converter model is represent in (5)

\[
\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{21} & \Xi_{22} \\ \Xi_{31} & \Xi_{32} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2\Gamma_{11} - \Gamma_{12} - \Gamma_{13} & 2\Gamma_{21} - \Gamma_{22} - \Gamma_{23} \\ -\Gamma_{11} + 2\Gamma_{12} - \Gamma_{13} & -\Gamma_{21} + 2\Gamma_{22} - \Gamma_{23} \\ -\Gamma_{11} - \Gamma_{12} + 2\Gamma_{13} & -\Gamma_{21} - \Gamma_{22} + 2\Gamma_{23} \end{bmatrix} \tag{4}
\]
This model will be used to control the dc voltage of capacitors, \( U_{dc} \).

### III. CONTROLLER DESIGN

This section defines the line compensation currents using the instantaneous p-q theory \([5, 6, 9]\) and the controllers synthesis using linear and non linear techniques.

#### A. Compensation of reactive power

Using \( \alpha\beta \) coordinates, the calculation of the references for the compensating currents can be obtained as a function of active or reactive power of the load (fig. 2).

\[
\begin{bmatrix}
p_T \\
q_T
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix} \begin{bmatrix}
t_{\alpha} \\
t_{\beta}
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix} \begin{bmatrix}
-t_{\alpha} + i_{\alpha C} \\
-t_{\beta} + i_{\beta C}
\end{bmatrix}
\]

Using the active power and \( \alpha\beta \) coordinates, the following relations hold:

\[
\begin{bmatrix}
p_T \\
q_T
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix} \begin{bmatrix}
t_{\alpha} \\
t_{\beta}
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix} \begin{bmatrix}
-t_{\alpha} + i_{\alpha C} \\
-t_{\beta} + i_{\beta C}
\end{bmatrix}
\]

Solving for the currents as functions of total active power, \( p_T \), and total reactive power \( q_T \):

\[
\begin{bmatrix}
t_{\alpha} \\
t_{\beta}
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix}^{-1} \begin{bmatrix}
p_T \\
q_T
\end{bmatrix}
\]

Since the system is conservative, the total active power equals the load active power \( p_L = p_T \). To achieve reactive power compensation, we must have \( q_T = q_L \), being:

\[
\begin{bmatrix}
t_{\alpha} \\
t_{\beta}
\end{bmatrix} = \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix}^{-1} \begin{bmatrix}
p_L \\
0
\end{bmatrix}
\]

Solving, it is obtained:

\[
\begin{bmatrix}
i_{\alpha L} - i_{\alpha C} \\
i_{\beta L} - i_{\beta C}
\end{bmatrix} = \frac{1}{u_{\alpha}^2 + u_{\beta}^2} \begin{bmatrix}
u_{\alpha} \\
u_{\beta}
\end{bmatrix} \begin{bmatrix}
p_L \\
0
\end{bmatrix}
\]

Where the currents are:

\[
\begin{cases}
i_{\alpha C} = i_{\alpha L} - \frac{u_{\alpha}}{u_{\alpha}^2 + u_{\beta}^2} p_L \\
i_{\beta C} = i_{\beta L} - \frac{u_{\beta}}{u_{\alpha}^2 + u_{\beta}^2} p_L
\end{cases}
\]

This model will be used to control the dc voltage of capacitors, \( U_{dc} \).
Applying Park’s transformation to (12) and using the reactive component, reference currents $i_{d_{\text{ref}}}$ and $i_{q_{\text{ref}}}$ are derived. However, the $i_d$ component must include the converter losses. Therefore, the active component $i_{d_{\text{ref}}}$ is obtained from the voltage control loop operating with the $U_{dc}$ variable (section III d).

The compensating currents in coordinates $\alpha\beta$ ($i_{\alpha_{\text{ref}}}$ and $i_{\beta_{\text{ref}}}$) can be obtained applying inverse Park’s transformation to $i_{d_{\text{ref}}}$ and $i_{q_{\text{ref}}}$.

### B. Sliding-mode control of the compensation currents

This section presents the sliding-mode approach, to enforce the compensation currents references ($i_{\alpha_{\text{ref}}}$, $i_{\beta_{\text{ref}}}$).

The NPC converter model is non-linear and also 123 and $\alpha\beta$ models are time variant. Variable structure closed-loop control and sliding-mode in particular, can solve these problems in spite of requiring the reading or estimation of the system state variables.

The control goal considered in this situation is defined by,

$$\begin{cases} i_a = i_{\alpha_{\text{ref}}} \\ i_\beta = i_{\beta_{\text{ref}}} \end{cases} \quad (13)$$

and control errors are,

$$\begin{cases} e_{\alpha} = i_{\alpha_{\text{ref}}} - i_{\alpha} \\ e_{\beta} = i_{\beta_{\text{ref}}} - i_{\beta} \end{cases} \quad (14)$$

Since the $\alpha\beta$ state space model (6) is in controllability canonical form with respect to $i_a$ and $i_\beta$ currents, it can be concluded that the converter ac currents show a strong relative degree of one [7] (as its first time derivative contains the control variables). Therefore, according to the strong relative degree of each output variable, and considering the feedback errors as state variables, the sliding surfaces ensuring the robustness of the closed loop controlled system [8], are:

$$\begin{cases} S(e_{\alpha},t) = k_{\alpha}e_{\alpha} \\ S(e_{\beta},t) = k_{\beta}e_{\beta} \end{cases} \quad (15)$$

Sliding-mode exists if $S(e_{\alpha\beta},t) = 0$. To ensure that the system remains in sliding-mode operation the controller should also guarantee $\dot{S}(e_{\alpha\beta},t) = 0$. These two conditions can only exist if an infinite switching frequency was used. In a practical power converter, the switching frequency is upper-bound limited by the semiconductors switching losses. Thus, a small current error $\varepsilon$ (ripple) will be present (fig. 3).

Since the controller is designed to stabilize the system, the following relation (sliding-mode stability condition) must be ensured:

$$\dot{S}(e_{\alpha\beta},t) S(e_{\alpha\beta},t) < 0 \quad (16)$$

Therefore, if $S(e_{\alpha\beta},t) > +\varepsilon$ then $\dot{S}(e_{\alpha\beta},t) < 0$, and if $S(e_{\alpha\beta},t) < -\varepsilon$ then $\dot{S}(e_{\alpha\beta},t) > 0$. The power converter control should increase or decrease the output voltage level to verify the precedent conditions.

Therefore the multilevel switching laws are:

$$\begin{cases} S(e_{\alpha},t) > +\varepsilon & \rightarrow \text{increase a level of } u(t) \text{ until } \dot{S}(e_{\alpha},t) < 0 \\ S(e_{\alpha},t) < -\varepsilon & \rightarrow \text{decrease a level of } u(t) \text{ until } \dot{S}(e_{\alpha},t) > 0 \end{cases} \quad (17)$$

These conditions can be accomplished, considering that the capacitors voltages $U_{C1}$ and $U_{C2}$ are balanced for guarantee $U_{C1} \approx U_{C2} \approx \frac{U_{dc}}{2}$. Then, the output current vector dynamics, in $\alpha\beta$ coordinates, is:

$$\begin{bmatrix} \frac{d}{dt}i_a \\ \frac{d}{dt}i_\beta \end{bmatrix} = \begin{bmatrix} \frac{R}{L} & 0 \\ 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_a \\ i_\beta \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u_{s_a} \\ u_{s_\beta} \end{bmatrix} + \frac{1}{L} \begin{bmatrix} 0 \\ 1 \end{bmatrix} U_{C1} + \frac{1}{L} \begin{bmatrix} 0 \\ 0 \end{bmatrix} U_{C2} \quad (18)$$

Using (15), the derivatives of the switching surfaces are:

$$\begin{cases} \dot{S}(e_{\alpha},t) = k_{\alpha} \left( \frac{d}{dt}i_{\alpha_{\text{ref}}} + \frac{R}{L} i_{\alpha} + \frac{1}{L} u_{s_a} - \frac{1}{L} U_{s_a} \right) \\ \dot{S}(e_{\beta},t) = k_{\beta} \left( \frac{d}{dt}i_{\beta_{\text{ref}}} + \frac{R}{L} i_{\beta} + \frac{1}{L} u_{s_\beta} - \frac{1}{L} U_{s_\beta} \right) \end{cases} \quad (19)$$

Therefore:

$$\begin{cases} \dot{S}(e_{\alpha},t) > 0 & \Rightarrow U_{S} < L \frac{d}{dt} i_a + R i_a + u_s \\ \dot{S}(e_{\alpha},t) < 0 & \Rightarrow U_{S} > L \frac{d}{dt} i_a + R i_a + u_s \end{cases} \quad (20)$$
Then, the switching laws are:

\[ S(e_{i\alpha \beta}, t) > +\varepsilon \land S(e_{i\alpha \beta}, t) > 0 \]  
\[ S(e_{i\alpha}, t) > +\varepsilon \land U_{sa} < L \frac{d}{dt}i_{a \text{ref}} + R i_{\alpha} + u_{sa} \]  
\[ S(e_{i\beta}, t) > +\varepsilon \land U_{sb} < L \frac{d}{dt}i_{\beta \text{ref}} + R i_{\beta} + u_{sb} \]  

(22)

\[ S(e_{i\alpha \beta}, t) < -\varepsilon \land S(e_{i\alpha \beta}, t) < 0 \]  
\[ S(e_{i\alpha}, t) < -\varepsilon \land U_{sa} > L \frac{d}{dt}i_{a \text{ref}} + R i_{\alpha} + u_{sa} \]  
\[ S(e_{i\beta}, t) < -\varepsilon \land U_{sb} > L \frac{d}{dt}i_{\beta \text{ref}} + R i_{\beta} + u_{sb} \]  

(23)

When conditions (22) hold, the NPC converter controller must increase the level of the \( U_c \) voltage. Conversely, if the conditions (23) hold, the NPC converter controller must decrease that level. This is done increasing or decreasing integer variables \( \lambda_\alpha, \lambda_\beta \in \{-2,-1,0,1,2\} \) and then selecting vectors in the \( \alpha\beta \) plane accordingly.

Considering again voltages \( U_{C1}=U_{C2}=U_{dc}/2 \), the application of the Concordia transformation to the \( U_{sk} \) voltages (fig. 1) gives these voltages in the \( \alpha\beta \) plane:

\[ \begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{3} & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \sqrt{3} & \frac{-2}{3} & \frac{-\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} y_1 U_{dc} \\ y_2 \\ y_3 \end{bmatrix} \]  

(24)

Being \( y_k \) given by (1), the converter presents 27 \( U_\alpha, U_\beta \) combinations or vectors (fig. 4). This multilevel converter shows nine levels for the \( \alpha \) component and five levels for \( \beta \) component. To select one out of 25 vectors (3 are null vectors) two \( 5 \times 5 \) tables (table I and table II) were created, with the integer variables \( \lambda_\alpha, \lambda_\beta \) as inputs.

C. DC link capacitor voltage equalization

The two DC-link capacitor voltages \( U_{C1}, U_{C2} \) must be balanced. The redundant vectors in tables I (2, 5, 6, 13, 17, 18) and II (10, 11, 15, 22, 23, 26) can provide an extra degree of freedom to allow dc capacitor voltage equalization. Therefore capacitor voltage equalization will enable the right choice between the center vectors of table I and table II, done according to the sign of the power flow in the two capacitors midpoint.

From the circuit of fig. 5, it is written:

\[ i_n = i_{C1} - i_{C2} \]  

(25)

Using (5):

\[ \begin{align*} 
\frac{dU_{C1}}{dt} &= \frac{i_{C1} + i_{o}}{C_1} = \frac{\sum_{k=1}^{3} I_{K} + i_{o}}{C_1} = \frac{-\sum_{k=1}^{3} I_{K} + i_{o}}{C_1} \\
\frac{dU_{C2}}{dt} &= \frac{i_{C2} + i_{o}}{C_2} = \frac{\sum_{k=1}^{3} I'_{K} + i_{o}}{C_2} = \frac{-\sum_{k=1}^{3} I_{2K} + i_{o}}{C_2} 
\end{align*} \]  

(26)
Therefore:

\[
\begin{align*}
\frac{dU_{C1}}{dt} &= -i_1\Gamma_{11} - i_2\Gamma_{12} - i_3\Gamma_{13} + i_o \\
\frac{dU_{C2}}{dt} &= -i_1\Gamma_{21} - i_2\Gamma_{22} - i_3\Gamma_{23} + i_o \\
&= -\frac{i_1\Gamma}{C_1} - \frac{i_2\Gamma}{C_2} + i_o
\end{align*}
\]

(27)

Calculating the first derivative of the control error \( e_{C12} = U_{C1} - U_{C2} \), and considering \( C_1 = C_2 = C \):

\[ i_o = C \frac{de_{C12}}{dt} = \left( -\sum_{k=1}^{3} i_k \frac{\gamma_k(\gamma_k+1)}{2} \right) + \left( \sum_{k=1}^{3} i_k \frac{1-\gamma_k}{2} \right) \]

The two capacitors midpoint current \( i_o \) is a function of \( i_1 \) and \( i_2 \):

\[ i_o = \frac{i_1(\gamma_3^2 - \gamma_1^2) + i_2(\gamma_3^2 - \gamma_2^2)}{C} \]

(28)

Therefore the switching law for \( e_{C12} \) is:

If \( (U_{C1} - U_{C2}) i_o < 0 \) \( \Rightarrow \) table I

If \( (U_{C1} - U_{C2}) i_o > 0 \) \( \Rightarrow \) table II

Figure 6 shows a MATLAB/Simulink implementation of this algorithm.

\[ F_d \quad \text{Linear Control of the } U_{dc} \text{ Voltage} \]

The linear control of the \( U_{dc} \) voltage provides the \( i_{d \text{ref}} \) component as said previously. The controller is designed from the converter model in dq0 coordinates (7), considering \( C_1 = C_2 = 2C \) and \( U_{C1} = U_{C2} = \frac{U_{dc}}{2} \).

From (7) the \( U_{dc} \) voltage dynamics is:

\[ \frac{dU_{dc}}{dt} = \frac{i_d}{C} \gamma_d + \frac{i_q}{C} \gamma_q + \frac{i_o}{C} \]

(30)

At unity power factor, this leads to:

\[ \frac{dU_{dc}}{dt} = \frac{i_d}{C} \gamma_d + \frac{i_o}{C} \]

(31)

Considering \( C \) and the output dc current \( i_o \) as a disturbance \( \left( i_o = \frac{U_{dc}}{R_{eq}} \right) \) the equivalent model (32) is obtained (fig. 7).

\[ \frac{U_{dc}(s)}{I_{d \text{ref}}(s)} = \frac{\gamma_d R_{eq}}{R_{eq}Cs + 1} \]

(32)

Assuming a small time delay \( T_d \) in the current controlled current source \( \gamma_d i_o \), the block diagram for the \( U_{dc} \) control is shown in fig. 8.

\[ U_{dc ref} \quad e \quad C(s) \quad I_{d \text{ref}} \quad \gamma_d \quad I_d \quad \frac{R_c}{R_{eq}Cs + 1} \quad U_{dc} \]

Fig. 8. Block diagram of the linear control.

The \( C(s) \) controller is a PI controller \( C(s) = k_p + \frac{k_i}{s} \) or \( C(s) = \frac{k_c(\tau_c s + 1)}{s} \) with \( k_c = k_l \) and \( \tau_c = k_p / k_l \). The zero of the controller is used to cancel the system dominant pole, resulting in a 2nd order system with transfer function (33).

\[ \frac{U_{dc}(s)}{U_{dc \text{ref}}(s)} = \frac{k_c \gamma_d R_{eq}}{s^2 + \frac{1}{T_d} + \frac{k_c \gamma_d R_{eq}}{T_d}} \]

(33)
Comparing with 2\textsuperscript{nd} order standard transfer function (34):

$$\frac{Y(s)}{X(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$  \hspace{0.5cm} (34)

The following relations hold:

$$\begin{align*}
2\xi\omega_n &= \frac{1}{T_d} \\
\omega_n^2 &= \frac{k_C}{\gamma_d \frac{R_{eq}}{T_d}} \\
k_C &= k_I \\
\tau_C &= \frac{R_{eq}}{C} = \frac{k_p}{k_I}
\end{align*}$$  \hspace{0.5cm} (35)

If the controller is designed with a damping factor $\xi<1$ a strong voltage unbalance can arise. Therefore, a slower response speed is selected ($\xi=9$), so that the $U_{dc}$ voltage error decreases to zero. Using the NPC converter parameters, $C=2.35 \text{ mF}$; $R_{eq} = 4.4 \text{ k}\Omega$; $T_d = 0.1 \text{ ms}$; $\gamma_d = 0.44$, the PI parameters are $k_I=0.02$ and $k_p=0.16$.

IV. SIMULATION RESULTS

The simulation results of shunt VAr compensator were obtained using Simulink environment in accordance with figure 9.

IV. EXPERIMENTAL RESULTS

A NPC multilevel inverter laboratory prototype was built using 1200V, 50A IGBT Transistors (MG50Q2YS50), switching frequencies near 5 kHz, neutral clamp diodes (IRKD56/16A, (fig. 11), $C_1=C_2=4.7\text{ mF}$ and inductive load RL (3x25mH, 10Ω). The three level converter was controlled using DSpace DSP 1104 (fig. 12 and 13).
VI. CONCLUSIONS

This paper presented NPC multilevel converter model, as well as the instantaneous p-q theory, suitable to obtain the compensation currents for reactive power compensation of three-phase inductive loads. To enforce the line compensation currents a robust sliding-mode approach was presented. The dc link capacitor voltage control was designed using a linear controller, although dc link capacitor voltage equalization used sliding-mode. Presented simulation and experimental results shown the effectiveness of the proposed strategy.

VII. REFERENCES


VIII. BIOGRAPHIES

Luís Encarnação was born in Portimão, Portugal, in 1973. He received the B.S and electrical engineering degrees from Instituto Superior de Engenharia de Lisboa, Lisbon, Portugal in 1995 and 1999, respectively. Presently, he is an Assistant at the ISEL where he teaches Control Theory and a researcher at Centro de Automática. He is currently working towards M.Sc degree. His research interests include control and modeling applied to energy systems and power electronics systems.

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